Beyond the Higgs

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Abstract

Particle physics lived a decade of great experimental successes that have strengthened the Standard Model (SM) as a successful description of nature. Yet, these results also concluded that the SM matter only represents about 5% of the energy density of the Universe and therefore they called for a physics beyond the SM, albeit direct evidence for such physics is desperately missing. The sector at the origin of the spontaneous breaking of the SM electroweak symmetry could well provide us with the first hints of this new physics in a detector. The aim of this review is to give a survey of recent approaches that have been proposed to address the dynamics responsible for the breaking of the electroweak symmetry. An extended version of these notes can be found in Ref. [1] along with a complete and detailed bibliography. Due to page limitation, only few references will be given here.

1 Introduction

The Standard Model can be divided into three sectors: the gauge sector, the flavour sector and the electroweak symmetry-breaking sector. While the first two have been well tested in accelerator experiments (such as LEP, SLD, BABAR, BELLE, etc.), the sector of electroweak symmetry breaking (EWSB) is currently the attention of intense scrutiny not only because particle physicists hope to discover the Higgs boson at the Large Hadron Collider (LHC) soon to be operational at CERN, but also because this sector could well provide us with the first hints in a detector of new physics beyond the Standard Model. Indeed the usual Higgs mechanism jeopardizes our current understanding of the SM at the quantum level and requires the existence of additional structures (new particles, new symmetries, new dimensions, . . . ) to stabilize the weak scale. Better than a long introduction, the following tautology reveals that an understanding of the dynamics of EWSB is still missing.

Why is EW symmetry broken?

Because the Higgs potential is unstable at the origin.
Why is the Higgs potential unstable at the origin?
Because otherwise EW symmetry wouldn’t be broken.

One should understand here that the Higgs mechanism is only a description of EWSB and not an explanation of it, since in particular there is no dynamics to explain the instability of the Higgs potential at the origin.

The Higgs sector involves two experimentally unknown parameters, namely the Higgs boson mass ($M_h$) and the cutoff scale ($\Lambda$) of the SM itself, i.e. the scale at which new physics will show up. Yet, these two parameters are subject to theoretical consistency constraints — the well-known unitarity, triviality, stability and naturality bounds — as well as indirect experimental constraints through the electroweak precision data.

Any heavy particle, when integrated out, will generate new non-renormalizable interactions between the light SM particles. Would the SM make any sense as an effective theory at low energy, these new interactions would have to leave the SM gauge symmetry unbroken. Yet,

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they can break some (accidental, approximate) SM global symmetries and, depending on which
global symmetry is actually broken, these new interactions can manifest themselves at rather
low energy (see Table 1). The EWSB sector seems a good place to look for direct manifestations
of new physics in the energy range that will be explored at the LHC.

<table>
<thead>
<tr>
<th>Broken symmetry</th>
<th>Operators</th>
<th>Scale Λ</th>
</tr>
</thead>
<tbody>
<tr>
<td>B, L</td>
<td>$(QQQL)/\Lambda^2$</td>
<td>$10^{13}$ TeV</td>
</tr>
<tr>
<td>Flavor (1,2nd family), CP</td>
<td>$(\bar{d}s\bar{d}s)/\Lambda^2$</td>
<td>1000 TeV</td>
</tr>
<tr>
<td>Flavor (2,3rd family)</td>
<td>$m_b(\bar{s}\sigma_{\mu\nu}F^{\mu\nu}b)/\Lambda^2$</td>
<td>50 TeV</td>
</tr>
<tr>
<td>Custodial $SU(2)$</td>
<td>$(h^\dagger D_{\mu}h)^2/\Lambda^2$</td>
<td>5 TeV</td>
</tr>
<tr>
<td>None</td>
<td>$(h^\dagger h)^2/\Lambda^2$</td>
<td>–</td>
</tr>
</tbody>
</table>

Table 1: Examples of non-renormalizable interactions between SM particles obtained after
integrating out some heavy degrees of freedom, and bounds on the corresponding scales that
suppress them. These interactions can be classified according to the global symmetries they
break. Rather low scales can affect the EWSB sector.

This raises three important questions, which we will try to address in this review:

- Given the experimental results from LEP, SLD, Tevatron, what are the constraints on
  new physics in the EWSB sector?
- Is it possible to add new physics around the TeV scale that stabilizes the EW scale?
- What are the potentials to discover new physics in the EWSB sector at the LHC?

## 2 New Physics and EWSB

### 2.1 Stabilization of the Higgs potential by symmetries

The description of EWSB with a Higgs suffers from several instabilities (triviality, stability and
naturality bounds) at the quantum level. Extra structures (particles and/or symmetries) are
needed to stabilize the Higgs potential. To keep radiative corrections under control, a theorist
can make use of two tools:

- The **spin trick**: in general, a particle of spin $s$ has $2s+1$ degrees of polarization with the
  only exception of a particle moving at the speed of light, in which case fewer polarizations
  may be physical. And conversely if a symmetry decouples some polarization states then
  the particle will necessarily propagate at the speed of light and thus remain massless.
  For instance, gauge invariance ensures that the longitudinal polarization of a vector field
  is non-physical, and chiral symmetry keeps only one fermion chirality: both spin-1 and
  spin-1/2 particles are protected from dangerous radiative corrections. Unfortunately, this
  spin trick cannot be used for a spin-0 particle such as the SM Higgs scalar boson.

- The **Goldstone theorem**: when a global symmetry is spontaneously broken, the spectrum contains a **massless** spin-0 particle. However, here again, it seems difficult to invoke
  this trick to protect the SM Higgs boson from radiative corrections since a Nambu–
  Goldstone boson can only have some derivative couplings, unlike the Higgs field. Little
Higgs models have been constructed to circumvent these difficulties and they provide realistic examples of Higgs as a (pseudo-)Nambu–Goldstone boson. A short account of these models is given in Section 3.

In the late 60’s, the Coleman–Mandula and Haag–Lopuszanski–Sohnius theorems taught us how to apply the spin trick to spin-0 particles: the four-dimensional Poincaré symmetry has to be enlarged. The first construction of this type consists in embedding the 4D Poincaré algebra into a superalgebra. Then the supersymmetry between fermion and boson extends the spin trick to scalar particles. Actually there exists an even simpler way of enlarging the Poincaré symmetry, which is going into extra dimensions: the 5D Poincaré algebra obviously contains the 4D Poincaré algebra as a subalgebra. After compactification of the extra dimensions, from a 4D dimensional point of view, the higher-dimensional gauge field decomposes into a 4D gauge field (the components along our 4D world) and 4D scalar fields (the components along the extra dimensions). The symmetry between vectors and scalars allows us to extend the spin trick to spin-0 particles.

Neither supersymmetry nor higher-dimensional Poincaré symmetry are exact symmetries of nature. Therefore, if they ever have a role to play, they have to be broken. In order not to lose any of their benefits, this breaking has to proceed without reintroducing any strong UV dependence into the renormalized scalar mass square: we need a soft breaking. This question has been well studied in supersymmetric theories and we would like, in Section 4, to discuss a soft breaking of higher-dimensional gauge theories. In Section 5, we will briefly report on Higgsless models, where the EWSB is no longer achieved through a Higgs mechanism but results from non-trivial boundary conditions for the gauge fields at the boundaries of a fifth dimension.

2.2 EW precision tests

New particles are needed to stabilize the weak scale. They have to be massive to evade direct searches. They still influence SM physics and they can be “detected” through precision measurements.

2.2.1 An example of EW corrections induced by a heavy particle

As an example, let us take an extra-heavy $B'$ gauge boson. The full Lagrangian is

\[
\mathcal{L} = -\frac{1}{4} W_3^2 (p^2 - M_W^2) W_3 - t_0 M_W^2 W_3 B - \frac{1}{2} B (p^2 - t_0^2 M_W^2) B \\
+ g J_3 W_3 + g' J_y B - \frac{1}{2} B' (p^2 - M'^2) B' + g' J_y B'
\]

(1)

where $t_0 = g'/g$ (later on, we will also use $c_0 = g/\sqrt{g^2 + g'^2}$ and $s_0 = g'/\sqrt{g^2 + g'^2}$). $J_y$ and $J_3$ are the usual fermion currents coupled to $B_\mu$ and $W_{3\mu}$: $J_y^\mu = \sum_i y_i \bar{f}_i \sigma^\mu f_i$ and $J_3^\mu = \sum_i T_{3L} f_i \bar{f}_i$. Let us now integrate out the heavy particle, which means that we freeze its dynamics and replace $B'$ by its equation of motion

\[
\frac{\partial \mathcal{L}}{\partial B'} = 0 \iff B' = \frac{g' J_y}{p^2 - M^2}.
\]

(2)

Plugging back this expression into the original Lagrangian, and after an expansion for $M \gg p$, we obtain the effective Lagrangian

\[
\mathcal{L} = -\frac{1}{4} W_3^2 (p^2 - M_W^2) W_3 - t_0 M_W^2 W_3 B - \frac{1}{2} B (p^2 - t_0^2 M_W^2) B \\
+ g J_3 W_3 + g' J_y B - \frac{(g' J_y)^2}{2M^2}.
\]

(3)
Using the equation of motion for $B$, $g' J_y = t_0 M_W^2 W_3 + (p^2 - t_0^2 M_W^2) B$, we can actually write the four-Fermi interaction as a modification of the gauge boson propagators:

$$
\mathcal{L} = -\frac{1}{2} W_3 \left( p^2 - M_W^2 \left(1 - \frac{t_0^2 M_W^2}{M^2}\right)\right) W_3
- t_0 M_W^2 \left(1 + \frac{p^2 - t_0^2 M_W^2}{M^2}\right) W_3 B
- \frac{1}{2} B \left(p^2 \left(1 - 2 \frac{t_0^2 M_W^2}{M^2}\right) - t_0^2 M_W^2 \left(1 - \frac{t_0^2 M_W^2}{M^2}\right) + \frac{p^4}{M^2}\right) B
+ g J_3 W_3 + g' J_y B + \mathcal{O}(p^6) + \mathcal{O}(1/M^4).$$

(4)

The mass matrix in the $(W_3, B)$ basis thus reads

$$
\begin{pmatrix}
1 - \frac{t_0^2 M_W^2}{M^2} & -t_0 M_W^2 \\
-t_0 M_W^2 & t_0^2 M_W^2
\end{pmatrix}
\begin{pmatrix}
M_W^2 \\
-t_0 M_W^2
\end{pmatrix}
.$$ 

(5)

Note that the determinant of this mass matrix is vanishing, as it should to maintain the masslessness of the photon. Furthermore, we also note that the weak mixing angle is unaffected

$$Z = c_0 W_3 - s_0 B \quad \text{and} \quad \gamma = s_0 W_3 + c_0 B.$$ 

(6)

This is essential here to ensure that the photon actually couples to the electric charge $T_{3L} + Y$. The photon remains massless but the heavy $B'$ modifies the mass of the $Z$

$$M_Z^2 = \frac{1}{c_0^2} M_W^2 \left(1 - \frac{t_0^2 M_W^2}{M^2}\right) \quad \text{and} \quad M_\gamma^2 = 0.$$ 

(7)

So, at low energy, we obtain a deviation to $\rho = 1$ since we now have

$$\rho \equiv \frac{M_W^2}{M_Z^2 c^2} \approx 1 + \frac{t_0^2 M_W^2}{M^2}.$$ 

(8)

The deviation to $\rho = 1$ is usually called the $T$ parameter

$$\rho \equiv 1 + \alpha_{em} T.$$ 

(9)

So in our example, we get

$$T = \frac{t_0^2 M_W^2}{\alpha_{em} M^2}.$$ 

(10)

The upper bound [2] on the $T$ parameter, $T \leq 0.2$, gives a lower bound on the mass of the heavy $B'$, $M \geq 1.1$ TeV. That is a rather generic result: the bound on new physics needed to stabilize the weak scale is at least one order of magnitude above the weak scale. This has been dubbed the little hierarchy problem, or LEP paradox.

2.2.2 General structure of the EW corrections

Under mild assumptions (universality, heaviness of the new physics, flavour universality, CP invariance), we can obtain the general form of the corrections induced by new physics. The most general $U(1)_{em}$-invariant quadratic Lagrangian for the SM gauge bosons

$$\mathcal{L} = -\frac{1}{2} W_3^\mu \Pi_{33}(p^2) W_3^\mu - W_3^\mu \Pi_{3B}(p^2) B_\mu - \frac{1}{2} B^\mu \Pi_{BB}(p^2) B_\mu
- W_3^\mu \Pi_{-}(p^2) W_-^\mu.$$ 

(11)
involves four vacuum polarizations that we expand in powers of momentum
\[ \Pi_V(p^2) = \Pi_V(0) + p^2 \Pi'_V(0) + \frac{1}{2} (p^2)^2 \Pi''_V(0) + \mathcal{O}(p^6). \] (12)

So 12 coefficients should describe the most general low-energy effective Lagrangian. But 3 of them can actually be removed by normalizing the gauge bosons (which corresponds to the identification of the three SM parameters \( g, g' \) and \( v \))
\[ \Pi'_{+-}(0) = \Pi'_{BB}(0) = 1, \quad \Pi_{+-}(0) = -M^2_W = -(80.425 \text{ GeV})^2. \] (13)

The remaining 9 parameters are not yet fully independent, since we need to impose the masslessness of the photon and its coupling to \( Q \). These two consistency constraints explicitly read \( g^2 \Pi_{33} + g^2 \Pi_{00} + 2gg' \Pi_{30} = 0 \) and \( g \Pi_{00} + g' \Pi_{30} = 0 \). So we are left with a total of 7 arbitrary coefficients [3]. They are given in Table 2, along with the example of the dimension-6 operators that generate them.

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>Dim.-6 operators</th>
<th>SU(2)_c</th>
<th>SU(2)_L</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{S} = \frac{g}{g} \Pi_{BB}(0) )</td>
<td>((H^\dagger \tau^a H)W^a_{\mu\nu}B_{\mu\nu}/gg')</td>
<td>+</td>
<td>−</td>
</tr>
<tr>
<td>( \hat{T} = \frac{1}{M^2_W} (\Pi_{33}(0) - \Pi_{+-}(0)) )</td>
<td>(</td>
<td>H^\dagger D_{\mu}H</td>
<td>^2 )</td>
</tr>
<tr>
<td>( \hat{U} = \Pi'<em>{+-}(0) - \Pi'</em>{33}(0) )</td>
<td>Dim. 8</td>
<td>−</td>
<td>−</td>
</tr>
<tr>
<td>( \hat{V} = \frac{M^2_W}{2} (\Pi''<em>{33}(0) - \Pi''</em>{+-}(0)) )</td>
<td>Dim. 10</td>
<td>−</td>
<td>−</td>
</tr>
<tr>
<td>( \hat{X} = \frac{M^2_W}{2} \Pi_{33}(0) )</td>
<td>Dim. 8</td>
<td>+</td>
<td>−</td>
</tr>
<tr>
<td>( \hat{Y} = \frac{M^2_W}{2} \Pi_{BB}(0) )</td>
<td>( (\partial_{\mu}B_{\mu\nu})^2/2g^2 )</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>( \hat{W} = \frac{M^2_W}{2} \Pi''_{33}(0) )</td>
<td>( (D_{\mu}W^a_{\mu\nu})^2/2g^2 )</td>
<td>+</td>
<td>+</td>
</tr>
</tbody>
</table>

Table 2: Seven coefficients parametrize the most general low-energy Lagrangian beyond the SM. \( SU(2)_L \times U(1)_Y \) invariant higher-dimensional operators can give rise to these corrections. Notice that they have definite symmetry properties under the gauge \( SU(2)_L \) and the \( SU(2)_c \) custodial symmetry. The more usual \( S, T \) and \( U \) coefficients are obtained by \( S = 4s^2_w \hat{S}/\alpha_{em} \approx \frac{119}{2} \hat{S} \), \( T = \hat{T}/\alpha_{em} \approx 129 \hat{T} \) and \( U = -4s^2_w \hat{U}/\alpha_{em} \approx -119 \hat{U} \). From [3] (where a non-canonical normalization of the gauge bosons is used, hence the different factors of \( g \) and \( g' \) appearing in the definition of \( \hat{S}, \hat{T}, \ldots, \hat{W} \)).

For instance, in the example of a heavy \( B' \) discussed in the previous section, we obtain
\[ t_0^2 \hat{S} = \hat{T} = t_0^2 \hat{Y} = \frac{t^2_0 M^2_{B'}}{M^2} \quad \text{and} \quad \hat{U} = \hat{V} = \hat{X} = \hat{W} = 0. \] (14)

In universal models, i.e. when new physics couples to the SM fermions only through the combinations that appear in SM fermionic currents:
\[ J^\mu_y = \sum_i y_i \tilde{f}_i \sigma^\mu f_i, \quad J^\mu_d = \sum_d \tilde{f}_d \sigma^\mu d, \] (15)

all the corrections induced by heavy particles are encoded in the oblique parameters, while for non-universal models, more effects might appear as vertex corrections. In universal models, 4 oblique parameters are dominant over the other ones, which can also be understood from the fact that \( \hat{U}, \hat{V} \) and \( \hat{X} \) are not generated by dimension-6 operators, so we generically expect
them to be further suppressed with respect to the other four coefficients: \( \hat{U} \sim \frac{M^2}{\Lambda^2} \hat{T}, \hat{V} \sim \frac{M^4}{\Lambda^2} \hat{T} \) and \( X \sim \frac{M^2}{\Lambda^2} \hat{S} \). In non-universal models, this is no longer true, and \( \hat{U}, \hat{V}, \hat{X} \) can be of the same order as \( \hat{S}, \hat{T}, W, Y \).

The electroweak precise measurements can be analysed using the parametrization just described; the results of the fits (assuming the existence of a light/heavy Higgs) are:

<table>
<thead>
<tr>
<th>Fit</th>
<th>( 10^3 \hat{S} )</th>
<th>( 10^3 \hat{T} )</th>
<th>( 10^3 Y )</th>
<th>( 10^3 W )</th>
</tr>
</thead>
<tbody>
<tr>
<td>115 GeV Higgs</td>
<td>0.0 ± 1.3</td>
<td>0.1 ± 0.9</td>
<td>0.1 ± 1.2</td>
<td>−0.4 ± 0.8</td>
</tr>
<tr>
<td>800 GeV Higgs</td>
<td>−0.9 ± 1.3</td>
<td>2.0 ± 1.0</td>
<td>0.0 ± 1.2</td>
<td>−0.2 ± 0.8</td>
</tr>
</tbody>
</table>

2.2.3 An example of EW corrections induced by a higher dimensional operator

As a concrete example of the above analysis, we would like to find out the effects of the higher-dimensional operator \([4]\)

\[
\mathcal{L}_T = \frac{a}{\Lambda^2} |H^\dagger D_\mu H|^2,
\]

where \( a \) is a dimensionless coefficient. After EWSB, \( \langle H \rangle = (0, v/\sqrt{2}) \), the operator \( \mathcal{L}_T \) simply appears as an additional mass term for the \( Z \) gauge boson

\[
\mathcal{L}_T = \frac{av^4}{16\Lambda^2} \left( g' B_\mu - g W^3_\mu \right)^2.
\]

Since the \( W \) mass and the weak mixing angle remained untouched, we easily obtain the correction to the \( \rho \) parameter induced by \( \mathcal{L}_T \)

\[
\rho = 1 + \frac{av^2}{2\Lambda^2},
\]

which corresponds to a non-vanishing \( T \) parameter

\[
T = \frac{\rho - 1}{\alpha_{em}} = -\frac{av^2}{2\alpha_{em}\Lambda^2}.
\]

This is also what we could derive using the formalism of Table 2 since

\[
\Pi_{+-} = -\frac{1}{4} g^2 v^2 \quad \text{and} \quad \Pi_{33} = -\frac{1}{4} g^2 v^2 - \frac{ag^2 v^4}{8\Lambda^2}.
\]

3 Little Higgs models

In analogy with the pions of QCD, the lightness of the Higgs could be explained if it were a Nambu–Goldstone boson (NGB) corresponding to a spontaneously broken global symmetry of the new strongly interacting sector. This is not a new idea per se but a new ingredient; the notion of collective breaking \([5]\) has been added in the past few years to construct realistic models, allowing large non-derivative couplings that, at the same time, are still free of quadratic divergences at one loop. The idea is that some interaction terms are introduced to break the global symmetries from which the Nambu–Goldstone boson originates, but two or more such interactions should be turned on simultaneously for the would-be NGB to acquire a mass. The one-loop radiative corrections should then involve two symmetry-breaking interactions to generate a mass term. The absence of quadratic divergences now follows from the fact that there are no quadratically divergent diagrams involving two symmetry-breaking couplings: therefore
the corrections to the NGB Higgs mass are logarithmic only. This way, we can obtain a light composite Higgs compatible with a strong coupling scale around 10 TeV.

Diagrammatically, the cancellation of the quadratic divergences is due to a set of new TeV-scale particles: gauge bosons, vector-like quarks, and extra massive scalars, which are related to the SM particles by the original global symmetry. It is noteworthy, and contrary to supersymmetry, that the cancellation of the divergences is achieved by same-spin particles. These new particles around one TeV, with definite couplings to ordinary particles as dictated by the global symmetries of the theory, are perfect goals for the LHC.

Very good reviews are already available on Little Higgs models [6], and the reader is referred to them for further details. We will just mention that the compatibility of Little Higgs models with experimental data is significantly improved when the global symmetry of the models involves a custodial symmetry as well as a $T$-parity under which, in analogy with $R$-parity in SUSY models, the SM particles are even and their partners are odd.

4 Gauge–Higgs Unification Models

The components of the gauge fields along some extra dimensions are seen from the 4D point of view as some 4D scalar fields (we will call them gauge-scalars). It is only above the compactification scale, when the extra dimensions open up, that the higher-dimensional gauge structure reveals itself. We will now describe models of gauge–Higgs unification where the Higgs is identified as some gauge-scalars. This approach is actually quite old [7] but it is only recently, within the context of orbifolds that it has been implemented in realistic models. A series of questions immediately pops up:

- Which gauge group will contain the Higgs?
- How many extra dimensions do we need? How are they compactified?
- What are the radiative corrections?
- How can matter be incorporated? How are the Yukawa couplings generated?

It is interesting to note that the deconstruction versions of these gauge–Higgs unification models led to the idea of Little Higgs models [5]. The symmetry protecting the Higgs mass is there a discrete shift symmetry and the construction is much less constrained by the absence of 5D Lorentz invariance.

4.1 Orbifold breaking. A 5D $SU(3)$ model.

Both 4D vectors and 4D scalars originating from higher-dimensional gauge fields belong to an adjoint representation of the gauge group, while the SM Higgs boson is a fundamental representation of the weak symmetry. In order to identify the Higgs as a component of a gauge field in extra dimensions, we thus need to enlarge the $SU(2)_L \times U(1)_Y$ gauge symmetry into a bigger group $G$. This bigger group can be broken in different ways: $(i)$ by introducing a higher-dimensional Higgs field; $(ii)$ by a Green–Schwarz mechanism; $(iii)$ by compactification on a non-trivial background manifold; $(iv)$ by compactification on an orbifold. This last method is not only well motivated in a stringy context, but it also offers the advantage of easily accommodating the presence of 4D chiral matter.

The simplest example of an orbifold is $S^1/\mathbb{Z}_2$, i.e. a circle ($-\pi R \leq y < \pi R$) with a parity identification ($y \sim -y$). The identification of the points $y$ and $-y$ means that the values of
any field evaluated at these points have to be physically equivalent, i.e. equal up to a global symmetry transformation: $\phi(x, -y) = U\phi(x, y)$. For consistency, $U$ has to be a $\mathbb{Z}_2$ symmetry, $U^2 = 1$. Note that there are two special points of the circle, 0 and $\pi R$, which are identified with themselves: they are fixed points of the orbifold. The invariance of the kinetic term dictates the transformation of the various components of the gauge field:

$$A_\mu(x, -y) = UA_\mu(x, y)U^{-1} \quad \text{and} \quad A_5(x, -y) = -UA_5(x, y)U^{-1}. \quad (21)$$

In a Kaluza–Klein (KK) decomposition, the 4D mass is related to the derivative of the field along the extra dimension; thus a massless mode should be independent of $y$. From the orbifold boundary conditions (21), we obtain that the 4D massless vectors correspond to the generators of the gauge group that commute with the orbifold matrix $U$, while the 4D massless gauge-scalars correspond to the generators that anticommute with $U$. Let us consider the example of an $SU(3)$ gauge group broken by the orbifold projection $U = \text{diag}(-1, -1, 1)$ down to $SU(2) \times U(1)$; from the eight gauge components of $A_5$, only a $SU(2)$ scalar doublet remains massless:

$$\begin{array}{c|c}
\text{SU(3)} & \text{SU(2)} \times \text{U(1)} \\
 U = \text{diag}(-1, -1, 1) & \\
 [A_\mu, U] = 0 & A_\mu = \frac{1}{2} \\
 \{A_5, U\} = 0 & A_5 = \frac{1}{2} \\
 & \left( \begin{array}{cc}
 A_5^+ + iA_5^- & A_5^+ - iA_5^- \\
 A_5^+ + iA_5^- & A_5^+ - iA_5^- \\
 \end{array} \right) = \frac{1}{2} \left( \begin{array}{cc}
 A_5^+ + iA_5^- & A_5^+ - iA_5^- \\
 A_5^+ + iA_5^- & A_5^+ - iA_5^- \\
 \end{array} \right) \\
 & \left( \begin{array}{cc}
 A_5^+ + iA_5^- & A_5^+ - iA_5^- \\
 A_5^+ + iA_5^- & A_5^+ - iA_5^- \\
 \end{array} \right) \left( \begin{array}{cc}
 A_5^+ + iA_5^- & A_5^+ - iA_5^- \\
 A_5^+ + iA_5^- & A_5^+ - iA_5^- \\
 \end{array} \right) \\
 & \left( \begin{array}{cc}
 A_5^+ + iA_5^- & A_5^+ - iA_5^- \\
 A_5^+ + iA_5^- & A_5^+ - iA_5^- \\
 \end{array} \right) \left( \begin{array}{cc}
 A_5^+ + iA_5^- & A_5^+ - iA_5^- \\
 A_5^+ + iA_5^- & A_5^+ - iA_5^- \\
 \end{array} \right) \\
 & \left( \begin{array}{cc}
 A_5^+ + iA_5^- & A_5^+ - iA_5^- \\
 A_5^+ + iA_5^- & A_5^+ - iA_5^- \\
 \end{array} \right) \left( \begin{array}{cc}
 A_5^+ + iA_5^- & A_5^+ - iA_5^- \\
 A_5^+ + iA_5^- & A_5^+ - iA_5^- \\
 \end{array} \right)
\end{array}$$

It is tempting to identify the massless $SU(2)$ doublet contained in $A_5$ as the Higgs doublet: $H_0 = (A_5^0 - iA_5^7)/2$ and $H_+ = (A_5^0 - iA_5^7)/2$. To do this, we need to know its $U(1)$ charge. Under any transformation of $SU(3)$, $A_5$ transforms as $\delta_T A_5 = g[T, A_5]$. In particular, under the $U(1)$ of $SU(2) \times U(1)$, $T = T_8 = \text{diag}(1, 1, -2)/(2\sqrt{3})$, we obtain

$$\delta_T \left( \begin{array}{cc}
 H_+ & H_0 \\
 H_0^* & H_+ \\
 \end{array} \right) = g \left( \frac{3}{2\sqrt{3}} \right) \left( \begin{array}{cc}
 H_+ & H_0 \\
 H_0^* & -H_+ \\
 \end{array} \right). \quad (22)$$

So the $U(1)$ charge of the doublet is equal to $3\sqrt{3}/2$. We need to change the normalization of the $U(1)$ for the charge of the doublet to be 1/2, which is achieved by picking up $U(1)_Y = T_8/\sqrt{3}$. The gauge coupling of $U(1)_Y$ is thus $g' = \sqrt{3}g$. Since we embedded $SU(2)_L \times U(1)_Y$ in a simple group, we get a prediction for the weak mixing angle

$$\sin^2 \theta_W = \frac{g'^2}{g'^2 + g'^2} = \frac{3g^2}{g^2 + 3g^2} = \frac{3}{4}. \quad (23)$$

This value is quite far from the experimental one ($\sin^2 \theta_W \approx 0.23$), which certainly invalidates this simple $SU(3)$ gauge–Higgs unification model. Furthermore, with this embedding of $SU(2)_L \times U(1)_Y$ into $SU(3)$, there is no way to get the quarks and leptons from $SU(3)$ irreducible representations.

At this point, we can envision at least two ways of proceeding: (i) add another $U(1)$ factor to $SU(3)$; (ii) examine other embedding of $SU(2)_L \times U(1)_Y$ into simple groups. Although the former gives up one nice aspect of the gauge–Higgs unification models, namely the prediction of the weak mixing angle, recent developments seem to indicate that it is the right direction to
follow while, as we are going to see it, a radiative instability spoils the most promising models of the second class. Finally a third way to go is to modify the geometry of the extra-dimensional space.

Before going on with the construction of gauge–Higgs unification models, we would like to mention that the orbifold projection can be reinterpreted as simple boundary conditions on an interval:

\[
\begin{array}{c}
G \rightarrow H \text{ orbifold breaking} \\
A^H_\mu (-y) = A^H_\mu (y) \quad \text{equivalent to} \quad \partial_5 A^H_\mu |_{y=0,\pi R} = 0 \\
A^G/H_\mu (-y) = -A^{G/H}_\mu (y) \quad \text{equivalent to} \quad A^{G/H}_\mu |_{y=0,\pi R} = 0 \\
A^{G/H}_5 (-y) = A^{G/H}_5 (y) \quad \text{equivalent to} \quad \partial_5 A^{G/H}_5 |_{y=0,\pi R} = 0
\end{array}
\]

It is also possible to accommodate a Scherk–Schwarz twist, i.e. \( \phi (y + 2\pi R) = T \phi (y) \). The twist will manifest itself by different boundary conditions at both ends of the interval.

Let us also mention that orbifold breaking has been applied to Grand Unified symmetries (see [8] for a review). In that latter case, the compactification is close to the GUT scale, while in gauge–Higgs models it is of the order of the weak scale.

### 4.2 Radiative corrections

There are two types of operators involving the 4D gauge-scaler fields that can be generated radiatively [9]:

(i) some bulk operators ;

(ii) some operators localized at the fixed points of the orbifold.

The higher-dimensional gauge invariance acts on the gauge bosons and the gauge-scalars as

\[
\delta A^A_M = \partial_M \epsilon^A + g f^{ABC} A^B_M \epsilon^C ,
\]

where \( f^{ABC} \) are the structure constant. For consistency, the gauge transformation parameters, \( \epsilon^C \), obey the same boundary conditions as \( A_\mu \):

\[
G \rightarrow H : \quad \partial_5 \epsilon^H |_{\text{fixed point}} = 0 \quad \text{and} \quad \epsilon^{G/H} |_{\text{fixed point}} = 0.
\]

So this is really gauge invariance that protects the first kind of operators: indeed above the compactification scale, the gauge-scaler fields really appear as some components of the higher-dimensional gauge field and the Slavnov–Taylor identities forbid, for instance, the appearance of any mass term. Below the compactification scale, however, we have to deal with ordinary scalars, which pick up some radiative but finite —since cut off at the compactification scale— mass. All the bulk operators are thus generated by IR effects and are finite. Another way to see it is that the only gauge-invariant operator that can give rise to a Higgs potential must be non-local in the extra dimensions and expressed in term of the Wilson line \( \mathcal{P} e^{\int dx^5 A_5} \). Being a non-local operator, the Higgs potential is finite to all orders in perturbation theory, it is UV-insensitive and calculable once the degrees of freedom around the weak scale are known.
As far as the brane-localized operators are concerned, the situation is more complicated. At the fixed point, the bulk gauge group is partially broken: there is only a $SU(2) \times U(1)$ subgroup left unbroken. And acting on the gauge-scalar doublet, the unbroken gauge group acts linearly and certainly does not forbid any quadratically divergent localized mass term to be generated:\footnote{Since $[H, H] \subset H$ and $[H, G/H] \subset G/H$, the only non-vanishing structure constants are $f^{HHH}$ and $f^{G/H G/H H}$ and cyclic permutations.}

\begin{equation}
\delta_H A^H_\mu = \partial_\mu \epsilon^H + g f^{HHH} A^H_\mu \epsilon^H \quad \delta_H A^{G/H}_\mu = 0
\end{equation}

\begin{equation}
\delta_H A^G_5 = 0 \quad \delta_H A^{G/H}_5 = g f^{G/H G/H H} A^{G/H}_5 \epsilon^H.
\end{equation}

Even though $H$ is the only unbroken gauge symmetry at the fixed points, there is still some residual symmetry left over from the full $G$ gauge symmetry in the bulk: indeed the broken generators of the higher-dimensional gauge invariance act on the gauge-scalars as a shift symmetry proportional to the derivative of the gauge parameters \cite{9}:

\begin{equation}
\delta_{G/H} A^H_\mu = 0 \quad \delta_{G/H} A^{G/H}_5 = 0
\end{equation}

\begin{equation}
\delta_{G/H} A^G_5 = 0 \quad \delta_{G/H} A^{G/H}_5 = \partial_5 \epsilon^{G/H}.
\end{equation}

This Peccei–Quinn-like symmetry is sufficient to prevent the appearance of a local mass counter-term. To construct an invariant, we need to use an object that transforms homogeneously under the gauge symmetry, such as the gauge field strength tensor $F_{MN}$. In 5D orbifolds, there is no possible local counter term involving the gauge field strength, since it is an antisymmetric object while we have only one index at our disposal. In 6D orbifolds, however, the brane localized operators \cite{10, 11}

\begin{equation}
\text{Tr}(U^k F_{56}) \quad k = 1, 2, 3, \ldots
\end{equation}

are perfectly allowed and are invariant under the local gauge transformations:

\begin{equation}
\text{Tr}(U F_{56}) \rightarrow \text{Tr}(U g(0) F_{56} g^{-1}(0)) = \text{Tr}(U F_{56} g^{-1}(0) g(0)) = \text{Tr}(U F_{56}).
\end{equation}

where in the first equality we used the fact that, \textit{at the fixed points}, $U$ and $g$ commute (for concreteness, we considered that the fixed point is at the origin). These operators are potentially quite dangerous, since they would correspond to a tadpole for some massive KK gauge-scalars along the unbroken $U(1)$’s directions and, through the non-Abelian part of $F_{56}$, to a mass for the massless gauge-scalars. And by power counting, these operators are \textit{quadratically divergent}.

### 4.3 Experimental signatures

The collider signals have not been studied in detail yet (may be for lack of a fully realistic model). We can still pin down some predictions of a generic gauge–Higgs unification model, namely that we should observe:

- KK excitations of the $W$ and $Z$ around 500 GeV – 1 TeV;
- spin-1 KK excitations of the $G/H$ coset, in particular, some gauge bosons with the EW quantum numbers of the Higgs doublet;
- extra scalar fields;
- some bulk fermions that mix with the SM fermions to generate their masses.
4.4 Recent developments and open issues

In view of the quadratic divergence for the localized tadpole in the most promising 6D models, recent studies of gauge–Higgs unification models have focused mainly on 5D. The main issue of 5D models is to accommodate the heaviness of the top quark and of the Higgs.

Regarding the top mass, since the Yukawa couplings are generated through gauge coupling, it is hard to engineer a setup with a top heavier than the $W$ mass. A possible way out is to embed the top in a large representation such that the effective Yukawa is enhanced by a group factor. For instance in the $SU(3)$ model of [12], the prediction $M_t = 2M_W$ has been reached at tree level. The main drawback of this possibility is that the large representation will lower the scale where the extra-dimensional theory becomes strongly coupled. Moreover, some rather large deviations in the coupling of the left bottom quark to the $Z$, $Zb_L\bar{b}_L$, will be introduced at tree level. Another possibility pursued in [13] is to explicitly give up Lorentz invariance along the extra dimension. In this case, each fermion will effectively feel an extra dimension of different length, alleviating the relation between the top and the $W$ masses. The strong coupling scale is also lowered in that case and the Lorentz breaking reintroduces a UV sensitivity of the Higgs potential at higher loop (as in Little Higgs models). And again, corrections to $Zb_L\bar{b}_L$ and four-Fermi operators generated by the KK gauge bosons pose a bound on the scale of the fifth dimension of the order of few TeV.

Regarding the Higgs mass, it generically turns out to be too small, below the value currently excluded by LEP, because the quartic interaction is now generated at one loop (contrary to the $G_2$ 6D model, where it was present at tree level). Since the entire potential (mass and quartic) is loop generated, the potential will also generically prefer large values of the Higgs vev relative to the compactification scale, so that the scale of new physics stays dangerously low. It was shown in [12] that the Higgs mass can be raised by the presence of several (twisted) bulk fermions. In the Lorentz-violating model of [13], the Higgs mass is set by the scale of the top.

Another direction that has been explored, in particular in [14], is to embed the idea of gauge–Higgs unification in a warped extra dimension. The nice thing is that the warping enhances both the Higgs and the top mass. However, the non-trivial background will also induce corrections to EW precision observables. Via the AdS/CFT correspondence, these models will now be reinterpreted as weakly coupled duals of the old composite Higgs models of Georgi–Kaplan. One highly valuable benefit of warped extra dimension is the ability to postpone the scale of new physics to very high energy with in particular the possibility to accommodate unification.

One final comment concerns the dynamics of the EW phase transition in these gauge–Higgs unification models. As in Little Higgs theories, the structure of the radiatively generated Higgs potential is richer than just the $\phi^4$ Mexican hat potential, with the presence of a series of non-renormalizable interactions. It was shown that we could then obtain a moderately first-order EW phase transition, even for reasonably large values of the Higgs mass [15]. This revives the possibility of EW baryogenesis to generate the asymmetry between matter and antimatter.

5 Higgsless models

5.1 Higgs mechanism localized on a boundary: scalar decoupling limit

Let us now consider a five-dimensional setup with gauge fields propagating in the bulk. Scalar fields that develop vev’s are added on the boundary. We want to see the effect of the vev on the boundary conditions satisfied by the various components of the gauge field. Let us consider
\[ \partial_5 A^a_\mu(0) = 0 \quad \partial_5 A^a_\mu(\pi R) = -\frac{1}{4}g^2 v^2 A^a_\mu(\pi R) \]

Figure 1: Example of a Higgs mechanism localized on a boundary. For a finite Higgs vev, we obtain a mixed BC which, in the infinite vev limit, simply becomes a Dirichlet BC: all the gauge bosons that couple to the Higgs have a wave-function that vanishes at the point where the Higgs is localized. In that limit, there is no scalar degree of freedom in the low energy effective action and the gauge symmetry is entirely broken by the BCs and the mass of the lightest KK state is simply inversely proportional to the size of the extra dimension.

for instance (see Fig. 1) a SU(2) gauge group with Newmann BCs for the \( A_\mu \) components at both ends of the interval. We then assume that at \( y = \pi R \), SU(2) is fully broken by the vev of a Higgs doublet. As for the scalar case, the boundary mass generated by the Higgs vev induces a mixed BC of the form

\[ \partial_5 A^a_\mu(\pi R) = -\frac{1}{4}g^2 v^2 A^a_\mu(\pi R). \]  

The canonically normalized KK modes are given by

\[ A^a_\mu(x, y) = \sum_{k=1}^{\infty} f_k(y)A^{(k)}(x), \]

with

\[ f_k(y) = \frac{\sqrt{2}}{\sqrt{\pi R(1 + 16M_k^2/(g_{5D}^2v^4)) + 4/(g_{5D}^2v^2)}} \cos(M_ky) \sin(M_k\pi R). \]

The BC at the origin, \( y = 0 \), is trivially satisfied while the condition at \( y = \pi R \) determines the mass spectrum through the equation:

\[ M_k \tan(M_k\pi R) = \frac{1}{4}g^2 v^2. \]

In the large vev limit, we obtain that the wave-functions at the \( y = \pi R \) boundary vanish like \( 1/v^2 \)

\[ f_k(\pi R) \sim 2\sqrt{\frac{2}{\pi R}} \frac{2k + 1}{g_{5D}^2 R v^2}, \]

while the KK masses remain finite

\[ M_k \sim \frac{2k + 1}{2R} \left( 1 - \frac{4}{g_{5D}^2 \pi R v^2} \right). \]
This limit exactly corresponds to a Dirichlet BC: in the large vev limit, the wave-functions of the gauge bosons that couple to the Higgs vanish. It can also be checked that, in that limit, $A_5$ actually obeys a Neumann BC. Though, in our example, because of the other Dirichlet BC at $y = 0$, there is still no physical massless mode for $A_5$, while the would-be massive ones are eaten to give the longitudinal polarizations of the massive $A_\mu$. What allows us to decouple the Higgs degree of freedom from the low energy action is that, contrary to 4D, the masses of the gauge bosons are not proportional to the Higgs vev.

5.2 Unitarity Restoration by KK Modes. Sum Rules of Higgsless Theories

$$\epsilon_\mu = \left( \frac{|\vec{p}|}{M_n}, \frac{E}{M_n}, \frac{\vec{p}}{M_n} \right)$$

$$p_\mu^{in} = (E, 0, 0, \pm \sqrt{E^2 - M_n^2})$$

$$q_\mu^{out} = \left( E, \pm \sqrt{E^2 - M_n^2 \sin \theta}, 0, \pm \sqrt{E^2 - M_n^2 \cos \theta} \right)$$

Figure 2: Elastic scattering of longitudinal modes of KK gauge bosons, $n + n \rightarrow n + n$, with the gauge index structure $a + b \rightarrow c + d$. The $E$-dependence can be estimated from $\epsilon \sim E, p_\mu \sim E$ and a propagator $\sim E^{-2}$.

Our aim is to build a Higgsless model of electroweak symmetry breaking using BC breaking in extra dimensions. However, there is a problem in theories with massive gauge bosons without a Higgs scalar: the scattering amplitude of longitudinal gauge bosons will grow with the energy and violate unitarity at a low scale [16]. What we would like to first understand is what happens to this unitarity bound in a theory with extra dimensions. For simplicity we will be focusing on the elastic scattering of the longitudinal modes of the $n^{th}$ KK mode. The energy dependence can be estimated from $\epsilon \sim E, p_\mu \sim E$ and a propagator $\sim E^{-2}$. This way we find that the amplitude could grow as quickly as $E^4$, and then for $E \gg M_W$, we can expand the amplitude in decreasing powers of $E$ as

$$A = A^{(4)} \frac{E^4}{M_n^4} + A^{(2)} \frac{E^2}{M_n^2} + A^{(0)} + O \left( \frac{M_n^2}{E^2} \right).$$

In the SM (and any theory where the gauge kinetic terms form the gauge-invariant combination $F_{\mu\nu}^2$) the $A^{(4)}$ term automatically vanishes, while $A^{(2)}$ is only cancelled after taking into account the Higgs exchange diagrams.

In the case of a theory with an extra dimension with BC breaking of the gauge symmetry, there are no Higgs exchange diagrams; however, one needs to sum up the exchanges of all KK modes. As a result we will find the following expression for the terms in the amplitudes that grow with energy:

$$A^{(4)} = i \left( g_{mnmn}^2 - \sum_k g_{nk}^2 \right) a^{(4)}(\theta).$$

with

$$a^{(4)}(\theta) = (f^{abe} f^{cde} (3 + 6 \cos \theta - \cos^2 \theta) + 2(3 - \cos^2 \theta) f^{ace} f^{bde}),$$

(37)
In order for the term $A^{(4)}$ to vanish it is sufficient to ensure that the following sum rule between the couplings of the various KK modes is satisfied [17]:

$$E^4 \text{ sum rule: } g_{nnn}^2 = \sum_k g_{nnk}^2. \quad (40)$$

Assuming $A^{(4)} = 0$ we get

$$A^{(2)} = \frac{i}{M_n^2} \left(4g_{nnn}M_n^2 - 3\sum_k g_{nnk}^2M_k^2 \right)a^{(2)}(\theta), \quad (41)$$

with

$$a^{(2)}(\theta) = (f^{ace}f^{bde} - \sin^2\frac{\theta}{2}f^{abe}f^{cde}). \quad (42)$$

Assuming that relation (40) holds, we can find a sum rule that ensures the vanishing of the $A^{(2)}$ term:

$$E^2 \text{ sum rule: } g_{nnn}M_n^2 = \frac{3}{4}\sum_k g_{nnk}^2M_k^2. \quad (43)$$

Here $g_{nnn}^2$ is the quartic self-coupling of the $n$th massive gauge field, while $g_{nnk}$ is the cubic coupling between the KK modes. In theories with extra dimensions, these are of course related to the extra dimensional wave-functions, $f_n(y)$, of the various modes as

$$g_{mnk} = g_5 \int dy f_m(y)f_n(y)f_k(y) \quad \text{and} \quad g_{mnkl}^2 = g_5^2 \int dy f_m(y)f_n(y)f_k(y)f_l(y). \quad (44)$$

Amazingly, higher-dimensional gauge invariance will ensure that both of the sum rules are satisfied as long as the breaking of the gauge symmetry is spontaneous. For example, it is easy to show the first sum rule via the completeness of the wave functions $f_n(y)$:

$$\int_0^{\pi R} dy f_n^4(y) = \sum_k \int_0^{\pi R} dy \int_0^{\pi R} dz f_n^2(y)f_n^2(z)f_k(y)f_k(z). \quad (45)$$

One can similarly show that the second sum rule will also be satisfied if the boundary conditions are natural and all terms in the Lagrangian (including boundary terms) are gauge-invariant. Let us insist on the particular case of a Higgs mechanism localized at the boundary: for finite Higgs vev, the cancellation of the $E^2$ term requires the exchange of the brane Higgs scalar degree of freedom; however, in the infinite vev limit, the contribution of the Higgs exchange to the scattering amplitude actually cancels out and we are left with simple Dirichlet BCs for which the scattering amplitude is unitarized by the sole exchange of spin-1 KK excitations.

At this point, it should be noted that the two sum rules cannot be satisfied with a finite number of KK modes. This is in full agreement with the old theorem by Cornwall et al., who established that the only way to restore perturbative unitarity in the scattering of massive spin-1 particles is through the exchange of a scalar Higgs boson. Our 5D theory is non-renormalizable anyway, so it is valid up to a finite cutoff. What our result really shows is that, through the exchange of the KK gauge bosons, the perturbative unitarity breakdown is postponed from an energy scale of the order of the mass of the lightest KK state to the true 5D cutoff of the order of the mass of the heaviest KK state (see Fig. 3).

---

3These expressions of the effective cubic and quartic couplings are valid in flat space. When the fifth dimension is curved, appropriate powers of the warp factor appear. The scalar product used in the completeness relation has to be modified accordingly. At the end of the day, the same sum rules still hold.
What we see from the above analysis is that in any gauge-invariant extra-dimensional theory the terms in the amplitude that grow with the energy will cancel. However, this will not automatically mean that the theory itself is unitary. The reason is that there are two additional worries: even if $A^{(4)}$ and $A^{(2)}$ vanish, $A^{(0)}$ could be too large and spoil unitarity. This is what happens in the SM if the Higgs mass is too large. A full analysis has been performed in [18], where it was shown that, after taking into account the opening up of the inelastic channels, the scattering amplitude will grow linearly with energy and will always violate unitarity at some energy scale. This is a consequence of the intrinsic non-renormalizability of the higher-dimensional gauge theory. It was found that the unitarity violation scale due to the linear growth of the scattering amplitude is equal (up to a small numerical factor of order 2–4) to the cutoff scale of the 5D theory obtained from naive dimensional analysis (NDA). This cutoff scale can be estimated in the following way. The one-loop amplitude in 5D is proportional to the 5D loop factor $g_5^2/(24\pi^3)$. The dimensionless quantity obtained from this loop factor is $g_5^2E/(24\pi^3)$, where $E$ is the scattering energy. The cutoff scale can be obtained by calculating the energy scale at which this loop factor will become of order 1 (that is the scale at which the loop and tree-level contributions become comparable). From this we get $\Lambda_{\text{NDA}} = 24\pi^3/g_5^2$. We can express this scale by using the matching of the higher-dimensional and the lower-dimensional gauge couplings. In the simplest theories this is usually given by $g_5^2 = \pi R g_4^2$, where $\pi R$ is the length of the interval, and $g_4$ is the effective 4D gauge coupling. So the final expression of the cutoff scale can be given as $\Lambda_{\text{NDA}} = 24\pi^3/g_4^2$. We will see that in the Higgsless models $1/R$ will be replaced by $M_W^2/M_{KK}^2$, where $M_W$ is the physical $W$ mass, and $M_{KK}$ is the mass of the first KK mode beyond the $W$. Thus the cutoff scale will indeed be lower if the mass of the KK mode used for unitarization is higher. However, this $\Lambda_{\text{NDA}}$ could be significantly higher than the cutoff scale in the SM without a Higgs, which is around 1.2 TeV.
5.3 Warped Higgsless Model with Custodial Symmetry

It is clear that in order to find a Higgsless model with the correct W/Z mass ratio one needs to find an extra-dimensional model that has the custodial SU(2) symmetry incorporated [19]. Therefore we need to somehow involve SU(2)_R in the construction. The simplest possibility is to put an entire SU(2)_L × SU(2)_R × U(1)_{B-L} gauge group in the bulk of an extra dimension [20]. In order to mimic the symmetry-breaking pattern in the SM most closely, we assume that on one of the branes the symmetry breaking is SU(2)_L × SU(2)_R → SU(2)_D, with U(1)_{B-L} unbroken. On the other boundary the bulk gauge symmetry must be reduced to that of the SM, and thus have a symmetry-breaking pattern SU(2)_R × U(1)_{B-L} → U(1)_Y, which is illustrated in Fig. 4. The custodial symmetry is broken on one boundary. To reduce the effect of this breaking on the KK modes, we need to engineer a setup such that all the KK wave-functions are localized away from the point where the custodial symmetry is broken. This is automatically achieved if the space is curved by a negative vacuum energy to an anti-de Sitter (AdS) background.

Figure 4: The symmetry-breaking structure of the warped Higgsless model [20]. We will be considering a 5D gauge theory in the fixed gravitational anti-de-Sitter (AdS) background. The UV brane (sometimes called the Planck brane) is located at z = R and the IR brane (also called the TeV brane) is located at z = R'. R is the AdS curvature scale. In conformal coordinates, the AdS metric is given by \( ds^2 = (R/z)^2 \left( \eta_{\mu\nu} dx^\mu dx^\nu - dz^2 \right) \).

The appropriate BCs are

\[
\begin{align*}
\text{at } z = R : & \quad \partial_z (g_5 R A_\mu^L) + \tilde{g}_5 A_\mu^{R_3} = 0, \quad \partial_z A_\mu^L = 0, \quad A_\mu^{R_1,2} = 0, \quad \tilde{g}_5 B_\mu - g_5 R A_\mu^{R_3} = 0, \quad (46) \\
\text{at } z = R' : & \quad \partial_z (g_5 L A_\mu^L) + g_5 L A_\mu^{R_a} = 0, \quad \partial_z B_\mu = 0, \quad g_5 L A_\mu^L - g_5 R A_\mu^{R_a} = 0, \quad (47)
\end{align*}
\]

We denoted by \( A_\mu^{R_a}, A_\mu^{L_a} \) and \( B_\mu \) the gauge bosons of SU(2)_R, SU(2)_L and U(1)_{B-L} respectively; \( g_5L \) and \( g_5R \) are the gauge couplings of the two SU(2)'s, and \( \tilde{g}_5 \) the gauge coupling of the U(1)_{B-L}. The corresponding KK decomposition is given by

\[
\begin{align*}
B_\mu &= g_5 a_0 \gamma_\mu(x) + \sum_{k=1}^{\infty} \psi_k^{(B)}(z) Z_\mu^{(k)}(x), \\
A_\mu^{L_3} &= g_5 a_0 \gamma_\mu(x) + \sum_{k=1}^{\infty} \psi_k^{(L_3)}(z) Z_\mu^{(k)}(x), \\
A_\mu^{R_3} &= \tilde{g}_5 a_0 \gamma_\mu(x) + \sum_{k=1}^{\infty} \psi_k^{(R_3)}(z) Z_\mu^{(k)}(x), \\
A_\mu^{L_\pm} &= \sum_{k=1}^{\infty} \psi_k^{(L_\pm)}(z) W_\mu^{(k)\pm}(x), \\
A_\mu^{R_\pm} &= \sum_{k=1}^{\infty} \psi_k^{(R_\pm)}(z) W_\mu^{(k)\pm}(x), \quad (48)
\end{align*}
\]
The wavefunctions, solutions of the bulk equation of motion in AdS space, involve some Bessel functions of order 1

\[ \psi_k^{(A)}(z) = z \left( a_k^{(A)} J_1(q_k z) + b_k^{(A)} Y_1(q_k z) \right). \]  

(49)

On top of a flat massless mode, corresponding to the photon of the unbroken \( U(1)_{\text{em}} \), the spectrum involves two light gauge bosons, naturally identified as the SM gauge bosons, with masses suppressed by \( \log R'/R \) compared with the rest of the KK towers (\( M_{KK} \sim O(1)/R' \)):

\[
M_W^2 \sim \frac{2g_{5L}^2}{g_{5L}^2 + g_{5R}^2} \frac{1}{R^2 \log R'/R} + \ldots, \quad M_Z^2 \sim \frac{2g_{5L}^2}{g_{5L}^2 + g_{5R}^2} \frac{g_{5R}^2 + g_{5L}^2(1 + g_{5R}^2/g_{5L}^2)}{g_{5R}^2 + g_{5L}^2} \frac{1}{R^2 \log R'/R} + \ldots
\]

(50)

where \( \ldots \) denote corrections in \( 1/\log^2(R'/R) \). The coupling of the photon allows us to identify the 4D SM gauge couplings as functions of the 5D parameters:

\[
\frac{1}{g^2} = \frac{R \log R'/R}{g_{5L}^2} \quad \text{and} \quad \frac{1}{g'^2} = \frac{1}{R^2 \log R'/R} \left( \frac{1}{g_{5R}^2} + \frac{1}{g_{5L}^2} \right). \]

(51)

The \( \rho \) parameter is thus equal to 1, as announced earlier. This equality would not occur if the extra dimension were flat, it is a consequence of the localization property of the KK wavefunctions, which ensure that the bulk gauge \( SU(2)_R \) symmetry acts as a custodial symmetry. The presence of this approximate global symmetry can also be easily understood from the AdS/CFT duality. From that perspective, our 5D warped Higgsless model appears as a weakly coupled dual of walking technicolour models [20].

Finally, after redshift due to the warping of the space, the NDA cutoff is estimated to be

\[
\Lambda_{\text{NDA}} \sim \frac{24\pi^3}{g^2} \frac{R}{R'} \sim \frac{24\pi^3}{g'^2} \frac{1}{R^2 \log R'/R} \sim \frac{50\pi^3}{g^2} \frac{M_W^2}{M_{KK}}.
\]

(52)

As dictated by intuition, the smaller \( M_{KK} \), the higher the scale where perturbative control is lost. Phenomenologically, the preferred range of \( M_{KK} \) will be around 500 GeV to 1 TeV.

### 5.4 Fermion Masses

In the Standard Model, quarks and leptons acquire a mass after electroweak symmetry breaking through their Yukawa couplings to the Higgs. In the absence of a Higgs, one cannot write any Yukawa coupling and one should expect the fermions to remain massless. However, as for the gauge fields, appropriate BCs will force the fermions to acquire a momentum along the extra dimension and this is how they will become massive from the 4D point of view.

The SM fermions cannot be completely localized on the UV boundary: since the unbroken gauge group on that boundary coincides with the SM \( SU(2)_L \times U(1)_Y \) symmetry, the theory on that brane would be chiral and there is no way for the chiral zero-mode fermions to acquire a mass. The SM fermions cannot live on the IR brane either since the unbroken \( SU(2)_D \) gauge symmetry will impose an isospin-invariant spectrum and the up-type and down-type quarks will be degenerate, as well as the electron and the electron neutrino. The only possibility is thus to embed the SM fermions into 5D fields living in the bulk and feeling the gauge symmetry breakings on both boundaries. Bulk fermions are generically Dirac fermions; however, on an interval in warped space only one of the chiralities will have a zero mode. The location of the zero mode in warped space depends on the bulk mass term, and can be localized close to the UV brane for all the fermions of the first two generations and the leptons of the third generation. For the right-handed top quark, one can localize the wave function of the zero mode closer to the IR brane. Since the theory on the IR brane is vector-like (only \( SU(2)_D \) is unbroken there),
a mass for the zero modes can be added on the IR brane (which corresponds to a dynamical isospin symmetric fermion mass in the CFT language). The size of the physical mass will then depend on the location of the zero mode and the value of the mass term on the IR brane. However, because of the unbroken $SU(2)_D$ symmetry on the IR brane, these masses must be isospin-symmetric, that is the mass for the up and down type quarks are equal at this point. Isospin splitting can be introduced by adding operators on the UV brane. For instance one can introduce different brane-localized kinetic terms for the up and down right-handed quarks. The full spectrum of quarks and leptons can be easily reproduced this way.

5.5 Electroweak Precision Constraints, Collider Signatures and Conclusions

Waiting for the LHC to reveal any signs of new physics, the major stumbling block for any theory beyond the SM is the level of corrections to electroweak precision measurements. And sharing so many resemblance with technicolour models, it is not a surprise that generically a large contribution to the $S$ parameter of order unity is found [21]. This contribution can be lowered by introducing a brane kinetic term on the IR brane for the $B-L$ gauge group, albeit at the price of lowering the mass of one of the $Z'$ to phenomenologically unacceptable levels. In Ref. [22], it was pointed out that one can in fact easily eliminate the large contributions to the $S$ parameter by changing the position of the light fermions. The reason behind this is simple: the oblique correction parameters on their own are meaningless until the normalization of the couplings between the fermions and the gauge bosons is fixed. An overall shift in the fermion–gauge boson couplings can be reabsorbed in the oblique correction parameters [23] and thus effectively change the predicted values of $S,T$. This is exactly what happens when one changes the localization parameters of the light fermions. When the fermions are strictly localized on the UV brane, one obtains a positive $S$ parameter. However, it has been known that if fermions are localized on the TeV brane then the $S$ parameter in the Randall–Sundrum model is in fact negative. Therefore it should be expected that there should be an intermediate position where $S$ exactly vanishes. This actually happens when the fermion wave-functions are “flat”. This is just a simple consequence of the orthogonality of the KK mode wave functions of the gauge bosons: when the fermion wave-functions are flat, the coupling of the KK gauge bosons to the fermions vanishes, eliminating any possible additional LEP or Tevatron constraints on this setup. This way, with an appropriate tuning of the localization of the fermions in the bulk, the model can pass the electroweak precision constraints.

A reason for localizing the light generations near the UV brane was that corrections to Flavour Changing Neutral Currents, coming from higher-order operators, should be suppressed by a large scale, of order $1/R$ rather than the strong coupling scale estimated earlier. If we delocalize the light fermions, such scale is red-shifted to a dangerously low energy. In order to escape experimental bounds, we need to implement a flavour symmetry in the bulk and on the IR brane. Moreover, the mechanism that generates masses for the fermions themselves will induce some distortions in the wave functions, thus modifying in a non-universal way the couplings with the SM gauge bosons.

A more serious problem arises when one tries to introduce the third family [24]: there is a tension between the heaviness of the top and the coupling of the left-handed $b_L$ to the $Z$ gauge boson. It has recently been argued that this problem can be alleviated by a suitable embedding [25] of the SM third generation into non-standard representations of $SU(2)_L \times SU(2)_R$. Many different realizations of Higgsless models have been proposed, differing in the way the SM fermions are introduced or even in the number of extra dimensions. All these models
will have different particular signatures. However, the fundamental mechanism by which $\Lambda$ is raised is a common feature to all these models: new massive spin-1 particles, with the same quantum numbers as the SM gauge bosons, appear at the TeV scale and their couplings to the $W, Z$ and $\gamma$ obey unitarity sum rules like (40) and (43), which enforce the cancellation of the energy-growing contributions to the scattering amplitudes of the longitudinal $W, Z$. Vector boson fusion processes will thus provide a model-independent test of the Higgsless scenario. The non-observation of a physical scalar Higgs would be the first indication for a Higgsless scenario. Yet, the absence of proof is not the proof of the absence and some other models exist in which the Higgs is unobservable at the LHC and we need to look for other distinctive features of Higgsless models, such as the presence of spin-1 KK resonances with the $W, Z$ quantum numbers, some slight deviations in the universality of the light fermion couplings to the SM gauge bosons, or some deviations in the gauge boson self-interactions compared with the SM. More than ever, experimental data are eagerly awaited to disentangle what may be the most pressing question faced by particle physics today: How is electroweak symmetry broken?

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