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Abstract.

The quasi-momentum representation for an arbitrary quantum system is constructed. In particular, the precise formula which express the Bloch functions through the elliptic Jakobe theta functions are obtained for the following dynamical systems: the relativistic and nonrelativistic electron in the uniform constant magnetic field; nonrelativistic electron in the time-dependent uniform electromagnetic field ($\vec{E} \cdot \vec{H} = 0$); the quantum oscillator; the general quadratic system. The mathematical expression of the completeness of the Bloch functions is given. The new connections between the theta functions are received.

COHERENT STATES AND BLOCH FUNCTIONS

S.G.Krivoshlykov, I.A.Malkin, V.I.Man'ko

Introduction

It is known that the Hamiltonian of the spinless charged particle moving in the constant and uniform magnetic field has no translational symmetry. But as it was shown by H.Fischbeck [1], E.Brown [2], J.Zak [3] there is the special symmetry in this problem coinciding with translational one when the magnetic field is going to zero. This symmetry has been called magnetic-translational symmetry and the operators, commuting with the Hamiltonian and coinciding with the usual translations in the limit $\vec{H} = 0$, magnetic translations operators. This symmetry gives the possibility to construct such solutions of the Schrodinger equation which realize the irreducible representations of the magnetic translations group, provided the definite translation restrictions, which lead to the flux quantization condition, have been made.

The magnetic translations operators are diagonal for these solutions and the solutions themselves are very similar to the usual Bloch wave functions which realize the irreducible representations of the space translations [1-3]. This kind of functions was considered also by A.Jannussis [4] and P.Jagannathan and N.Ranga-

nathan [5]. A complete classification of the magnetic translations groups in so far as translational symmetry is concerned is given in the paper by Opechowski and Tam [6,7], together with a discussion on their irreducible representations.

As it was shown in the papers [8,9], one can construct the Bloch functions of the Dirac electron in the uniform magnetic field (and in the fields $\vec{E} \vec{H} = 0$, $E < H$).

The coherent states of the electron in the magnetic field (relativistic and nonrelativistic cases) as the closest to the classical states were constructed in [10].

In the paper [8] it has been shown that the unitary Weil's operators, creating the coherent states from the "ground" state, coincide with the magnetic translations operators. Later, Jannussis also pointed out that the magnetic translations operators are the particular case of the Weil's operators [11].

This connection, as it has been shown by Malkin and Man'ko [12], gives the possibility to introduce the quasi-momentum representation, i.e. to find the Bloch functions in all physical problems where the coherent states representations have been used.

In the present paper the quasi-momentum representation for an arbitrary quantum system is constructed. In particular, the precise formulas which express the Bloch functions through the elliptic Jacobi theta functions are

obtained for the following dynamical systems: the relativistic and nonrelativistic electron in the uniform constant magnetic field; nonrelativistic electron in the time-dependent uniform electromagnetic field ($\vec{E} \vec{H} = 0$); the quantum oscillator; the general quadratic system.

The mathematical expression of the completeness of the Bloch functions is given. The condition of this completeness has been recently also given by Boon [13].

The new connections between the theta functions are received.

It should be noted that so far as the consideration of the physical phenomena in the semiconductors and semimetals in the frame of the two-band model (in fields $\vec{E} \vec{H} = 0$) is reduce to the solution of the Dirac equation [14], one can use the quasi-momentum representation to describe the two-dimensional periodic structure state of these semiconductors and semimetals with the quantized electromagnetic flux. Already there are the first experimental indications on the existence of these states [15]. One can utilize this representation also to describe the lattice structure of the type-II superconductor [16].

1. The construction of the Bloch functions for an arbitrary quantum system.

Let us consider an arbitrary quantum system with $\checkmark$ degrees of freedom. This system has $2\checkmark$ independent integ-

rals of the motion. This statement follows immediately the correspondence principle, since every classical system has $2N$ independent integrals of the motion. One can choose the initial values of coordinates and momenta as such independent integrals of the motion. If there is the unitary evolution operator U , the corresponding quantum integrals of the motion can be expressed as follows: $p_\alpha^{in} = U p_\alpha U^{-1}$, $x_\alpha^{in} = U x_\alpha U^{-1}$, where $\alpha = 1, 2, \dots, N$, and x_α and p_α - are the operators of the coordinates and momenta.

The commutation relations between operators of the initial coordinates and momenta have the form: $[x_\alpha^{in}, p_\beta^{in}] = i \delta_{\alpha\beta}$.

All the others integrals of the motion can be received from the initial coordinates and momenta as in the case of the classical mechanics. Any integrals of the motion have the property to transfer the solution of the corresponding quantum equation (Schrodinger, Dirac and others) again in the solution.

One can construct the coherent states of an arbitrary quantum system [17]. For this purpose let us introduce the integrals of the motion $a_\alpha^+ = \frac{1}{\sqrt{2}}(x_\alpha^{in} - i p_\alpha^{in})$, $a_\alpha = \frac{1}{\sqrt{2}}(x_\alpha^{in} + i p_\alpha^{in})$, satisfying the standard commutation relations $[a_\alpha, a_\beta^+] = \delta_{\alpha\beta}$. The coherent states can be constructed by the usual way as the eigenstate of nonhermitian operators a_α :

$$a_{\mu}|\vec{\alpha}\rangle = \alpha_{\mu}|\vec{\alpha}\rangle, \quad \vec{\alpha} = (\alpha^1, \alpha^2, \dots, \alpha^N)$$

where $\vec{\alpha}$ is the N -dimensional complex vector.

This system form a whole set

$$\pi^{-N} \int |\vec{\alpha}\rangle \langle \vec{\alpha}| d^2\vec{\alpha} = I$$

the scalar product of the coherent states has the form:

$$\langle \vec{\alpha} | \vec{\beta} \rangle = \exp\left(-\frac{|\vec{\alpha}|^2}{2} - \frac{|\vec{\beta}|^2}{2} + \vec{\alpha}^* \vec{\beta}\right) \quad (1.1)$$

The coherent states may be obtained from the ground states (for which $a_{\mu}|0\rangle = 0$) by means of the unitary Weil's operators:

$$D(\vec{\beta}) = \exp(\vec{\beta} \vec{a}^+ - \vec{\beta}^* \vec{a}) \quad (1.2)$$

as follows:

$$|\beta_1^1, \dots, \beta_N^N\rangle = \prod_{p=1}^N D(\beta^p) |0\rangle \quad (1.3)$$

The operators $D(\beta^p)$ satisfy the relations:

$$D(\beta_1^p) D(\beta_2^p) = D(\beta_1^p + \beta_2^p) \exp(i \text{Im} \beta_1^p \beta_2^{p*}) \quad (1.4)$$

where index P is $1, 2, \dots, N$.
Let now β^P be of the form $\beta_{m_1 m_2}^P = m_1^P \beta_1^P + m_2^P \beta_2^P$; $m_1, m_2 = 0, \pm 1, \dots$

We consider now the case of the finite dimensional representation of the commutational relations (1.4) that means the case of finite dimensional unitary representations of the discrete Weil's group.

If $D(\beta_{m_1 m_2}^P)$ is the matrix $N^P \times N^P$ the calculation of the determinant of the left and right sides of the relation (1.4) gives

$$\exp(i N^P \text{Im}(\beta_1^P \beta_2^{P*})) = \exp(-i N^P \text{Im}(\beta_1^P \beta_2^{P*}))$$

This condition leads to the "quantization" condition i.e. to the restriction condition for the translation parameters β_1^P and β_2^P in complex β^P -plane

$$\text{Im}(\beta_1^P \beta_2^{P*}) = \frac{\pi n^P}{N^P} \quad (n^P = 0, \pm 1, \dots) \quad (1.5)$$

Thus, we have the infinit lattice in the complex β^P -plane. The area of one cell of this lattice is equal to $\pi \frac{n^P}{N^P}$.

Let us designate the translation operators on this lattice by following way $D(m_1^P \beta_1^P + m_2^P \beta_2^P) \equiv D_{m_1 m_2}^P$

As it is easy to show from the relations (1.4), the

operators $D_{m_1 m_2}^P$ on the lattice satisfy the relation: $D_{m_1 m_2}^P D_{l_1 l_2}^P = D_{m_1 + l_1, m_2 + l_2}^P \exp\{i \pi n^P (m_1^P l_2^P - m_2^P l_1^P)\}$

Let us introduce a new operators $\tilde{D}_{m_1^p m_2^p} \equiv (-1)^{n^p m_1^p m_2^p}$ which are already submitted to the condition:

$$\tilde{D}_{m_1^p m_2^p} \tilde{D}_{\ell_1^p \ell_2^p} = \tilde{D}_{m_1^p + \ell_1^p, m_2^p + \ell_2^p} \quad (1.6)$$

As it is necessary for the translation operators.

In the present paper the Bloch functions i.e. the quasi-momentum representation shall be constructed for these new operators.

Let us consider the operator:

$$\hat{V}_{K_1 K_2}^{p,p} = \sum_{m_1^p m_2^p = -\infty}^{\infty} \prod_{p=1}^N \exp\{i(K_1^p m_1^p \beta_1 + K_2^p m_2^p \beta_2)\} \tilde{D}_{m_1^p m_2^p} \quad (1.7)$$

which is the integral of the motion by the construction.

Here m_1^p, m_2^p are the natural numbers, and generally speaking complex quasi-moments K_1^p and K_2^p satisfy the usual conditions $|K_1^p| \leq \frac{\pi}{\beta_1^p}, |K_2^p| \leq \frac{\pi}{\beta_2^p}$. The index p takes the value from 1 to N .

It is easy to verify that the operator $\hat{V}_{K_1 K_2}^{p,p}$ has the property:

$$\tilde{D}_{m_1^s m_2^s}^s \hat{V}_{K_1 K_2}^{p,p} = e^{-i K_1^s \beta_1 m_1^s} e^{-i K_2^s \beta_2 m_2^s} \hat{V}_{K_1 K_2}^{p,p} \quad (1.8)$$

where s is one from the indexes p .

One can also verify that:

$$\hat{V}_{K_1 K_2}^{\rho \rho} D(\vec{\beta}) = D(\vec{\beta}) \hat{V}_{K_1 - 2\gamma m \vec{\beta}, K_2 + 2Re \vec{\beta}}^{\rho \rho} \quad (1.9)$$

Applying the operator (1.7) to any solution of the corresponding quantum equation we obtain the solution of this equation in the quasi-momentum representation.

It is natural to construct the Bloch wave function an arbitrary quantum system from the coherent states $|\vec{\beta}\rangle$:

$$f_{K_1 K_2}^{\vec{\beta}} = D(\vec{\beta}) \hat{V}_{K_1 - 2\gamma m \vec{\beta}, K_2 + 2Re \vec{\beta}}^{\rho \rho} |\vec{0}\rangle \quad (1.10)$$

Here the relations (1.9) have been used.

Thus we have constructed the quasi-momentum representation for an arbitrary quantum system.

Let us begin discussing of the concrete problem.

2. The relativistic electrons in the uniform constant magnetic field.

Let's consider the Dirac equation for the electron in the uniform magnetic field. One can solve it by the usual method [18]

$$H_D \psi_E = [\vec{\alpha}(\vec{p} - e\vec{A}) + \beta m] \psi_E = E \psi_E \quad (2.1)$$

($\hbar = c = 1$)

$\vec{\alpha}$ and β are the usual Dirac matrices, the vector $\vec{A}$ being equal to $\frac{1}{2}[\vec{H} \times \vec{L}]$.

Multiplying the Dirac equation $(H_D - E)\psi_E = 0$ by the operator

$$O_- = \begin{pmatrix} E + m & \vec{\sigma}(\vec{p} - e\vec{A}) \\ \vec{\sigma}(\vec{p} - e\vec{A}) & E - m \end{pmatrix}$$

One can receive the nonrelativistic Pauli equation

$$\left[\frac{1}{2m}(\vec{p} - e\vec{A})^2 - \frac{e}{2m} \vec{H} \vec{\sigma}_z - \frac{1}{2m}(E^2 - m^2) \right] \varphi_E = 0 \quad (2.2)$$

where φ_E is the 4-component bispinor.

The one-to-one correspondence of the solutions φ_E of the equation (2.2) with the solutions ψ_E of the equation (2.1) is given [18] by the formula $\psi_E = O_- \varphi_E$, $\varphi_E = \begin{pmatrix} 0 \\ \varphi_E \end{pmatrix}$ where φ_E is two-component spinor satisfying the equation (2.2).

Then we have:

$$\psi_E = \begin{pmatrix} R \varphi_E \\ C \varphi_E \end{pmatrix}$$

The operators R and C is given by the formula

$R = \vec{\sigma}(\vec{p} - e\vec{A})$ and $C = H_D - m$, moreover, it is known that $R^2 = H_D^2 - m^2$ and $C^2 = H_D^2 + m^2 - 2H_D m$.

Let's introduce the new variable $\xi = \frac{x + iy}{2} \sqrt{m\omega}$, where $\omega = \frac{e\vec{H}}{m}$.

Then the unitary Weil's operators (1.2) have the form $D(\beta) = \exp(\beta \hat{b}^\dagger - \beta^* \hat{b})$ where according to [10] the creation and annihilation operators can be expressed as $\hat{b}^\dagger = \frac{1}{\sqrt{2}}(\hat{f} - \frac{\partial}{\partial \hat{f}^*})$ and $\hat{b} = \frac{1}{\sqrt{2}}(\hat{f}^* + \frac{\partial}{\partial \hat{f}})$.

It is easy to verify that the magnetic translations operator $T(\vec{R}) = \exp[-i\vec{R}(\vec{p} + e\vec{A})]$ which have been introduced in the papers [1-3] commute with the Hamiltonian H_D being consequently the integrals of the motion.

In the paper [8] it has been shown that in the case of the constant magnetic field the unitary Weil's operators, creating the coherent states from the "Ground" state, coincide with the magnetic translations operators when:

$$\beta = \gamma (R_x - i R_y) \quad (2.3)$$

where $\gamma = \sqrt{\frac{m\omega}{2}}$ and $\omega = \frac{e\hbar}{m}$.

The magnetic translations operators satisfy the relation

$$T(\vec{R}_1)T(\vec{R}_2) = T(\vec{R}_1 + \vec{R}_2) \exp\left\{\frac{ie\hbar}{2}[\vec{R}_1 \times \vec{R}_2]\right\} \quad (2.4)$$

According to relation (2.3) this formula coincide with (1.4).

The requirement of the commutation for the magnetic

translations $T(a\vec{i})$ and $T(b\vec{j})$ along the axis X and Y in a plane perpendicular to the direction of the magnetic field leads to the quantization conditions i.e. to the restriction conditions for the translation parameters a and b

$$eH a b = 2\pi n \quad (n = 0, \pm 1, \dots) \quad (2.5)$$

The operators $T(m_1 a \vec{i})$, $T(m_2 b \vec{j})$ realize the unitary infinite dimensional reducible representation of the discrete translation group $\vec{R} = m_1 \vec{a} + m_2 \vec{b}$. Since the abelian group has only ^{one} dimensional unitary representations the problem of finding the Bloch functions is reduced to the consideration of all the one-dimensional representation of the discrete translation group which are contained in the reducible representation under consideration.

Thus, in the case of the constant magnetic field the condition (1.5) when $N^P = 1$ has the form (2.5) and following physical sense: the flux of the magnetic field through a two-dimensional cell in a plane perpendicular to the direction of field is a natural multiple of a flux quantum.

It should be noted that Brown constructed the Bloch functions as the eigenfunctions of the operator $T_{m_1 m_2}$ [2]. These functions have the correct Bloch property

for the arbitrary translation vectors of a lattice. Only in the case of even numbers n in (2.5) when the operators $T_{m_1 m_2}$ and $\tilde{T}_{m_1 m_2} = (-1)^{nm_1 m_2} T_{m_1 m_2}$ coincide. It will be shown that just the case of the odd number $n=1$ corresponds to the condition of the completeness of the Bloch system being the most interesting.

In the present paper the Bloch functions will be constructed as the eigenfunctions of the operator $\tilde{T}_{m_1 m_2}$ in case of an arbitrary number n .

The coherent states of the nonrelativistic electrons in the uniform constant magnetic field have been constructed in the paper [19]

$$|\alpha\beta\rangle = \sqrt{\frac{m\omega}{2\pi}} \exp\left\{-\frac{1}{2}|\alpha|^2 + \sqrt{2}\beta\alpha + i\sqrt{2}\alpha\beta^* - i\alpha\beta - \frac{|\alpha|^2}{2} - \frac{|\beta|^2}{2}\right\} \quad (2.6)$$

The operator (1.7) in the present case according to (2.3) has the form

$$\hat{V}_{K_x K_y} = \sum_{m_1 m_2 = -\infty}^{\infty} (-1)^{nm_1 m_2} e^{iK_x m_1 a + iK_y m_2 b} D_{m_1 m_2}(m_1 \alpha - i m_2 \beta) \quad (2.7)$$

where the quasi-momenta have the usual restrictions:

$$|K_x| \leq \frac{\pi}{a}, \quad |K_y| \leq \frac{\pi}{b}, \quad \hat{V}_{K_x K_y} D(\alpha) =$$

$$= D(\alpha) \hat{V}_{K_x - 2\gamma J m_d, K_y - 2\gamma R e d} \quad \text{which is similar to}$$

(1.9) and construct the Bloch function from the states

$$\begin{aligned}
 (2.6) \quad f_{K_x K_y}^{\alpha\beta} &= D(\alpha) D(\beta) \left[\sqrt{\frac{m\omega}{2\pi}} e^{-1/2} \right] \sum_{m_1 m_2 = -\infty}^{\infty} (-1)^{m_1 m_2} \exp \{ i m_1 (K_x - 2\gamma) m \beta \} a \\
 &+ i m_2 (K_y - 2\gamma) m \beta \} b + \sqrt{2} \gamma (m_1 a - i m_2 b) - \frac{\gamma^2}{2} (m_1^2 a^2 + m_2^2 b^2) \} \quad (2.8)
 \end{aligned}$$

One can sum up this expression by using the definitions of the elliptic theta functions [19]

$$\vartheta_3(\eta/v) = \sum_{m=-\infty}^{\infty} q^{m^2} e^{2\pi i m v} \quad (2.9)$$

$$\vartheta_2(\eta/v) = \sum_{m=-\infty}^{\infty} q^{(m-1/2)^2} e^{i\pi v(2m-1)} \quad (2.10)$$

If n is the even number, then

$$f_{K_x K_y}^{\alpha\beta} = \langle \alpha\beta | \vartheta_3(\eta_1/v_1) \vartheta_3(\eta_2/v_2) \rangle \quad (2.11)$$

If n is the odd number, then

$$f_{K_x K_y}^{\alpha\beta} = |\alpha\beta\rangle \left\{ \mathcal{V}_3(q_1^4/2v_1) \mathcal{V}_3(q_2^4/2v_2) + \right. \\ \left. + \mathcal{V}_3(q_1^4/2v_1) \mathcal{V}_2(q_2^4/2v_2) + \mathcal{V}_2(q_1^4/2v_1) \mathcal{V}_3(q_2^4/2v_2) - \mathcal{V}_2(q_1^4/2v_1) \mathcal{V}_2(q_2^4/2v_2) \right\} \quad (2.12)$$

Here: $q_1 = e^{-\frac{\gamma^2 \alpha^2}{2}}$; $v_1 = \frac{\alpha}{2\pi} (K_x - i\sqrt{2}\gamma\{\} - \alpha\gamma + i\gamma\beta)$
 $q_2 = e^{-\frac{\gamma^2 \beta^2}{2}}$; $v_2 = \frac{\beta}{2\pi} (K_y - \sqrt{2}\gamma\{\} + i\alpha\gamma - \gamma\beta)$

In case of odd number n the sum (2.8) has been divided on four sum in which $m_1 = 2K_1, m_2 = 2K_2$; $m_1 = 2K_1, m_2 = 2K_2 + 1$; $m_1 = 2K_1 + 1, m_2 = 2K_2 + 1$; and $m_1 = 2K_1 + 1, m_2 = 2K_2$ accordingly.

In similar way one can divide the sum in the definition of the $\mathcal{V}_3$ - function (2.9) on p sum in which $m = K+1, m = K+2, \dots, m = K+p$ and prove the new connection between the $\mathcal{V}_3$ - function:

$$\mathcal{V}_3(q/v) = \sum_{\ell=0}^{p-1} q^{\ell^2} e^{2\pi i v \ell} \mathcal{V}_3(q^{p^2}/p v + \frac{\ell}{p} \tilde{r}) \quad (2.13)$$

where $q^{p^2} = e^{i\pi \tilde{r}}$.

The same formula for the $\mathcal{V}_4$ - function [19]

$$\mathcal{V}_4(q/v) = \sum_{m=-\infty}^{\infty} (-1)^m q^{m^2} e^{2\pi i v m} \quad (2.14)$$

has the form

$$\psi_4^q(\eta/v) = \sum_{\ell=0}^{\rho-1} (-1)^\ell q^{\rho^2} e^{2\pi i v \ell} \psi_3^q(q^{\rho^2}/\rho v + \frac{\ell}{\rho} \tau) \quad (2.15)$$

When $\rho = 2$ we have from (2.13) and (2.15):

$$\psi_3^q(\eta/v) = \psi_3^q(\eta^4/2v) + \psi_2^q(\eta^4/2v) \quad (2.16)$$

$$\psi_4^q(\eta/v) = \psi_3^q(\eta^4/2v) - \psi_2^q(\eta^4/2v) \quad (2.17)$$

For the case $v=0$ these relations were recently given by Zuker [20].

It is clearly from the relation (2.16) that

$$\begin{aligned} & \psi_3^q(\eta_1^4/2v_1) \psi_3^q(\eta_2^4/2v_2) + \psi_3^q(\eta_1^4/2v_1) \psi_2^q(\eta_2^4/2v_2) + \\ & + \psi_2^q(\eta_1^4/2v_1) \psi_3^q(\eta_2^4/2v_2) + \psi_2^q(\eta_1^4/2v_1) \psi_2^q(\eta_2^4/2v_2) = \\ & = \psi_3^q(\eta_1/v_1) \psi_3^q(\eta_2/v_2) \end{aligned} \quad (2.18)$$

Thus it is easy to represent the Bloch functions (2.12) for odd number η in more convenient form:

$$f_{K_x K_y}^{\alpha\beta} = |\alpha\beta\rangle \left\{ \psi_3^q(\eta_1/v_1) \psi_3^q(\eta_2/v_2) - 2 \psi_2^q(\eta_1^4/2v_1) \psi_2^q(\eta_2^4/2v_2) \right\} \quad (2.19)$$

One can construct the Bloch functions from the another kind of the coherent states $|N\beta\rangle = \frac{1}{\sqrt{N!}} (a^+)^N |0\beta\rangle$ [10].

It is easy to verify that $f_{K_x K_y}^{N\beta} = \frac{1}{\sqrt{N!}} (a^+)^N f_{K_x K_y}^{0\beta}$

We can construct the Bloch function in the constant magnetic field from the Landau function [21].

This function can be represented as:

$$|N, c\rangle = \sqrt{\frac{m\omega}{2\pi}} \frac{1}{\sqrt{2^N N!}} \exp\left\{icx + \frac{im\omega}{2} xy - \frac{m\omega}{2} \left(y + \frac{c}{m\omega}\right)^2\right\} H_N \left[\sqrt{m\omega} \left(y + \frac{c}{m\omega}\right)\right] \quad (2.20)$$

Where H_N - is the Hermit polinom.

According to ref. [2] and [9] the magnetic translation operator $T_{m_1 m_2}$ acts on the Landau function by the following way:

$$T_{m_1 0} |N, c\rangle = T(m_1 a i) |N, c\rangle = e^{-im_1 c a} |N, c\rangle \quad (2.21)$$

$$T_{0 m_2} |N, c\rangle = T(m_2 b j) |N, c\rangle = |N, c - m\omega b m_2\rangle$$

After the action of the operator $\hat{V}_{K_x K_y}$ on the function $|N, c\rangle$ one can obtain:

$$\begin{aligned} f_{K_x K_y}^{Nc} &= \sum_{m_1 m_2 = -\infty}^{\infty} e^{iK_x a m_1} e^{iK_y b m_2} \tilde{T}_{m_1 0} \tilde{T}_{0 m_2} |N, c\rangle = \\ &= \sum_{m_1 m_2 = -\infty}^{\infty} e^{i(K_x - c)m_1 a} e^{iK_y m_2 b} |N, c - m\omega b m_2\rangle \end{aligned} \quad (2.22)$$

One can sum this expression only in case of the "ground" state when $N=0$ and $H_N=1$.

The ground state of the Landau function in the ξ - variables can be represented as:

$$|0C\rangle = \sqrt{\frac{m\omega}{2\pi}} \exp\left\{-\frac{1}{2}\xi^2 + \left(\xi + \frac{ic}{\sqrt{m\omega}}\right)^2 + \frac{c^2}{2m\omega}\right\} \quad (2.23)$$

The Bloch function constructed from the Landau function $|0C\rangle$ is:

$$f_{K_x K_y}^{0C} = |0C\rangle \sqrt[3]{\frac{1}{2\pi}(K_x - c)} \sqrt[3]{\frac{1}{2\pi}(K_y - ic - 2\sqrt{m\omega})} \exp\left\{-\frac{m\omega b^2}{2}\left(\frac{K_x - c}{2\pi} + \frac{K_y - ic - 2\sqrt{m\omega}}{2\pi}\right)^2\right\} \quad (2.24)$$

In all these expressions for the Bloch functions the part of the wave functions which depend from the spin of the particle have not been taken into account. One can take into consideration the spin of the particle and the movement along axis Z by means of the factor $\chi_s^{1/2} e^{-iK_z Z}$. The time-dependence is given by the factor $\exp(-\frac{i\omega t}{2})$.

3. Nonrelativistic electrons in the time-dependent uniform magnetic field.

The Schrodinger equation for the problems of the electrons in time-dependant magnetic field has the form

$$i \frac{\partial \psi}{\partial t} = -\frac{1}{2} \left(\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} \right) + \frac{\omega^2}{8} (x^2 + y^2) \psi + \frac{i\omega}{2} \left(x \frac{\partial \psi}{\partial y} - y \frac{\partial \psi}{\partial x} \right) \quad (3.1)$$

The charge and mass of the electrons also equal to unity for simplicity.

The vector $\vec{A}$ being equal to $\frac{1}{2} [\vec{H} \vec{L} \vec{z}]$, where $\vec{H}(t) = (0, 0, \omega(t))$.

One can introduce the new variables which correspond to the transition to the moving system of the coordinates

$$Z(t) = \frac{1}{\sqrt{2}} (x + iy) \exp \left[\frac{i}{2} \int_0^t \omega(\tau) d\tau \right] \quad (3.2)$$

In new variables the equation (3.1) has the form

$$i \frac{\partial \psi}{\partial t} = -\frac{\partial^2 \psi}{\partial Z \partial Z^*} + \frac{\omega^2}{4} |Z|^2 \psi \quad (3.3)$$

The creating and annihilation operators [22]

$$B = \frac{1}{\sqrt{2}} \left(i \dot{E} Z^* - E \frac{\partial}{\partial Z} \right); B^+ = \frac{1}{\sqrt{2}} \left(-i \dot{E}^* Z + E^* \frac{\partial}{\partial Z^*} \right) \quad (3.4)$$

$$A = \frac{1}{\sqrt{2}} \left(\dot{E} Z + \frac{i E \partial}{\partial Z^*} \right); A^+ = \frac{1}{\sqrt{2}} \left(\dot{E}^* Z^* + \frac{i E^* \partial}{\partial Z} \right)$$

commute with the $i \frac{\partial}{\partial t} - H$ being consequently the integrals of motion.

Here the time-dependent function $\mathcal{E}(t)$ is the solution of the equation $\ddot{\mathcal{E}} + \frac{1}{4}\omega^2\mathcal{E} = 0$, moreover it is known that $\mathcal{E} = |\mathcal{E}| \exp[i\int |\mathcal{E}|^{-2} d\tau]$.

The magnetic translations operators $T(\vec{R}) = \exp[-i\vec{R}(\vec{p} + e\vec{A})]$ in the case of the time-dependent magnetic field already are not the integrals of the motion. But it is easy to show that they coincide with the integrals of the motion $D(\beta) = \exp(\beta B^+ - \beta^* B)$ when:

$$\beta = \frac{1}{\mathcal{E}}(R_x - iR_y) \exp\left[-\frac{i}{2} \int_0^t \omega(\tau) d\tau\right] \quad (3.5)$$

By this connection, we may express the Bloch function in the complex β -plane over the translations vectors $\vec{R}_x$ and $\vec{R}_y$ in the plane perpendicular to the direction of the magnetic field $\vec{H}(t)$ and thus construct the Bloch functions of the electron in the time-dependent magnetic field which coincide with (2.11) and (2.19) when the field is constant.

The quantization condition (1.5), $\text{Im } \beta_1 \beta_2^* = \pi n$ when according to (3.5) $\beta_1 = \frac{1}{\mathcal{E}} a \exp\left[-\frac{i}{2} \int_0^t \omega(\tau) d\tau\right]$ and $\beta_2 = -\frac{i}{\mathcal{E}} b \exp\left[-\frac{i}{2} \int_0^t \omega(\tau) d\tau\right]$ has the form

$$\frac{1}{|\mathcal{E}|^2} a b = \pi n \quad (n=0, \pm 1, \dots) \quad (3.6)$$

The $\mathcal{E}(t)$ is the time-dependent function. Therefor the perodes $a(t)$ and $b(t)$ are the time-dependent func-

tions too.

Thus we have the "breathing lattice" in the case of the time-dependent magnetic field. It is clearly from the physical consideration. In the case of the constant magnetic field it is should to take $\mathcal{E} = \mathcal{E}_c = \sqrt{\frac{2}{\omega_c}} \exp\left[\frac{i\omega_c t}{2}\right]$ according to [22].

In this limit the expressions (3.5) and (3.6) have the form (2.3) and (2.5) accordingly if one take into account that $\vec{H} = (0, 0, \omega)$.

The coherent states of the electrons in the time-dependent magnetic field [22] are given by the formula

$$|\alpha\beta\rangle = \pi^{-1/2} \mathcal{E}^{-1} \exp\left\{i \frac{\mathcal{E}}{\mathcal{E}} |Z|^2 - \frac{1}{2} (|\alpha|^2 + |\beta|^2) - i \mathcal{E}^{-1} 2^{1/2} (\alpha Z^* - \beta Z) - \alpha\beta \frac{\mathcal{E}^*}{\mathcal{E}}\right\} \quad (3.7)$$

The Bloch functions, according to (3.5) have the form:

$$f_{K_x K_y}^{\alpha\beta} = \mathcal{D}(\alpha) \mathcal{D}(\beta) \sum_{m_1 m_2 = -\infty}^{\infty} (-1)^{m_1 m_2} \exp\{i m_1 (K_x - 2C \operatorname{Im} \beta) \alpha + i m_2 (K_y - 2C \operatorname{Re} \beta) \beta\} |0, C(m_1 \alpha - i m_2 \beta)\rangle \quad (3.8)$$

where $\alpha = \alpha(t)$, $\beta = \beta(t)$ and $C = C(t) = \frac{1}{\mathcal{E}(t)} \exp\left[-\frac{i}{2} \int \omega(t) dt\right]$

For even number n we have

$$f_{K_x K_y}^{\alpha\beta} = |\alpha\beta\rangle \mathcal{V}_3^q(p_1/v_1) \mathcal{V}_2^q(p_2/v_2) \quad (3.9)$$

For odd number η we have

$$f_{K_X K_Y}^{2\beta} = |\alpha\beta\rangle \left\{ \psi_3^*(q_1/v_1) \psi_3^*(q_2/v_2) - \right. \\ \left. - 2 \psi_2^*(q_1^4/v_1) \psi_2^*(q_2^4/v_2) \right\} \quad (3.10)$$

$$\text{where } q_1 = e^{-\frac{c^2 a^2}{2}}; \quad v_1 = \frac{a}{2\pi} (K_X + i\epsilon^{-1/2} Z C + d \frac{\epsilon^*}{\epsilon} C + i\beta C) \\ q_2 = e^{-\frac{c^2 b^2}{2}}; \quad v_2 = \frac{b}{2\pi} (K_Y + \epsilon^{-1/2} Z C - id \frac{\epsilon^*}{\epsilon} C - \beta C)$$

The functions (3.9) and (3.10) coincide with (2.11) and (2.19) when the magnetic field is constant.

One can obtain the another kind of Bloch functions by means of the formula $f_{K_X K_Y}^{N\beta} = \frac{1}{\sqrt{N!}} (A^+)^N f_{K_X K_Y}^{0\beta}$ and take into consideration the spin of the particle by means of the factor $\chi_{S_Z} \exp[i\mu_0 \int_0^{\tau} \omega(\tau') S_Z d\tau']$.

4. The charged particle in a uniform electromagnetic field $\vec{E}(t), \vec{H}(t) = 0$.

Let us consider the motion of a charged particle in a uniform electromagnetic field $\vec{H}(t) = (0, 0, \omega(t))$, $\vec{E}(t) = (E_1(t), E_2(t), 0)$. We shall choose the vector potential in the form of $\vec{A} = \frac{1}{\omega} [\vec{H}(t) \times \vec{r}]$. In this case the creating and annihilation operators have the

form [23]

$$\begin{aligned}
 A &= \frac{1}{\sqrt{2}} \left[\dot{\mathcal{E}} \mathcal{Z} + \frac{i\mathcal{E}}{\partial \mathcal{Z}^*} + \dot{\mathcal{E}} \mathcal{Z}_0 - \mathcal{E} \dot{\mathcal{Z}}_0 \right] \\
 A^\dagger &= \frac{1}{\sqrt{2}} \left[\dot{\mathcal{E}}^* \mathcal{Z}^* + \frac{i\mathcal{E}^*}{\partial \mathcal{Z}} + \dot{\mathcal{E}}^* \mathcal{Z}_0^* - \mathcal{E}^* \dot{\mathcal{Z}}_0^* \right] \\
 B &= \frac{1}{\sqrt{2}} \left[i\dot{\mathcal{E}} \mathcal{Z}^* - \frac{\mathcal{E}}{\partial \mathcal{Z}} + i\dot{\mathcal{E}} \mathcal{Z}_0^* - i\mathcal{E} \dot{\mathcal{Z}}_0^* \right] \\
 B^\dagger &= \frac{1}{\sqrt{2}} \left[-i\dot{\mathcal{E}}^* \mathcal{Z} + \frac{\mathcal{E}^*}{\partial \mathcal{Z}^*} - i\dot{\mathcal{E}}^* \mathcal{Z}_0 + i\mathcal{E}^* \dot{\mathcal{Z}}_0 \right]
 \end{aligned} \tag{4.1}$$

The time-dependent function $\mathcal{Z}_0(t)$ is the solution of the equation $\ddot{\mathcal{Z}}_0 + \frac{1}{4}\omega^2 \mathcal{Z}_0 = 0$.

It was shown in [23] that the unitary operators

$$\begin{aligned}
 \mathcal{D} &= \exp \left[-i(\mathcal{Z} \dot{\mathcal{Z}}_0^* + \mathcal{Z}^* \dot{\mathcal{Z}}_0) \right] \exp \left\{ \mathcal{Z}_0 \frac{\partial}{\partial \mathcal{Z}} + \right. \\
 &\quad \left. + \mathcal{Z}_0^* \frac{\partial}{\partial \mathcal{Z}^*} + i \int_0^t \left(\frac{1}{4} \omega^2 / \mathcal{Z}_0'^2 - / \dot{\mathcal{Z}}_0'^2 \right) d\tau' \right\}
 \end{aligned} \tag{4.2}$$

transfer the solution of the Schrodinger equation for the charged particle in the time-dependent magnetic field when the electric field equal to zero in the same one when the electric field doesn't equal to zero.

Applying this operator to the Bloch functions (3.9), (3.10) for the pure time-dependent magnetic field, one can obtain the Bloch function in case when the electric field exist and $\vec{\mathcal{E}}(t) \vec{\mathcal{H}}(t) = 0$.

These functions are given by the expressions (3.9)

and (3.10) where the coherent states and the arguments have the form:

$$\begin{aligned}
 |\alpha\beta\rangle = & \varepsilon^{-1/\pi} \exp \left\{ i \frac{\varepsilon}{\varepsilon} (|\dot{z}|^2 + |\dot{z}_0|^2) - (i \dot{z}^* - i \frac{\varepsilon}{\varepsilon} \dot{z}_0^*) \dot{z} - \right. \\
 & - (i \dot{z}_0 - i \frac{\varepsilon}{\varepsilon} \dot{z}_0) \dot{z}^* + i \int_0^t \left(\frac{1}{4} \omega^2 |\dot{z}_0|^2 - |\dot{z}_0|^2 \right) d\tau - \\
 & \left. - \frac{1}{2} (|\dot{\alpha}|^2 + |\dot{\beta}|^2) - i \varepsilon^{-1/2} \left[\dot{\alpha} (\dot{z}^* + \dot{z}_0^*) - i \beta (\dot{z} + \dot{z}_0) \right] - \dot{\alpha} \beta \frac{\varepsilon}{\varepsilon} \right\} \\
 \eta_1 = & e^{-\frac{c^2 \alpha^2}{2}}; \quad \psi_1 = \frac{a}{2\pi} \left(k_\chi + i \varepsilon^{-1/2} c (\dot{z} + \dot{z}_0) + \dot{\alpha} c \frac{\varepsilon}{\varepsilon} + i \beta c \right)
 \end{aligned} \tag{4.3}$$

$$\eta_2 = e^{-\frac{c^2 \beta^2}{2}}; \quad \psi_2 = \frac{b}{2\pi} \left(k_\gamma + \varepsilon^{-1/2} c (\dot{z} + \dot{z}_0) - i \dot{\alpha} c \frac{\varepsilon}{\varepsilon} - \beta c \right) \tag{4.4}$$

$$c = \frac{1}{\varepsilon(t)} \exp \left[-\frac{i}{2} \int_0^t \omega(\tau) d\tau \right]$$

The quantization conditions for the electromagnetic field $\vec{E}(t) \cdot \vec{\mathcal{H}}(t) = 0$ are the same as in the case of the pure magnetic field (3.6) i.e. we as usual have the "breathing lattice" in the plane perpendicular to the direction of the magnetic field.

Let us discuss the case of the constant electromagnetic field $\mathcal{H} = \mathcal{H}_z = \text{const.}$ and $\mathcal{E} = \mathcal{E}_y = \text{const.}$ The

charge and mass of the electron is equal to unity, i.e.

$$\hbar z = \omega.$$

One can obtain the Bloch functions for this case from the Bloch functions for the time-dependent case in the limit of the constant fields, i.e. when

$$\mathcal{E} = \mathcal{E}_0 = 2^{\frac{1}{2}} \omega^{\frac{1}{2}} \exp\left(\frac{i\omega t}{2}\right); Z_0 = \frac{i\mathcal{E}_0}{2^{\frac{1}{2}} \omega^2} \exp\left(\frac{i\omega t}{2}\right) \quad (4.5)$$

according to [24].

One can obtain these functions also by applying to the functions (2.11) and (2.19) the operator

$$\mathcal{D} = \exp\left\{-\frac{i\mathcal{E}_0}{2\omega^{\frac{3}{2}}}(\xi + \xi^*) - \frac{i\mathcal{E}_0}{2\omega^{\frac{3}{2}}}\left(\frac{\partial}{\partial \xi} - \frac{\partial}{\partial \xi^*}\right)\right\} \quad (4.6)$$

which is the operator (4.2) in the limit of the constant fields (4.5).

These functions have the standard form (3.9) and (3.10) where:

$$\begin{aligned} |\alpha\beta\rangle &= \sqrt{\frac{\omega}{2\pi}} \exp\left(-\frac{i\omega t}{2}\right) \exp\left\{-\frac{1}{2}\alpha^2 + \sqrt{2}\beta\left(\xi - \frac{i\mathcal{E}_0}{2\omega^{\frac{3}{2}}}\right) + \right. \\ &+ i\sqrt{2}\alpha\left(\xi^* + \frac{i\mathcal{E}_0}{2\omega^{\frac{3}{2}}}\right) - i\alpha\beta - \frac{1}{8}\alpha^2 - \frac{1}{2}\beta^2 - \frac{i\mathcal{E}_0}{\omega^{\frac{3}{2}}}\xi - \frac{\mathcal{E}_0^2}{4\omega^3}\left.\right\} \quad (4.7) \\ p_1 &= \ell^{\frac{2}{2}} \frac{\alpha^2}{2}; \quad v_1 = \frac{\alpha}{2\pi} \left(k_x - i\sqrt{2}\beta\left(\xi - \frac{i\mathcal{E}_0}{2\omega^{\frac{3}{2}}}\right) - \alpha\gamma + i\beta\right) \end{aligned}$$

$$q_2 = e^{-\gamma^2 \beta^2 / 2}; \quad \psi_2 = \frac{b}{2\pi} \left(k_y - \sqrt{2} \gamma \left(\xi - \frac{i \mathcal{E}_y}{2\omega^{3/2}} \right) + i \alpha \gamma - \gamma \beta \right)$$

In the case of the constant fields the system of the coordinates K' in which the electric field equal to zero and magnetic field equal to $\mathcal{H}_2' = \sqrt{\mathcal{H}_2^2 - \mathcal{E}_y^2}$ always exist. This system K' moves with the velocity $V = \frac{\mathcal{E}_y}{\mathcal{H}_2}$ along the axis X . The area of the one cell of the lattice in this moving system K' is equal to $a'b' = ab \frac{1}{\sqrt{1-V^2}} = \frac{ab \mathcal{H}_2}{\sqrt{\mathcal{H}_2^2 - \mathcal{E}_y^2}}$.

And the quantization condition in this system K' have the form

$$a'b' \mathcal{H}_2' = \frac{ab \mathcal{H}_2}{\sqrt{\mathcal{H}_2^2 - \mathcal{E}_y^2}} \sqrt{\mathcal{H}_2^2 - \mathcal{E}_y^2} = ab \mathcal{H}_2 = 2\pi n \quad (4.8)$$

$$(n = 0, \pm 1 \dots)$$

Therefore, the quantization conditions for the electromagnetic field are the same as in the case of the pure magnetic field. It is obvious that this is correctly in the case of the time-dependent electromagnetic fields too. The constructed functions have the same form and in the case, $\mathcal{E}_y/\mathcal{H}_2 > 1$, i.e. when the velocity of the system is more then the velocity of the light.

Here there is not any contradiction since we consider the nonrelativistic equation.

One can take into account the preceding discussion and conclude that the operator (4.6) correspond to the passage to the new system, moving with respect to the initial one. The operator (4.6) realize the ray infinite dimensional irreducible unitary representation of the Gallileian group.

This operator can be considered as the unitary Weil's translation operator creating the coherent states of the electron in the uniform electromagnetic fields $\vec{E} \cdot \vec{H} = 0$ from the "ground" state which correspond to the case of the pure magnetic field ($\vec{E} = 0$). Otherwise, the operator (4.6) has the form $D(\beta) = \exp(\beta \hat{b}^\dagger - \beta^* \hat{b})$ where the translation parameter β is connected with the electric field by the relation $\beta = -\frac{i e \mathcal{E}}{\sqrt{2} \omega^{3/2}}$. Here $\hat{b}^\dagger$ and $\hat{b}$ are the creation and annihilation operators in the case of the constant magnetic field (see the part 2).

It should be noted that the connection of the wave functions for the different systems of the coordinates for the charge particle in the electromagnetic fields is another then one for the particle in the given potential field.

5. The Bloch functions of the oscillator.

The coherent states of the oscillator in q - representation have the form [25]

$$\begin{aligned} \langle q|\alpha\rangle &= \mathcal{D}(\alpha)\langle q|0\rangle = \\ &= \left(\frac{\omega}{\pi}\right)^{1/4} \exp\left(-\frac{i\omega t}{2}\right) \exp\left\{-\left(\frac{\sqrt{\omega}}{2}q-\alpha\right)^2 + \frac{\alpha^2 - |\alpha|^2}{2}\right\} \quad (5.1) \\ &\quad (\hbar = m = 1) \end{aligned}$$

but in the coherent states representation have the form

$$\langle \beta|\alpha\rangle = \exp\left(\alpha\beta^* - \frac{|\alpha|^2}{2} - \frac{|\beta|^2}{2}\right) \quad (5.2)$$

and the time-dependence of the coherent states are given

by the law $|\alpha(t)\rangle = |\alpha_0\rangle e^{-i\omega t}$

Here

$$\mathcal{D}(\alpha) = \exp(\alpha a^\dagger - \alpha^* a); \quad \alpha = \frac{\omega^{1/2}q + i\omega^{-1/2}p}{\sqrt{2}} \quad (5.3)$$

The coherent states are more close to the classical states. The movement of the classical oscillator, which convenient to represent as the movement on the phase plan (p, q) or on the complex plan $\alpha = 2^{-1/2}(\sqrt{\omega}q + \frac{i}{\sqrt{\omega}}p)$ is the circular movement with the radius $\sqrt{2}|\alpha|$ and frequency ω .

Therefore, the "quantization" condition (1.5) for the this problem take the form: $p^0 q^0 = 2\pi\hbar$, $\hbar = 0, \pm 1, \dots$. Thus in phase plane we have the lattice with the periods q^0 and p^0 . The area of one cell of this lattice is

equal to $2\pi\eta$, according to the quasiclassical theorem of the quantum mechanics.

The Bloch functions of the oscillator will be constructed as the quasi-momentum representation in the phase lattice with the periods q^0 and p^0 .

The operator (1.7) has the form, if $\alpha m_1 m_2 = \alpha_1 m_1 + i \alpha_2 m_2 =$
 $= 2^{-1/2} \omega^{1/2} q_0 m_1 + i (2\omega)^{-1/2} p^0 m_2$

$$V_{k_p k_q} = \sum_{m_1 m_2 = -\infty}^{\infty} (-1)^{nm_1 m_2} \exp\{i(k_q m_1 q_0 + k_p m_2 p^0)\} \times \quad (5.4)$$

$$\times D_{m_1 m_2} \left(\frac{q_0 \omega^{1/2}}{\sqrt{2}} m_1 + i \frac{p^0}{\sqrt{2\omega}} m_2 \right)$$

Applying the operator (5.4) to the coherent states (5.2) we have the Bloch functions in the coherent states representation.

For the even number η :

$$\langle \beta | \alpha k_q k_p \rangle = \langle \beta | \alpha \rangle \mathcal{V}_3(q_1/\sqrt{2}) \mathcal{V}_3(q_2/\sqrt{2}) \quad (5.5)$$

For the odd number η :

$$\langle \beta | \alpha k_q k_p \rangle = \langle \beta | \alpha \rangle \left\{ \mathcal{V}_3(q_1/\sqrt{2}) \mathcal{V}_3(q_2/\sqrt{2}) - \right. \quad (5.6)$$

$$\left. - 2 \mathcal{V}_2^4(q_1^4/2\sqrt{2}) \mathcal{V}_2^4(q_2^4/2\sqrt{2}) \right\}$$

$$q_1 = e^{-\frac{(q^0)^2 \omega}{4}}; \quad \psi_1 = \frac{q^0}{2\pi} \left(K_q - \frac{i\sqrt{\omega}}{\sqrt{2}} (\beta^* - \alpha^*) \right)$$

$$q_2 = e^{-\frac{(p^0)^2}{4\omega}}; \quad \psi_2 = \frac{p^0}{2\pi} \left(K_p + \frac{1}{\sqrt{2}\omega} (\beta^* - \alpha^*) \right)$$

One can construct the Bloch function in the q - representation applying the operator (5.4) to the coherent states in this representation (5.1).

In this representation the expressions for the even and odd numbers η coincide

$$\begin{aligned} \langle q | \alpha K_p K_q \rangle &= \langle q | \alpha \rangle \psi_3 \left(1 / \frac{p^0}{2\pi} (K_p + q) \right) \times \\ &\times \psi_3 \left(e^{-\frac{(q^0)^2 \omega}{2}} / \frac{q^0}{2\pi} (K_q + i\sqrt{2}\omega\alpha - i\omega q) \right) \end{aligned} \quad (5.7)$$

The function (5.7) is connected with the functions (5.5) and (5.6) by the relation

$$\int \langle \beta | q \rangle \langle q | \alpha K_p K_q \rangle dq = \langle \beta | \alpha K_p K_q \rangle \quad (5.8)$$

From this relation one can see, that the diverging theta function with the argument $q = 1$ should be realized in the general sense, thus the integral of this function with the square exponent (5.1) gives the quite good theta functions (5.5) or (5.6) for the even and odd numbers η accordingly.

Here the situation in the same sense similar to the well-known fact: the kernel of the unity operator in the coordinat representation is the singular δ - function but it is the analytic exponent $\exp(\alpha \beta^*)$ in the coherent states representation.

The functions (5.5), (5.6) and (5.7) have been constructed as the eigenfunctions of the translation operator for the lattice in phase plane $\tilde{D}_{m_1 m_2} = (-1)^{nm_1 m_2} \exp(\alpha_{m_1 m_2} a^\dagger - \alpha_{m_1 m_2} a)$.

This operator as the function of the translation periods in phase plane p^0 and q^0 , and according to (5.3) has the form:

$$\tilde{D}_{m_1 m_2} = (-1)^{nm_1 m_2} \exp(-i q^0 p m_1 + i p^0 q m_2)$$

6. The construction of the Bloch function for the general quadratic system.

The hamiltonian of the most general N -dimensional quadratic system can be written as follows

$$H = \frac{1}{2} \vec{q} B(t) \vec{q} + \vec{C}(t) \vec{q} + \varphi(t) \quad (6.1)$$

Here $2N$ -dimensional $\vec{q}$ is $\vec{q} = \begin{pmatrix} -i\hbar \frac{\partial}{\partial \vec{x}} \\ \vec{x} \end{pmatrix}$.

Evidently, one can always believe the matrix B to be

symmetrical

$$\vec{C}\vec{q} = \sum_{\alpha=1}^{2N} C_{\alpha} q_{\alpha}; \quad \vec{q} B \vec{q} = \sum_{\alpha, \beta=1}^{2N} q_{\alpha} B_{\alpha\beta} q_{\beta}$$

One can easily verify that $2N$ - dimensional vector

$$J_{\alpha} = \sum_{\beta=1}^{2N} \Lambda_{\alpha\beta} q_{\beta} + \Delta_{\alpha}; \quad \Lambda = \begin{vmatrix} \lambda_1 & \lambda_2 \\ \lambda_3 & \lambda_4 \end{vmatrix}$$

is the integral of the motion coinciding with q at the initial moment, provided the $2N$ - dimensional matrix $\Lambda(t)$ and vector $\Delta(t)$ satisfy the following system of ordinary differential equations of the first order:

$$\dot{\Lambda} = \Lambda \Sigma B; \quad \Lambda(0) = E_{2N}; \quad \dot{\Delta} = \Lambda \Sigma C, \quad \Delta(0) = 0$$

$$\Sigma = \begin{vmatrix} 0 & E_N \\ -E_N & 0 \end{vmatrix}; \quad E_N \text{ is the } N \times N \text{ unity matrix.}$$

Then, the integral of the motion, N - dimensional vector $\vec{\alpha}$ have the form:

$$\vec{\alpha} = \frac{1}{\sqrt{2}h} \{ \lambda_p (-i\hbar \frac{\partial}{\partial \vec{x}}) + \lambda_q \vec{x} \}$$

$$\lambda_p = \lambda_3 + i\lambda_1; \quad \lambda_q = \lambda_4 + i\lambda_2; \quad \lambda_p \lambda_q^+ - \lambda_q \lambda_p^+ = 2iE$$

The properties of the matrices λ_i , λ_p and λ_q follow from the condition that the matrix Λ must be symplectic, i.e. $\Lambda \Sigma \tilde{\Lambda} = \Sigma$. This condition is necessary in order to that $[q_{\alpha}, q_{\beta}] = [J_{\alpha}, J_{\beta}]$.

The ground state satisfy the Schrodinger equation, condition $\vec{\alpha}|0\rangle = 0$ and has the form [26]

$$\begin{aligned}
 |\vec{0}\rangle = & (\pi\hbar)^{-N/4} (\det \lambda_p)^{-1/2} \exp \left\{ -\frac{i}{2\hbar} \vec{x} \lambda_p \lambda_p^{-1} \vec{x} - \right. \\
 & -\frac{i}{\hbar} \vec{x} \lambda_p \vec{\delta} + \frac{1}{4\hbar} \vec{\delta} \lambda_p^* \lambda_p^{-1} \vec{\delta} - \frac{1}{4\hbar} |\vec{\delta}|^2 - \\
 & \left. -\frac{i}{2\hbar} \int_0^t \text{Im}(\vec{\delta}^* \dot{\vec{\delta}}) d\tau \right\}; \quad \delta_i = \Delta_{N+j} + i\Delta_j
 \end{aligned} \quad (6.2)$$

It is easy to construct the coherent state by action on the ground state $|\vec{0}\rangle$ the integral of the motion $\mathcal{D}(\vec{\alpha})$

$$|\vec{\alpha}\rangle = \mathcal{D}(\vec{\alpha}) |\vec{0}\rangle = \exp(\vec{\alpha} \vec{a} - \vec{\alpha}^* \vec{a}^\dagger) |\vec{0}\rangle$$

and

$$\begin{aligned}
 |\vec{\alpha}\rangle = & |\vec{0}\rangle \exp \left\{ \frac{1}{2} \vec{\alpha} \lambda_p^* \lambda_p^{-1} \vec{\alpha} - \frac{|\alpha|^2}{2} + \right. \\
 & \left. + \frac{1}{\sqrt{2}\hbar} \vec{\alpha} (\vec{\delta}^* - \lambda_p^* \lambda_p^{-1} \vec{\delta} + 2i\lambda_p^T \vec{x}) \right\} \quad (6.3)
 \end{aligned}$$

One can construct the Bloch function by usually way by applying the operator (1.7) to the coherent states (6.3)

$$\begin{aligned}
 \int_{K_1^1 K_2^1 \dots K_1^N K_2^N} \hat{V}_{K_1^1 K_2^1 \dots K_1^N K_2^N} \mathcal{D}(\vec{\alpha}) |\vec{0}\rangle = \\
 = \mathcal{D}(\vec{\alpha}) \sum_{\substack{p=1 \\ m_1^1 m_2^1, \dots, m_1^N m_2^N = -\infty}}^{\infty} \left[\prod_{p=1}^N (-1)^{n^p(m_1^p, m_2^p)} \exp \left\{ i(K_1^p - 2\text{Im} \alpha^p) m_1^p \beta_1^p + \right. \right. \\
 \left. \left. + i(K_2^p + 2\text{Re} \alpha^p) m_2^p \beta_2^p \right\} \right] |\beta_{m_1 m_2}\rangle \quad (6.4)
 \end{aligned}$$

where $\beta_{m_1 m_2}^p = m_1^p \beta_1^p + i m_2^p \beta_2^p$.

In general case the sum (6.4) can't be expressed by means of the known special functions and represent the new many dimensional special function which is defining by its translation property (1.8).

The sum (6.4) can be easily represented by means of the production one-dimensional theta functions in particular case when in expression (6.3) the matrix $\lambda_p^* \lambda_p^{-1}$ is in the diagonal form. One must take into account the quantization condition $\beta_1^p \beta_2^p = \pi n^p$, $n^p = 0, \pm 1, \dots$

The Bloch function in this case can be represented

as

$$f_{k_1^p k_2^p \dots k_1^p k_2^p}^{\vec{\lambda}} = |\vec{\lambda}\rangle \prod_{p=1}^N \psi_1^p(k_1^p / \delta_1^p) \psi_2^p(k_2^p / \delta_2^p)$$

$$\begin{aligned} q_1^p &= e^{\frac{\lambda-1}{2}(\beta_1^p)^2}; \quad \psi_1^p = \frac{\beta_1^p}{2\pi} \left(k_1^p + 2i\delta_1^p - \frac{i}{\sqrt{2}h} (\vec{\delta}^* - \lambda_p^* \lambda_p^{-1} \vec{\delta} + 2i\lambda_p^{-1} \vec{x})^p \right) \\ q_2^p &= e^{-\frac{\lambda+1}{2}(\beta_2^p)^2}; \quad \psi_2^p = \frac{\beta_2^p}{2\pi} \left(k_2^p + \frac{1}{\sqrt{2}h} (\vec{\delta}^* - \lambda_p^* \lambda_p^{-1} \vec{\delta} + 2i\lambda_p^{-1} \vec{x})^p \right) \\ \lambda &= \det \lambda_p^* \lambda_p^{-1} \end{aligned}$$

7. On the completeness of the system of the Bloch functions.

The problem of the completeness of the Bloch func-

tion is not simple.

Let's investigate this problem for the system of the Bloch function of the oscillator.

One can see from (5.4) that the Bloch function of the oscillator have been constructed from the system of the coherent states of the oscillator on the lattice in the complex plan $|\alpha m_1 m_2\rangle = |\alpha_1 m_1 + i \alpha_2 m_2\rangle$.

The problem of the completeness of this system have been investigated earlier. In papers [27-28] there is the proof of the following theorem: the system of the coherent states $|\alpha m_1 m_2\rangle$, $\alpha m_1 m_2 = \alpha_1 m_1 + i \alpha_2 m_2$ with the area of the one cell $S = \alpha_1 \alpha_2$ can't be complete if $S > \pi$, it is supercomplete and remain supercomplete, when the finit number of the states have been removed, if $S < \pi$; at last, if $S = \pi$ the system $|\alpha m_1 m_2\rangle$ is complete, it remains, complete when one state have been removed, but it becomes uncomplete when two any states have been removed.

The condition of the completeness of the system $|\alpha m_1 m_2\rangle$ when $S = \pi$ have been essentially used by Von-Neumann ([29] p.298) when he discuss the problem of the most precise measuring of the coordinate and impulse.

As have been shown earlier, the complex α - plan is similar to the phase plan for the classical oscilla-

tor and the cell of the phase plan with the area $2\pi\hbar$ correspondsto the cell of the complex α - plan with the area $\tilde{\pi}_1$.

It is clear from this, that the system $|\alpha_{m_1 m_2}\rangle$ is complete if in the cell of the phase plan with area $2\pi\hbar$ we have in average more then one, or one coherent state.

Further we shall discussed just the case when $\hbar = 1$, i.e. $S = \alpha_1 \alpha_2 = \tilde{\pi}_1$.

In this case, one can take the system of the coherent states $|\alpha_{m_1 m_2}\rangle$ without the state $|\alpha_{00}\rangle = |0\rangle$ as the complete and minimum system.

It is known [28], that in case of the complete and minimum system one can always construct the biortogonal complete and minimum system.

To find this biortogonal system $|\tilde{\alpha}_{m_1 m_2}\rangle$ let's consider the whole function which equal to zero in the all knots of the lattice [19]

$$\tilde{G}(\alpha) = \alpha \prod'_{m_1 m_2} \left(1 - \frac{\alpha}{\beta_{m_1 m_2}} \right) \exp \left\{ \frac{\alpha}{\beta_{m_1 m_2}} + \frac{1}{2} \frac{\alpha^2}{\beta_{m_1 m_2}^2} \right\} \quad (7.1)$$

where $\beta_{m_1 m_2} = \alpha_{m_1 m_2}^*$, and symbol $(\prod')$ signifies that in the product (7.1) there are all points of the lattice without point $m_1 = 0, m_2 = 0$.

The function $\tilde{G}(\alpha^*) = \tilde{G}(\alpha^*) e^{-\alpha^*} \alpha^{*2}$ according to [19] is the whole analytical function of the order of

two and of the minimum possible type for the given zeros of the type $\mu = \frac{\pi}{2S} = \frac{1}{2}$ [28].

This function satisfy the functional equation:

$$\tilde{\sigma}(\alpha + \alpha_{m_1 m_2}^*) = (-1)^{m_1 m_2 + m_1 + m_2} \frac{|\alpha_{m_1 m_2}|^2}{e^{\frac{1}{2}}} \alpha_{m_1 m_2} \tilde{\sigma}(\alpha) \quad (7.2)$$

Here: $\nu = \frac{1}{4\pi} (\eta_2 \alpha_1 - \eta_1 \alpha_2)$; $\eta_1 = \xi\left(\frac{\alpha_1}{2}\right)$, $\eta_2 = \xi\left(-\frac{i\alpha_2}{2}\right)$

and $\xi(\alpha)$ is given by

$$\xi(\alpha) = \frac{\sigma'(\alpha)}{\sigma(\alpha)} = \frac{1}{\alpha} + \sum_{m_1 m_2 = -\infty}^{\infty} \left(\frac{1}{\alpha - \beta_{m_1 m_2}} + \frac{1}{\beta_{m_1 m_2}} + \frac{\alpha}{\beta_{m_1 m_2}^2} \right) \quad (7.3)$$

Let's consider the function $\frac{\gamma e^{\gamma \beta^*}}{\tilde{\sigma}(\gamma)}$ which has the poles in all points of the lattice $\gamma_{m_1 m_2}$, since the function $\tilde{\sigma}(\gamma)$ is zero in all these points.

According to the well-known Mittag-Leffler's theorem, we have:

$$\frac{\gamma e^{\gamma \beta^*}}{\tilde{\sigma}(\gamma)} = \sum_{m_1 m_2 = -\infty}^{\infty} \frac{\text{Res}(\tilde{\sigma}(\gamma))}{\gamma - \alpha_{m_1 m_2}} = \sum_{m_1 m_2 = -\infty}^{\infty} \frac{\alpha_{m_1 m_2} e^{\alpha_{m_1 m_2} \beta^*}}{\tilde{\sigma}'(\alpha_{m_1 m_2}) (\gamma - \alpha_{m_1 m_2})} \quad (7.4)$$

The symbol (') signify, that in sum (7.4) there are all points of the lattice without the point $m_1 = 0, m_2 = 0$.

One can rewrite the relation (7.4) in form:

$$\sum_{m_1 m_2 = -\infty}^{\infty} \frac{d m_1 m_2}{\tilde{\sigma}'(d m_1 m_2)} \frac{\tilde{\sigma}(\gamma)}{\gamma(\gamma - d m_1 m_2)} e^{d m_1 m_2 \beta^*} = e^{\gamma \beta^*} \quad (7.5)$$

Or, if one multiply this equation on

$$\sum_{m_1 m_2 = -\infty}^{\infty} \frac{d m_1 m_2}{\tilde{\sigma}'(d m_1 m_2)} \frac{\tilde{\sigma}(\gamma) e^{\frac{|d m_1 m_2|^2 - |\gamma|^2}{2}}}{\gamma(\gamma - d m_1 m_2)} e^{d m_1 m_2 \beta^*} e^{-\frac{|\gamma|^2}{2}} e^{-\frac{|\beta|^2}{2}} =$$

$$= e^{\gamma \beta^*} e^{-\frac{|\gamma|^2}{2}} e^{-\frac{|\beta|^2}{2}} \quad (7.6)$$

In the right part of this equation there is the coherent state (5.2) of the oscillator: $\langle \beta | \gamma \rangle$.

The last three multipliers under the sum in the left part of the equation also give this coherent state on the lattice: $\langle \beta | d m_1 m_2 \rangle$.

Therefore the equation (7.6) has the form:

$$\sum_{m_1 m_2 = -\infty}^{\infty} \langle \beta | d m_1 m_2 \rangle \langle d m_1 m_2 | \gamma \rangle = \langle \beta | \gamma \rangle \quad (7.7)$$

where

$$\langle \gamma | d m_1 m_2 \rangle = \frac{d m_1 m_2}{\tilde{\sigma}'(d m_1 m_2)} \frac{\tilde{\sigma}(\gamma^*)}{\gamma^*(\gamma^* - d m_1 m_2)} e^{\frac{|d m_1 m_2|^2 - |\gamma|^2}{2}} \quad (7.8)$$

The condition of the completeness for the system of the coherent states $|\alpha_{m_1 m_2}\rangle$ follows from the equation (7.7) and can be represented as:

$$\sum_{m_1 m_2 = -\infty}^{\infty} |\alpha_{m_1 m_2}\rangle \langle \tilde{\alpha}_{m_1 m_2}| = \hat{I} \quad (7.9)$$

Thus $\langle \tilde{\alpha}_{m_1 m_2}|$ - is the our biorthogonal system and for this system is true:

$$\langle \alpha_{m_1 m_2} | \tilde{\alpha}_{n_1 n_2} \rangle = \delta_{m_1 n_1} \delta_{m_2 n_2} \quad (7.10)$$

It is easy to see, that the biorthogonal system $|\tilde{\alpha}_{m_1 m_2}\rangle$ is not the solution of the Schrodinger equation. This system has the such evolution that the condition of the completeness (7.9) whill be remained in time. Further, let's decompose the removed state $|\alpha_{00}\rangle$ over the others states $|\alpha_{m_1 m_2}\rangle$ by using the condition (7.9) and the condition (7.10)

$$\begin{aligned} |\alpha_{00}\rangle &= \sum_{m_1 m_2} \langle \tilde{\alpha}_{m_1 m_2} | \alpha_{00} \rangle |\alpha_{m_1 m_2}\rangle = \\ &= \sum_{m_1 m_2} \langle \tilde{\alpha}_{m_1 m_2} | \alpha_{00} \rangle |\alpha_{m_1 m_2}\rangle - |\alpha_{00}\rangle \end{aligned}$$

And after the solving of this equation we have:

$$|\alpha_{00}\rangle = \frac{1}{2} \sum_{m_1 m_2} \langle \tilde{\alpha}_{m_1 m_2} | \alpha_{00} \rangle |\alpha_{m_1 m_2}\rangle \quad (7.11)$$

Thus, if one introduces the factor $\frac{1}{2}$ in the expression for the system $|\tilde{\alpha}_{m_1 m_2}\rangle$, that the condition of the completeness can be represented as:

$$\sum |\alpha_{m_1 m_2}\rangle \langle \tilde{\alpha}_{m_1 m_2}| = \hat{I} \quad (7.12)$$

Here $|\tilde{\alpha}_{m_1 m_2}\rangle = \frac{1}{2} |\alpha_{m_1 m_2}\rangle$, and in the sum there are already all knots of the lattice.

Now we return to the problem of the completeness for the system of the Bloch function of the oscillator.

According to (5.5) this system is connected with the system $|\alpha_{m_1 m_2}\rangle$ when $n=1$ by the relation:

$$|\alpha_{k_1 k_2}\rangle = \sum_{m_1 m_2 = -\infty}^{\infty} (-1)^{m_1 m_2} e^{i(k_1 - 2\pi m_1 \alpha)} |\alpha_{m_1}\rangle e^{i(k_2 + 2\pi m_2 \alpha)} |\alpha_{m_2}\rangle \quad (7.13)$$

Let's determine the function $|\tilde{\alpha}_{k_1 k_2}\rangle$ in the following way:

$$|d\tilde{k}_1 k_2\rangle = \frac{1}{4\pi} \sum_{n_1 n_2 = -\infty}^{\infty} (-1)^{n_1 n_2} e^{i(k_1 - 2\gamma m d)d_1 n_1} \times$$

$$\times e^{i(k_2 + 2R e d)d_2 n_2} \mathcal{D}(d) |\tilde{d} n_1 n_2\rangle \quad (7.14)$$

Then it is easy to see, that:

$$\begin{aligned} & \int_{-\pi/d_1}^{\pi/d_1} \int_{-\pi/d_2}^{\pi/d_2} |d k_1 k_2\rangle \langle d \tilde{k}_1 k_2| d k_1 d k_2 = \\ & = \sum_{m_1 m_2 n_1 n_2} \int \int |d m_1 m_2\rangle \langle \tilde{d} n_1 n_2| e^{i(k_1 - 2\gamma m d)d_1(m_1 - n_1)} e^{i(k_2 + 2R e d)d_2(m_2 - n_2)} d k_1 d k_2 \\ & = \frac{(2\pi)^2}{d_1 d_2} \frac{1}{4\pi} \sum_{m_1 m_2 = -\infty}^{\infty} |d m_1 m_2\rangle \langle \tilde{d} m_1 m_2| = \hat{I} \quad \text{therefore } d_1 d_2 = \pi. \end{aligned}$$

Thus the relation of the completeness for the Bloch functions has the form:

$$\int_{-\pi/d_1}^{\pi/d_1} \int_{-\pi/d_2}^{\pi/d_2} |d k_1 k_2\rangle \langle d \tilde{k}_1 k_2| d k_1 d k_2 = \hat{I} \quad (7.15)$$

One can substitute in (7.14) the expression for

$|\tilde{d} n_1 n_2\rangle = \frac{1}{\mathcal{D}} |\tilde{d} n_1 n_2\rangle$ (7.8) in the coherent states representation and obtain the function $|d \tilde{k}_1 k_2\rangle$ in this representation.

$$\langle \beta | \tilde{\alpha} K_1 K_2 \rangle = \frac{1}{8\pi} \sum_{n_1 n_2 = -\infty}^{\infty} (-1)^{n_1 n_2} e^{i(K_1 - 2\pi m \alpha) \alpha_1 n_1} \times$$

$$\times e^{i(K_2 + 2\pi R \alpha) \alpha_2 n_2} \frac{d n_1 n_2}{2(\alpha)} \frac{\tilde{G}(\beta^*)}{\tilde{G}'(\alpha n_1 n_2) \beta^* (\beta^* - \alpha n_1 n_2)} e^{\frac{|\alpha n_1 n_2|^2}{2}} e^{-\frac{|\beta|^2}{2}}$$

And after the using the relation

$$\tilde{G}'(\alpha n_1 n_2) = (-1)^{n_1 n_2 + n_1 + n_2} e^{\frac{|\alpha n_1 n_2|^2}{2}} \quad (7.16)$$

which easy to get by means of the differentiate the relation (7.2) when $\alpha = 0$, one can simplify this expression

$$\langle \beta | \tilde{\alpha} K_1 K_2 \rangle = \frac{1}{8\pi} 2(\alpha) e^{-\frac{|\beta|^2}{2}} \frac{\tilde{G}(\beta^*)}{\beta^*} \times$$

$$\times \sum_{n_1 n_2 = -\infty}^{\infty} (-1)^{n_1 + n_2} e^{i(K_1 - 2\pi m \alpha) \alpha_1 n_1} e^{i(K_2 + 2\pi R \alpha) \alpha_2 n_2} \frac{d n_1 n_2}{\beta^* - \alpha n_1 n_2} \quad (7.17)$$

Thus, the system of the Bloch function in case when in the quantisation condition (5.3) $n=1$ is complete, but in case, when $n=\pm 2, \pm 3 \dots$ this system is not complete.

In last case one can obtain the complete system by adding the systems of the Bloch functions which was constructed over the others sublattice.

For the Bloch function of the electron in the magnetic field the condition of the completeness is the existence of one quantum of the flux through the elementary cell of the lattice, so far as this problem is reduced to the case of the simple oscillator. There are not the generalization the condition of the completeness in the many dimensional case, so far as there are not the generalization of the Mittag-Leffler's theorem in this case.

8. The some properties of the theta functions.

It is easy to verify, that coherent states of the electron in the magnetic field $|0\beta\rangle$ and the Landau function $|0C\rangle$ are connected by the transformation:

$$|0\beta\rangle = \int |0C\rangle K(C\beta) dC \quad (8.1)$$

where

$$K(C, \beta) = \frac{1}{\sqrt{\pi\omega}} \exp\left(-\frac{|B|^2}{2} + \frac{\beta^2}{2} - \frac{C^2}{2\omega} - \frac{i\sqrt{2}\beta C}{\sqrt{\omega}}\right) \quad (8.2)$$

One can apply the operator (2,7) to the equation (8.1)

and verify that the connection between the Bloch functions of the different kinds for the electron in the constant magnetic field has the same form:

$$f_{k_x k_y}^{0\beta} = \int f_{k_x k_y}^{0c} K(c, \beta) dc \quad (8.3)$$

One can substitute in this relation the expressions (2.11), (2.19) and (2.24), put for simplicity $m=\ell=\omega=1$ and obtain the connection between the theta functions.

When in quantization condition $\alpha\beta = 2\pi n$, n - is even number, we have:

$$\begin{aligned} & \int_{-\infty}^{\infty} \mathcal{V}_3 \left(1 / \frac{a}{2\pi} (k_x - c) \right) \mathcal{V}_3 \left(e^{-\frac{b^2}{4}} / \frac{b}{2\pi} (k_y - i\xi - \frac{\beta}{\sqrt{2}}) \right) \exp \left\{ - \left[c + i \left(\frac{\beta}{\sqrt{2}} - \xi \right) \right]^2 \right\} dc = \\ & = \mathcal{V}_3 \left(e^{-\frac{a^2}{4}} / \frac{a}{2\pi} (k_x - i\xi + i \frac{\beta}{\sqrt{2}}) \right) \mathcal{V}_3 \left(e^{-\frac{b^2}{4}} / \frac{b}{2\pi} (k_y - \xi - \frac{\beta}{\sqrt{2}}) \right) \end{aligned} \quad (8.4)$$

Let's introduce the new variables:

$$\begin{aligned} x &= c + i \left(\frac{\beta}{\sqrt{2}} - \xi \right); & \theta_1 &= \frac{a}{2\pi} (k_x - i\xi + i \frac{\beta}{\sqrt{2}}) \\ \theta_2 &= \frac{b}{2\pi} (k_y - \xi - \frac{\beta}{\sqrt{2}}); & \gamma_1 &= e^{-\frac{a^2}{4}} = e^{i\pi\tau_1} \\ & & \gamma_2 &= e^{-\frac{b^2}{4}} = e^{i\pi\tau_2} \end{aligned}$$

and represent this expression as:

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$$\begin{aligned}
 & \int \psi_3^*(1/\theta_1 - \sqrt{\frac{\pi_1}{i\pi}} x) \psi_3(q_2^2/\theta_2 - \sqrt{\frac{i\pi_2}{\pi}} x) e^{-x^2} dx = \\
 & = \psi_3^*(q_1/\theta_1) \psi_3(q_2/\theta_2) \quad (8.5)
 \end{aligned}$$

When in quantization condition. n is odd number, we have in the similar way:

$$\begin{aligned}
 & \int \psi_3^*(1/\frac{a}{2\pi}(k_x - i\xi + i\frac{b}{\sqrt{2}}) - x) \psi_3(e^{-\frac{b^2}{2}}/\frac{b}{2\pi}(k_y - \xi - \frac{b}{\sqrt{2}} - \\
 & - ix)) e^{-x^2} dx = \psi_3^*(e^{-\frac{a^2}{4}}/\frac{a}{2\pi}(k_x - i\xi - i\frac{b}{\sqrt{2}})) \times \\
 & \times \psi_3(e^{-\frac{b^2}{4}}/\frac{b}{2\pi}(k_y - \xi - \frac{b}{\sqrt{2}})) - 2 \psi_3^*(e^{-\frac{a^2}{4}}/\frac{a}{\pi}(k_x - i\xi + \\
 & + i\frac{b}{\sqrt{2}})) \psi_3^*(e^{-\frac{b^2}{4}}/\frac{b}{\pi}(k_y - \xi - \frac{b}{\sqrt{2}})) \quad (8.6)
 \end{aligned}$$

or

$$\int \psi_3^*(1/\theta_1 - \sqrt{\frac{\pi_1}{i\pi}} x) \psi_3(q_2^2/\theta_2 - \sqrt{\frac{i\pi_2}{\pi}} x) e^{-x^2} dx =$$

$$= \vartheta_3^4(q_1/q_1) \vartheta_3^4(q_2/q_2) - 2 \vartheta_2^4(q_1^4/2q_1) \vartheta_2^4(q_2^4/2q_2) \quad (8.7)$$

The relations (8.4) and (8.6) can be easily verified if we substitute the definitions of the theta functions in the left part of these expressions and sum up after the integration.

In the same way we may verify the similar relations for the ϑ_3 and ϑ_4 - functions of the real argument ϑ

$$\begin{aligned} \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \vartheta_3^4(q/\vartheta) e^{-\vartheta^2} d\vartheta &= \frac{1}{\sqrt{\pi}} \sum_{h=-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-\vartheta^2} e^{2\pi i \vartheta h} q^{h^2} d\vartheta = \\ &= \sum_{h=-\infty}^{\infty} q^{h^2} e^{-\pi^2 h^2} = \vartheta_3^4(q e^{-\pi^2}/0) \end{aligned} \quad (8.8)$$

and

$$\frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \vartheta_4^4(q/\vartheta) e^{-\vartheta^2} d\vartheta = \vartheta_4^4(q e^{-\pi^2}/0) \quad (8.9)$$

The relations (8.4), (8.6), (8.8) and (8.9) are very interesting so far as the expressions for the integration of the theta functions are not known in the mathematics. The relations of this kind (5.8) already have been obtained by us. From all these relations one can see that the integral of the theta functions with the square exponent e^{-x^2} gives also the theta functions.

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