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MESON MASS

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ON THE VACUUM INSTABILITY
AND THE HIGGS MESON MASS

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ABSTRACT

A theory of the decay of a metastable vacuum state both in quantum field theory and in quantum statistics at a non-vanishing temperature is considered. As an application it is shown that if the Universe is not enormously charge-asymmetric, then the Higgs meson mass in the Weinberg model should exceed 6.9 Gev.

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Last years it was discovered [1-5] that at certain relations between coupling constants radiative corrections may lead to the absence of symmetry breaking in the theories of the type of the Weinberg model. The reason for such a dynamical symmetry restoration is that in the one-loop approximation the effective potential $V(\varphi)$ of these theories is given by

$$V(\varphi) = (2A - B)\varphi^2\varphi^2 - A\varphi^4 + B\varphi^4 \ln \frac{\varphi^2}{\varphi_0^2}, \quad (1)$$

where φ is the classical part of the Higgs field, φ_0 is the value of φ in the minimum of $V(\varphi)$ at $\varphi \neq 0$, A and B are some constants. In the Weinberg model $\varphi_0 = 248$ GeV, and $B = 9.7 \cdot 10^{-5}$ for the empirical value of the mixing angle $\theta_w = 35^\circ$. The Higgs meson mass is given by

$$M_H^2 = \left. \frac{d^2 V}{d\varphi^2} \right|_{\varphi=\varphi_0} = 4\varphi_0^2 (3B - 2A), \quad (2)$$

and the effective coupling constant of the Higgs field self-interaction is equal to

$$\lambda = \left. \frac{d^4 V}{d\varphi^4} \right|_{\varphi=\varphi_0} = 4(25B - 6A). \quad (3)$$

The state $\varphi = \varphi_0$ is a minimum of $V(\varphi)$ only if $M_H^2 > 0$. In this case from (2) and (3) it follows that $\lambda > 64B$ [1,2], which implies that in the Weinberg model [2,4]

$$\lambda > 6.2 \cdot 10^{-3} \quad (4)$$

At $A > \frac{B}{2}$ ($M_H < 6.91$ GeV) the effective potential (1) has an extra minimum at $\varphi = 0$. Thus at $M_H < 6.91$ GeV there are two possible vacuum states: $\varphi = 0$ and $\varphi = \varphi_0$, with the energy gap between them

$$\epsilon = V(\varphi_0) - V(0) = (A - B)\varphi_0^4 \quad (5)$$

The state $\varphi = \varphi_0$ is energetically favorable ($\epsilon < 0$) only if

the Higgs meson mass in the Weinberg model satisfies the inequality [2,3,4]

$$M_H > 2\sqrt{B}G = 4.91 \text{ GeV} \quad (6)$$

The corresponding constraint on λ is [2,4]

$$\lambda > 7.4 \cdot 10^{-3} \quad (7)$$

As it was mentioned in [5-7], the constraints (7), (8) (but not the inequality (5)) would be incorrect if the Universe initially was in the metastable state $\varphi = G$, and the lifetime of this state is much greater than the age of the Universe [8]. The decay of this vacuum state proceeds by spontaneous generation of "bubbles" with $\varphi < G$. The bubbles spread up with the velocity $\sim C$, the field φ inside the bubbles decreases and gradually the field φ vanishes in all the Universe.

The generation of the bubbles is a tunneling process, since only the bubbles above some critical size can spontaneously appear in the vacuum [8]. Following Coleman [9], we shall treat the barrier tunneling as a classical motion in the imaginary time direction. To calculate the probability of the tunneling one should find $O(4)$ -symmetric solution of equation

$$\frac{d^2\varphi}{d\tau^2} + \Delta\varphi = \frac{dV}{d\varphi} \quad (8)$$

with the boundary condition $\varphi = G$ at $\vec{x}^2 + \tau^2 \rightarrow \infty$. The number of such tunnelings that have already occurred in the past is given by

$$N = F \cdot e^{-S_4} \quad (9)$$

where S_4 is the action, corresponding to the $O(4)$ -symmet-

ric solution of eq. (8), and $\bar{F}$ is some factor, proportional to the space-time volume of the backward light cone; in our case $\bar{F} \sim 10^{170}$ [7]. For the Universe to be now in the energetically favorable state, it is sufficient that at least one such tunneling have occurred in the past ($N \gtrsim 1$).

The first attempt to calculate N as a function of M_H has been performed by Frampton [7]. He has neglected the bubble surface thickness as compared with the bubble radius and has obtained in this approximation that $N \ll 1$ for $M_H > 3.5$ GeV, $N \gg 1$ for $M_H < 3.45$ GeV. He has concluded therefore that the constraint (6) should be replaced by

$$M_H > 3.5 \text{ GeV} \quad (10)$$

However the surface thickness is small only if the energy difference ϵ between the two vacua is much less than the energetical barrier between them [9]. Therefore the approximation used in [7] is applicable only for $M_H \sim 4.9$ GeV, but not for $M_H \sim 3.5$ GeV. To overcome this difficulty we have performed the computer calculations and solved eq.(8) numerically without any assumptions concerning the surface thickness, and the result is that $N \ll 1$ for $M_H > 250$ MeV, $N \gg 1$ for $M_H < 250$ MeV. Therefore if the Universe initially was in the state $\varphi = \psi$, then the improved constraint on M_H would be

$$M_H > 250 \text{ MeV} \quad (11)$$

while the corresponding constraint on λ would practically coincide with (4).

However, as it was shown in [10], the symmetry in the

Weinberg model was broken at $t \rightarrow 0$ (t is the time from the beginning of the Universe expansion) only if

$$L \geq 10^8 B, \quad (12)$$

where L and B are respectively the leptonic and barionic charges of the Universe. Such a large charge asymmetry would not contradict cosmological data only if there is a large excess of neutrinos over antineutrinos in the Universe ("neutrino sea") [10].

If the Universe is not so enormously charge asymmetric, then at $t \rightarrow 0$ the symmetry in the Weinberg model was restored due to the high-temperature effects: At $T \gtrsim U$ the state $\varphi = 0$ is the only minimum of the temperature-dependent effective potential $V^T(\varphi)$ [6,11]. In this case the problem becomes in some sense opposed to that considered above. Namely, at $M_H < 4.91$ GeV the phase transition from the initial vacuum state $\varphi = 0$ to the state $\varphi = U$ cannot take place. Moreover, at $4.91 \text{ GeV} < M_H < 6.91 \text{ GeV}$ the vacuum state $\varphi = 0$ with the decrease of temperature becomes metastable, and the spontaneous symmetry breaking can take place only if the lifetime of this metastable state is less than the age of the Universe.

To calculate the barrier tunneling in this case one should take into account that at high temperature the tunneling proceeds mainly not due to the quantum fluctuations, but due to the thermodynamic ones. Quantum statistics at $T \neq 0$ is equivalent to the Euclidean quantum field theory, but with the periodicity condition with the period $\frac{1}{T}$ in one of the four coordinates. Therefore at $T \neq 0$ instead of the $O(4)$ -symmetric bubble

considered above one has $O(3)$ -symmetric bubble with periodically changed size. At $T \gg M_H$ the corresponding field configuration becomes the $O(3) \times U(1)$ cylinder, and the exponential factor in (9) should be replaced by $e^{-\frac{S_3}{T}}$, where S_3 is the three-dimensional bubble action. *) (The subexponential factor F in (9) also should be modified, but this modification appears inessential for our final results). To calculate S_3 one should find the $O(3)$ -symmetric solution of equation

$$\Delta \varphi = \frac{dV}{d\varphi} \quad (13)$$

with the boundary condition $\varphi = 0$ at $\vec{X}^2 \rightarrow \infty$. In this case computer calculations show, that the phase transition to the vacuum state $\varphi = 0$ occurs only if

$$M_H > 6.9 \text{ GeV} \quad (14)$$

Note that this lower bound is very close to the value

$M_H = 6.91 \text{ GeV}$, for which $\frac{d^2 V}{d\varphi^2}|_{\varphi=0} = 0$ (Coleman-Weinberg case [13]). The corresponding constraint on λ is

$$\lambda > 8.5 \cdot 10^{-3} \quad (15)$$

Let us now summarize our results.

1) For the symmetry in the Weinberg model to be broken it is necessary that $M_H > 260 \text{ MeV}$, $\lambda > 6.2 \cdot 10^{-3}$.

*) If the bubble surface thickness is small, then $F \cdot e^{-\frac{S_3}{T}} = F \exp\left(-\frac{16\pi a^3}{3\epsilon T}\right)$, where a is the surface energy. This is a well known expression for the probability of the bubble formation in a boiling liquid [12].

2) If, as we expect, the Universe is not enormously charge-asymmetric, then the corresponding constraints are much stronger: $M_H > 6.9 \text{ GeV}$, $\lambda > 8.5 \cdot 10^{-3}$.

3) If the Higgs meson mass is in the range $260 \text{ MeV} < M_H < 6.9 \text{ GeV}$, then the symmetry breaking in the Weinberg model is completely determined by the charge asymmetry of the Universe. The discovery of the Higgs meson in this mass interval would be of great importance for cosmology, since this would imply the existence of a "neutrino sea" with an excess of neutrinos over antineutrinos $\Delta n_\nu = n_\nu - n_{\bar{\nu}} \geq 10^3 \text{ cm}^{-3}$.

The last statement may seem rather unexpected, since the characteristic value of Δn_ν , which may affect the symmetry breaking in the Weinberg model, is $\Delta n_\nu \sim 10^{50} \text{ cm}^{-3}$ [6,10]. The physical reason, why nevertheless the symmetry breaking in the Weinberg model may be connected with the present small excess of neutrinos over antineutrinos, is the large lifetime of metastable states in quantum field theory, or, in other words, the good "memory of vacuum": The vacuum state "remembers" the situation at the early stages of the Universe evolution, at which this vacuum state was formed and Δn_ν was large. This fact by itself seems to be of a much greater importance than the constraints on the Higgs meson mass obtained above.

Two further remarks are in order.

1) The Euclidean bubbles considered in this paper correspond to the extrema of the effective action (with the account taken of radiative corrections) and in this sense they differ from the previously discussed instantons [14], which are

the solutions of classical Euclidean equations of motion.

2) As it was mentioned above, the factor e^{-S_4} , which strongly suppresses the tunneling, at high temperature is replaced by $e^{-\frac{S^2}{T}}$. At $T \gg m$, where m is a characteristic mass scale, this factor becomes ~ 1 , and thus at a very high temperature there is no exponential suppression of the tunneling. This fact is of relatively small importance for the problem discussed in our paper, since the minimum of $V(\varphi)$ at $\varphi \neq 0$ appears only at not very high temperature. However the high-temperature effects may appear to be of a crucial importance for the problem of the decay of barions into leptons in the Weinberg model [15]. At $T = 0$ the decay of barions is suppressed by the factor $e^{-8\pi/g^2}$, where $8\pi/g^2$ is the instanton action at $T = 0$ [14, 15]. However one may expect that such a strong suppression was absent in the early Universe at a temperature $T \sim T_c$, where T_c is the critical temperature in the Weinberg model, at which the masses of all the particles vanish [6, 11]. The possible consequences of this effect will be discussed in a separate publication.

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