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ASYMPTOTICALLY FREE E_6 MODEL OF UNIFIED INTERACTION

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An asymptotically free E_6 model of unified interaction is constructed. For the model to exist quite definite relations between coupling constants are necessary, the gauge coupling constant being equal to the electromagnetic one. The strong interaction and the quark confinement are provided by gluonic mechanism. Some physical consequences of the model are briefly discussed.

At present in the elementary particle physics the models of unified interaction which are built on the basis of the exceptional groups are considered as promising [1], [2]. The matter is that, with these groups, the hope appears to achieve, due to the special octanionic multiplication rules, the unitarity of the theory in the singlet sector of the colour group already at the kinematical level. Another attractive feature of the exceptional groups is the existence, apart from the colour group, of the unitary spin group what makes possible to include the necessary symmetry group of the strong interacting particles at the very beginning. The problem of the quark confinement is expected to be solved here, as well as we hope to solve it in the usual models of unified interaction [3], [4] due to infrared divergencies of the unbroken gauge group of massless coloured gluons. Therefore the attempt becomes of especial interest now to build, within the exceptional groups, asymptotically free models of unified interactions ^{x)} as examples of the theory which would possess the due mathematical consistency (the "zero-charge" difficulty [5] will be absent from here) and would describe the realistic interaction of elementary particles.

In the present paper an example of such model is built within the group E_6 . It proves to be sufficiently convenient for description of the realistic interactions since it is the

^{x)} See ref. [4] for example of such models within nonexceptional groups.

only exceptional group which has complex representations (moreover the latter are free of anomalies). Here, by using the Higgs mechanism, one may easily extract, with the maximal subgroup of the E_6 -group exhausted, i) the $SU^c(3)$ subgroup of colour responsible for the strong interaction and quark confinement (this is the subgroup of the G_2 -group of the automorphisms of octonions), ii) the usual $SU(3)$ subgroup of the unitary spin and iii) the $SU(3)$ subgroup which unites the weak and electromagnetic interactions. The physics of low energies is in our model mainly the same as in ref. [1]. The peculiarity of the present model manifests itself in the region of high and superhigh energies where (contrary to the models built in refs [1] and [2]) all the interactions vanish asymptotically thus providing the logical and mathematical consistency of the theory.

The Lagrangian of the model splits into three structural blocks:

$$\mathcal{L} = \mathcal{L}_g + \mathcal{L}_\gamma + \mathcal{L}_3 \quad (1)$$

$\mathcal{L}_g$ displays the fields involved in the Lagrangian (1) and their interaction with the gauge fields:

$$\begin{aligned} \mathcal{L}_g = & -\frac{1}{4}(\partial_\mu V_\nu^\alpha - \partial_\nu V_\mu^\alpha + g_0 f^{\alpha\beta\gamma} V_\mu^\beta V_\nu^\gamma)^2 \quad (2) \\ & -\frac{1}{2}[(\partial_\mu \delta^{\alpha\gamma} + g_0 f^{\alpha\beta\gamma} V_\mu^\beta) \phi^\gamma]^2 - [(\partial_\mu \delta^{\alpha d} - ig_0 (\Gamma V_\mu)_a^d) M_d]^2 \\ & - [(\partial_\mu \delta^{\alpha d} - ig_0 (\Gamma V_\mu)_a^d) N_d]^2 + \bar{B}^d [\delta_\mu (\partial_\mu \delta^{\alpha\gamma} + g_0 f^{\alpha\beta\gamma} V_\mu^\beta) - m_B \delta^{\alpha\gamma}] B_\gamma \\ & + \bar{\eta}^\alpha \gamma_\mu [\partial_\mu \delta^{\alpha d} - ig_0 (\Gamma V_\mu)_a^d] \eta_d + \bar{\psi}_\alpha \gamma_\mu [\partial_\mu \delta^{\alpha d} + ig_0 (\Gamma V_\mu)_a^d] \psi^d. \end{aligned}$$

Here $f^{\alpha\beta\gamma}$ are the structural constants $[(\Gamma^a)^d_a]$ are the generators of the E_6 group (see for example [6]); $g_{\Gamma\Gamma} = (-, +, +, +)$; $\{\gamma_\mu, \gamma_\nu\} = 2\delta_{\mu\nu}$. The choice of such representations for the spinor and scalar particles is needed for solving, under the condition that the superfluous massless vector particles are absent, the problem of the asymptotical freedom for model (1) as a whole.

$\mathcal{L}_Y$ is the part of the Lagrangian (1) which describes the Yukawa-like interaction

$$\mathcal{L}_Y = -k F_{\alpha\beta}^{\rho} \bar{B}^{\alpha} \phi_{\rho} B^{\beta} \quad (3)$$

$$- [f(\bar{B}^{\rho} \Gamma_{\rho})^i_j K^{+j} \eta_i + h D_{ijk} \bar{\eta}^i K^{+j} \psi^k + f(\bar{B}^{\rho} \Gamma_{\rho})^i_j K_i \psi^j +$$

where $K = \frac{1-\gamma_5}{2} M + \frac{1+\gamma_5}{2} N$ and $F_{\alpha\beta}^{\rho} = i f^{\alpha\beta\rho}$.

$\mathcal{L}_S$ represents the interaction potential of the scalar fields

$$\mathcal{L}_S = -\frac{\lambda_{\phi}^2}{8} (\phi^{\alpha} \phi_{\alpha})^2 \quad (4)$$

$$- \left[\frac{\lambda_{\phi}^2 M^2}{2} (\phi^{\alpha} \phi_{\alpha}) (M^{+d} M_d) + \frac{\delta_{\phi}^2 M^2}{4} \{ \Gamma^{\alpha} \Gamma^{\beta} \}_a^{\rho} (\phi_{\alpha} \phi_{\rho}) (M^{+a} M_a) \right.$$

$$+ \frac{\lambda_{M^+}^2}{2} (M^{+d} M_d)^2 + \frac{\delta_{M^+}^2}{4} D_{\alpha\beta\rho} D^{fcd} (M^{+a} M^{+b} M_c M_d) + (M \rightarrow N) \text{ etc.}$$

$$- \delta_{M^+ N^+}^2 (M^+ N) (N^+ M) - \lambda_{M^+ N^+}^2 (M^+ M) (N^+ N) - \gamma_{M^+ N^+}^2 D_{\alpha\beta\rho} D^{fcd} (M^{+a} N^b N_c) |$$

In eq.(4) D_{abc} is the absolutely symmetric tensor on the E_6 group [6].

The asymptotical freedom of the model (1) is proved within the set of equations of the renormalisation group.

In the "one-loop" approximation these equations are easily obtained by usual methods [7], [8] and are combined into three groups. Here first of all, one has an equation for the effective coupling constant of the gauge fields

$$16\pi^2 \dot{g} = -16g^3 \quad (5)$$

then the set of equations for the effective coupling constants of the Yukawa-like interactions

$$\begin{aligned} 16\pi^2 \dot{k} &= 48k^3 + 6f^2k - 72g^2k, \\ 16\pi^2 \dot{f} &= \frac{148}{6}f^3 + 5h^2f - 62g^2f + 6k^2f, \\ 16\pi^2 \dot{h} &= 20h^3 + \frac{26}{3}f^2h - 52g^2h, \end{aligned} \quad (6)$$

and, at last, the set of equations for the effective constants of the scalar interaction

$$\begin{aligned} 16\pi^2 \dot{\lambda}_{\phi^4}^2 &= 86\lambda_{\phi^4}^4 + 108\lambda_{\phi^2M^2}^4 + 24\lambda_{\phi^2N^2}^2 \delta_{\phi^2M^2}^2 + 3\delta_{\phi^2M^2}^4 + 54g^4 \\ &\quad - 72k^4 + 4(24k^2 - 36g^2)\lambda_{\phi^4}^2, \\ 16\pi^2 \dot{\lambda}_{\phi^2M^2}^2 &= 4\lambda_{\phi^2M^2}^2 \left[\lambda_{\phi^4}^2 + 20\lambda_{\phi^4}^2 + 14\lambda_{M^4}^2 + 5\delta_{M^4}^2 + \frac{27}{2}\lambda_{M^2N^2}^2 + \frac{1}{2}\delta_{M^4N^2}^2 \right. \\ &\quad \left. + 5\delta_{M^2N^2}^2 \right] + \delta_{\phi^2M^2}^2 \left(\delta_{\phi^4M^2}^2 + \frac{26}{3}\lambda_{\phi^4}^2 + 6\lambda_{M^4}^2 + 2\delta_{M^4}^2 + 6\lambda_{M^4N^2}^2 \right. \\ &\quad \left. + 2\delta_{M^2N^2}^2 \right) + 12g^4 - 16k^2f^2 + 2(24k^2 + \frac{52}{3}f^2 + 10h^2 - 62g^2)\lambda_{\phi^2M^2}^2, \\ 16\pi^2 \dot{\delta}_{\phi^2M^2}^2 &= \delta_{\phi^2M^2}^2 \left(\frac{88}{6}\delta_{\phi^2M^2}^2 + 2\lambda_{\phi^4}^2 + 8\lambda_{\phi^2M^2}^2 + 2\lambda_{M^4}^2 + 2\delta_{M^4}^2 + 2\delta_{M^2N^2}^2 + 2\delta_{M^4N^2}^2 \right) \\ &\quad + 36g^4 - 48k^2f^2 + 2(24k^2 + \frac{52}{3}f^2 + 10h^2 - 62g^2)\delta_{\phi^2M^2}^2, \\ 16\pi^2 \dot{\lambda}_{M^4}^2 &= \lambda_{M^4}^2 (62\lambda_{M^4}^2 + 40\delta_{M^4}^2) + 20\delta_{M^4}^4 + \lambda_{\phi^2M^2}^2 (78\lambda_{\phi^2M^2}^2 + \frac{52}{3}\delta_{\phi^2M^2}^2) \\ &\quad + \frac{22}{9}\delta_{\phi^2M^2}^2 + 20\delta_{M^2N^2}^2 (\delta_{M^2N^2}^2 + 2\lambda_{M^2N^2}^2) + 2\delta_{M^4N^2}^2 (\delta_{M^4N^2}^2 + 2\lambda_{M^4N^2}^2) \\ &\quad + 54\lambda_{M^2N^2}^4 + \frac{88}{3}g^4 - \frac{320}{9}f^4 - 40h^4 + 4(\frac{52}{3}f^2 + 10h^2 - 26g^2)\lambda_{M^4}^2, \end{aligned}$$

$$16\pi^2\delta_{M^4}^2 = -6\delta_{M^4}^4 + 12\lambda_{M^4}^2\delta_{M^4}^2 + \frac{2}{3}\delta_{\phi^2 M^2}^4 + 8\chi_{M^2 N^2}^2(\delta_{M^2 N^2}^2 - 2\chi_{M^2 N^2}^2) + 8 - \frac{208}{3}f^2 h^2 + \frac{56}{3}f^4 + 32h^4 + 4\left(\frac{52}{3}f^4 + 10h^2 - 26g^2\right)\delta_{M^4}^2,$$

$$16\pi^2\lambda_{M^2 N^2}^2 = 4\lambda_{M^2 N^2}^4 + 78\lambda_{\phi^2 M^2}^4 + \frac{52}{3}\lambda_{\phi^2 M^2}^2\delta_{\phi^2 M^2}^2 + \frac{7}{9}\delta_{\phi^2 M^2}^4 + 10\chi_{M^2 N^2}^4 + 112\lambda_{M^4}^2\lambda_{M^2 N^2}^2 + 4\lambda_{M^4}^2\delta_{M^2 N^2}^2 + 40\lambda_{M^4}^2\chi_{M^2 N^2}^2 + 40\delta_{M^4}^2\lambda_{M^2 N^2}^2 + 20\delta_{M^4}^2\chi_{M^2 N^2}^2 + 2\delta_{M^2 N^2}^4 + \frac{28}{3}g^4 - \frac{20}{9}f^4 + 4\left(\frac{52}{3}f^2 + 10h^2 - 26g^2\right)\lambda_{M^2 N^2}^2,$$

$$16\pi^2\delta_{M^2 N^2}^2 = 54\delta_{M^2 N^2}^4 + \frac{5}{3}\delta_{\phi^2 M^2}^4 + 4\lambda_{M^4}^2\delta_{M^2 N^2}^2 + 20\delta_{M^4}^2\chi_{M^2 N^2}^2 + 10\chi_{M^4 N^2}^4 + 8\lambda_{M^2 N^2}^2\delta_{M^2 N^2}^2 + 40\delta_{M^2 N^2}^2\chi_{M^2 N^2}^2 + 20g^4 - \frac{4}{3}f^4 + 4\left(\frac{52}{3}f^2 + 10h^2 - 26g^2\right)\delta_{M^2 N^2}^2,$$

$$16\pi^2\chi_{M^2 N^2}^2 = 12\chi_{M^2 N^2}^4 + \frac{1}{3}\delta_{\phi^2 M^2}^4 + 4\lambda_{M^4}^2\chi_{M^2 N^2}^2 + 4\delta_{M^4}^2(\delta_{M^2 N^2}^2 - 4\chi_{M^2 N^2}^2) + 8\left(\lambda_{M^2 N^2}^2 + \frac{1}{2}\delta_{M^2 N^2}^2\right)\chi_{M^2 N^2}^2 + 4g^4 - \frac{44}{3}f^4 + 4\left(\frac{52}{3}f^2 + 10h^2 - 26g^2\right)\chi_{M^2 N^2}^2,$$

The boundary conditions of the ordinary type are meant in every case.

The solutions of the set of equations (6) and (7) will be obtained here in some particular form

$$\omega_i(t) = g(t)\bar{\omega}_i \quad (8)$$

where $\omega_i(t)$ is one of the effective coupling constants,

$\bar{\omega}_i$ are merely numbers giving the fixed points of the set of equations (6) and (7) modified with the help of eq.(8) and $g(t)$ is the exact solution of eq. (5)

$$g^2(t) = g^2 \left[1 + 32(g^2/16\pi^2)t \right]^{-1} \quad (9)$$

The investigation of solution of eqs. (6) and (7) in a more general form is difficult and not needed here since the asymptotical freedom of the model (I) is completely proved already within eq. (8).

All the sets of fixed points of eqs. (6) are easily obtained analytically. The set of the fixed points suggested below

$$\bar{k}^2 = 0.994253, \bar{f}^2 = 1.379311, \bar{h}^2 = 1.202299 \quad (10)$$

is more convenient for some reasons. All the other solutions lead either to unstable potential of scalar interaction or to the loss of asymptotical freedom when the set of eq. (7) is solved. The set of fixed points of eqs. (7) when eq. (11) is taken into account is obtained numerically

$$\begin{aligned} \bar{a}_{\phi^4}^2 &= 0.526893, \bar{a}_{\phi^2 M^2}^2 = 0.021569, \bar{\delta}_{\phi^2 M^2}^2 = 0.728620 \\ \bar{a}_{M^4}^2 &= 0.698427, \bar{\delta}_{M^4}^2 = 0.353515 \\ \bar{a}_{M^2 M^2}^2 &= -0.111609, \bar{\delta}_{M^2 M^2}^2 = -0.311164, \bar{\gamma}_{M^2 M^2}^2 = 0.341257 \end{aligned} \quad (11)$$

This set of fixed points satisfies all requirements of the model. It provides a stable form of the potential of the scalar interaction and asymptotical freedom for the needed coupling constants. Other solutions of eqs. (7) which do not satisfy those requirements are not discussed here.

Thus the model possesses the asymptotical freedom on the whole. In the region of the large transferred momentum the coupling constants satisfy eq. (8) wherein the $\bar{\omega}$'s are determined by eqs. (10) and (11). The arbitrariness remains for the gauge coupling constant g only. The fixed points

obtained here are the ultraviolet-unstable ones. The importance of such solutions has already been mentioned in papers [8], [9] and [4].

The Higgs mechanism which breaks the initial E_6 symmetry by reducing it to the level of the observed symmetry in low energy physics and gives masses to the needed vector fields is associated here with the nonvanishing vacuum expectation value of the scalar fields as follows

$$\omega_i \equiv \langle \Phi_{R_i}^P \rangle |_{i=1,2} ; v_i \equiv \langle (M_{\frac{n}{2}}^n)_{ii}^P \rangle |_{n=3} \quad (12)$$

The symmetry breaking thus chosen enables one to keep unbroken

$3U(3)$ subgroup of gauge fields and to provide, due to the infrared divergencies of this unbroken subgroup, the strong interaction between quarks. Matrices (12) must satisfy the set of equations for extremum of potential

$$\begin{aligned} \beta \omega_1 + [\omega_1 (v_1^+ v_1 + v_2^+ v_2) + (v_1^+ v_1 + v_2^+ v_2) \omega_1 - 2(v_1^+ \omega_2 v_1 + v_2^+ \omega_2 v_2)] = \lambda_1 \cdot I \\ \alpha_1 v_1 + \lambda (v_1 v_1^+ v_2) + \delta (v_1 v_2^+ v_2 + v_2 v_2^+ v_1) + \\ + \tau \beta p (v_1 v_2^+) v_2 + [\omega_1^2 v_1 + v_1 \omega_2^2 - 2\omega_1 v_1 \omega_2] = 0 \end{aligned} \quad (13)$$

which impose strong restrictions (in a number of cases define in unique manner) on these matrices (matrices $\omega_i(v_i)$ satisfy eqs. (13) after exchanging $\omega_1(v_1) \rightarrow \omega_2(v_2)$ and $v_1 \rightarrow v_1^+$, $\lambda \rightarrow -\lambda$ on equation for (1) only). In (13) the parameters β and α_i are free from ^{any} restriction (since they contain the square of mass parameter of the appropriate scalar fields); the other quantities are tied firmly with the effective coupling constants of scalar interaction.

We do not consider here all the solutions of eqs. (13) but

restrict ourselves to the one which gives, on the classical level, minimum of the potential $\mathcal{L}_3$ with the mass terms added. This solution with the gauge chosen earlier has the following form

$$\omega_1 = \omega_2 = \begin{bmatrix} -2t \\ t \\ t \end{bmatrix} \quad \vec{v}_1 = \begin{bmatrix} \tau \\ q \\ p \end{bmatrix} \quad \vec{v}_2 = \begin{bmatrix} d \\ b & c \\ c & a \end{bmatrix} \quad (14)$$

where $q \neq p$ and the independent parameters are t and two other parameters which define the matrices $\vec{v}_i$. All the parameters are real and nonzero.

The solution (14) keeps two subgroups unbroken: $SU(3)$ subgroup of gauge fields (the group of massless coloured gluons mediating the strong interaction) and $U(1)$ subgroup which is associated with the usual electromagnetic field

$$\frac{A}{e} = \frac{1}{g} \left[\left(V_{R_1}^{(3)} + \frac{1}{\sqrt{3}} V_{R_2}^{(3)} \right) + \left(V_{R_1}^{(3)} + \frac{1}{\sqrt{3}} V_{R_2}^{(3)} \right) \right] \quad (15)$$

All the other vector bosons are provided with masses. By appropriate choice of parameters one may easily obtain the needed masses for vector bosons responsible for the usual weak decays, the other $SU(3)$ -singlet vector bosons which give the additional weak decays being made at the same time more heavy. The masses of the lepton-quark bosons, with this solution, are not free parameters and take values of the same order as the masses of the vector bosons responsible for the weak decays. However, these lepton-quark bosons are not singlets under group of the massless coloured gluons, therefore they, as well as quarks, must not be met in free state or, at least, they must be provided with very large masses.

Now a few words must be said about the physical contents of the model. In total, the model contains 132 spinor particles [26 leptons ($2 \times 9 + 8$) which are singlets under $3U^c(3)$ coloured group, and 106 quarks ($2 \times 3 \times 6 + 8 + 8 + 3 \times 18$)] which are combined here into two fundamental and one adjoint representations. This set of leptons and quarks determines the whole physics of elementary particles up to superhigh energies, therefore in this view the large number of the superfluous leptons and quarks must not be considered here as the failure of the model. The low energy physics may be easily determined nevertheless with the usual small set of quarks and the known leptons which are the lightest spinor particles within the primary set of the leptons and quarks. The masses of heavy and superheavy leptons and quarks are fixed mainly by choosing the bare masses of the spinor multiplets, these masses being free parameters of the model. All vector bosons (except one photon) are provided with masses via the spontaneous symmetry breaking. As for the scalar particles, all of them, apart from the nonphysical Goldstone modes, will be also massive. Here we mean certainly that the appropriate mass term must be added in the Lagrangian (2) and the interaction potential of the scalar fields (4) must be completed with the following terms

$$D_{ijk} \left(\frac{\rho_M^2}{3!} M_i M_j M_k + \frac{\rho_{MN}^2}{2} M_i N_j N_k + (M \leftrightarrow N) \right) + h.c. \\ + (\phi^0 \Gamma^0)_i \left(\rho_{\phi M}^2 M^{+j} M_j + \rho_{\phi MN}^2 M^{+j} N_j + (M \leftrightarrow N) \right) \quad (16)$$

These terms have not any influence on the asymptotical freedom of the model [8] and do not affect the masslessness of vector particles. However they give some contribution to eqs. (13) thus adding some free parameters to the mass matrices of the vector and spinor fields what is usefull for the physical interpretation of this model. In fact the low energy physics in our model is the same as in ref. [1]. The essential differences appear in the region of high and superhigh energies only where in our model (contrary to the model built in refs [1] and [2]) all the interactions vanish asymptotically thus providing the logical and mathematical consistency of the theory.

To summarise, the present model demonstrates the possibility to construct (see also Ref. [4]), within the exceptional group, the asymptotically free models of unified interaction, i.e. the models which possess the due mathematical rigour (absence of the "zero-charge" difficulty [5]) and realistic content. The fact of using the exceptional group provides here the hope to achieve, due to the special octanionic multiplication rules, the unitarity of the theory in the singlet sector of the colour group already at the kinematical level.

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