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MODIFIED FEYNMAN RULES IN SUPERGRAVITY

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MODIFIED FEYNMAN RULEY IN SUPERGRAVITY

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The new modified Feynman rules are obtained in supergravity theory. They include an additional quartic ghost coupling. New modified Ward-Slavnov-Taylor identities are presented and a theory constructed in this way is shown to be gauge-invariant on the mass shell in spite of the absence of a closed algebra.

I. INTRODUCTION

We consider the theory of supergravity suggested by Freedman, Nieuwenhuizen and Ferrara and also by Deser and Zumino [1,2], which describes gravitons, spin $3/2$ particles and photons connected by local supersymmetry transformations. This theory has the renormalizability properties which are much better than in the usual gravity theory [3]. There is some hope that a gauge supersymmetry will make it possible to formulate a renormalizable theory of gravity.

In constructing a quantum theory of supergravity there arises, however, a number of unsolved problems. One of them is that local supersymmetry transformations do not correspond to any group with a closed algebra [2] as was the case in all up-to-now studied gauge theories - in the Yang-Mills theory and in the usual theory of gravity.

The first attempt to formulate Feynman rules in super-

gravity has been made in Ref. [4]. It has been shown that in supergravity there appears the 14-component ghost field which consists of a commuting spinor (corresponding to the spinor supertransformations), an anticommuting vector (for vector general coordinate transformations) and an anticommuting tensor (for local tensor Lorentz transformations). It was assumed in [4] that in the derivation of the supergravity Feynman rules one can apply the usual Faddeev-Popov and De Witt [5] method. However it is crucial for this method of constructing an effective quantum action that the classical action invariance transformations form a group with a closed algebra. In supergravity the corresponding graded algebra is closed only on the classical equations of motion [2]. One could think that the absence of a closed algebra off shell would be inessential for all on-shell processes, i.e. for the S-matrix. However the quantum gauge field is an arbitrary quantity, being the integration variable in the functional integral, i.e. the quantum field does not satisfy the classical equations of motion. Therefore the absence of a closed algebra is actually important and one may expect that the Feynman rules being obtained by the usual methods [4,5], the gauge invariance of the supergravity theory will be lost.

The simplest way to investigate the gauge-invariance of a quantum theory is connected with the transformations of Becchi-Rouet-Stora-Tyutin [6]. They have noticed that although the gauge invariance of the classical action is broken when the gauge condition and the ghost-particle action are added [5],

the resulting effective quantum action has a new nonlinear global invariance with an anticommuting parameter of transformation. From the effective action invariance under BRST transformations it is easy to derive Ward-Slavnov-Taylor identities [7] and to prove that the theory is gauge-invariant on mass shell. In supergravity it was shown by Townsend and Nieuwenhuizen [8] that an effective quantum action [4,5] is not invariant under the usual BRST transformations. From this fact one may conclude that the Feynman rules in supergravity actually should be modified.

The necessary modification of the Feynman rules was obtained independently by canonical quantization in the Hamiltonian approach by Fradkin and Vasiliev [9] and from the gauge invariance requirement in the Lagrangian approach by the present author [10].

The aim of this paper is to present a detailed derivation of the Feynman rules in supergravity [9,10] by the method used in Ref. [10].

The paper is organized as follows. In Sec.2 the main notations and an ordinary quantum action of gauge theories are presented. The necessity for a closed algebra for the invariance of a quantum action under BRST transformations is explicitly demonstrated. The term which gives a deviation from the gauge invariance in supergravity is calculated. In Sec.3 new transformations and a new effective action for theories with an algebra closed only on the equations of motion are presented. The quantum action modification suggested above is shown to provide

invariance under modified BRST transformations when some concrete properties of supergravity [1,2] are used. In Sec.4 modified Ward-Slavnov-Taylor identities are derived, and the theory is proved to be gauge independent on mass shell.

2. ORDINARY BRST TRANSFORMATIONS IN SUPERGRAVITY

In this section we consider first an arbitrary gauge theory in which a classical action $S_{cl}[\Phi]$ is invariant under some local transformations of the gauge field Φ^i (including supertransformations)

$$\Phi^i \rightarrow \Phi^i + R^i_{\alpha}[\Phi] \xi^{\alpha}(x) \quad (2.1)$$

where $\xi^{\alpha}(x)$ is an infinitesimal transformation parameter, i is a field index and α is a group index. The invariance condition has the form (since $\xi^{\alpha}(x)$ is arbitrary)

$$S_{,i} R^i_{\alpha} = 0 \quad (2.2)$$

where

$$S_{,i} \equiv \frac{\delta S}{\delta \Phi^i} \quad (2.3)$$

i.e. the index after the comma implies differentiating with respect to the functional argument. Besides, summation over the index in these condensed notations used first by De Witt [5] includes also integration over the continuous spatial variable x .

The field Φ^i may contain both commuting boson and anticommuting fermion variables.

The ordinary expression for the generating functional of quantum theory is given by [5]

$$Z = \int d\Phi d\bar{c} dc \exp i \{ S_{\text{eff}} + \int_i \Phi^i + \bar{K}_\rho c^\rho + \bar{c}^r K_r \} \quad (2.4)$$

where S_{eff} consists of three parts

$$S_{\text{eff}} = S_{\text{cl}} + S_{\text{gauge}} + S_{\text{ghost}} \quad (2.5)$$

Thus beside the classical action S_{cl} the effective quantum action includes the gauge-fixing term

$$S_{\text{gauge}} = \frac{1}{2} \Phi^i F_{ij} \tilde{r}^{ij} F_{ij} \Phi^i \quad (2.6)$$

In eq. (2.6) we restricted ourselves to the class of gauge conditions linear in the quantum field Φ^i . In eq. (2.6) F_{ij} and $\tilde{r}^{ij}$ are some constant (or dependent on the external field) matrices, they may contain also differential operators. At

$\tilde{r}^{ij} \rightarrow \infty$ we obtain the degenerate gauge $\delta(F_{ij} \Phi^i)$. Equation (2.5) for S_{eff} includes also the third term describing the action of ghost particles S_{ghost} . In the class of gauges (2.6) S_{ghost} has the form

$$\begin{aligned} S_{\text{ghost}} &= \bar{c}^r F_{ij} R^i_\rho c^\rho \equiv \\ &\equiv \bar{c}^r F_{ij} \Omega^i \Phi^i \end{aligned} \quad (2.7)$$

where $\Omega \Phi^i$ is defined in eq. (2.8).

The field $\bar{c}^r$, c^β contains both anticommuting variables connected with general covariance and Lorentz transformations and commuting ones connected with supertransformations, (commuting antighosts correspond to a complex Dirac conjugation). Besides the effective action eq. (2.4) includes also the source J_i of the gauge field Φ^i , the source $\bar{K}_\beta$ of the ghost particles c^β and the source K_γ of the antighosts $\bar{c}^r$.

Let us consider the ordinary BRST transformations of gauge and ghost fields that leave the effective action invariant.

$$\Phi^i \rightarrow \Phi^i + \delta^i{}_\alpha c^\alpha \Lambda \equiv \Phi^i + \Omega \Phi^i \Lambda \quad (2.8)$$

$$c^\beta \rightarrow c^\beta + (-1)^{\alpha(c^r)} \frac{1}{2} f_{\alpha\gamma}^\beta c^\alpha c^\gamma \Lambda \quad (2.9)$$

$$\bar{c}^r \rightarrow \bar{c}^r + \Phi^i F_{ij} \tilde{f}^{ir} \Lambda (-1)^{\alpha(\bar{c}^r)} \quad (2.10)$$

where Λ is an infinitesimal constant anticommuting parameter. In (2.9) $\alpha(c^r) = 1$ if c^r is a commuting field and $\alpha(c^r) = 0$ for anticommuting c^r . Let us substitute (2.8-2.10) into (2.6), (2.7), taking into account that S_{cl} is invariant under these transformations. We obtain

$$\delta S_{gauge} = \Phi^i F_{ij} \tilde{f}^{ir} F_{ir} \Omega \Phi^i \Lambda \quad (2.11)$$

The variation of the action of ghost particles consists of three terms. The first one is connected with the variation of the antighost field (2.10)

$$\delta S_{gh}^1 = (\epsilon 1)^{\alpha(\bar{c}^r)} \Phi^j F_{j\alpha} \tilde{\gamma}^{\alpha r} \Lambda F_{ir} \Omega \Phi^i \quad (2.12)$$

The second term is connected with the variation of the gauge field (2.8)

$$\delta S_{gh}^2 = \bar{c}^r F_{ir} R_{\beta,j}^i R_{\alpha}^j c^{\alpha} \Lambda c^{\beta} \quad (2.13)$$

The third term is connected with the variation of the ghost field (2.9)

$$\delta S_{gh}^3 = \frac{1}{2} (\epsilon 1)^{\alpha(c^{\beta})} \bar{c}^r F_{ir} R_{\alpha}^i f_{\beta\alpha}^{\alpha} c^{\alpha} c^{\beta} \Lambda \quad (2.14)$$

Suppose that in the theory under consideration the invariance transformations form a group, i.e. the generalized Lee bracket has the form

$$[R_{\beta,j}^i, R_{\alpha}^j] \equiv R_{\beta,j}^i R_{\alpha}^j - (\epsilon 1)^{\alpha\beta} R_{\alpha,j}^i R_{\beta}^j = R_{\alpha}^i f_{\beta\alpha}^{\alpha} \quad (2.15)$$

where $a_{\alpha\beta} = 1$, when α, β correspond to supertransformations, in the other cases $a_{\alpha\beta} = 0$, and $f_{\alpha\beta}^{\alpha}$ are the structure group constants. Equation (2.15) corresponds to a closed graded algebra since the two consequent transformations (2.1) give again the transformation of the same type. As is seen from (2.11) the variation of the gauge condition

(2.11) is compensated by the variation δS_{gh}^4 of the antighost (2.12) and the variation δS_{gh}^2 due to the change of the gauge field (2.13) is compensated by the variation of the ghost δS_{gh}^3 (2.14), provided that the gauge transformations of the theory form a group with the closed algebra (2.15). The Jacobian of the BRST transformations (2.8)-(2.10) is equal to

$$1 - (-1)^{a(\varphi^i)} R_{\alpha,i}^i c^\alpha \Lambda - (-1)^{a(c^r)} f_{\alpha\gamma}^r c^\alpha c^\gamma \Lambda \quad (2.17)$$

In the supergravity [1,2] it is equal to

$$1 - 4\kappa^2 \int \bar{\hat{c}} \gamma^\mu \psi_\mu \wedge \delta^4(0), \quad (2.18)$$

where ψ_μ is spin 3/2 field and $\hat{c}$ is a Majorana conjugate ghost field. This term can be compensated by adding to the generating functional a local measure g^{-2} , since $\delta g_{\mu\nu} = \kappa (\bar{\hat{c}} \gamma_\mu \psi_\nu + \bar{\hat{c}} \gamma_\nu \psi_\mu) \Lambda$. Let us note, however, that in the dimensional regularization the local measure terms are trivial since

$\delta^4(0) = 0, \delta^4(0) = 0 \dots$ in this regularization. Therefore for practical calculations of the Feynman diagrams they are not essential and further on we shall omit the local measure terms, as in Ref. [10].

Note also that the Jacobian (2.17) in the dimensional regularization is equal to unity not only for supergravity, but for arbitrary theory due to the locality of the functions

$$R_{\alpha,i}^i, f_{\alpha\gamma}^r. \text{ Indeed in the dimensional regularization} \quad R_{\alpha,i}^i = f_{\alpha\gamma}^r = 0 \quad (2.19)$$

since $R^i_{\alpha,i}$, $f^r_{\alpha r}$ contain also the trace in the spatial variable x , i.e. they are proportional to $\delta^r(0)$, $\delta^{rr}(0)$ etc.

Thus in the generating functional (2.4) only the terms with sources undergo variation under the change of the integration variables (2.8)-(2.10). This property is sufficient to derive Ward-Slavnov-Taylor identities and to show that on mass shell Z does not depend on the choice of the gauge functions F_{ij} and $\tilde{f}^{rr}$. This will be demonstrated for the case of supergravity [1,2] where a modification of the above theory will be necessary.

In supergravity instead of (2.15) one has

$$\{R^i_{\beta,j}, R^i_{\alpha}\} = R^i_{\gamma} f^r_{\beta\alpha} + \eta^{ij}_{\beta\alpha} S_{,j} \quad (2.20)$$

where $\eta^{ij}_{\beta\alpha}$ is some function of field variables ^{x)}. Besides, as distinct from the gauge theories studied before the structure "constants" $f^r_{\beta\alpha}$ in supergravity are functions of the fields. As is seen from eq. (2.20) now δS^2_{gh} in eq. (2.13) is not compensated completely by δS^3_{gh} eq. (2.14) and we obtain

$$\delta S_{\text{eff}} = \frac{1}{2} \bar{c}^r F_{ij} \eta^{ij}_{rs} S_{,j} c^r c^s \Lambda \quad (2.21)$$

x) The function $\eta^{ij}_{\beta\alpha} S_{,j}$ cannot be represented in the form of some gauge transformation $R^i_{\gamma} \tilde{f}^{rr}_{\beta\alpha}$ where $\tilde{f}^{rr}_{\beta\alpha}$ is an arbitrary function.

In (3.21) we have taken into account that $\eta_{\alpha\beta}^{ij} \neq 0$ only if C^α, C^β correspond to commuting ghosts connected with supertransformations.

Equation (2.21) can be represented in the form

$$\delta S_{eff} = - S_{,j} \bar{\Omega} \Phi^j \Lambda \quad (2.22)$$

where

$$\bar{\Omega} \Phi^j \equiv -\frac{1}{2} \bar{c}^r F_{ij} \eta_{\alpha\beta}^{ij} C^\alpha C^\beta \quad (2.23)$$

Equation (2.22) suggests that the gauge-field transformation (2.8) can be modified by supplementing it with the transformation $\bar{\Omega} \Phi^j \Lambda$ which is not already a classical action invariance transformation. In this case the variation of the first term in (2.5) compensates (2.22). One should however take into account that the variation of the gauge fixing term and also of the ghost term will change and therefore one should verify the invariance of the quantum action under the modified gauge field transformations all over again.

3. MODIFICATION OF BRST TRANSFORMATIONS IN SUPERGRAVITY

Let us consider now a new gauge-field transformation

$$\tilde{\delta} \Phi^i \equiv (\Omega + \bar{\Omega}) \Phi^i \Lambda, \quad (3.1)$$

see (2.8), (2.23).

For the further investigation it will be convenient also to introduce the quantity

$$\tilde{\Omega} \Phi^i \equiv (\Omega + \frac{1}{2} \bar{\Omega}) \Phi^i \quad (3.2)$$

The variation of the gauge fixing term under the transformation (3.1) takes the form

$$\delta S_{\text{gauge}} = \Phi^i F_{j\alpha} \tilde{f}^{\alpha r} F_{ir} \tilde{\delta} \Phi^i \quad (3.2)$$

For the variation (3.3) of the gauge fixing term to be cancelled as usual by the antighost variation, the new ghost action

$$\tilde{S}_{gh} \quad \text{should be constructed as follows}$$

$$\tilde{S}_{gh} = \bar{c}^r F_{ir} \tilde{\Omega} \Phi^i \equiv \bar{c}^r F_{ir} R_{\rho}^i c^{\rho} - \quad (3.4)$$

$$- \frac{1}{4} \bar{c}^r c^{\alpha} F_{ir} \eta^{\alpha\beta} F_{js} c^{\beta} \bar{c}^s$$

Then the variation of $\tilde{S}_{gh}$ (3.4) under the antighost transformation (2.10) is given by

$$\delta \tilde{S}_{gh} = - \Phi^i F_{j\alpha} \tilde{f}^{\alpha r} F_{ir} \tilde{\delta} \Phi^i \quad (3.5)$$

and therefore (3.5) cancels with (3.3). Note that besides the usual first term the new ghost action (3.4) contains a quartic ghost coupling ^{x)} whose vertex is determined by the

^{x)} The fact that in some theories canonical quantization may lead to the appearance of a quartic ghost coupling has been pointed out in Ref. [11]. However in all theories except supergravity such terms resulted in a modification of the local measure and vanished when dimensional regularization was used.

structure function $\eta_{\mu\nu}^{ij}$, which gives the deviation from the closed algebra in (2.20), and also by the form of the gauge-fixing condition $F_{i\gamma}$. That $\tilde{\Omega}\Phi^i$ and not $(\Omega + \bar{\Omega})\Phi^i$ enters in $\tilde{S}_{gh}$ is due to the fact that the second term in eq. (3.4) is quadratic both in C and $\bar{C}$ as distinct from the first term which depends on C and $\bar{C}$ only linearly. Therefore the variation of the second term ^{in (3.4)} under the variation of C or $\bar{C}$ contains an additional factor 2 as compared with the variation of the first term.

Now we shall consider the variation of the new ghost-particle action under the variation of the gauge field (3.1)

$$\delta \tilde{S}_{gh}^2 = \bar{C}^r F_{i\gamma} \delta \tilde{\Omega} \Phi^i = \bar{C}^r F_{i\gamma} \{ (R_{\mu,j}^i C^\mu + \quad (3.6)$$

$$+ \frac{1}{2} (\bar{\Omega} \Phi^i)_{,j} (R_{\rho}^j C^\rho + \bar{\Omega} \Phi^j) \} \Lambda$$

Equation (3.6) contains 4 terms. Let us consider them in detail.

Due to the commutativity (anticommutativity) property of C^μ , $\bar{C}^r$ the first term in (3.6) contains the known generalized Lee bracket (2.20) of the two gauge transformations

$$\bar{C}^r F_{i\gamma} R_{\mu,j}^i R_{\rho}^j C^\mu C^\rho \Lambda = \frac{1}{2} \bar{C}^r F_{i\gamma} (R_{\gamma}^i f_{\mu\rho}^i + \quad (3.7)$$

$$+ \eta_{\mu\rho}^{ij} S_{,j}) C^\mu C^\rho \Lambda$$

The second term in (3.6) contains two subsequent transformations, the new one $\bar{\Omega}\Phi$ and the gauge transformation R_{μ}^i

$$\frac{1}{2} \bar{C}^r F_{i\gamma} (\bar{\Omega} \Phi^i)_{,j} R_{\rho}^j C^\rho \Lambda = -\frac{1}{4} \bar{C}^r F_{i\gamma} \cdot \quad (3.8)$$

$$\cdot \bar{c}^i F_{\kappa s} \gamma_{\alpha\beta, j}^{\kappa i} R_{\beta}^j c^{\alpha} c^{\beta} c^{\gamma} \mathcal{L}$$

In the third term the new transformation $\bar{\Omega} P^i$ follows the gauge one R_{α}^i

$$-\frac{1}{2} \bar{c}^{\alpha} F_{ij} R_{\alpha, j}^i \gamma_{\beta\gamma}^{\kappa j} \bar{c}^{\beta} F_{\kappa s} c^{\alpha} c^{\beta} c^{\gamma} \mathcal{L} \quad (3.9)$$

And finally the fourth term contains two subsequent new transformations $\bar{\Omega} P$

$$\frac{1}{2} \bar{c}^{\alpha} F_{ij} (\bar{\Omega} P^j)_{\beta} \bar{\Omega} P^i \mathcal{L} = \frac{1}{8} \bar{c}^{\alpha} F_{ij} \bar{c}^{\beta} F_{\kappa s} \quad (3.10)$$

$$\cdot \gamma_{\alpha\beta, j}^{\kappa i} \gamma_{\beta\gamma}^{\mu j} \bar{c}^{\alpha} F_{\mu\lambda} c^{\beta} c^{\gamma} c^{\lambda} \mathcal{L}$$

Till now we have not used any concrete properties of the supergravity model [1,2], but we shall need them for the analyses of equations (3.8)-(3.10). An explicit expression for the function (2.20) for supergravity is presented in Ref. [2].

The multiplet of gauge fields $\mathcal{P}^i$ in supergravity contains a vierbein field e_{μ}^a connected with the graviton, a field ψ_{μ}^{α} of spin 3/2 and also may contain an electromagnetic field A_{μ}^x .

A characteristic property of the function $\gamma_{\alpha\beta}^{ij}$ is that it does not vanish only when the anticommutator of two super-

x) We use here a second-order formalism in which the connection $\omega_{\mu\alpha\beta}$ is expressed through the other fields by means of the equation of motion.

transformations of the spinor field ψ_μ^α is considered. For the remaining fields e_μ^a and A_μ the algebra is closed, i.e. the corresponding components of $\eta_{\alpha\beta}^{ij}$ are equal to zero. The function $\eta_{\alpha\beta}^{ij}$ for the spinor field depends on the field e_μ^a only, therefore the quantity that enters in (3.10) is zero

$$\eta_{\alpha\beta, j}^{\kappa i} \eta_{\beta\gamma}^{mj} = 0 \quad (3.11)$$

In eq. (3.9) instead of

$$-\frac{1}{2} R_{\alpha, j}^i \eta_{\beta\gamma}^{\kappa j} \quad (3.12)$$

one can write,

$$-\frac{1}{4} R_{\alpha, j}^i \eta_{\beta\gamma}^{\kappa j} - \frac{1}{4} R_{\alpha, j}^{\kappa} \eta_{\beta\gamma}^{ij} \quad (3.13)$$

due to the symmetry $i \leftrightarrow \kappa$ of the factors $\bar{c}^i F_{ij}$ and $\bar{c}^\kappa F_{\kappa j}$. Then if we take into account eq. (3.11), the expression, which we should analyse to determine $\delta \tilde{S}_{gh}^2$ in (3.6), is equal to

$$-\frac{1}{4} (\eta_{\alpha\beta, j}^{\kappa i} R_\rho^j + R_{\alpha, j}^i \eta_{\beta\gamma}^{\kappa j} + R_{\alpha, j}^{\kappa} \eta_{\beta\gamma}^{ij}) \quad (3.14)$$

To analyse this expression we use the Jacoby identity for a cyclic permutation of three gauge transformations $R_\mu^i C^\mu$,

which in our case can be presented in the form

$$\{(R^i_{\alpha,j}, R^j_{\beta})_{,\kappa} R^{\kappa}_{\gamma} + R^i_{\gamma,j} (R^i_{\alpha,\kappa} R^{\kappa}_{\beta})\}, \quad (3.15)$$

$$*C^{\alpha} C^{\beta} C^{\gamma} = 0$$

due to the commutativity (anticommutativity) properties of C^{α} .

After the substitution of (2.20) into (3.15) one gets

$$\{A^i_{\gamma\beta\gamma} + B^ij_{\gamma\beta\gamma} S_{,j}\} C^{\alpha} C^{\beta} C^{\gamma} = 0 \quad (3.16)$$

where

$$A^i_{\gamma\beta\gamma} = R^i_{\lambda} f^{\lambda}_{\gamma\beta} f^{\gamma}_{\gamma} + R^i_{\gamma} f^{\gamma}_{\gamma\beta,\kappa} R^{\kappa}_{\gamma} \quad (3.17)$$

$$B^ij_{\gamma\beta\gamma} = -\eta^{ij}_{\gamma\beta} f^{\gamma}_{\gamma} + \eta^{ij}_{\gamma\beta,\kappa} R^{\kappa}_{\gamma} + \eta^{i\kappa}_{\gamma\beta} R^j_{\gamma,\kappa} + R^i_{\gamma,\kappa} \eta^{\kappa j}_{\gamma\beta} \quad (3.18)$$

The most general solution of equation (3.16) for the function $B^ij_{\gamma\beta\gamma}$ has the form

$$C^{\alpha} C^{\beta} C^{\gamma} \{B^ij_{\gamma\beta\gamma} - S_{,\kappa} \tilde{\eta}^{ij\kappa}_{\gamma\beta\gamma} + R^i_{\lambda} K^{j\lambda}_{\gamma\beta\gamma} + (i \leftrightarrow j)\} = 0 \quad (3.19)$$

A direct analysis of the expression $B^ij_{\gamma\beta\gamma}$ (3.18) in supergravity shows that

$$\tilde{\eta}^{ij\kappa}_{\gamma\beta\gamma} = 0 \quad (3.20)$$

and that

$$K_{\alpha\beta}^{j\lambda} = 0 \quad (3.21)$$

for all λ except those corresponding to the Lorentz transformations.

Now we substitute (3.19), (3.20), (3.21) into (3.14) and (3.6) and obtain

$$\begin{aligned} \delta \tilde{S}_{gh}^2 = & -S_{,i} \bar{\Omega} \Phi^i \Lambda + \frac{1}{2} \bar{c}^r F_{ij} (a_{,i}^j - \frac{1}{2} \eta_{ij}^r) \\ & + F_{ij} \bar{c}^i c^j f_{\alpha\beta}^{'j} c^\alpha c^\beta \mathcal{L}(1)^{a(c^r)} \end{aligned} \quad (3.22)$$

where for the Lorentz λ component of $f_{\alpha\beta}^{'j}$ we have

$$f_{\alpha\beta}^{'j} = f_{\alpha\beta}^j + F_{j\alpha} K_{\beta\gamma}^{i\lambda} c^i c^\beta \quad (3.23)$$

and for other components $f_{\alpha\beta}^{'j} = f_{\alpha\beta}^j$ according to (3.21).

Let us slightly modify the BRST transformations (2.9) of the ghost field c^β (only of its Lorentz component)

$$c^\beta \rightarrow c^\beta + (-1)^{a(c^r)} \frac{1}{2} f_{\alpha\gamma}^{'\beta} c^\alpha c^\gamma \Lambda \quad (3.24)$$

Now eq. (3.22) shows that the ghost action variation $\delta \tilde{S}_{gh}^2$ due to the variation of the gauge field (3.1) can be exactly cancelled by two terms. The first term in the r.h.s. of (3.22) is cancelled by the classical action variation

$$\delta S_{cl} = S_{,i} \bar{\Omega} \Phi^i \Lambda \quad (3.25)$$

The second term in the r.h.s. of (3.22) is compensated by the ghost action variation $\delta \tilde{S}_{gh}$ connected with the modified ghost field variation (3.24)

$$\delta \tilde{S}_{gh}^3 = -\frac{1}{2} \bar{c}^r F_{ir} (R^i_{\lambda} - \frac{1}{2} \eta^{ij}_{\lambda\tau} F_{j\tau} \bar{c}^s c^s).$$

$$+ f'^{\lambda}_{\mu} c^r c^s \Lambda(-1)^{a(c^r)}$$

Thus a modified effective action of supergravity (2.5), (2.6), (3.4) is shown to be invariant under the modified BRST transformations (3.1), (3.24), (2.10). The Jacobian of these modified transformations coincides in supergravity with the old one (2.17), (2.18) since a condition

$$\eta^{ij}_{\mu,i} = 0$$

holds in supergravity.

4. MODIFIED WARD-SLAVNOV-TAYLOR IDENTITIES IN SUPERGRAVITY

Consider the generating functional of the theory (2.4) where $\tilde{S}_{gh}$ (3.4) is used instead of S_{gh} , i.e. the theory is defined by the modified effective action

$$\tilde{S}_{eff} = S_{cl} + S_{gauge} + \tilde{S}_{ghost} \quad (4.1)$$

Let us make a change in the integration variables (3.1), (3.24), (2.10). Since $\tilde{S}_{eff}$ is invariant under these transformations and the transformation Jacobian is equal to unity, in the dimensional regularizations, it is only the terms with sources that undergo variation

$$\delta Z = \int d\Phi d\bar{c} dc \exp i \{ \tilde{S}_{eff} + \Phi^i J_i + \\ + c^\rho \bar{K}_\rho + \bar{c}^r K_r \} + i [\delta \Phi^i J_i + \delta c^\rho \bar{K}_\rho +$$

$$+ \delta \bar{c}^r K_r] \equiv$$

$$\equiv i \langle \bar{\delta} \Phi^i \mathcal{J}_i + \delta c^a \bar{K}_a + \delta \bar{c}^r K_r \rangle = 0_{(4.2)}$$

After the differentiation of (4.2) with respect to Λ one gets the modified Ward-Slavnov-Taylor identities in the following form

$$\begin{aligned} & \langle (R^i_\alpha c^\alpha - \frac{1}{2} \bar{c}^r F_{ij} \gamma^{ij}_{\alpha\beta} c^\alpha c^\beta) \mathcal{J}_i (-1)^{a(\Phi^i)} \\ & + \frac{1}{2} f_{\alpha\gamma}^\beta c^\alpha c^\gamma \bar{K}_\beta + \Phi^i F_{ij} \tilde{\gamma}^{jr} K_r \rangle = 0 \end{aligned} \quad (4.3)$$

To obtain the modified Ward-Slavnov-Taylor identities in a more usual form one should differentiate (4.3) with respect to K_r and at $\bar{K}_a = K_r = 0$ this gives

$$\langle \Phi^i F_{ij} \tilde{\gamma}^{jr} + i \bar{c}^r (R^i_\alpha c^\alpha - \frac{1}{2} \bar{c}^s \dots) \rangle \quad (4.4)$$

$$\langle F_{\kappa\delta} \gamma^{\kappa i}_{\alpha\beta} c^\alpha c^\beta \mathcal{J}_i \rangle_{K=\bar{K}=0} = 0$$

Relation (4.4) represents a complete set of Ward-Slavnov-Taylor identities [7]. Their difference from the case of usual gauge theories, where transformations have a group pro-

perty, is in the existence of the third term in the left-hand side of (4.4) which contains four ghost-particle operators.

The simplest of the identities for a gauge-field propagator is obtained from (4.4) by differentiating it with respect to

$\bar{J}^i$ (one should bear in mind that the notation $\langle \dots \rangle_{\kappa=\bar{\kappa}=0}$ means averaging with the weight containing $\exp i\{\bar{S}_{eff} + \bar{P}^i \bar{J}_i\}$)

$$\langle \bar{P}^j F_{j\alpha} \tilde{\gamma}^{\alpha r} \bar{P}^i + \bar{c}^r R_{\alpha}^i c^{\alpha} - \quad (4.5)$$

$$- \frac{1}{2} \bar{c}^{\beta} c^{\beta} F_{\kappa\beta} \gamma_{\beta}^{\kappa i} c^{\alpha} \bar{c}^r + i(\Omega + \bar{\Omega}) \bar{P}^j \bar{J}_j; \bar{c}^r \bar{P}^i \rangle_{\kappa=\bar{\kappa}=0} = 0$$

i.e. now the "longitudinal" part of the gauge-field propagator is expressed not only via the ghost-particle propagator but also via a four-particle Green function of ghosts.

Note that eq. (4.5) can be represented in the form

$$\langle \frac{\partial \bar{Z}}{\partial F_{i\gamma}} - (\Omega + \bar{\Omega}) \bar{P}^j \bar{J}_j; \bar{c}^r \bar{P}^i \rangle_{\kappa=\bar{\kappa}=0} = 0 \quad (4.6)$$

where $\partial \bar{Z} / \partial F_{i\gamma}$ is the derivative of the generating functional

$\bar{Z}$ with respect to the explicitly entering gauge condition

$F_{i\gamma}$. It can be verified as usual in gauge theories that

the second term in (4.6) gives no contribution on mass shell.

Therefore it follows from eq. (4.6) that $\bar{Z}$ does not depend on $F_{i\gamma}$ on mass shell.

From eq. (4.4) another ward identity can be obtained

$$\langle \Phi^i F_{j\alpha} F_{i\beta} \Phi^i + i(\Omega + \bar{\Omega}) \Phi^i J_j \bar{c}^r \tilde{\gamma}_{r\alpha}^{-1} F_{i\beta} \Phi^i + i \tilde{\gamma}_{\alpha\beta}^{-1} \rangle = 0 \quad (4.7)$$

When deriving (4.7) we used the identity

$$\langle \bar{c}^r F_{i\alpha} (Q_{\alpha}^i - \frac{1}{2} \bar{c}^s c^{\beta} F_{\beta\alpha} \eta_{\alpha\beta}^{\kappa i}) c^{\alpha} + i \delta_{\alpha}^r \rangle = 0 \quad (4.8)$$

Note that in the case when $\tilde{\gamma}^{\alpha\beta}$ contains differential operators and depends on external fields the known modification of the generating functional [5] should be carried out even in dimensional regularization: the term

$$\frac{1}{2} \text{Tr} \ln \tilde{\gamma}^{\alpha\beta} \quad (4.9)$$

should be added to the effective quantum action. In this case eq. (4.7) can be represented as

$$\langle \frac{\partial \tilde{Z}}{\partial \tilde{\gamma}_{\alpha\beta}^{-1}} (\Omega + \bar{\Omega}) \Phi^i J_j \bar{c}^r \tilde{\gamma}_{r\alpha}^{-1} F_{i\beta} \Phi^i \rangle = 0 \quad (4.10)$$

and it can be verified that the second term in (4.10) gives no contribution on shell. Therefore eq. (4.10) shows that the generating functional $\tilde{Z}$ with the effective quantum action $\tilde{S}_{\text{eff}}$ (4.1) with the additional term (4.9) is $\tilde{\gamma}^{\alpha\beta}$ -independent on shell, which provides the remaining step in our proof of the gauge independence of S-matrix in supergravity.

CONCLUSION

In this paper we have presented a more detailed derivation of the modified Feynman rules of supergravity recently obtained by Fradkin and Vasiliev [9] and by the present author [10], and of the modified BRST transformations for this theory [10]. It was shown, that in the effective quantum action of supergravity besides the usual quadratic ghost coupling, which appears in the gauge theories in the quantization by the usual method of Faddeev-Popov and of De Witt [5], an additional quartic ghost term $\sim \bar{C}^2 C^2$ (3.4) also appears. Thus supergravity provides the first example of a gauge theory in which the usual Faddeev-Popov determinant is insufficient and a new considerable modification of the Feynman rules is needed due to the off shell non-closure of the algebra of the gauge transformations. As has been noted above (see also [9]) in supergravity a local measure g^{-2} also appears, but for practical calculations by means of dimensional regularization corresponding corrections to the Feynman rules vanish.

It is worth noting that in general the absence of the closed algebra in gauge theories can lead to the infinitely increasing number of the non-linear couplings of ghosts, to the terms $\bar{C}^3 C^3$, $\bar{C}^4 C^4$ and so on. It is a gratifying and surprising property of supergravity that this procedure proves to be closed and no additional terms except the quartic one actually appear. This property is provided by a special form of the structure functions $2_{\mu\nu}^i$, $f_{\mu\nu}^2$ and transformation ge-

nerators R'_α , which results in eqs. (3.11), (3.20), (3.21).

The modified effective action (3.4) is invariant under some new modified BRST transformations (3.1), (3.24), (2.10).

These new BRST transformations have the following properties. The gauge field Φ' transformation (3.1) is modified essentially. It is no more a classical action invariance transformation. The antighost field $\bar{C}$ transformation (2.10) is identical to the usual one. The absence of the deviation from the old rule for the antighost field is connected directly with the necessity to obtain the Ward identities (4.5) which provide the gauge invariance of the theory. The ghost field C variation (3.24) differs from the old one (2.9) very slightly. Only the Lorentz λ -component of the structure "constant" $f_{\alpha\beta}^\lambda$ has some additional term $f_{\alpha\beta}^\lambda \rightarrow f_{\alpha\beta}'^\lambda$ see (3.23).

The interesting conclusion of the above investigation is the following. In the previously studied gauge theories (Yang-Mills theory, quantum gravity) the BRST transformations were only a convenient method of investigation. It seems to us that in the gauge theories with a non-closed algebra the BRST method of investigation is the only possible one in the Lagrangian approach.

The analysis of the modified Ward-Slavnov-Taylor identities (4.4) and of the restrictions that they impose upon the divergences seems to be a nontrivial problem which deserves further investigation.

Note added. An interesting paper by Sterman, Townsend and Nieuwenhuizen "Unitarity, Ward Identities, and new Quantization Rules of Supergravity" ITP-SB-77-65 has appeared recently. They have obtained the new Feynman rules up to $\mathcal{K}^V$ order by means of BRST transformations and also proved the unitarity of the theory in the Lagrangian approach.

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