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SPONTANEOUS SYMMETRY BREAKING AND GAUGE INTERACTION
HIERARCHY.

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Following the calculations of the effective potential in the one-loop approximation it is shown that within gauge theories a spontaneous symmetry breaking may provide a gauge hierarchy of any desired magnitude.

One of the main points in constructing realistic models of elementary particle physics is the realization within the theory of the needed spontaneous symmetry breaking [1]. The latter is to provide appropriate masses for gauge and spinor fields keeping at the same time the interaction hierarchy established phenomenologically. For such theories as a rule it is needed *after all* to provide the existence, *besides* the gauge fields of comparatively small masses of a number of gauge fields with arbitrarily heavy masses *as well* in this case initiated by them undesirable contributions in real processes may be not taken into account effectively and thus we have a possibility to make the theory physically meaningful.

The statement [2] has recently appeared however that such a hierarchy of gauge field masses cannot be attained, at least in perturbation theory. It is asserted namely in ref. [2] that if in a theory all the coupling constants are of the order of the electromagnetic one, the ratio of the vector meson masses cannot exceed $1/\sqrt{2}$ by the order of magnitude. If it were really so, the possibility of constructing models of unified interactions would be hampered very much.

The urgency of this problem requires more detailed investigations. However, to ^{or} answer this question in a general form is not easy because of computational difficulties. In this connection a simple model is built in this paper as a first step. Within this model the assertion of ref. [2] is shown to be incorrect. So, even if the statement made in ref. [2] proves to be

correct, it is in any case of limited applicability. Some explanations of this point which are irrespective of the model are presented at the end of the paper.

The calculations will be carried out in the framework of the $SU(2)$ gauge model with a Lagrangian of the following form:

$$\mathcal{L} = -\frac{1}{4}(\partial_\mu V_\nu^a - \partial_\nu V_\mu^a + g \cdot \epsilon^{abc} V_\mu^b V_\nu^c)^2 - \frac{1}{2}[(\partial_\mu \delta^{ab} - ig \cdot (\frac{\tau^c}{2})_a^b V_\mu^c) \Phi_a]^2 - \frac{1}{2}[(\partial_\mu \delta^{ab} + g \cdot \epsilon^{abc} V_\mu^c) A^b]^2 \quad (1) \\ + m_\phi^2 \phi^\dagger \phi + \frac{M}{2} A^2 - \frac{f}{8} (A^2)^2 - \frac{\lambda}{2} (\phi^\dagger \phi)^2 - \frac{h}{2} A^2 (\phi^\dagger \phi).$$

Here V_μ^a is a triplet of gauge fields with usual quantization rules; ϕ^a and A^a are scalar fields which are transformed by fundamental and adjoint $SU(2)$ group representations, respectively. Spinor fields are omitted from (1) since they do not affect the essence of the matter, although we know after all that it is due to them that the model can possess asymptotical freedom and has the needed physical content. The coupling constants which determine the interaction potential of scalar fields in (1) are not independent and we consider here the renormalized scalar particle masses only as free parameters of the model as distinct from [2].

The gauge interaction hierarchy is established here both in the tree and in the one-loop approximation. All the calculations are carried out in an explicit form. It is shown that the ratio of the gauge field masses within the one-loop approximation as well as within the tree approximation may be arbitrary and is determined by the initial model parameters only.

The calculations in the tree approximation are trivial.

Spontaneous symmetry breaking which is chosen as a solution of the equations for the effective potential extreme is determined here through nonzero vacuum expectation values of the scalar fields of the following form:

$$\langle \Phi^c \rangle|_{c=2} = \chi ; \quad \langle A_3 \rangle|_{3=3} = Z \quad (2)$$

where the quantities χ and Z are expressed through the scalar field masses by simple formulae

$$\chi^2 = \frac{-h M^2 + f m^2}{f\lambda - h^2} ; \quad Z^2 = \frac{2(-h m^2 + \lambda M^2)}{f\lambda - h^2} \quad (3)$$

The corresponding vector fields acquire the following masses

$$M_{V_1}^2 = M_{V_2}^2 = M_1^2 = g^2 \left(Z^2 + \frac{\chi^2}{2} \right) ; \quad M_{V_3}^2 = M_3^2 = \frac{g^2 \chi^2}{2} \quad (4)$$

and their ratio may be arbitrarily large if e.g. $\chi^2 \rightarrow 0$ at $Z^2 = \text{const.}$ For it according to (3) it is necessary that $m^2 \rightarrow (\frac{h}{f}) M^2$. Occasional zeroing of the denominator in (3) is not taken into consideration.

Thus, within the tree approximation the needed hierarchy of gauge fields is easily attained. Now we are in a position to show that this result for the model (1) is valid in the one-loop approximation too.

At this stage two problems should be solved. Firstly, the correction of the one-loop approximation to formulae (4), determining the vector particle masses, should be calculated; secondly, one should find the change of the very quantities χ and Z by solving the equations for the effective potential extreme calculated in the one-loop approximation. For

definiteness we consider that $\mu_1^2/\mu_H^2 \gg 1$, which in the tree approximation corresponds to $m^2 \rightarrow (1/4)M^2$.

The vector particle masses are determined by the corresponding singularities of the D-function of gauge fields. For this one must calculate here the polarization operator in the one-loop approximation where first of all it should be shown that the correction to the "light" particle mass $\delta\mu_H^2$ is not determined by the "heavy" masses. It is just in this point that the statement of ref. [2] is easily understood if the corresponding corrections to the "light" particle mass were not cancelled. Indeed, all undesirable contributions are cancelled exactly if one takes into account all the set of dangerous diagrams

$$\begin{aligned}
 \text{Diagram 1} &= \frac{9g^2}{32\pi^2} \mu_1^2 \ln \frac{\Lambda^2}{\mu_1^2}; & \text{Diagram 2} &= -\frac{3g^2}{8\pi^2} \mu_1^2 \ln \frac{\Lambda^2}{\mu_1^2}; \\
 \text{Diagram 3} &= -\frac{3g^2}{32\pi^2} \left(\frac{2}{3}\right) \left[\frac{\mu_1^2}{M_1^2 - \mu_1^2} \ln \frac{\Lambda^2}{\mu_1^2} - \frac{M_1^2}{M_1^2 - \mu_1^2} \ln \frac{\Lambda^2}{M_1^2} \right]
 \end{aligned}
 \tag{6}$$

corresponding to a propagation in a virtual state of a "heavy" gauge particle. Here we consider that $M_1^2 = 0$ (Goldstone particle mass) and μ_1^2 is determined by formula (4). The contribution of the remaining diagrams are not determined by the mass μ_1^2 and as a result $\delta\mu_H^2$ becomes vanishingly small as soon as $x^2 \rightarrow 0$.

$$\delta\mu_1^2 = -\frac{3g^2\mu_1^2}{32\pi^2} \ln \frac{\mu_1^2}{\mu^2} - \frac{3g^2}{64\pi^2} \frac{\mu_1^4(\mu_1^2 - M_{11}^2)}{(\theta_+^2 - \mu_1^2)(\theta_-^2 - \mu_1^2)} \ln \frac{\mu_1^2}{\mu^2} + \frac{3g^2}{64\pi^2} \frac{\mu_1^2}{\theta_+^2 - \theta_-^2} \left[\frac{\theta_+^2(M_{11}^2 - \theta_+^2)}{\theta_+^2 - \mu_1^2} \ln \frac{\theta_+^2}{\mu^2} - \frac{\theta_-^2(M_{11}^2 - \theta_-^2)}{\theta_-^2 - \mu_1^2} \ln \frac{\theta_-^2}{\mu^2} \right] \quad (7)$$

In (7) μ^2 is the normalization point; $\theta_{\pm}^2$ is the spectrum of elementary excitations of scalar particles

$$\theta_{\pm}^2 = \frac{(m_2^2 + M_{11}^2) + \lambda \alpha^2}{2} \pm \sqrt{\left[\frac{(m_2^2 - M_{11}^2) + \lambda \alpha^2}{2} \right]^2 + 2(h\alpha z)^2} \quad (8)$$

after the spontaneous symmetry breaking; the other designations are as follows

$$m_2^2 = \frac{1}{2} h z^2 + 2\lambda \alpha^2 - m^2; \quad M_{11}^2 = \alpha^2 h + \frac{3}{2} f z^2 - M^2. \quad (9)$$

It should be noted that this result could have been foreseen if one had followed the general considerations also. Indeed, if $\alpha^2 \neq 0$, it can easily be shown that the symmetry is being broken up to $U(1)$ group only leaving one gauge field massless.

So, it is already obvious here that the statement of ref.[2] within the given model proves to be incorrect. Moreover, it is just in this point that we see the possibility to extend the result of this paper to more realistic models (see e.g. lectures [3]).

Now we are in a position to calculate the change of the quantities α and z , as compared with the results of the tree approximation. For this purpose it is necessary to inves-

tigate the equations which determine the extreme of the effective potential found in the one-loop approximation. The needed correction to effective potential is calculated here according to the usual formula [4]

$$\Delta V = \int \frac{d^4 p}{(2\pi)^4} \ln \text{Det } \bar{D}_{ab}^{-1}(p; \phi) \quad (10)$$

with the $\bar{D}^{-1}$ function which must be found in the tree approximation for a theory with a broken symmetry. Renormalization is carried out with the aid of a standard Z-factor theory and is reduced here practically to a charge renormalization and to elimination of mass divergencies. The result of the calculation is as follows:

$$\begin{aligned} \Delta V = & \frac{3g^4}{32\pi^2} \left(x^2 + \frac{x^2}{2} \right)^2 \left[\ln \frac{g^2 \left(x^2 + \frac{x^2}{2} \right)}{\mu^2} - \frac{1}{2} \right] + \frac{3g^4}{256\pi^2} x^4 \left[\ln \frac{g^2 x^2}{2\mu^2} - \frac{1}{2} \right] \\ & + \frac{3[m^2 + (x^2 + \frac{1}{2}z^2)]^2}{64\pi^2} \left[\ln \frac{m^2 + (x^2 + \frac{1}{2}z^2)}{\mu^2} - \frac{1}{2} \right] + \frac{\theta^4}{64\pi^2} \left[\ln \frac{\theta^2}{\mu^2} - \frac{1}{2} \right] \quad (11) \\ & + \frac{[M^2 + (hx^2 + \frac{f}{2}z^2)]^2}{32\pi^2} \left[\ln \frac{M^2 + (hx^2 + \frac{f}{2}z^2)}{\mu^2} - \frac{1}{2} \right] + \frac{\theta^4}{64\pi^2} \left[\ln \frac{\theta^2}{\mu^2} - \frac{1}{2} \right]. \end{aligned}$$

In (11) x and z are to be considered as the independent variables, variation over which must provide for the effective potential extreme.

Such calculations within (11) with the account taken of the effective potential of the tree approximation lead to the following equations

$$\begin{aligned}
 0 = & -M^2 + (h\alpha^2 + \frac{f}{2}z^2) + \frac{3h[-m^2 + (\lambda\alpha^2 + \frac{h}{2}z^2)]}{32\pi^2} \ln \frac{-m^2 + (\lambda\alpha^2 + \frac{h}{2}z^2)}{\mu^2} \\
 & + \frac{3g^4}{8\pi^2} (z^2 + \frac{\alpha^2}{2}) \ln \frac{g^2(z^2 + \frac{\alpha^2}{2})}{\mu^2} + \frac{f[-M^2 + (h\alpha^2 + \frac{f}{2}z^2)]}{16\pi^2} \ln \frac{-M^2 + (h\alpha^2 + \frac{f}{2}z^2)}{\mu^2} \\
 & + \frac{3f+h}{64\pi^2} [\theta_+^2 \ln \frac{\theta_+^2}{\mu^2} + \theta_-^2 \ln \frac{\theta_-^2}{\mu^2}] + \frac{K}{64\pi^2} [\theta_+^2 \ln \frac{\theta_+^2}{\mu^2} - \theta_-^2 \ln \frac{\theta_-^2}{\mu^2}];
 \end{aligned}$$

$$\begin{aligned}
 0 = & -m^2 + (\lambda\alpha^2 + \frac{h}{2}z^2) + \frac{3\lambda[-m^2 + (\lambda\alpha^2 + \frac{h}{2}z^2)]}{32\pi^2} \ln \frac{-m^2 + (\lambda\alpha^2 + \frac{h}{2}z^2)}{\mu^2} \\
 & + \frac{3g^4}{32\pi^2} (z^2 + \frac{\alpha^2}{2}) \ln \frac{g^2(z^2 + \frac{\alpha^2}{2})}{\mu^2} + \frac{h[-M^2 + (h\alpha^2 + \frac{f}{2}z^2)]}{16\pi^2} \ln \frac{-M^2 + (h\alpha^2 + \frac{f}{2}z^2)}{\mu^2} \\
 & + \frac{h+3\lambda}{64\pi^2} [\theta_+^2 \ln \frac{\theta_+^2}{\mu^2} + \theta_-^2 \ln \frac{\theta_-^2}{\mu^2}] + \frac{F}{64\pi^2} [\theta_+^2 \ln \frac{\theta_+^2}{\mu^2} - \theta_-^2 \ln \frac{\theta_-^2}{\mu^2}] + \frac{3g^4}{128\pi^2} \alpha^2 \ln \frac{g^2\alpha^2}{\mu^2}.
 \end{aligned}$$

In (12) K and F are convenient designations which simplify the final result

$$\begin{aligned}
 F = & \frac{1}{\theta_+^2 - \theta_-^2} \left\{ [(m_i^2 - M_i^2) + \lambda\alpha^2](3\lambda - h) + 4h^2 z^2 \right\}, \\
 K = & \frac{1}{\theta_+^2 - \theta_-^2} \left\{ [(m_i^2 - M_i^2) + \lambda\alpha^2](h - 3f) + 8h^2 z^2 \right\} \quad (13)
 \end{aligned}$$

The gauge interaction hierarchy suggested in the tree approximation implicates here that the condition $\alpha^2 \rightarrow 0$ at $z^2 \rightarrow \text{const}$ must agree with the solution of Eqs. (12) when the parameters are chosen appropriately. This fact is easily verified by linearization of Eqs. (12) by the following substitution

$$z^2 = \frac{2M^2}{f}(1-\delta); \quad m^2 = \frac{M^2 h}{f}(1-\Delta) \quad (14)$$

Obtained in such a way the set of linearized equations for δ, Δ

$$\begin{aligned} \delta &= \frac{3g^4}{4\pi^2 f} \ln\left(\frac{2M^2 g^2}{f \mu^2}\right) + \frac{3f}{16\pi^2} \ln\left(\frac{2M^2}{\mu^2}\right); \\ \Delta &= \delta - \left[\frac{3g^4}{16\pi^2 h} \ln\left(\frac{2M^2 g^2}{f \mu^2}\right) + \left(\frac{f}{2} + h\right) \frac{1}{8\pi^2} \ln\left(\frac{2M^2}{\mu^2}\right) \right], \end{aligned} \quad (15)$$

shows that solution (14) does exist, i.e. the gauge interaction hierarchy suggested does not contradict the complete set of Eqs. (12).

As a conclusion we would like to make some model-independent remarks on the problem under consideration. We have shown here that the results of ref [2] within the above $SU(2)$ gauge model are incorrect or at least they are of a limited applicability. Every particular case requires additional investigation of this problem. However some general considerations may be expressed. Indeed, it may be stated that the hierarchy of the gauge fields which correspond to "independent" parameters determining spontaneous symmetry breaking always takes place. "Independent" parameters are here the ones whose vanishing leads to the appearance in the theory of additional massless gauge fields. In this sense the statement of ref [2] concerns only the mass differences within each group of such gauge fields and is not connected with the mass hierarchy among "independent" groups. The latter may be arbitrary.

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