the possible existence of a short-wavelength limit to the continuous spectrum.⁴ Hence, in all the exposures but one, the electrons were accelerated to the peak energy of the synchrotron (310 Mev) so that the spectrum corresponds to the integral of the instantaneous power spectrum over an acceleration cycle. In one run, by turning off the rf voltage early in the acceleration cycle, the maximum energy was lowered to 220 Mev. The exposure times, under the particular operating conditions of the synchrotron, were of the order of 30 minutes except for the low-energy run. With Ilford QI plates, good photographic densities were obtained with the above exposure times.

The following observations are derived by inspection of the photographic records dealing with various runs at the peak energy.

a. A strong continuum is present in the soft x-ray region, which for 310-Mev electrons appears to extend at least down to the instrumental short-wavelength cutoff which is observed to set in at about 55A.

b. The photographic negatives over the 50 to 400A spectral range show no marked density variations, with no sudden drop over the short-wavelength region available under the particular conditions of detection.

c. When thin metallic foils of Be and Al are introduced into the path of the undispersed radiation, the characteristic *K* and *L*_{2,3} absorption edges of Be and Al, respectively, are strongly recorded in two orders. (The Be *K* edge is located at 110A, while the *L*_{2,3} edge of Al is at 170A.)

d. For the 220-Mev run, the microphotometer trace shows a gradual but definite increase in density, from about 60A on, and attains a peak in the vicinity of 170A, diminishing gradually towards longer wavelengths.

The height of the recorded continuous spectrum here is much smaller than the height of images of line spectra photographed with the same highly astigmatic spectrograph. This characteristic is probably the result of the fact that the space distribution of the radiation is very different from that of conventional light sources.

Various tests have definitely established the absence of fogging of the spectral plates by gamma rays or other sources such as the light from the injector filament. An attempt will be made to carry out a photometric reduction of the plates in order to obtain more quantitative information concerning the spectral energy distribution, at least over that part of the spectrum involving no overlap of the various orders of the instrument.

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The Theory of Coulomb Excitation of Nuclei

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THE possibility of exciting nuclear states by the electric field of impinging charged particles has been discussed by several authors. We shall here follow the formulation of the theory of the Coulomb excitation process as given by Ter-Martirosyan.^{1,2}

The feasibility of performing experiments of this type has recently been demonstrated,³ and we here report numerical results of use in the application of the theory to the interpretation of the experiments.

The basis for the theory is that the trajectory of the impinging

particle can be described by classical mechanics. This is possible if

$$\kappa = 2Z_1Z_2e^2/\hbar v \gg 1 \quad (1)$$

and if the particle loses only a small fraction of its energy in the collision. In the expression (1), *Z*₁ and *Z*₂ refer to the charge numbers of the projectile and the target nucleus respectively, and *v* means the relative velocity.

Usually one will bombard with energies below the Coulomb barrier to ensure that no nuclear interaction takes place, and thus $\kappa \gg 1$ is fulfilled. As the excitation probability is small, one may use quantum-mechanical perturbation theory to find the cross section. The perturbation energy is

$$V(t) = \sum_{p=1}^A \frac{Z_1 e_p^2}{|\mathbf{r}_p - \mathbf{r}(t)|}, \quad (2)$$

where *r*_{*p*} are the coordinates and *e*_{*p*} the charges of the nucleons in the target nucleus, and *r*(*t*) is the coordinate of the projectile in the classical hyperbolic orbit. It is now convenient to expand (2) in multipole components. One gets in this way an expression for the excitation probability as a sum over multipole components. Each term will be proportional to the square of the usual nuclear matrix element for emission of electric 2^{*n*}-pole radiation and a definite function *S*^(*n*) of the orbit of the projectile.

Particularly well suited for Coulomb excitation are the rotational states in rather heavy nuclei for which the quadrupole component of the nuclear matrix elements are especially big.⁴ Also the conditions for the semi-classical treatment are well satisfied here.

By taking *n* equal to 2 and performing an integration over the different orbits, one gets

$$\sigma_{\text{inel}} = \frac{4\pi^2}{25} \cdot \frac{m}{Z_2^2 e^2 \hbar} E B_e(2) g_2(\xi) \quad (3)$$

for the total inelastic cross section for excitation by the electric quadrupole field. The reduced mass is denoted by *m*, and *E* is the kinetic energy in the center-of-mass system. The energy-independent quantity

$$B_e(2) = \sum_{\mu, M_f} \left| \langle i | \sum_{p=1}^A e_p r_p^2 Y_{2,\mu}(\theta_p, \varphi_p) | f \rangle \right|^2 \quad (4)$$

is the reduced transition probability for the electric quadrupole excitation from the initial state *i* to the final state *f* with the magnetic quantum number *M_f*. Further, the *Y*_{2,μ} are the normalized spherical harmonics depending on the polar angles *θ_p*, *φ_p* of the nucleons in the target nucleus. The function *g*₂(*ξ*) depends on the energy through the variable

$$\xi = \frac{\Delta E Z_1 Z_2 e^2}{2E \hbar v}, \quad (5)$$

which measures the ratio of the collision time to the nuclear period, *ΔE* being the nuclear excitation.

The function *g*₂(*ξ*) is given by

$$g_2(\xi) = \sum_{\mu} \int_1^{\infty} |S_{\mu}^{(2)}|^2 \epsilon d\epsilon, \quad (6)$$

where *ε* is the eccentricity of the orbit, and where the non-vanishing components of *S*⁽²⁾ are

$$\begin{aligned} S_0^{(2)} &= \int_{-\infty}^{\infty} \exp[i\xi(\epsilon \sinh \omega + \omega)] \frac{1}{(\epsilon \cosh \omega + 1)^2} d\omega, \\ S_{\pm 2}^{(2)} &= -\sqrt{\frac{2}{5}} \int_{-\infty}^{\infty} \exp[i\xi(\epsilon \sinh \omega + \omega)] \\ &\quad \times \frac{[\cosh \omega + \epsilon \mp i(\epsilon^2 - 1)^{1/2} \sinh \omega]^2}{(\epsilon \cosh \omega + 1)^4} d\omega. \end{aligned} \quad (7)$$

These integrals have been evaluated numerically, and the resultant function *g*₂ is given in Fig. 1.

The yield of *γ* quanta following the Coulomb excitation provides a measure of the cross section for the excitation. The direction of these quanta is, however, correlated with the direction of the incident particles and the angle in which the exciting projectile is scattered. If one measures only the angle of emission of the *γ* rays

FIG. 1. The function $g_2(\xi)$ plotted on a logarithmic scale, for $0 < \xi < 2$.

with respect to the incident particles, one obtains an angular distribution which is very similar to the angular correlation between two γ 's in cascade. It turns out that the distribution function is

$$W(\theta) = 1 + B_2 a_2(\xi) P_2(\cos\theta) + B_4 a_4(\xi) P_4(\cos\theta), \quad (8)$$

which should be compared with the angular correlation in the γ - γ cascade,

$$\begin{matrix} E2 & \gamma \\ I_i \rightarrow I_f & \rightarrow I_{ff}, \end{matrix}$$

given by

$$W'(\theta) = 1 + B_2 P_2(\cos\theta) + B_4 P_4(\cos\theta). \quad (9)$$

The I 's are the spins of the target nucleus, the Coulomb excited state, and the final state after the γ -emission, respectively. The first γ -transition, being an electric quadrupole radiation, corresponds to the electric quadrupole excitation. The coefficients B_K are given, e.g., by Biedenharn and Rose.⁵

The energy-dependent coefficients a_k are determined by

$$a_2(\xi) = \int_1^\infty \left[S_2^2 - S_0^2 + S_{-2}^2 + \sqrt{6} S_0 (S_2 + S_{-2}) \left(1 - \frac{2}{\epsilon^2} \right) \right] d\epsilon \times \left\{ \int_1^\infty [S_2^2 + S_0^2 + S_{-2}^2] d\epsilon \right\}^{-1},$$

$$a_4(\xi) = \int_1^\infty \left[\frac{3}{32} S_2^2 + \frac{9}{16} S_0^2 + \frac{3}{32} S_{-2}^2 + \frac{5\sqrt{6}}{16} S_0 (S_2 + S_{-2}) \left(1 - \frac{2}{\epsilon^2} \right) + \frac{35}{16} S_2 S_{-2} \left(1 - \frac{8}{\epsilon^2} + \frac{8}{\epsilon^4} \right) \right] d\epsilon \times \left\{ \int_1^\infty [S_2^2 + S_0^2 + S_{-2}^2] d\epsilon \right\}^{-1}.$$

FIG. 2. The angular distribution coefficients $a_2(\xi)$ and $a_4(\xi)$ in the region $0 < \xi < 2$.

The result of a numerical calculation of these functions is shown in Fig. 2.

As it is seen from this figure, a_4 is so small that P_4 will give almost no contribution. The P_2 term may however be quite big. In the important case of the excitation of the lowest state in even-even nuclei, where $I_i = I_{ff} = 0$ and $I_f = 2$, the anisotropy amounts to 30 percent. In the case of Ta^{181} , studied by Huus and Zupančič,³ where $I_i = I_{ff} = 7/2$ and $I_f = 9/2$, the anisotropy is however only about 0.5 percent.

A more detailed discussion of the theory of Coulomb excitation is being prepared.

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$\text{Li}^7(\gamma, p)\text{He}^6$ Cross Section*

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THE (γ, p) reaction in Li^7 produces β -active He^6 . This activity gives a convenient measure of the photonuclear reaction. The β half-life of 0.9 sec allows a typical yield curve to be obtained by a scintillation counter gated to operate during the betatron's dead time. Normalization of the curve was made by direct comparison with copper under the same geometry. The cross section was derived from the yield curve by the photon-difference method.

Figure 1 shows the experimental setup. Stilbene was adopted as a scintillator to avoid a troublesome neutron-capture activity in the NaI crystal previously employed. The discriminator was set on the 625-kev cesium conversion line. Identical geometry with

FIG. 1. Experimental arrangement.

comparable self-absorption was obtained by using a laminated copper sample of the same weight as the lithium disk.

The usual self-absorption and discriminator corrections were applied to each yield curve. Then the copper cross section $[\text{Cu}^{63}(\gamma, n)\text{Cu}^{62}]$ of Katz *et al.*¹ was used to calibrate the absolute scale.

The beam transmission monitor was checked against copper at 13, 15, and 17 Mev and was found to be energy-independent. Under apparently constant betatron trapping conditions, the monitor reading showed no variation with beam energy between 9 and 17 Mev, with only a 5 percent decline at 18 Mev and a 12 percent decline at 19 Mev. These variations were incorporated in the calculated cross section and in the probable errors.

A contaminating neutron-induced 5-min activity in the copper was separated by observing the decay of the copper activity over several half-lives.

The only troublesome competing activity in Li^7 would be neutron capture resulting in radioactive Li^8 , which has a β decay