Magnetized completion of the Λ CDM paradigm

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(Received 12 December 2007; published 7 March 2008)

The standard Λ CDM paradigm is complemented with a magnetized contribution whose effects on the anisotropies of the cosmic microwave background are assessed by means of a dedicated numerical approach. The accuracy on the temperature and polarization correlations stems from the inclusion of the large-scale magnetic fields both at the level of the initial conditions and at the level of the Einstein-Boltzmann hierarchy which is consistently embedded in a generalized magnetohydrodynamical framework. Examples of the calculations of the temperature and polarization angular power spectra are illustrated and discussed. The reported results and the described numerical tools set the groundwork for a consistent inclusion of a magnetized contribution in current strategies of cosmological parameter estimation.

DOI: [10.1103/PhysRevD.77.061301](https://doi.org/10.1103/PhysRevD.77.061301)

PACS numbers: 98.70.Vc, 78.20.Ek, 98.62.En, 98.80.Cq

Current analyses of cosmological data sets (see, for instance, [1]) are customarily performed using a variety of theoretical models which represent diverse completions (i.e. delicate improvements) of the pivotal Λ cold dark matter (Λ CDM) paradigm. As an example, if we ought to know how large could be the contribution of a stochastic background of gravitational waves to the anisotropies of the cosmic microwave background (CMB), the possible presence of tensor modes should be added, as a supplementary feature, to the basic list of cosmological parameters of the standard Λ CDM paradigm whose updated version (improved by a tensor contribution) will then be compared with the experimental data. From the latter comparison likely values of the ratio between the tensor and scalar power spectra can be inferred. Yet another class of delicate improvements of the Λ CDM paradigm contemplates the inclusion of one (or more) nonadiabatic modes which can be either correlated or anticorrelated with the standard adiabatic component (see, for instance, [2]). For specific choices of the nonadiabatic amplitude and spectral index the fit to the experimental data may even improve (see, for instance, the last reference in [2]).

In this paper we are going to describe another completion of the Λ CDM paradigm. The possibility we are going to present will be dubbed as magnetized Λ CDM (m Λ CDM) scenario. Large-scale magnetic fields arise over different scales ranging from galaxies to clusters [3]. Superclusters have also been claimed to have magnetic fields [3] at the μ G level even if firmer evidence is still lacking. Hopefully some of the findings of the Auger project [4] could be used for a magnetic “tomography” of the local Universe, say, within a cocoon of 60 Mpc.

If the present magnetized structures emanate from pre-existing cosmological relics, they must be present prior to matter-radiation equality, affecting, in this way, the physics of photon decoupling and, ultimately, the formation of the CMB anisotropies. In the m Λ CDM scenario the CMB

anisotropies are computed in the presence of a stochastic magnetic field¹ which will affect both the initial conditions and the dynamical evolution of the Einstein-Boltzmann hierarchy:

$$\begin{aligned} \langle B_i(\vec{k}) B_j(\vec{p}) \rangle &= \frac{2\pi^2}{k^3} \delta^{(3)}(\vec{k} + \vec{p}) \mathcal{P}_B(k) P_{ij}(k), \\ P_{ij}(k) &= \delta_{ij} - \frac{k_i k_j}{k^2}, \quad \mathcal{P}_B(k) = A_B \left(\frac{k}{k_L} \right)^{n_B-1}. \end{aligned} \quad (1)$$

The minimal m Λ CDM scenario contains, on top of the (six) parameters of the Λ CDM paradigm, two new parameters, namely, the magnetic spectral index n_B and the amplitude of the magnetic power spectrum $\mathcal{P}_B(k)$ evaluated at the magnetic pivot scale k_L . Nonminimal extensions of the m Λ CDM scenario include the correlation (or the anticorrelation) between the adiabatic mode and the magnetic field, the simultaneous presence of a magnetized adiabatic mode, together with one (or more) magnetized isocurvature modes. For the sake of simplicity we will stick here to the minimal situation reporting elsewhere on the complementary cases. There have been lately theoretical and semianalytical works along this direction [6], but the moment has now come to tailor, for the first time, a consistent numerical approach to treat the effects of a fully inhomogeneous magnetic field on the scalar modes of the geometry.

Since, after neutrino decoupling, the concentration of electrons and protons is roughly 10 orders of magnitude smaller than the concentration of the photons, the Debye length scale will be approximately 20 orders of magnitude

¹A stochastic magnetic field does not break, by definition, the overall spatial isotropy of the background geometry and allows, as a consequence, for angular power spectra just expressed in terms of their dependence upon the multipole ℓ without any further preferred direction. For the opposite situation see, for instance, [5].

smaller than the Hubble radius (for instance at matter-radiation equality). The conductivity of the globally neutral plasma, i.e. σ_c , will be dominated by Coulomb scattering so that $\alpha_{\text{em}}\sigma_c \simeq 4\pi(T/m_e)^{1/2}T(\ln\Lambda_C)^{-1}$ where Λ_C is the argument of the Coulomb logarithm. Ohmic electric fields are then suppressed with respect to the total Ohmic current, as is the case in a good terrestrial conductor:

$$\vec{E} + \vec{v}_b \times \vec{B} = \frac{\vec{J}}{\sigma} \simeq \frac{\vec{\nabla} \times \vec{B}}{4\pi\sigma}, \quad \sigma = a\sigma_c, \quad (2)$$

where $\vec{E} = a^2\vec{\mathcal{E}}$ and $\vec{B} = a^2\vec{\mathcal{B}}$ are the electric and magnetic fields rescaled through the second power of the scale factor $a(\tau)$ of a (conformally flat) Friedmann-Robertson-Walker geometry. The large-scale description of our problem is, physically, the curved-space version of magnetohydrodynamics (MHD) where the electric field, the magnetic field and the Ohmic current are all solenoidal quantities [7]. The effects of the magnetic field on the scalar modes of the geometry are far from trivial: the evolution of the bulk velocity of the plasma is given by²

$$\theta'_b + \mathcal{H}\theta_b = \frac{4}{3}\frac{\rho_\gamma}{\rho_b}\epsilon'(\theta_\gamma - \theta_b) + \frac{\vec{\nabla} \cdot [\vec{J} \times \vec{B}]}{a^4\rho_b}, \quad (3)$$

$$\theta_b = \vec{\nabla} \cdot \vec{v}_b.$$

Given the hierarchy between the electron and proton masses the bulk velocity of the plasma θ_b will be essentially the proton velocity. Because of the Coulomb-dominated conductivity, the magnetic fields will be present and not diffused for typical length scales larger than the magnetic diffusivity length L_σ whose ratio to the Hubble radius is so small (i.e. $HL_\sigma \simeq 3.9 \times 10^{-17} (\text{T/eV})^{1/4}$) that the $k_\sigma \simeq L_\sigma^{-1}$ will provide effectively an ultraviolet cutoff to the magnetic power spectrum of Eq. (1). In the presence of diffusion damping (i.e. shear viscosity) the Silk wave number will be the dominant dissipative scale for the baryon-photon fluid. Under these conditions the magnetic flux (and helicity) will be effectively conserved to a very good approximation.

Large values of the conductivity break explicitly Lorentz invariance. The plasma frame, where the electric fields are suppressed in comparison with the magnetic fields, arises naturally. The scalar fluctuations of the geometry, still relativistic, will be treated, numerically, in the synchronous gauge (S gauge) where the only nonvanishing entries of the perturbed metric are, in Fourier space,

$$\delta_s g_{ij}(k, \tau) = a^2(\tau) \left[\hat{k}_i \hat{k}_j h(k, \tau) + 6\xi(k, \tau) \left(\hat{k}_i \hat{k}_j - \frac{1}{3} \delta_{ij} \right) \right], \quad (4)$$

²A prime denotes a derivation with respect to the conformal time coordinate τ and $\mathcal{H} = (\ln a)'$. The notation $\theta_X = \vec{\nabla} \cdot \vec{v}_X$ denotes the three divergence of the peculiar velocity of the species X . Throughout the paper $\epsilon' = x_e n_e \sigma_{\text{Th}} n_e a / a_0$ denotes the optical depth. Finally, with similar notations, $\delta_X = \delta_s \rho_X / \rho_X$ is the density contrast of the species X .

where the symbol δ_s emphasizes that we are dealing here with *scalar* (as opposed to vector or tensor [8]) fluctuations of the geometry. The $m\Lambda\text{CDM}$ code is an extension of the CMBFAST package [9] which is, in turn, based on the COSMICS package [10]. The evolution of the scale factor is integrated numerically from the usual Friedmann-Lemaître equations. Equation (3) will then be supplemented by the governing equations for the magnetic fields as well as by the CDM particles and by the neutrino component:

$$\delta'_c = -\theta_c + \frac{h'}{2}, \quad \theta'_c + \mathcal{H}\theta_c = 0, \quad (5)$$

$$\delta'_\nu = -\frac{4}{3}\theta_\nu + \frac{2}{3}h', \quad \theta'_\nu = \nabla^2\sigma_\nu - \frac{1}{4}\nabla^2\delta_\nu, \quad (6)$$

$$\sigma'_\nu = \frac{4}{15}\theta_\nu - \frac{2}{15}h' - \frac{4}{5}\xi', \quad (7)$$

where $\sigma_\nu = \mathcal{F}_{\nu 2}/2$ is the quadrupole of the perturbed phase space distribution. Higher multipoles of the Boltzmann hierarchy (like the octupole $\mathcal{F}_{\nu 3}$) follow by setting initial conditions on the lower multipoles. The governing equations for baryons and photons are given by Eq. (3) together with

$$\theta'_\gamma = -\frac{1}{4}\nabla^2\delta_\gamma + \epsilon'(\theta_b - \theta_\gamma), \quad \delta'_\gamma = -\frac{4}{3}\theta_\gamma + \frac{2}{3}h',$$

$$\delta'_b = -\theta_b + \frac{h'}{2} + \frac{\vec{E} \cdot (\vec{\nabla} \times \vec{B})}{4\pi\sigma a^4\rho_b}, \quad (8)$$

where the last term in the evolution of δ_b is negligible at finite conductivity. The plasma and the magnetic fields all gravitate and contribute to the Hamiltonian and momentum constraints whose specific form is, respectively,

$$2\nabla^2\xi + \mathcal{H}h' = -8\pi Ga^2[\delta_s\rho_t + \delta_s\rho_B], \quad (9)$$

$$\nabla^2\xi' = 4\pi Ga^2 \left\{ (p_t + \rho_t)\theta_t + \frac{\vec{\nabla} \cdot [\vec{J} \times \vec{B}]}{4\pi a^4\sigma} \right\}, \quad (10)$$

where $\delta_s\rho_t$ is the total density fluctuation in the S gauge and $(p_t + \rho_t)\theta_t = \sum_a (p_a + \rho_a)\theta_a$ is the total peculiar velocity. Since the conductivity σ is always large, the contribution of the MHD Poynting vector is, in practice, always negligible. The (ij) components of the perturbed Einstein equations read

$$h'' + 2\mathcal{H}h' + 2\nabla^2\xi = 24\pi Ga^2[\delta_s p_t + \delta_s p_B], \quad (11)$$

$$(h + 6\xi)'' + 2\mathcal{H}(h + 6\xi)' + 2\nabla^2\xi$$

$$= 24\pi Ga^2[(p_\nu + \rho_\nu)\sigma_\nu + (p_\gamma + \rho_\gamma)\sigma_B], \quad (12)$$

where $\delta_s p_t$ is the fluctuation of the total pressure. In MHD, $4\pi\vec{J} = \vec{\nabla} \times \vec{B}$ so that the Lorentz force and the magnetic anisotropic stress (associated with σ_B) are related by

$$\nabla^2 \sigma_B = \frac{3}{16\pi a^4 \rho_\gamma} \vec{\nabla} \cdot [(\vec{\nabla} \times \vec{B}) \times \vec{B}] + \frac{\nabla^2 \Omega_B}{4}, \quad (13)$$

$$\Omega_B(\vec{x}) = \frac{\delta \rho_B(\tau, \vec{x})}{\rho_\gamma(\tau)}.$$

The simplest set of initial conditions to be imposed on the hierarchies of the fluctuations in the intensity and of the polarization is the magnetized adiabatic mode. When Coulomb and Thompson couplings are both tight (i.e. $\theta_b \simeq \theta_\gamma = \theta_{\gamma b}$), the whole system of the governing equations can be solved in the limit when the relevant wavelengths are larger than the Hubble radius prior to matter-radiation equality (i.e. in terms of the wave number k , $k\tau \ll 1$). To lowest order in $k\tau$ the magnetized adiabatic mode reads, in Fourier space,

$$\xi(k, \tau) = -2C(k) + \left[\frac{4R_\nu + 5}{6(4R_\nu + 15)} C(k) + \frac{R_\gamma(4\sigma_B(k) - R_\nu \Omega_B(k))}{6(4R_\nu + 15)} \right] k^2 \tau^2, \quad (14)$$

$$h(k, \tau) = -C(k)k^2 \tau^2 - \frac{1}{36} \left[\frac{8R_\nu^2 - 14R_\nu - 75}{(2R_\nu + 25)(4R_\nu + 15)} C(k) + \frac{R_\gamma(15 - 20R_\nu)}{10(4R_\nu + 15)(2R_\nu + 25)} (R_\nu \Omega_B(k) - 4\sigma_B(k)) \right] k^4 \tau^4, \quad (15)$$

$$\delta_\gamma(k, \tau) = -R_\gamma \Omega_B(k) - \frac{2}{3} \left[C(k) - \sigma_B(k) + \frac{R_\nu}{4} \Omega_B(k) \right] k^2 \tau^2, \quad (16)$$

$$\delta_\nu(k, \tau) = -R_\gamma \Omega_B(k) - \frac{2}{3} \left[C(k) + \frac{R_\gamma}{4R_\nu} (4\sigma_B(k) - R_\nu \Omega_B(k)) \right] k^2 \tau^2, \quad (17)$$

$$\delta_c(k, \tau) = -\frac{3}{4} R_\gamma \Omega_B(k) - \frac{C(k)}{2} k^2 \tau^2, \quad (18)$$

$$\delta_b(k, \tau) = -\frac{3}{4} R_\gamma \Omega_B(k) - \frac{1}{2} \left[C(k) - \sigma_B(k) + \frac{R_\nu}{4} \Omega_B(k) \right] k^2 \tau^2, \quad (19)$$

$$\theta_{\gamma b}(k, \tau) = \left[\frac{R_\nu}{4} \Omega_B(k) - \sigma_B(k) \right] k^2 \tau - \frac{1}{36} \left[2C(k) + \frac{R_\nu \Omega_B(k) - 4\sigma_B(k)}{2} \right] k^4 \tau^3, \quad (20)$$

$$\theta_\nu(k, \tau) = \left[\frac{R_\gamma}{R_\nu} \sigma_B(k) - \frac{R_\gamma}{4} \Omega_B(k) \right] k^2 \tau - \frac{1}{36} \times \left[\frac{2(4R_\nu + 23)}{4R_\nu + 15} C(k) + \frac{R_\gamma(4R_\nu + 27)}{2R_\nu(4R_\nu + 15)} \times (4\sigma_B(k) - R_\nu \Omega_B(k)) \right] k^4 \tau^3, \quad (21)$$

$$\theta_c(k, \tau) = 0, \quad (22)$$

$$\sigma_\nu(k, \tau) = -\frac{R_\gamma}{R_\nu} \sigma_B(k) + \left[\frac{4C(k)}{3(4R_\nu + 15)} + \frac{R_\gamma(4\sigma_B(k) - R_\nu \Omega_B(k))}{2R_\nu(4R_\nu + 15)} \right] k^2 \tau^2, \quad (23)$$

where $R_\nu = r/(r+1)$ with $r = 0.681(N_\nu/3)$ (and $R_\gamma = 1 - R_\nu$). The constant mode of h leads to a gauge mode and should be projected out of the solution. The second potentially dangerous gauge mode is fixed by setting to 0 the CDM peculiar velocity θ_c . The spectrum of $C(k)$ is simply related to the spectrum of $\mathcal{R}$ which parametrizes the curvature perturbations on comoving orthogonal hypersurfaces [6]. Deep in the radiation epoch, according to Eq. (14), $\mathcal{R}(k, \tau) = -2C(k)$. Consequently the initial conditions for the adiabatic component will be given in terms of the spectrum of $\mathcal{R}$, i.e. $\mathcal{P}_\mathcal{R}(k) = \mathcal{A}_\mathcal{R}(k/k_p)^{n_s-1}$ where n_s is the adiabatic spectral index and $\mathcal{A}_\mathcal{R}$ is the amplitude of the scalar power spectrum at the pivot scale $k_p = 0.002 \text{ Mpc}^{-1}$. The spectrum of the magnetized contribution will be encoded in the spectra of σ_B and Ω_B , i.e., respectively, $\mathcal{P}_\sigma(k) = \mathcal{G}(n_B) \tilde{\Omega}_{BL}^2(k/k_L)^{2(n_\sigma-1)}$ and $\mathcal{P}_\Omega(k) = \mathcal{F}(n_B) \tilde{\Omega}_{BL}^2(k/k_L)^{2(n_B-1)}$ where k_L denotes the magnetic pivot scale³ which will be taken 1 Mpc^{-1} . Note that $\tilde{\Omega}_{BL} = B_L^2/(8\pi\bar{\rho}_\gamma)$ is the magnetic energy density in units of the photon background. In the minimal $m\Lambda$ CDM scenario $n_\sigma = n_B$. The spectra $\mathcal{P}_\Omega(k)$ and $\mathcal{P}_\sigma(k)$ are not arbitrary but rather computed from Eq. (1) through a rather standard procedure (see, for details, [11]):

$$\mathcal{G}(n_B) = \frac{(2\pi)^{2(n_B-1)}}{\Gamma^2(\frac{n_B-1}{2})} \left[\frac{n_B + 29}{15(5 - 2n_B)(n_B - 1)} \right], \quad (24)$$

$$\mathcal{F}(n_B) = \frac{20(7 - n_B)}{n_B + 29} \mathcal{G}(n_B).$$

In Fig. 1 (plot at the left) the effects of the magnetized contribution on the temperature (TT) autocorrelations are illustrated for different values of the spectral index and of

³The range of magnetic spectral indices discussed here is $1 < n_B < 5/2$. In this case, regularizing the magnetic field with a Gaussian window function, the factors $\mathcal{F}(n_B)$ and $\mathcal{G}(n_B)$ are the ones reported in Eq. (24). For values outside the mentioned range cutoffs are required either in the infrared or in the ultraviolet [11]. The value of the magnetic pivot scale implies that B_L coincides effectively with the proper amplitude of the magnetic field regularized over a Mpc window at the onset of protogalactic collapse [6].

FIG. 1 (color online). The magnetized TT autocorrelations stemming from the magnetized adiabatic mode (left plot) and in the absence of an adiabatic component (plot at the right). The other parameters are fixed to the central values of the best fit of WMAP 3 yr data alone (plot at the left) and to the central values of the best fit of the WMAP data supplemented by the gold sample of supernovae (see [1]).

the magnetic field intensity. If the strength of the magnetic field is augmented (or the value of the spectral index becomes bluer), a distortion of the second and of the third peaks is correlated with an increase of the first peak. This spectral distortion fits with the results of a semianalytical calculation conducted in the longitudinal gauge (see [6], last reference). In spite of the correct spectral distortion on the shape, the semianalytical argument is intrinsically less accurate. The predicted height of the first peak varies in a nonmonotonic way with the variation of the spectral index and of the magnetic field intensity [11]. If the adiabatic mode is absent from the initial conditions (Fig. 1, plot at the right), the amplitude of the TT autocorrelations is always much smaller than in the case of the magnetized adiabatic mode. Furthermore a hump appears at intermediate multipoles. In Fig. 2 we illustrate the angular power spectrum of the temperature-polarization cross correlations (for short, the TE correlations) as well as the angular power spectrum for the polarization auto-correlations (for short, the EE correlations). According to Fig. 3, for high ℓ (where, hopefully, there will be in the near future more precise data from the Planck explorer mission) the TT autocorrelation is definitely sensitive, for $\ell > 1500$, to a nG magnetic field. The Planck sensitivity to high multipoles just stems directly from its angular resolution.

The putative Planck sensitivity to a given model can be obtained by estimating the 1-sigma errors using as reference model, for instance, the extrapolated 3 yr best fit to the Wilkinson Microwave Anisotropy Probe (WMAP) data alone. By doing this exercise [11], with the specifications of Planck bluebook [12], we found that the combination of the three low frequency channels of the High Frequency Instrument, i.e. 100, 143 and 217 GHz, will be sensitive to distortions induced by a magnetic field between 2 nG and 10 nG in the TT autocorrelations. The TE correlation is also sensitive to the nG range (see Fig. 3, plot at the right) especially as soon as the spectral index increases from the nearly scale-invariant limit. We did not estimate the analog quantity for the TE and EE power spectra. The Low Frequency Instrument seems to be now also more sensitive to polarization than previously thought. Our exercises will have to be complemented in the near future by more accurate estimates based, possibly, on more than three channels. The final warning on this matter is that, of course, the nominal Planck sensitivity might not be the one achieved by the High Frequency Instrument and Low Frequency Instrument.

The magnetized CMB anisotropies have never been computed consistently and systematically for the scalar modes of the geometry which are the ones observationally

FIG. 2 (color online). The magnetized TE and EE correlations are illustrated for different values of the spectral indices and different values of the magnetic field intensities.

FIG. 3 (color online). The TT autocorrelation and TE correlation for $\ell \gg 1500$. The full curve is the extrapolation of the 3 yr best fit of the WMAP data alone.

more relevant. In this paper we built a consistent numerical approach to the problem and presented a magnetized completion of the Λ CDM paradigm. Our approach is accurate enough to resolve nG magnetic fields. The usual custom of setting a bound on the magnetic field becomes too simplistic in the light of the present results and according to the standards of CMB physics. In a similar perspective, for instance, the bound on the ratio between tensor and scalar power spectra can only stem from an appropriate strategy of parameter extraction where the tensor contribution is included in the CMB anisotropy calculations and fits. The same must be done in the case of the $m\Lambda$ CDM scenario. If the correct strategy is not enforced, potentially interesting degeneracies between the parameters of the magnetized

background and other cosmological parameters can be totally overlooked. Consequently, our program is, in the short run, to extend (and complete) our code to the case of tensor and vector modes (which are known to be far less relevant at large scales) and to the case of the various magnetized isocurvature modes. In parallel we ought to test the $m\Lambda$ CDM scenario against the various data sets eagerly waiting for Planck data and its claimed high accuracy for large multipoles.

K. E. K. is supported by the “Ramón y Cajal” program and Grant Nos.FPA2005-04823 and FIS2006-05319 of the Spanish Science Ministry.

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