New framework for multijet predictions and its application to Higgs boson production at the Large Hadron Collider

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We present a new framework for calculating multijet observables through resummation. The framework is based on the factorization of scattering amplitudes in an asymptotic limit. By imposing simple constraints on the analytic behavior of the result when applied away from this limit, we get good agreement with the known lowest order perturbative behavior of the scattering amplitude, and predictions for the behavior to all orders in the perturbative series. As an example of application we study predictions for Higgs boson production through gluon fusion at the LHC in association with at least two jets.

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Events with many energetic particles will form the backbone of many search strategies at the LHC for physics beyond the standard model (SM) of particle physics, but will simultaneously test our understanding of SM processes. Energetic particles of QCD charge will be detected as jets, and events with multiple jets require the calculation of scattering amplitudes with a high number of external quark and gluon (parton) legs. However, the calculation of the SM contribution to such processes beyond even the lowest order in perturbation theory is notoriously difficult. Despite recent impressive progress in the calculation of many multileg scattering amplitudes at both tree and one-loop level, the first two radiative corrections in the perturbative series (i.e. full next-to-next-to-leading accuracy) for loop level, the first two radiative corrections in the pertur-
effects of further soft and collinear radiation were studied in Ref. [14]. In the limit of infinite top mass, the coupling of the Higgs boson to gluons through a top-quark loop can be described as a point interaction [15–17]. This approximation was applied in all these studies, and will be applied also in the present one, although this is not essential to the approach.

We will start motivating the need for considering the hard radiative corrections to higher orders by studying the leading order predictions in full QCD for Higgs boson production in association with hard jets. We apply similar leading order predictions in full QCD for Higgs boson production in association with hard jets. We apply similar event selection cuts to the study of Ref. [13] as detailed in Table I [18]. All of the following results are obtained by choosing renormalization and factorization scales in accordance with the study of Ref. [14], and the following values for the Higgs boson mass, vacuum expectation value of the Higgs boson field, and top quark mass, respectively:

\[ m_H = 120 \text{ GeV}, \quad \langle \phi \rangle = \frac{v}{\sqrt{2}}, \]
\[ v = 246 \text{ GeV}, \quad m_t = 174 \text{ GeV}. \]

We also include a factor multiplying the effective Higgs boson vertices, accounting for finite top-mass effects [19]:

\[ K(\tau) = 1 + \frac{7\tau}{30} + \frac{2\tau^2}{21} + \frac{26\tau^3}{525}, \quad \tau = \frac{m_H^2}{4m_t^2}. \]

We choose the \( k_t \)-jet algorithm as implemented in Ref. [20] with \( R = 0.6 \), and the parton distribution functions of Ref. [21]. For the strong coupling \( \alpha_s \), we choose renormalization scales as in Ref. [14] [22]. With this, we find (using matrix elements from MADEvent/MADGraph [23]) the tree-level cross section from the QCD generated \( hjj \) channel to be \( 281^{+210}_{-111} \) fb, where the uncertainty is obtained by varying the common factorization and renormalization scale by a factor of 2. For the three-jet sample, all jets must satisfy the left-most cuts of Table I, but we require that there exist two jets \( c, d \) satisfying all the right-most cuts in Table I. We then find the leading order cross section for the production of Higgs boson plus three jets (\( hjjj \)) to be \( 257^{+262}_{-120} \) fb. The requirement of an extra hard jet with an accompanying \( \alpha_s \), suppression leads only to a 9% suppression compared to the leading order prediction for \( hjj \). The \( \alpha_s \) suppression of the matrix element is compensated by the integration over a large phase space for the third jet. The large size of the three-jet rate (which obviously depends on the event selection cuts) was already reported in Ref. [24] and clearly demonstrates the necessity of considering hard multiparton emissions.

We will now describe a method for approximating the perturbative scattering matrix elements for multiparticle production to any order. We start by recalling the result of Fadin and collaborators (FKL) [25], that in the limit of infinite invariant mass between all produced particles [the Multi-Regge-Kinematic (MRK) limit], the leading contribution is given by processes of the form

\[ \alpha(p_a) + \beta(p_b) \to \alpha(p_0) + \sum_{i} g(p_i) + \beta(p_n) + h(p_h), \]

where \( \alpha, \beta \in \{ q, q, g \} \), and the partons are ordered according to rapidity in both the initial and final states. These processes allow neighboring particles to be connected by gluon propagators of momentum \( q_i \), such that \( p_i = q_i - q_{i+1} \). We have explicitly checked that at leading order, partonic configurations which are not captured in this framework account for only 0.8 fb (0.3%) and 24 fb (< 10%) of the two- and three-jet rate, respectively, even when there is no requirement of large invariant mass between all particles.

In the MRK limit, the scattering amplitude for the remaining configurations factorizes, and the results of Ref. [25] can straightforwardly be modified to include also the production of a Higgs boson. In the case of the Higgs boson being produced between the jets (in rapidity) these amplitudes take the form

\[
\int \mathcal{M}_{FKL}^{p_a p_b \to p_0 \cdots p_{n+1}} = 2i\delta(i g_s f^{a b c} g_{\mu \nu \rho}) \cdot \prod_{i=1}^{j} \left( \frac{1}{q_i^2} \exp[\hat{\alpha}(q_i)(y_{i-1} - y_i)](i g_s f^{c d i r_{i+1}} C_{\mu_i}(q_i, q_{i+1})) \right) \cdot \prod_{i=1}^{n} \left( \frac{1}{q_i^2} \exp[\hat{\alpha}(q_i)(y_{j-1} - y_j)](i g_s f^{c d i r_{i+1}} C_{\mu_i}(q_i, q_{i+1})) \right) \cdot \frac{1}{q_{n+1}^2} \exp[\hat{\alpha}(q_{n+1})(y_{n-1} - y_n)](i g_s f^{b d e i r_{n+1}} g_{\mu \nu \rho}),
\]

where \( g_s \) is the strong coupling constant \( (\alpha_s = \frac{g^2}{4\pi}) \); \( f^{a b c} \) color structure constants; \( y_i, y_j \) are the rapidities of the emitted particles; \( s = (p_a + p_b)^2 \) is the partonic center of mass energy. The factor \( C_{\mu_i} \) is a Lipatov effective vertex describing the emission of gluon \( i \). This has the explicit form

\[
C_{\mu_i}(q_i, q_{i+1}) = \left[ (q_i + q_{i+1})^\mu_i - \left( \frac{\hat{s}_{ai}}{\hat{s}} + 2 \frac{q_i^2}{\hat{s}_{bi}} \right) p_b^\mu_i \right] + \left( \frac{\hat{s}_{bi}}{\hat{s}} + 2 \frac{q_i^2}{\hat{s}_{ai}} \right) p_a^\mu_i,
\]
denotes the projection of a 4-momentum onto its transverse components. Also, \( C_H \) is an effective vertex coupling the Higgs to off-shell gluons via a top-quark loop, whose form has been calculated in [26]. The exponential factors \( \hat{\alpha}(q_i) \) encode the leading virtual corrections (see e.g. Ref [27]):

\[
\hat{\alpha}(q_i) = -\frac{g^2 N_c \Gamma(1 - \varepsilon)}{(4\pi)^{2 + \varepsilon}} \frac{2}{\varepsilon} \left( |q_{i\perp}|^2 / \mu^2 \right) \varepsilon, \tag{7}
\]

where singularities have been regularized using dimensional regularization in \( D = 4 + 2\varepsilon \) dimensions, where \( \mu \) is the renormalization scale and \( N_c \) the number of colors. The color factors in Eq. (1) are for incoming gluons. The form of the amplitude is the same for initial state quarks, apart from different color factors.

Equation (1) formally applies in the so-called Multi-Regge-kinematic (MRK) limit. Thus, it is clear why this may be a good starting point for describing matrix elements with many hard partons. First, it can be applied at any order in the perturbation expansion. Second, it does not rely upon soft and collinear approximations.

The multigluon emissions have in fact not previously been studied directly as implemented in Eq. (1). Instead, a simplified version has been used, which is equivalent in the MRK limit. In this limit, the squared 4-momenta fulfil \( q_i^2 = -|q_{i\perp}|^2 \), and the squared Lipatov vertices \(-C_\mu, C^{\mu_i} \rightarrow \frac{4 |q_{i\perp}|^2 |q_{j\perp}|^2}{|k_{i\perp}|^2} \). This means that in the products of Eq. (1), only squares of transverse momenta appear. Extending these kinematic approximations to all of phase space (not just the MRK limit), the sum over \( j, n \) and the phase space integral over the emitted gluons can be approximated by solving the BFKL (Balitsky-Fadin-Kuraev-Lipatov) equation [28]. In this form, the framework has previously been extensively applied to other processes. In the present context (after implementing local 4-momentum conservation, which is strictly subleading in the BFKL approach), we find by expanding the solution in powers of \( \alpha_s \) that the lowest order BFKL results for the \( hjj \) (554 fb) and \( hjjj \) (775 fb) cross sections differ from their full leading order counterparts by 97% and 200%, respectively. The kinematic approximations are clearly inadequate in describing amplitudes in general at the LHC.

Instead, we define a set of amplitudes based on Eq. (1) as written, supplemented by the following guidelines:

1. Use of full virtual 4-momenta: Rather than substituting \( q_i^2 = -|q_{i\perp}|^2 \) as in the BFKL equation, we keep the dependence on the full 4-momenta of all particles. This ensures that outside of the MRK limit, the singularity structure of the approximate amplitudes coincides with known singularities of the full fixed-order scattering amplitude.
2. Positivity of the squared Lipatov vertex: The square of the amplitudes in Eq. (1) are not positive definite, when the effective Lipatov vertex is applied to momentum configurations very far from the MRK limit. It is here possible to obtain \(-C_\mu, C^{\mu_i} < 0\), where the minus sign arises from the contraction of the gluon polarization tensor. We choose to remove the contribution from the small region of phase space where this happens.

These modifications combine known analytic behavior of the full scattering amplitudes and the factorized expressions obtained in the MRK limit to any order in perturbation theory.

When these modifications are made, the resulting amplitudes do indeed approximate well the known perturbative results at low orders, and thus can be reliably used at higher orders, where full results are unknown or computationally unfeasible. Using our approach, we find an \( \alpha_s^3 \) contribution to \( hjj \) of 321 fb and \( \alpha_s^2 \) contribution to \( hjjj \) of 217 fb, within 16% and 7% of the full results, respectively. In general, we find good agreement in a large region of phase space, and the level of accuracy reported here does not require any fine-tuning of cuts. This is summarized in Fig. 1, and one sees that the approximate results are well within the uncertainty associated with the full results, obtained by varying a common renormalization and factorization scale by a factor of 2.

Having validated the approximation at low orders in \( \alpha_s \) (where it can be compared with known fixed-order results), we now consider results obtained using matrix elements with any number of final state partons, which at present cannot be calculated using standard perturbation theory. The divergence in Eq. (1) arising when any \( p_i \rightarrow 0 \) is regulated by the divergence of the virtual corrections encoded in \( \hat{\alpha} \). Thus the resulting formalism is efficiently implemented in a Monte Carlo generator following the method for phase space generation outlined in Ref. [29]. Given that one knows the full tree-level results for 2 and 3 partons, however (and they are also computationally quick), we have combined these results with the approxi-
mate matrix elements using a suitable matching procedure. We find a total cross section of $499^{+527}_{-307}$ fb. The large uncertainty in the total cross section due to scale variations, however, is not reflected in the distribution of the number of hard jets as shown in Fig. 2. One sees a significant number of events with more than 3 hard jets.

The transverse momentum spectrum of the Higgs boson when produced in association with at least two hard jets is shown in Fig. 3. To the best of our knowledge, this is the first report of this quantity, in contrast to the completely inclusive Higgs boson $p_T$ spectrum, which has previously been studied in the literature. The tree-level 2-parton final state predicts a bimodal structure, which ultimately arises from the azimuthal correlation between the jets. This structure disappears when extra radiation is added, giving a qualitatively different behavior. The significant difference between the fixed-order spectra emphasizes the importance of considering yet higher order corrections.

In summary, we have outlined a technique, not relying on a soft and collinear approximation, for estimating scattering amplitudes with multiple partons to all orders in perturbation theory, and demonstrated its application to Higgs-boson production (via GGF) in association with at least two jets. Our technique is based on the FKL factorization formula of Ref. [25], with important modifications which ensure that the singularity structure of the amplitudes coincides with known all-order analytic properties of the perturbation expansion. At low orders in $\alpha_s$, where the full fixed-order result can be obtained, our description agrees well, which verifies the trustworthiness of the approach. It captures both real and virtual corrections and can be applied at any order in $\alpha_s$ in a computationally efficient manner.

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[18] If further a rapidity veto on jets is applied, the effects studied in Ref. [30] need to be taken into account.
[22] For the resummed results presented later, some freezing of the coupling $\alpha_s$ is necessary below a suitable low scale $Q_0$. However, the results are fairly insensitive to this choice.