A Bayesian Approach to the Constrained MSSM

Leszek Roszkowski,1 Roberto Ruiz de Austri2 and Roberto Trotta3
1Department of Physics and Astronomy, University of Sheffield, Sheffield S3 7RH, UK
2Departamento de Física Teórica C-XI and Instituto de Física Teórica C-XVI, Universidad Autónoma de Madrid, Cantoblanco, 28049 Madrid, Spain
3Astrophysics Department, Oxford University, Denys Wilkinson Building, Keble Road, Oxford OX1 3RH, UK

Abstract
We present a new analysis of the Constrained MSSM in terms of Bayesian statistics. We illustrate our results with the light Higgs boson whose inferred mass range one should be able to exclude at the Tevatron with high confidence.

1 Introduction
Softly-broken low-energy supersymmetry (SUSY) offers a promising framework within which many questions challenging particle physics and cosmology, such as the hierarchy problem or the nature of dark matter, can be addressed. Despite many attractive features, without a reference to grand (or string) unification, SUSY models suffer from the lack of predictivity due to a large number of free parameters (e.g., over 120 in the Minimal Supersymmetric Standard Model (MSSM)). The MSSM with one particularly popular choice of universal boundary conditions at the unification scale is called the Constrained MSSM, or CMSSM [1]. The CMSSM is defined in terms of five free parameters: common scalar ($m_0$), gaugino ($m_{1/2}$) and tri-linear ($A_0$) mass parameters (all specified at the unification scale), plus the ratio of Higgs vacuum expectation values $\tan \beta$ and sign($\mu$), where $\mu$ is the Higgs/higgsino mass parameter. The economy of parameters makes the CMSSM a useful framework for exploring SUSY phenomenology.

Many studies have explored the CMSSM or other SUSY models, mostly by evaluating the goodness-of-fit of points scanned using fixed grids in parameter space. However, this approach has a number of severe limitations. Firstly, the number of points required scales as $k^N$, where $N$ is the number of a model’s parameters and $k$ the number of points for each of them, making the approach highly inefficient for exploring with sufficient resolution parameter spaces of even modest dimensionality, say $N > 3$. Secondly, narrow “wedges” and similar features of parameter space can easily be missed by not setting a fine enough resolution (which, on the other hand, may be completely unnecessary outside such special regions). Thirdly, extra sources of uncertainties (e.g., those due to the lack of precise knowledge of SM parameter values) and relevant external information (e.g., about the parameter range) are difficult to accommodate in this scheme.

Here we present a different approach, encoded in the publicly available package SuperBayes [2]. It is based on Bayesian statistics and Markov Chain Monte Carlo scanning methods. After introducing our procedure we will present our results obtained in the framework of the CMSSM. In particular we focus on the lightest Higgs boson $h^0$. We also comment on prospects for superpartner searches at the LHC and on direct neutralino dark matter detection. We refer the reader to [3, 4, 5] for a detailed presentation. The Bayesian approach has several technical and statistical advantages over the more traditional fixed-grid scan technique, the most important being perhaps the ability to incorporate all relevant sources of uncertainties, e.g., the residual uncertainty in the value of SM parameters. This means that the inferred high probability regions of the CMSSM parameters (or resultant observables) take fully into account all sources of uncertainty relevant to the problem. For other recent works applying a similar approach to the CMSSM, see [6, 7, 8, 9].
2 Parameter space, priors and data used

We consider the 8 dimensional parameter space \( m = (\theta, \psi) \), where \( \theta = (m_0, m_{1/2}, A_0, \tan \beta) \) is a vector of CMSSM parameters, while \( \psi = (M_1, m_b(m_b)_{\overline{MS}}, \alpha_{em}(M_Z)_{\overline{MS}}, \alpha_s(M_Z)_{\overline{MS}}) \) is a vector of relevant SM parameters, where \( M_1 \) is the pole top quark mass, \( m_b(m_b)_{\overline{MS}} \) is the bottom quark mass at \( m_b \), and \( \alpha_{em}(M_Z)_{\overline{MS}} \) and \( \alpha_s(M_Z)_{\overline{MS}} \) are the electromagnetic and the strong coupling constants at the \( Z \) pole mass \( M_Z \). The last three quantities are evaluated in the \( \overline{MS} \) scheme. Since we are only interested in the effect of the residual uncertainty in the experimental determination of the SM parameters on our observables (see below), we treat them as “nuisance parameters” and at the end we integrate them out from our probability distribution function (pdf). It turns out that including them has an important impact in widening high probability regions of the CMSSM parameters.

In Bayesian statistics the posterior probability distribution \( p(m|d) \) is computed using the Bayes theorem, \( p(m|d) = p(d|m, f(m)) \pi(m)/p(d) \). The likelihood \( p(d|m, f(m)) \) supplies the information provided by the data, by comparing the base parameters \( m \) or any derived function \( f(m) \) to the data \( d \). The quantity \( \pi(m) \) denotes a prior probability density function (hereafter called simply a prior) which encodes our state of knowledge about the values of the parameters before we see the data. We here first take the prior to be flat (i.e., constant) in the variables \( m \); below we specify their ranges. If the constraining power of the likelihood is strong enough to override the choice of the prior, then the latter does not matter in the nal inference based on the posterior pdf. We have adopted a wide prior region of up to 4 TeV for \( m_0, m_{1/2} \) (in order to include the so-called “focus point” (FP) region at large \( m_0 \), \( |A_0| \leq 7 \) TeV and \( 2 \leq \tan \beta \leq 62 \). The prior range on the nuisance parameters does not influence the nal results, since the SM parameters are rather tightly constrained by the data: \( M_t = 171.4(2.1) \) GeV , \( m_b(m_b)_{\overline{MS}} = 4.20(0.07) \) GeV, \( \alpha_s(M_Z)_{\overline{MS}} = 0.1176(0.0002) \) and \( 1/\alpha_{em}(M_Z)_{\overline{MS}} = 127.955(0.018) \).

In our analysis, for each choice of \( m \) we compute a series of derived observable quantities \( f(m) \). We list them here along with their experimental values and (estimated) theoretical errors, which are added in quadrature: the \( W \) gauge boson mass \( M_W = 80.392(0.029)(0.015) \) GeV, the effective leptonic weak mixing angle \( \sin^2 \theta_{\text{eff}} = 0.23153(0.00016)(0.00015) \), a SUSY contribution to the anomalous magnetic moment of the muon, \( \delta a_\mu^{\text{SUSY}} = a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = 28(8.1)(1) \times 10^{-10} \), the branching ratio \( BR(B \to X_s \gamma) = 3.55(0.26)(0.21) \times 10^{-4} \) and the cosmological neutralino relic abundance \( \Omega_\chi h^2 = 0.104(0.009)(0.1 \Omega_\chi h^2) \). For existing limits we take: \( BR(B_s \to \mu^+ \mu^-) < 1.0 \times 10^{-7} \), the light Higgs mass \( m_h > 114.4(3 \text{ th. error only}) \) GeV (91.0 GeV) and superpartner masses; see [5] for a complete list. The above data are included in the likelihood and used to constrain high posterior probability regions of the model. The likelihood is modified in such a way that it includes estimated theoretical errors in the mapping from CMSSM and SM parameters to derived quantities, another major advantage of employing a Bayesian approach (see [3, 5] for details).

3 Numerical results

First, in the left panel of Fig. 1 (from [5]) we present the 2-dim posterior pdf for \( m_{1/2} \) and \( m_0 \), with all other parameters marginalized over. The 68% total probability region lies mostly at large \( m_0 \gtrsim 1 \) TeV and not as large \( m_{1/2} \), predominantly in the FP region. This is caused mostly by a recent downwards shift of the SM value of \( BR(B \to X_s \gamma) \) [10], below the current experimental world average, as explained in [5]. Surprisingly enough, with this new value, SUSY predictions from our analysis fit the experimental distribution of \( BR(B \to X_s \gamma) \) better for \( \mu < 0 \) (the case which we have also explored) than for \( \mu > 0 \). (Despite this, the case of \( \mu < 0 \) shows a rather poor overall fit to the data - for details see [5].) Most other observables fit the data well (or even very well), except for the anomalous magnetic moment of the muon. The overall preference for large \( m_0 \) makes \( \delta a_\mu^{\text{SUSY}} \) rather small. As a result, for both signs of \( \mu \) the peaks of the relative probability of \( \delta a_\mu^{\text{SUSY}} \) are far below the central experimental value (about 3.2\( \sigma \) for \( \mu > 0 \) versus about 3.7\( \sigma \) for \( \mu < 0 \)), and close to each other. Clearly, while the 95% total probability region lies well within the assumed prior of \( m_{1/2} \) (as well
as $A_0$ and $\tan \beta$, see [5]), this is not the case for $m_0$. This should be kept in mind in deriving conclusions from the pdfs of any observables that depends directly on $m_0$, such as sfermion masses. There are also sizable uncertainties associated with the FP region, in particular with the reliability of existing numerical codes in computing mass spectra. Also, the SM value of $BR(B \to X_s \gamma)$ may still change somewhat after the NNLO calculation is completed. Thus this result should still be treated with a pinch of salt.

Note also that the above high-probability regions do not necessarily coincide with the best fitting points in parameter space if the pdf is strongly non-Gaussian, as in the present case. See [3, 5] for a detailed description of the discrepancy and a discussion of its meaning in terms of probabilistic inference.

Despite these outstanding issues, some results seem fairly robust. One is the properties of the lightest Higgs boson $h^0$. In the right panel of Fig. 1 (from [5]) we present, for each sign of $\mu$, the 1-dim relative pdf of the $h$ mass, obtained after marginalizing over all other parameters. (A previous plot, obtained in [4] with the previous value of $BR(B \to X_s \gamma)$ is nearly identical, and also agrees rather well with ref. [8].) It is clearly well confined, with the ranges of posterior probability given by $115.4 \text{ GeV} < m_h < 120.4 \text{ GeV}$ (68%) and $112.5 \text{ GeV} < m_h < 121.9 \text{ GeV}$ (95%). A finite tail on the l.h.s. of the 1-dim pdf for $m_h$, below the final LEP-II lower bound of $114.4 \text{ GeV}$ (95% CL) is a consequence of the fact that our likelihood function does not simply cut off points with $m_h$ below some arbitrary CL, but instead it assigns to them a lower probability. On the other hand, a sharp drop-off on the r.h.s. of the relative probability density is mostly caused by the assumed upper bound on $m_0 < 4 \text{ TeV}$. For instance, adopting a much more generous upper limit $m_0 < 8 \text{ TeV}$ would lead to changing the above ranges to $120.4 \text{ GeV} < m_h < 124.4 \text{ GeV}$ (68% CL) and $115.4 \text{ GeV} < m_h < 125.6 \text{ GeV}$ (95% CL). Other properties of the lightest Higgs boson, including its couplings to $ZZ$ and $WW$ pairs, for the most part closely resemble those of the SM Higgs with the same mass [4]. This means that ongoing SM Higgs searches at the Tevatron almost directly apply to $h^0$. According to ref. [11], with about 2 fb$^{-1}$ of integrated luminosity per experiment (with around 3 fb$^{-1}$ already on tape), a 95% CL exclusion limit can be set for the whole 95% posterior probability light Higgs mass range given derived for $m_0 < 4 \text{ TeV}$ ($\sim 2.5 \text{ fb}^{-1}$ for $m_0 < 8 \text{ TeV}$). It is remarkable that negative Higgs searches at the Tevatron should allow one to make definitive conclusions about the ranges of CMSSM parameters, in particular $m_0$, which extend well beyond the reach of even the LHC in direct searches for superpartners.

We have also studied in detail prospects for dark matter detection, both direct [5] and indirect [12].

Fig. 1: Left panel: The 2-dimensional probability density in the $m_{1/2}$ and $m_0$ plane (with all other parameters marginalized), with the contours containing 68% and 95% probability also marked. Right panel: The 1-dim relative probability density for the light Higgs boson mass $m_h$ for $\mu < 0$ (dotted red) and $\mu > 0$ (dashed blue).
To give some highlights, for \( \mu > 0 \) the neutralino dark matter direct detection elastic scattering cross section \( \sigma_{SI}^{p} \) shows two main features (see Fig. 11 in [5]). Firstly, there is a strong probability peak around \( 10^{-8} \) pb, mostly due to a contribution from the FP region. Secondly, there is another high probability region of \( \sigma_{SI}^{p} \) which extends between about \( 10^{-10} \) pb and about \( 10^{-7} \) pb (which is roughly today’s experimental sensitivity) and which shows a strong anticorrelation with \( m_\chi \). The largest values of \( \sigma_{SI}^{p} \) correspond, for the most part, to the FP region of large \( m_0 \). Thus this region will soon be tested in DM searches while remaining inaccessible to the LHC, except for smallest values of \( m_0 \) in the FP region.

4 Conclusions

We have presented a new method of exploring the CMSSM parameters using a state-of-the-art Bayesian method, encoded in the package SuperBayes [2]. The power and flexibility of the approach allows one to probe many previously unexplored choices of parameters and to fully incorporate the effects of remaining uncertainties in relevant SM parameters and other theoretical uncertainties in computing observables.

Using the method, we derived high probability ranges of the CMSSM parameters and showed that current data (most notably the SM value prediction for \( BR(B \to X_s \gamma) \)) favour the focus point region. Despite some theoretical uncertainties in that region, we delineated high probability ranges of \( m_h \) which one should be able to rule out with high confidence on the basis of the data already collected at the Tevatron (although not yet fully analyzed). Prospects for dark matter detection in the CMSSM also look very promising. So far, as a starting point, we only assumed at priors in the CMSSM and SM parameters - studies using different priors are in progress. Higgs properties are fairly robust with respect to changes in the \textit{a priori} allowed range for the parameters or to the exclusion of the anomalous magnetic moment of the muon measurement from the analysis. The observables which depend on \( m_0 \) are not.

5 Acknowledgements

R.RdA is supported by the program “Juan de la Cierva” of the Ministerio de Educación y Ciencia of Spain. R.T. is supported by the Royal Astronomical Society through the Sir Norman Lockyer Fellowship and by St Anne’s College, Oxford. The authors would like to thank the European Network of Theoretical Astroparticle Physics ILIAS/N6 under contract number RII3-CT-2004-506222 for financial support.

References