Primordial Fluctuations and Non-Gaussianities in Multifield Dirac-Born-Infeld Inflation

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We study Dirac-Born-Infeld inflation models with multiple scalar fields. We show that the adiabatic and entropy modes propagate with a common effective sound speed and are thus amplified at the sound horizon crossing. In the small sound speed limit, we find that the amplitude of the entropy modes is much higher than that of the adiabatic modes. We show that this could strongly affect the observable curvature power spectrum as well as the amplitude of non-Gaussianities, although their shape remains as in the single-field Dirac-Born-Infeld case.

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The past decade has seen an accumulation of cosmological data of increasing precision. Together with future experiments planned to measure the cosmic microwave background (CMB) fluctuations with yet further accuracy, we may be able to piece together more clues about early Universe physics. In parallel with this observational effort, there has been tremendous progress in recent years in the construction of early Universe models in the framework of high energy physics and string theory.

A particularly interesting class of models based on string theory is known as Dirac-Born-Infeld (DBI) inflation [1,2], associated with the motion of a D3-brane in a higher-dimensional background spacetime. The characteristic of DBI inflation, and that which gives it its name, is that the action is of the Dirac-Born-Infeld type and thus contains nontrivial kinetic terms. Most studies of DBI inflation models (or even of string-based inflationary models) have so far concentrated on a single-field description meaning, in the DBI case, that the inflaton corresponds to a radial coordinate of the brane in the extra dimensions. Taking into account the “angular” coordinates of the brane naturally leads to a multifield description since each brane coordinate in the extra dimensions gives rise to a scalar field from the effective four-dimensional point of view. This setup has started to be explored only very recently [3,4].

In this Letter, we show that the multifield DBI action contains some terms, higher order in spacetime gradients and vanishing in the homogeneous case, which have been overlooked. The inclusion of these terms leads to drastic consequences on the primordial fluctuations generated in these types of models. The scalar-type perturbations in multifield models can be divided into (instantaneous) adiabatic modes, fluctuations along the trajectory in field space, and entropy modes which are orthogonal to the former [5]. In contrast with previous expectations, we show that, in DBI models, these two classes of modes propagate with the same speed, namely, an effective speed of sound $c_s$ smaller than the speed of light. As a consequence, the amplification of quantum fluctuations occurs at the sound horizon crossing for both types of modes. Moreover, when $c_s \ll 1$, this leads to an enhancement of the amplitude of the entropy modes with respect to that of the usual adiabatic modes. As primordial non-Gaussianities—potentially detectable in forthcoming experiments if strong enough—discriminate between various models, we also study the impact of the entropy modes on non-Gaussianity in the DBI case.

Our starting point is the DBI Lagrangian governing the dynamics of a D3-brane:

$$L_{DBI} = -\frac{1}{f} \sqrt{-\det(g_{\mu\nu} + f G_{IJ} \partial_\mu \phi^I \partial_\nu \phi^J)}, \quad (1)$$

where $f = f(\phi^I)$ is a function of the scalar fields $\phi^I (I = 1, 2, \ldots)$ and $G_{IJ}(\phi^K)$ is a metric in field space. From a higher-dimensional point of view, (1) is proportional to the square root of the determinant of the induced metric on the brane, meaning that the $\phi^I$ correspond to the brane coordinates in the extra dimensions, $f$ embodies the warp factor, and $G_{IJ}$ is (up to a rescaling) the metric in the extra dimensions. We also allow for the presence of a potential and hence consider a full action of the form

$$S = \int d^4x \sqrt{-g} \left( \frac{\mathcal{R}}{2} + P \right), \quad (2)$$

where we have set $8\pi G = 1$. The determinant $\mathcal{D} = \det(\delta^\mu_\nu + f G_{IJ} \partial_\mu \phi^I \partial_\nu \phi^J)$ coming from Eq. (1) can be rewritten as

$$\mathcal{D} = \det(\delta^I_\ell - 2f X^I_\ell) = 1 - 2f G_{I\ell} X^I_\ell + 4f^2 X^I_\ell X^I_\ell - 8f^3 X^I_\ell X^I_\ell X^K_\ell + 16f^4 X^I_\ell X^I_\ell X^K_\ell X^K_\ell, \quad (3)$$

where we have defined

$$X^I_\ell \equiv -\frac{1}{2} \partial_\nu \phi^I \partial_\nu \phi^J, \quad X^I_\ell \equiv G_{JK} X^K_\ell, \quad (4)$$

and where the brackets denote antisymmetrization of the field indices. In the single-field case ($I = 1$), the terms in
extra terms, one can show [7] that the second-order action
of this form, but despite that it can be rewritten as
where
Here
is the cosmological scale factor; the effective
space, \( \perp_{ij} \equiv G_{ij} - e_{\sigma i} e_{\sigma j} \) is the projector orthogonal to
the vector \( e_{\sigma} \), and
\[
c_i^2 = \frac{\tilde{P}_{,\dot{\phi}}}{\tilde{P} + 2X\tilde{P}_{,\dot{\phi}}} = 1 - f \dot{\sigma}^2. \tag{9}
\]
Let us stress that the only difference between action (6)
and the corresponding expression in Ref. [6] is the term in
spatial gradients, with coefficient \( c_i^2 \tilde{P}_{,\phi} \tilde{G}_{ij} \) instead of
\( \tilde{P}_{,\phi} \). This crucial difference implies that all perturbations,
both adiabatic and entropic, propagate with the same speed
of sound in multifield DBI inflation, in contrast with Refs. [3,4,6],
where they have different speeds. Finally, one
should recall that the above expressions apply to the DBI
group where \( \tilde{P} \) is given in (5) so that \( \tilde{P}_{,\phi} = 1/c_\sigma \).

For simplicity, let us now restrict our attention to two
fields (\( I = 1, 2 \)). The perturbations can then be decomposed
into \( Q^I = Q^I_\sigma e^\sigma_I + Q^I_\epsilon e^\epsilon_I \), where \( e^\sigma_I \), the unit vector
orthogonal to \( e^\epsilon_I \), characterizes the entropy direction. [For
\( N \) fields, the entropy modes would span an \( (N - 1) \)-dimensional
subspace in field space.] As in standard inflation, it is more convenient, after
going to conformal time \( \tau = \int dt/a(t) \), to work in terms of the canonically
normalized fields
\[
v_\sigma = \frac{a}{c_\sigma} \sqrt{\tilde{P}_{,\phi} Q_{\sigma}}, \quad v_\epsilon = \frac{a}{c_\sigma} \sqrt{\tilde{P}_{,\phi} Q_{\epsilon}}.
\]
Note that the adiabatic and entropy coefficients differ
because \( \tilde{G}_{ij} \) is anisotropic. The equations of motion for
\( v_\sigma \) and \( v_\epsilon \) then follow from the action (6), reexpressed in
terms of the rescaled quantities (10). One finds
\[
v_\sigma'' - \xi v_\sigma' + \left( c_\sigma^2 k^2 - \frac{z''}{z} \right) v_\sigma - \frac{(z\xi')'}{z} v_\sigma = 0, \tag{11}
v_\epsilon'' + \xi v_\epsilon' + \left( c_\epsilon^2 k^2 - \frac{\alpha''}{\alpha} + a^2 \mu_\epsilon^2 \right) v_\epsilon - \frac{z'}{z} \xi v_\epsilon = 0, \tag{12}
\]
where
\[
\xi = \frac{a}{c_\epsilon \tilde{P}_{,\phi} c_\sigma} \left[ (1 + c_\epsilon^2) \tilde{P}_{,\phi} - c_\epsilon^2 \dot{\sigma}^2 \tilde{P}_{,\phi} \right], \tag{13}
\]
and \( \mu_\epsilon^2 \) follows from the mass matrix (7) (see [6,7]
details). We will assume that the effect of the coupling \( \xi \)
can be neglected when the scales of interest cross out the
sound horizon, so that the two degrees of freedom are
decoupled and the system can easily be quantized. In the slow-varying
regime, where the time evolution of \( H, c_\sigma \),
and \( \dot{\sigma} \) is slow with respect to that of the scale factor, one
gets \( z''/z \approx 2/\tau^2 \) and \( \alpha''/\alpha \approx 2/\tau^2 \). The solutions of (11)
and (12) corresponding to the Minkowski-like vacuum on
small scales are thus
\[
v_{\sigma k} \approx v_{\epsilon k} \approx \frac{1}{\sqrt{2kc_\sigma}} e^{-ikc_\sigma\tau} \left( 1 - \frac{i}{kc_\sigma \tau} \right), \tag{15}
\]
when \( \mu_\epsilon^2 / H^2 \) is negligible for the entropic modes (if
\( \mu_s^2/H^2 \) is large, the entropic modes are suppressed. The power spectra for \( \nu_s \) and \( \nu_i \) after sound horizon crossing therefore have the same amplitude \( P_\nu = (\kappa^3/2\pi^2)|\nu_i|^2 \).

The power spectra for \( Q_\sigma \) and \( Q_s \) are thus
\[
P_{Q_\sigma} \approx \frac{H^2}{4\pi^2c_s T^*}, \quad P_{Q_s} \approx \frac{H^2}{4\pi^2c_s^3 T^*},
\]
evaluated at the sound horizon crossing. One recognizes the familiar result of \( k \) inflation for the adiabatic part \([8,9]\), while for small \( c_s \), the entropic modes are amplituded with respect to the adiabatic modes: \( Q_s \approx Q_\sigma/c_s \).

These results can be reexpressed in terms of the comoving curvature perturbation \( \mathcal{R} = (H/\dot{\sigma})Q_\sigma \) which is useful to relate the perturbations during inflation to the primordial fluctuations during the standard radiation era. We recover the usual single-field result for the power spectrum of \( \mathcal{R} \) at the sound horizon crossing:
\[
P_{\mathcal{R}_s} \approx \frac{H^4}{8\pi^2c_s^3 T^*}, \quad P_{\mathcal{R}_s} \approx \frac{H^2}{8\pi^2 c_s^2},
\]
where \( \epsilon = -\dot{H}/H^2 = \dot{X}/X/H^2 \) (the subscript * indicates that the corresponding quantity is evaluated at the sound horizon crossing). It is then convenient to define an entropy perturbation \( S = c_s H/\dot{\sigma}Q_\sigma \), such that \( P_{S_s} \approx P_{\mathcal{R}_s} \). The power spectrum for the tensor modes is, as usual, governed by the ground trajectory in field space, but it can be parametrized by a transfer coefficient \([11]\) which appears in the formal solution \( \mathcal{R} = \mathcal{R}_s + \mathcal{T}_{RS}S_\sigma \) of the first-order evolution equations for \( \mathcal{R} \) and \( S \) which follow from (11) and (12) in the slow-varying regime on large scales. Equivalently, one can define the "transfer angle" \( \Theta \) by \( \sin\Theta = \mathcal{T}_{RS}/\sqrt{1 + \mathcal{T}_{RS}^2} \) (so that \( \Theta = 0 \) if there is no transfer and \( |\Theta| = \pi/2 \) if the final curvature perturbation is mostly of entropic origin). This implies in particular that the final curvature power spectrum can be formally expressed as \( P_{\mathcal{R}} = (1 + \mathcal{T}_{RS})P_{\mathcal{R}_s} = P_{\mathcal{R}_s}/\cos^2\Theta \). Therefore, the tensor to scalar ratio is given by
\[
r = \frac{P_{\mathcal{R}}}{P_{\mathcal{R}_s}} = 16c_s^3 \cos^2\Theta.
\]
This expression combines the result of \( k \) inflation \([9]\), where the ratio is suppressed by the sound speed \( c_s \) and that of standard multifield inflation \([5]\).

We finally turn to primordial non-Gaussianities, whose detection would provide an additional window on the very early Universe. This aspect is especially important for DBI models since it is well known that (single-field) DBI inflation produces a (relatively) high level of non-Gaussianity for small \( c_s \) \([2]\). How, therefore, do the entropic modes, whose amplitude is much larger than that of the adiabatic fluctuations, affect the primordial non-Gaussianity? In the small \( c_s \) limit, one can estimate the dominant contribution by extracting from the third-order Lagrangian the analogue of the terms giving the dominant contribution in the single-field case, but including now the entropy components. These terms are \([7]\)
\[
S^{(\text{main})}_{(3)} = \int dt d^3x \left( \alpha_1^3 a^3 \dot{Q}_\sigma + c_s^2 \dot{Q}_s \right) - \frac{a}{2c_s^2} \left[ \dot{Q}_\sigma \nabla Q_\sigma \right] - c_s^2 \dot{Q}_s \nabla Q_s + 2c_s^2 \dot{Q}_s \nabla Q_\sigma \nabla Q_s,
\]
where we have replaced \( f \) by \( 1/\dot{\sigma}^2 \) since, for \( c_s \ll 1, \dot{\sigma}^2 \approx 1 \). Following the standard procedure \([12,13,14]\), one can compute the 3-point functions involving adiabatic and entropy fields. The purely adiabatic 3-point function is naturally the same as in single-field DBI \([15,16]\). The new contribution is
\[
\langle Q_\sigma(k_1)Q_\sigma(k_2)Q_s(k_3) \rangle = -2a^3 \delta(\sum_i k_i) \frac{H^4}{4\sqrt{2\epsilon_s c_s^4}} \left[ k_1^2 k_2^3 k_3^2 - k_1^2 k_2 k_3^3 (2k_2 k_3 - k_1 K + 2K^2)
\right.
\]
\[
+ k_3^2 (k_1 \cdot k_2) (2k_1 k_2 - k_3 K + 2K^2) + k_2^2 (k_1 \cdot k_3) (2k_1 k_3 - k_2 K + 2K^2) \right],
\]
where \( K = \sum_i k_i \). By using \( \mathcal{R} = \mathcal{A}_\sigma Q_\sigma + \mathcal{A}_s Q_s \), with \( \mathcal{A}_\sigma = (H/\dot{\sigma}) \), and \( \mathcal{A}_s = T_{RS} c_s \mathcal{A}_\sigma \), one can express the 3-point function of the curvature perturbation \( \mathcal{R} \), which is the observable quantity, in terms of the correlation functions of the scalar fields. We find
\[
\langle \mathcal{R}(k_1)\mathcal{R}(k_2)\mathcal{R}(k_3) \rangle = (\mathcal{A}_\sigma)^3 \langle Q_\sigma(k_1)Q_\sigma(k_2)Q_s(k_3) \rangle + \mathcal{A}_s (\mathcal{A}_s)^2 \langle (Q_\sigma(k_1)Q_s(k_2)Q_s(k_3)) + \text{perm.} \rangle
\]
\[
= (\mathcal{A}_\sigma)^3 \langle Q_\sigma(k_1)Q_\sigma(k_2)Q_s(k_3) \rangle (1 + T_{RS}^2),
\]
where we have implicitly assumed that the vectors \( k_i \) are of the same order of magnitude (so that the slowly varying background parameters are evaluated at about the same time). As we see, the above quantity depends on the symmetrized
version of the 3-point function (19), which has exactly the same shape as in single-field DBI. Note that the enhancement of the mixed correlation \( \langle Q_\sigma Q_\sigma Q_\sigma \rangle \) by a factor of \( 1/c_s^2 \) is compensated by the ratio between \( A_\sigma \) and \( A_1 \), so that the adiabatic and mixed contributions in (20) are exactly of the same order. In principle, there are other contributions to the observable 3-point function, in particular, those coming from the 4-point function of the scalar fields, which can be reexpressed in terms of the power spectrum via Wick’s theorem [17]. The amplitude of this contribution will depend on the specific models. We implicitly ignore them in the following.

The non-Gaussianity parameter \( f_{\text{NL}} \) is defined by

\[
\langle R(k_1)R(k_2)R(k_3) \rangle = -(2\pi)^4 \delta \left( \sum_i k_i \right) \left[ \frac{3}{10} f_{\text{NL}}(R)^2 \right] 
\times \sum_i k_i^3 f_{\text{NL}}^2, \tag{21}
\]

from which we obtain, for the equilateral configuration,

\[
f_{\text{NL}}^{(3)} = -\frac{35}{108} \frac{1}{c_s^2} \frac{1}{1 + T_{RS}^2} = -\frac{35}{108} \frac{1}{c_s^2} \cos^2 \Theta. \tag{22}
\]

One can easily understand this result. The curvature power spectrum is amplified by a factor of \( (1 + T_{RS}^2) \) due to the feeding of curvature by entropy modes. Similarly, the 3-point correlation function for \( \dot{R} \) resulting from the 3-point correlation functions of the adiabatic and entropy modes is enhanced by the same factor \( (1 + T_{RS}^2) \). However, since \( f_{\text{NL}} \) is roughly the ratio of the 3-point function with respect to the square of the power spectrum, one sees that \( f_{\text{NL}} \) is now reduced by the factor \( (1 + T_{RS}^2) \). The so-called UV model of DBI inflation is under strong observational pressure because it generates a high level of non-Gaussianities that exceed the experimental bound [18,19]. We stress that their reduction by multiple-field effects may be very important for model building.

We end by revisiting the consistency condition relating the non-Gaussianity of the curvature perturbation, the tensor to scalar ratio \( r \), and the tensor spectral index \( n_T = -2\epsilon \), given in Ref. [20] for single-field DBI. In our case, substituting \( f_{\text{NL}}^{(3)} \approx -\frac{1}{3} c_s^2 \cos^2 \Theta \) in (18) gives

\[
r + 8n_T = -r(\sqrt{-3f_{\text{NL}}^{(3)} c_s^2 \cos^{-3} \Theta - 1}). \tag{23}
\]

As we can see from (22) and (23), violation of the standard inflation consistency relation [corresponding to a vanishing right-hand side in (23)] would be stronger in multifield DBI than in single-field DBI and thus easier to detect. In the multifield case, the consistency condition is only an inequality (unless \( \Theta \) is observable when the entropy modes survive after inflation).

To summarize, we have shown that both adiabatic and entropy modes propagate with the same speed of sound \( c_s \), in multifield DBI models. Both modes are thus amplified at the sound horizon crossing, with an enhancement of the entropy modes with respect to the adiabatic ones in the small \( c_s \) limit. The amplitude of the non-Gaussianities, which are important in DBI models, is also strongly affected by the entropy modes, although their shape remains as in the single-field case. All of these features are generic in any model governed by the multifield DBI action. The model-specific quantity (depending on the field metric, the warp factor, and the potential) is the transfer coefficient between the initial entropy modes and the final curvature perturbation between the time when the fluctuations cross out the sound horizon and the end of inflation. Recent analyses (see, e.g., [21]) in slightly different contexts show that this transfer can be very efficient, leading to a final curvature perturbation of entropic origin (as in the curvaton scenario). More generally, our results show that multifeid effects, common in string theory-motivated inflation models, deserve close attention as the entropy modes produced could significantly affect the cosmological observable quantities.

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