Continuous global symmetries and hyperweak interactions in string compactifications


Abstract: We revisit general arguments for the absence of exact continuous global symmetries in string compactifications and extend them to D-brane models. We elucidate the various ways approximate continuous global symmetries arise in the 4-dimensional effective action. In addition to two familiar methods - axionic Peccei-Quinn symmetries and remnant global abelian symmetries from Green-Schwarz gauge symmetry breaking - we identify new ways to generate approximate continuous global symmetries. Two methods stand out, both of which occur for local brane constructions within the LARGE volume scenario of moduli stabilisation. The first is the generic existence of continuous non-abelian global symmetries associated with local Calabi-Yau isometries. These symmetries are exact in the non-compact limit and are spontaneously broken by the LARGE volume, with breaking effects having phenomenologically interesting sizes ($\sim 0.01$) for plausible choices for underlying parameters. Such approximate flavour symmetries are phenomenologically attractive and may allow the fermion mass hierarchies to be connected to the electroweak hierarchy via the large volume. The second is the possible existence of new hyper-weak gauge interactions under which Standard Model matter is charged, with $\alpha_{HW} \sim 10^{-9}$. Such groups arise from branes wrapping bulk cycles and intersecting the local (resolved) singularity on which the Standard Model is supported. We discuss experimental bounds for these new gauge bosons and their interactions with the Standard Model particles.

Keywords: Flux compactifications, Intersecting branes models, Global Symmetries.
Symmetry principles are one of the deepest and most powerful concepts in theoretical physics. They underly both the forces and matter content of the Standard Model, these being nothing more than the local symmetry group and its representations. The presence or absence of symmetries determines the decay rate of many Standard Model particles, and can also be used to give an analytic handle on the strongly coupled physics of QCD through chiral perturbation theory. Symmetries are also integral to many ideas for physics beyond the Standard Model (for example supersymmetry) and also for understanding the structure of Yukawa couplings within the Standard Model.

String theory is a powerful set of ideas and is very promising as a conceptual structure that goes beyond effective field theory by providing a self-consistent ultraviolet completion. However the large number of string vacua make it difficult to find a decisive low-energy way to confront string theory with experiment. This diversity of vacua does not prevent some model-independent statements. In particular, while in field theory local and global symmetries are both equally natural, in string theory this is not so: global symmetries are in almost all cases gauge symmetries.

In this paper we revisit the arguments for the absence of exact global symmetries in string compactifications [1]. Our goal is to explore the reach and limitations of these arguments as a constraint on the low-energy effective actions that are relevant to phenomenology and cosmology. Owing to the importance of symmetries for understanding
low-energy physics, we regard this study as timely given the significant recent progress made in string phenomenology through the understanding of moduli stabilisation.

As the original argument of [1] was made using the heterotic string, while much current model building involves D-branes, we first analyse the effects of the open string sector keeping localised D-brane models in mind. Our main conclusion here is that, as generally believed, it is the closed string sector that forbids exact global symmetries and contains the corresponding gauge field in its massless spectrum. The open string sector does not by itself require the gauging of global symmetries and can admit continuous global symmetries. However, open string theories necessarily contain closed strings and so the result still holds.

This however opens the possibility of having approximate continuous global symmetries, which become exact in the limit where closed string modes decouple. We argue this can be achieved in local models of D-branes with very large extra dimensions. To do so we first enumerate the three possible ways of obtaining approximate global symmetries at low-energies when the microscopic theory only admits local symmetries. We then describe two distinct and new ways that string models can generate approximate global symmetries.

1. **Approximate isometries.** Although compact Calabi-Yau metrics have no isometries, non-compact local Calabi-Yau metrics do. In compact embeddings such local isometries can remain as approximate symmetries. If the localised Standard Model sector is embedded into a large bulk, approximate isometries of the localised region of the Calabi-Yau where the Standard Model resides can manifest themselves as approximate global (flavour) symmetries of the low-energy field theory describing the Standard Model interactions.

2. **Hyperweak interactions.** Gauge symmetries acting on the matter sector become indistinguishable from global symmetries in the limit of vanishing gauge couplings. In local brane constructions with large bulk volumes, branes that wrap bulk cycles and intersect with Standard Model matter can give hyperweakly coupled gauge groups that in the extreme large volume limit represent approximate global symmetries.

We explore how both possibilities arise in the phenomenologically appealing LARGE\(^1\) volume scenario developed in [2, 3]. This combines \(\alpha'\) and non-perturbative corrections in IIB flux vacua to stabilise the compactification volume at values that are exponentially large in the string coupling,\(^2\)

\[
V = \frac{\text{Volume}}{l_s^6} \sim e^{c/g_s}, \quad \text{for } c \text{ an } \mathcal{O}(1) \text{ constant.} \tag{1.1}
\]

The volume is thus essentially arbitrary and runs through an enormous range of values for small changes in the underlying parameters. The geometry resembles a Swiss cheese: in addition to the LARGE bulk volume (\(V \sim \tau_b^{3/2}\) (‘size of the cheese’), there are also small blow-up cycles of size \(\tau_s \sim \ln V\) (‘holes in the cheese’). Standard Model matter and interactions are assumed to be supported on branes wrapping a small cycle.

\(^1\)The capital ‘LARGE’ emphasises that the volume is *enormously* large compared to the string scale.

\(^2\)Unless otherwise specified, throughout this paper numerical values of lengths and volumes will all be in units of the fundamental (string) scale, \(l_s = 2\pi \sqrt{\alpha'}\).
The resulting LARGE volume is a powerful tool to generate hierarchies and predicts an interestingly complicated pattern of mass scales. Phenomenologically the most attractive value is $V \sim 10^{15}$. This brings many of the attractive features of intermediate scale string scenarios [4, 5], such as a solution to the hierarchy problem through TeV supersymmetry breaking using $m_{3/2} \sim \frac{M_{PP}}{V}$, an axion decay constant $f_a \sim 10^{11}$GeV in the allowed window [3], and a potentially natural scale for neutrino-mass generating operators $\Lambda \sim 10^{14}$GeV [1]. On the IIB side aspects of this scenario have been studied in [8 – 17].

The mirror IIA description of these models was recently constructed in [18]. While much of our discussion will be general, throughout this paper we picture the Standard Model as residing on a localised stack of branes within an exponentially large volume Calabi-Yau. In the context of large extra dimensions, flavour symmetries have been explored in [19] in the context of the ADD model, by postulating a flavour symmetry broken on a brane far away from the Standard Model brane.

This paper will focus on the conceptual aspects of realising continuous global symmetries in string theory. We will not try and construct specific singularities and brane configurations realising either the MSSM or some extension of it, nor will we discuss the bounds that FCNCs may pose for concrete models of flavour in the MSSM. One reason for this is that it is far from clear what field theory and matter content we should aim at - there are no strong theoretical reasons why there should not be either new gauge groups or new exotic matter at the TeV scale. Examples of papers focusing on the model-building aspects of local constructions are [20 – 24]. We instead look for generic features which we may hope to be phenomenologically relevant even if any particular model is not.

The paper is organised as follows. We start by enumerating in section 2 the three ways in which a global symmetry can emerge in the low-energy limit of a microscopic theory which has only local invariance. We then review and extend the Banks-Dixon argument for the non-existence of continuous global symmetries, including the effects of the open string sector. Since this latter discussion uses the language of worldsheet conformal field theory, some readers may wish to omit it. Our main results are in sections 3 and 4. In section 3 we describe how local metric isometries can generate approximate continuous (non-)Abelian global symmetries for local brane constructions in Calabi-Yau compactifications. We describe how the quality of the symmetry is set by the size of the volume and relate our analysis to more phenomenological discussions of flavour symmetries. Section 4 is dedicated to the existence of hyperweak gauge fields and their mixing with the Standard Model U(1), for which we also summarise some of the experimental bounds.

2. General arguments

This section starts by describing the general mechanisms whereby global symmetries can appear to arise in the low-energy limit of UV completions having only local symmetries. It then outlines the arguments for the absence of global symmetries in closed-string theory, together with their extensions to include the open-string sector.
2.1 Obtaining approximate low energy global symmetries

There are three ways through which (approximate) global symmetries may arise within the low energy limit of a microscopic theory which has none. On the one hand, it can happen that the low energy symmetry cannot be extended in any way to be also a symmetry of its full UV completion. Alternatively, if such an extension does exist the symmetry must be a local one for which there is a gauge boson in the spectrum. This case then subdivides into two further cases, depending on whether or not this gauge boson is lighter or heavier than the cutoff of the low-energy theory. These considerations lead to the following three categories.

1. **Emergent symmetries.** Symmetries are emergent if they arise as approximate symmetries in the low energy limit, but cannot be extended to a symmetry once the full UV particle content is included. Such symmetries (also known as ‘accidental’ symmetries [25]), can arise when the gauge invariance and particle content of the low-energy theory restrict the possible low-energy interactions so severely as to automatically ensure invariance under additional global symmetries. For instance, conservation of baryon and lepton number would automatically emerge in this way for any string vacuum whose low-energy particle content consists purely of the Standard Model particle content, even if this vacuum did not contain any combination of $B$ or $L$ as gauge symmetries.

2. **Gauge symmetries with heavy gauge bosons.** Under certain circumstances a local symmetry can resemble a global symmetry at low energies if it is spontaneously broken, with the mass of the corresponding gauge boson being too heavy to allow it to be included in the low-energy effective theory. Perhaps the simplest example consists of a U(1) gauge boson, $A_\mu$, coupled to matter fields, $\psi$ of charge $q$, as well as a Stueckelberg field, $\sigma$, describing the would-be Goldstone mode whose consumption by the gauge field gives it its mass, $M$. The lagrangian for this system is constructed from the two gauge invariant combinations, $V_\mu = A_\mu - \partial_\mu \sigma$, and $\chi = e^{-iq\sigma} \psi$, such as

$$L = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{M^2}{2} V_\mu V^\mu + \mathcal{L}_m(\chi) + ej^\mu(\chi)V_\mu + \cdots,$$

(2.1)

with $F_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu = \partial_\mu A_\nu - \partial_\nu A_\mu$, and the local symmetry allows the gauge choice $\sigma = 0$. The $\chi$-dependent quantities, $\mathcal{L}_m$ and $j^\mu$, appearing here are unconstrained by the local symmetry, and so in general can be arbitrary and in particular need not be U(1) symmetric. However it can happen that their lowest-dimension contribution (which dominates at low energy) can nonetheless have such a symmetry, such as if the only matter field is described by a Dirac spinor, for which the lowest-dimension terms in both are $\mathcal{L}_m = -m \bar{\chi} \chi - \tilde{m} \bar{\chi} \chi^c - \bar{\chi} \not\partial \chi + \cdots$. This enjoys the accidental symmetry $\delta \chi = i\omega \chi$ if $\tilde{m} = 0$, such as might perhaps be ensured by a discrete symmetry. Superconductors provide examples along these lines, for which spontaneous breaking of electromagnetic gauge invariance does not imply electric charge non-conservation.
3. **Gauge symmetries with light gauge bosons.** It can also happen that the gauge boson of
the local symmetry is lighter than the cutoff, but is so weakly coupled that its presence
could go unremarked in the low-energy theory. This resembles a global symmetry
because gauge symmetries acting on the matter sector become indistinguishable from
global symmetries in the limit of vanishing gauge couplings. So one might expect to
find string vacua having approximate global symmetries acting on particular sectors
of the low energy theory, provided the spectrum also includes very weakly coupled,
light spin-one gauge bosons.

**Previous examples from string theory.** Despite the absence of continuous global
symmetries in string theory, there are some well known ways that exploit the above options
to obtain approximate continuous global symmetries of the low-energy effective field theory.

1. **Peccei-Quinn symmetries.** The massless spectrum of all string theories include RR
and/or NS-NS forms that lead to axion fields in the 4D effective field theory. These
resemble Stueckelberg fields, with a corresponding shift symmetry. This is a pertur-
bative abelian symmetry and is broken by non-perturbative effects.

2. **Nonlinearly realised Abelian gauge symmetries.** String theory also provides examples
of gauge bosons that are massive at the string scale and with abelian global sym-
metries below the scale of the gauge boson mass. The low-energy description in this
case involves coupling the electromagnetic gauge boson to a (Stueckelberg) Goldstone
boson, \( \sigma \), as above. In string models the origin of these low-energy abelian global
symmetries is usually the \( B \wedge F \) Green-Schwarz coupling induced by anomalous or
non-anomalous U(1)'s, which upon dualisation induces the \( V^\mu V_\mu \) coupling above. The
corresponding U(1) becomes massive by the Stueckelberg mechanism without nes-
cessarily having a Higgs field to break the symmetry. The corresponding U(1) therefore
survives in the low energy effective theory as a remnant global symmetry. As with
the Peccei-Quinn symmetry, this symmetry is also broken by non-perturbative ef-
fects. In brane models examples of these U(1)s can be combinations of baryon and
lepton numbers.

### 2.2 Continuous global and gauge symmetries in string theory

It is well-known that except in very special circumstances, in string theory no continuous
global symmetries exist and all spacetime symmetries are gauged \[ \square \]. We now review the
closed string arguments presented in \[ \square \textbf{20} \text{3} \text{ and discuss their extension to open strings.}

#### 2.2.1 Closed strings

In string theory spacetime symmetries necessarily originates from world-sheet symmetries,
which then have a Noether current. Consider the case where the holomorphic and anti-
holomorphic parts of the Noether currents are independently conserved. The symmetry

---

\[ \text{The proof in } \square \text{ assumes that the Noether current does not contain world-sheet ghosts fields. This}
assumption is removed in the discussion in } \square .\]
comes in a pair and the corresponding generator of the world-sheet symmetry has the form
\[ Q = \frac{1}{2\pi i} \oint dz \frac{j(z) - d\bar{z} j(\bar{z})}{z - \bar{z}}, \] (2.2)
assuming that we have integrated over the superspace Grassmann coordinates in a super-
string theory. Since \( Q \) generates a physical symmetry, it must commute with the BRST
charge and additionally be conformally invariant, implying \( j \) and \( \bar{j} \) must transform as con-
formal fields with conformal dimensions \((1, 0)\) and \((0, 1)\). Consider for concreteness bosonic
string theory\(^4\) and the pair

\[ V = \int d^2 z \, j \, \partial X^\mu \exp(ik.X), \quad \bar{V} = \int d^2 z \, \bar{j} \, \partial X^\mu \exp(ik.X). \] (2.3)

The integrands have conformal dimension \((1, 1)\) for \( k^2 = 0 \). Both \( V \) and \( \bar{V} \) are conformally
invariant, and inherit BRST invariance from \( j \) and \( \bar{j} \). They are thus a pair of valid vertex
operators corresponding to massless vectors which gauge the pair of symmetries.

There are two exceptions to the above discussion. The first concerns axionic symme-
tries. All worldsheet fields are uncharged under these and so the charge \( Q \) vanishes. In
this case the axionic symmetry exists as a perturbative symmetry of the low energy action,
broken non-perturbatively by instantons, under which matter fields do not transform.

A second exception is the Lorentz symmetry of uncompactified space-time. The holo-
morphic and anti-holomorphic parts of the corresponding world-sheet Noether currents are
not conserved independently. This stems from the non-compact nature of spacetime, which
gives rise to an infrared divergence in the Green’s function. The Ward identity
\[ \partial \langle T(z)j(w)j(0) \rangle = \delta^2(z-w)(z\partial_z + 1)\langle j(z)j(0) \rangle + \delta^2(z)\langle j(z)j(0) \rangle \] (2.4)
cannot be integrated [1]. In other words, the corresponding \( j \) and \( \bar{j} \) do not transform as
conformal fields and the resultant \( V \) and \( \bar{V} \) are not valid vertex operators. The existence
of the global Lorentz symmetry is therefore tied to the presence of a non-compact target
space in the sigma model.

2.2.2 Open strings in a general system of D-branes

Open strings are described by boundary conformal field theories (for a review see for
example [27]). The boundary is a collection of points where the open string ends and the
conformal boundary condition is given by
\[ T_{ab} n^a t^b = 0, \] (2.5)
where \( T_{ab} \) is the world-sheet energy-momentum tensor, and \( n^a, t^b \) are world-sheet vectors
that are respectively parallel and transverse to the boundary. The boundary condition
states that the off-diagonal component of \( T_{ab} \), with mixed parallel and perpendicular in-
dices, should vanish. This requirement does not distinguish Neumann or Dirichlet boundary

\(^4\)In a superstring theory we could replace \( \partial X^\mu \) by \( \psi^\mu e^{-\phi} \), where \( \psi^\mu \) are the world-sheet fermions and \( \phi \)
the super-ghost and the discussion would follow.
conditions. For a Neumann boundary, for example, the condition (2.3) implies that there is no momentum flowing across the boundary, as is expected of a free end of an open string. It is convenient to work with Euclidean world-sheets and define complex coordinates \( z, \bar{z} \).

Using world-sheet re-parametrization and conformal invariance, the boundary can be generally taken to be the real axis, and the CFT defined only in the upper half-plane. The doubling trick can be applied \([26, 27]\), which essentially means that the (anti)-holomorphic modes are reflected off the boundary. The important difference with closed strings is that open string vertex operators are inserted at the boundary. Therefore the integral is one dimensional. In our coordinates it takes the form

\[
V_{\text{open}} = \int_{\text{Im}(z)=0} dz \, V(z). \tag{2.6}
\]

Conformal invariance of \( V_{\text{open}} \) requires that \( V(z) \) has conformal dimension \((1,0)\). An open string vertex operator is thus half of a closed string vertex operator. Any operator that takes the form

\[
A(z) = \partial X^\mu : \exp (ik . X) : \tag{2.7}
\]

therefore already has conformal dimension \((1,0)\) for \( k^2 = 0 \), and there is no room for the insertion of extra world-sheet currents as in (2.3). The rotational symmetry of the Chan-Paton factors does not have world-sheet currents - in fact, this is not a symmetry of the world-sheet as the open string end-points are charged under the Chan-Paton rotation and are shifted around. We can construct a gauge field vertex operator by attaching the Chan-Paton labels, which do not carry conformal dimension, to \( A \),

\[
V_{\text{gauge}}(z) = \lambda_{\alpha\beta} A(z). \tag{2.8}
\]

For other world-sheet symmetries with non-trivial local world-sheet currents, the charges and currents have the same properties as in (2.2) and the currents \( j \) must have conformal dimension \((1,0)\) and be BRST invariant. These symmetries are not gauged in the open string sector. An example is the rotational symmetries of the transverse directions of a D-brane, which are ungauged independent of whether or not these directions are compactified. This rotational symmetry is however shared with the closed strings which can supply the gauge fields if the directions are compact.

One slightly different case also deserves a comment. As each open string has two endpoints, there are up to two boundaries that satisfy different boundary conditions. These can be taken as the positive and negative real axis, with the boundary changing vertex operator \( V_{\text{BC}} \) inserted at the origin. This scenario corresponds to open strings connecting different D-branes. The previous discussion concerning open strings ending on the same brane will continue to apply, except that now the insertion of \( V_{\text{BC}} \) in every vertex operator takes up further conformal dimension (see \([28]\) for explicit examples with intersecting branes). It is now not even possible to form a Lorentz vector by inserting \( A \) as in (2.8), and these open strings describe matter fields transforming in bi-fundamental representations \((P, Q)\).

To summarise, world-sheet symmetries that give rise to non-trivial local world-sheet currents are not gauged in the open string sector. In fact the presence of the boundary reduces the symmetry in the open string sector compared to the closed string sector,
as e.g. D-branes break translational symmetries. Since boundary conditions do not affect local properties of the world-sheet, these symmetries are automatically shared with closed strings, which can supply the gauge fields. In the low energy limit where closed strings decouple, these symmetries become approximately global - for example the SU(4) R-symmetry on the world-volume theory of D3 branes.

We conclude that in string theory the closed string sector is the source of massless vectors that gauge continuous global symmetries. This fits well with the standard claim that gravitational theories naturally break continuous global symmetries as the corresponding charges are not conserved by the process of black hole evaporation. As it is the closed string sector that gives gravity it is only this sector that is expected to forbid global symmetries.

Local D-brane models can have approximate global symmetries, and these are broken only by the interaction of the branes with the geometry of the extra dimensions. As the extra dimensions become very large and the bulk decouples, the quality of the symmetry increases. In the limit that the bulk decompactifies, these approximate symmetries take the same status as the Lorentz symmetries of 4-dimensional spacetime and become exact.

Remarks on non-CFT constructions

The above argument relies on world-sheet CFT techniques appropriate to weakly coupled string theory. To discuss the strongly coupled regimes, one would have to use string theory duality relations, and non-CFT models such as M and F-theory. Internal structures and symmetries of the lower dimensional theories are encoded as geometrical data in M/F theory, by specifying the compactification manifolds. Symmetries are realised as symmetries of the manifolds. While we have far less understanding and control of M/F theories, it is generally believed that they are gravity theories in the low energy limit. Therefore, at least in the low energy limit where concrete statements can be made about these theories, the geometrical symmetries are gauged by gravity. In the limit where gravity is essentially decoupled\footnote{This is the regime discussed in recent F-theory constructions of quasi-realistic models - see for example recent discussions in \cite{23, 24, 29}.}, these symmetries remain as approximate global symmetries as in perturbative string theory.

3. Continuous global flavour symmetries

One of the main phenomenological applications of flavour symmetries is to the problem of understanding the masses and mixings of the Standard Model fermions. The fermion masses, plotted in figure 1, are strongly hierarchical with inter-generational mass ratios approximately $O(100) : 1$. This structure strongly suggests a deeper organising principle, and arguably the most appealing idea is the existence of approximate flavour symmetries under which Standard Model is charged.

The previous section has reviewed some ways to obtain approximate global symmetries. However these methods cannot generate realistic flavour models: Standard Model matter is uncharged under the axionic symmetries and in brane models bifundamental quark doublets must have identical charges under anomalous $U(1)$s. This is because the $U(1)$ charges of
the quark doublets are fixed by the branes they are connect, which are identical, and so the three generations necessarily have the same U(1) charge assignments. This is too restrictive for obtaining realistic Yukawa textures, using (for example) the Froggatt-Nielsen mechanism \[35\].

The arguments excluding exact global symmetries do not preclude the existence of approximate global symmetries. For local brane constructions of the Standard Model - this is necessarily the case within the LARGE volume models - physics is determined by the local geometry and metric. From the viewpoint of the local model, the metric is that of a non-compact Calabi-Yau. There are many known examples of such metrics and they often have isometries. This is familiar from the AdS/CFT correspondence, where isometries of the local metric corresponds to global flavour or R-symmetries in the field theory. Examples of local Calabi-Yau metrics and their isometries are:

1. The local geometry $\mathbb{C}^3$ with isometry $\text{SO}(6)$,
2. The conifold, $\sum_i z_i^2 = 0$ with local isometry $\text{SU}(2) \times \text{SU}(2) \times \text{U}(1)$ \[30\],
3. The resolved $\mathbb{C}^3/\mathbb{Z}_3$ singularity, $\mathcal{O}_{\mathbb{P}^2}(-3)$ with local isometry $\text{SU}(3)/\mathbb{Z}_3$ \[31, 32\].

For local brane models embedded in these geometries, isometries correspond to flavour symmetries. For D3/D7 magnetised brane models we can view flavour as arising from different solutions of the Dirac equation. Isometries rotate solutions of the Dirac equation into each other thereby acting as a flavour symmetry. We note that such symmetries can only hold for local models, as global models rely on the whole Calabi-Yau and compact Calabi-Yau metrics have no continuous isometries. Even for local models, the local isometry is not expected to survive if the size of the compact space is $O(1)$ in string units - this includes conventional GUT unification models - as the local metric will then receive large corrections from the bulk. However in a non-compact limit,\footnote{Here we refer to a limit in which the scales of the local geometry are left unaltered while the bulk volume is taken to infinity.} the local metric is exact and the field theory has an exact global flavour symmetry.

This is attractive for LARGE volume, as in this case the full local metric can be sensibly viewed as a perturbation on the non-compact result and the exact isometry of the
non-compact case will survive as an approximate isometry of the LARGE volume limit. As it is generic for local Calabi-Yau metrics to have continuous isometries, this implies that in the context of the LARGE volume scenario, one should generically expect the low-energy matter couplings to be governed by approximate continuous flavour symmetries. We stress that this argument says nothing about the exact nature of the local gauge group or matter content, or precisely how the flavour symmetries act. These depend on the details of the singularity - different singularities have different isometries - and the local brane construction.

The flavour symmetries act as selection rules on the superpotential. This is familiar from the AdS/CFT correspondence and the study of branes at singularities. As a well known example, the theory of D3 branes on the conifold has a global SU(2) × SU(2) × U(1) symmetry [33]. In cases where the bulk is compact, the flavour symmetry ceases to be exact. This will manifest itself as corrections to the Kähler potential (the flavour symmetry breaking is carried by the Kähler moduli which cannot appear in the perturbative superpotential). We can estimate the size of the breaking parameter. In local brane models two scales parametrise the geometry, the length scale of the local metric ($R_s$) and the length scale of the bulk ($R_b$). These are related to the sizes of 4-cycles by $\tau_s = R_s^4$ and $\tau_b = R_b^4$.

As stringy effects will modify the classical geometry at small volume, we impose a cutoff $R_s \geq l_s$ on the effective local length scale. A dimensionless measure of breaking is set by the ratio of local and global length scales, $R_s/R_b$. Roughly, $R_b$ is the distance one needs to go to feel the bulk metric, and so $R_s/R_b$ measures the effect of the bulk on the local geometry. Global rescalings of the metric, $g_{i\bar{j}} \rightarrow \lambda^2 g_{i\bar{j}}$, leave solutions to the Dirac equation unaltered and thus do not modify the flavour structure. The order parameter for flavour symmetry breaking is then $R_s/R_b$.

In phenomenological models based on the LARGE volume scenario, the size of the overall volume generates the weak scale using $m_{3/2} = M_P W_0 / V$. Assuming that the flux superpotential $W_0 \sim 1$,

$$V \sim \frac{R_b^6}{l_s^6} \sim 10^{14}.$$  \hspace{1cm} (3.1)

In contrast the small cycle supports the Standard Model and has a size $\tau_s \sim R_s^4 \sim 20 l_s^4$. As the bulk radius is $R_b \sim 100 l_s$ and the local radius $R_s \sim l_s$, the expansion parameter for the breaking of local isometries is $R_s/R_b \sim 0.01$. While the absence of any explicit construction of a local Standard Model configuration precludes more detailed analysis, such an expansion parameter is not inconsistent with the observed pattern of fermion masses.

An attractive feature of this scenario is that it may allow the fermion mass hierarchies of the Standard Model to be generated by the same physics, namely the exponentially large volume, that is used to generate the weak hierarchy. The approximate flavour symmetry is geometric in origin and comes from the same geometric feature - LARGE volume - used to generate the weak hierarchy. Note that this is somewhat similar to what occurs in phenomenological models of Randall-Sundrum scenarios with fermions in the bulk [34], where

\footnote{As we are interested in flux compactifications involving D7 branes, it is 4-cycles that are the natural unit in which to describe the Kähler moduli.}

\footnote{We note however that it is unclear what power of the expansion parameter ($R_s/R_b$) is appropriate for the Yukawa couplings.}
extra-dimensional fermion profiles are separated using the same warping that generated the weak scale hierarchy.

We finally relate our analysis to the discussion of [1] and section 2 of global flavour symmetries in string theory. As seen above, the proof that string compactifications have no continuous global flavour symmetries relies on the compactness of the internal 2d worldsheet conformal field theory, and specifically on the fact that the spectrum of operator conformal dimensions is bounded from below. If the spectrum is non-compact (for example Minkowksi space time) then global symmetries can and do exist (for example the Lorentz group). However, and as described in [1], in cases where the radius of the internal CFT is much larger than the string scale approximate continuous global symmetries can exist as to the sigma model the space looks locally non-compact. For the weakly coupled heterotic string then in vogue, this large-radius limit is incompatible with a viable phenomenology. However, this limit is perfectly viable with the advent of D-branes and local brane constructions.

**Connection to phenomenological discussions**

The above discussion of flavour symmetries has been geometric and we would like to connect it to the phenomenological literature [35 – 37]. One approach [35] starts by postulating a new gauged flavour symmetry $G_F$ (a global symmetry would give rise to a unobserved familon Goldstone boson), and new flavon fields $\Phi$ that are charged under $G_F$ and neutral under the Standard Model. Standard Model matter $X_i$ is charged under both $G_F$ and $G_{SM} = SU(3) \times SU(2) \times U(1)$. The superpotential is

$$W = \sum_{\alpha, i} Y_{\alpha \beta ... ijk} \frac{\Phi_\alpha \Phi_\beta}{M_X} X_i X_j X_k. \quad (3.2)$$

The flavon fields acquire vevs, spontaneously breaking the flavour symmetry and generating the fermion mass hierarchy. The expansion parameter for flavour symmetry breaking is $< \Phi >/M_X$.

What are the flavons in our case? $\tau_s$ and $\tau_b$ are the only low-energy 4-dimensional fields which affect the local geometry. As flavour symmetry breaking is controlled by the ratio $\tau_s/\tau_b$ we would like to identify the flavons with this ratio. However this is not possible: the isometry group may be non-Abelian - for example $SU(3)/\mathbb{Z}_3$ for the $\mathbb{C}^3/\mathbb{Z}_3$ singularity - while this ratio is a real singlet and can therefore only be in the trivial representation of the flavour group.

The answer is somewhat subtle. The presence of the flavour symmetry depends on an approximate isometry of the local higher-dimensional metric. Whatever the full local metric is, we can regard it as a perturbation on the local metric applicable to the non-compact case.\footnote{For local models of branes at (resolved) singularities, we again stress that by the non-compact limit we refer to a limit which keeps the sizes of local cycles fixed while taking the bulk volume to infinity.}

$$g_{MN,\text{local}}(y) = g_{MN,\text{local}},_{V \to \infty}(y) + \delta g_{MN,\text{local}},_{V \text{ finite}}(y). \quad (3.3)$$
Here $g_{MN,\text{local}}$ is invariant under the flavour symmetry while $\delta g_{MN}$ carries the form of flavour symmetry breaking. For example, if we imagine a D7 brane wrapping the $\mathbb{C}P^2$ of the $\mathcal{O}_{\mathbb{P}^2}(-3)$ resolution of $\mathbb{C}^3/\mathbb{Z}_3$, then $g_{MN,\text{local}}(\mathbb{C}P^2)$ is the Fubini-Study metric and $\delta g_{MN}(\mathbb{C}P^2)$ are deformations away from the Fubini-Study metric that scale as $(R_s/R_b)^k$ for some $k$. From a four dimensional perspective, the fields that excite $\delta g_{MN} \neq 0$ are higher-dimensional modes of the metric, namely Kaluza-Klein modes, and it is these modes that are charged under the flavour symmetry.

It is not the size of the bulk per se that breaks the local SU(3)/$\mathbb{Z}_3$ flavour symmetry, but instead the form of the local metric. However, by Yau’s theorem the full Calabi-Yau metric is completely specified by the moduli. The complete form of the local metric is therefore fully determined by the relative sizes of the bulk and local cycles. The IR and UV degrees of freedom are inter-related - although the moduli are light IR modes, the vevs of the moduli determines the masses, vevs and mixings of the Kaluza-Klein modes. The KK modes are the UV degrees of freedom which act as flavons and whose vevs break the flavour symmetry. A notable difference compared to conventional flavour models is that while there are an infinite number of flavons in total, there are in fact no flavons within the four dimensional effective field theory. Once the flavons are integrated out, flavour symmetry breaking is parametrised only by the ratio $\frac{R_s}{R_b}$ measuring the relative sizes of bulk and local cycles.

Isometries and gauge symmetries

We would also like to relate our discussion of isometries to that in Kaluza-Klein reduction. It is well known that in traditional Kaluza-Klein reduction isometries of the extra dimensions give gauge symmetries in lower dimensions, with the lower dimensional gauge group determined by the isometry group. In our case we have approximate local isometries, which we might expect to be associated to approximately massless gauge bosons. We would expect that in the limit that the isometry becomes exact, the gauge boson should become massless. As the symmetry is geometric in nature such fields should be geometric modes that are increasingly massless in the large volume limit.

Large volume. Notice that the isometries are isometries only of the local geometry and not of the full global metric. Indeed, the isometries are not respected in any way by the bulk geometry. The local metric has characteristic length scale $R_s$ whereas the bulk metric has characteristic scale $R_b$. Local excitations of the geometry with length scale $R_s \lesssim R < R_b$, corresponding to energy scales $(R_s^2\alpha')^{-1} < E^2 < (R_b^2\alpha')^{-1}$, see the isometry as a good symmetry and on these energy scales there is still an approximately massless mode. However when the characteristic length scale of an excitation reaches $R_b$, the isometry ceases to be a good symmetry and the mode acquires a mass,

$$M \sim M_{KK,bulk} \sim \frac{M_{\text{string}}}{R_b} \sim \frac{M_{\text{string}}}{\tau_b^{1/4}}. \quad (3.4)$$

Such modes are bulk KK modes, which are hierarchically lighter than the characteristic
scale of localised geometric modes,
\[ M_{KK, local} = \frac{M_{\text{string}}}{R_s} \sim \frac{M_{\text{string}}}{\tau_s^{1/4}}, \tag{3.5} \]
set by the curvature scale of the local geometry.

In that we wish to interpret the approximate local isometry as a higher-dimensional gauge symmetry, this seems to single out bulk vector KK modes as the approximately massless gauge bosons. These modes correspond to extending the local isometry to transformations of the full compact space including the bulk region. The mass scale of these modes is hierarchically lighter than those of the local geometry,
\[ \frac{M_{KK, \text{bulk}}}{M_{KK, \text{local}}} \sim \frac{R_s}{R_b} \ll 1 \text{ for } R_b \gg 1. \tag{3.6} \]

On the other hand the low-energy Standard Model fields have masses bounded by the gravitino mass \( m_{3/2} \sim M_{\text{string}}/R_b^3 \) and therefore their splittings due to the breaking of the approximate global symmetry are much smaller than the mass of the vector KK modes, justifying the existence of the approximate global symmetry in the low-energy effective theory.

However there are several reasons to be cautious as to the utility of interpreting massive bulk KK modes as gauge bosons. Most notably, these are not symmetries for compact Calabi-Yau spaces: there is no locus in parameter space where there actually exist massless gauge fields with finite coupling. Regarded as a symmetry, the isometry never extends to the bulk, which sets the compactification scale.\(^\text{10}\) This contrasts with for example KK reduction on \( M_4 \times T^6 \). The flat \( T^6 \) has isometries and massless gauge bosons. However even if the isometry is not manifest and the \( T^6 \) metric is highly curved, there still exists a regime of parameter space where the metric is flat, and so every point in parameter space can be viewed as a (possibly large) perturbation on a limit containing massless gauge bosons at finite coupling.

The second difficulty is that even if we do identify bulk KK modes with approximately massless gauge bosons, there is never a gauge symmetry in the four-dimensional effective theory. While from a higher-dimensional perspective it may make sense to describe KK modes as approximately massless in the limit \( R_b/R_s \gg 1 \), within 4d effective field theory such a description is rather eccentric. From the viewpoint of the 4-dimensional effective action, the isometry is simply an approximate global symmetry of the field theory.

We can cast this in the terminology of the list of section 2.1. From the viewpoint of the higher-dimensional effective theory, the vector bosons associated with approximate isometries are the bulk KK modes. These are in the higher dimensional effective field theory, but have couplings to the observable sector of interest that are suppressed (as in option 3 of the list of §2.1). By contrast, in the low energy 4D effective theory all KK modes are integrated out. The global symmetries can be viewed as an examples of the second category of section 2.1. In the low energy spectrum the approximate isometry simply acts as an approximate, possibly nonabelian, global symmetry.

\(^{10}\text{It is a mathematical question whether, irrespective of considerations of supersymmetry, there exists any metric on a space that is topologically a compact Calabi-Yau that has the local isometry group manifest as a isometry of the entire compact metric. These statements assume that this is not possible.}\)
Warped throats. We note that warped throats can provide similar examples of low-energy symmetries whose origins ultimately lie in the approximate isometries of the strongly warped regions. In this case although the bulk breaks these isometries, the resulting symmetry-breaking interactions only arise suppressed by powers of the small warp factor for observers in the throat. Powers of the warp factor also enter into the mass of the corresponding KK gauge boson, but these need not be the same powers, allowing symmetry breaking scales to be suppressed relative to KK scales for brane modes within the throat. It should be possible to quantify these statements using a careful analysis of the low energy effective action describing the interactions of the Standard Model fields at the tip of the throat.

It is interesting to observe that both the above examples of approximate isometries occur for geometries which naturally generate scales hierarchically lower than the Planck scale. One can speculate whether this may in fact be a generic feature, and whether the systematic presence of approximately global geometric symmetries can be tied to the presence of new scales hierarchically lower than the 4-dimensional Planck scale.

4. Hyper-weakly coupled gauge groups

We now describe another phenomenon that can arise in local brane models, namely hyper-weak interactions. While these are related to the above discussion of approximately global flavour symmetries, they offer novel phenomenological possibilities and are interesting in their own right.

In string constructions it is necessary that all Standard Model gauge factors are supported on cycles with sizes close to the string scale. This follows from the known size and running of the Standard Model gauge couplings,

\[ \alpha^{-1}(m_s) = \frac{4\pi}{g^2(m_s)} \sim 20, \]

and that for a gauge group supported on (for concreteness) D7 branes wrapping a 4 cycle \( \Sigma \),

\[ \frac{4\pi}{g^2(m_s)} = \text{Re}(T_s), \]

where \( \text{Re}(T_s) = e^{-\phi} (\text{Vol}(\Sigma)/l_s^4) \).

However, generally we expect additional branes in the compactification beyond simply those of the Standard Model, leading to additional hidden sectors. One particularly interesting possibility is the existence of bulk branes that wrap cycles supported on the entire Calabi-Yau. On dimensional grounds, the size of such a 4-cycle is \( \tau_b \sim \mathcal{V}^{2/3} \) which for LARGE volume leads to extremely weak gauge couplings. Numerically, for the case of \( \mathbb{P}^4_{[1,1,1,6,9]} \) (this Calabi-Yau has featured prominently in studies of the LARGE volume scenario) \( \tau_b = 162^{-1/3}\mathcal{V}^{2/3} \sim 0.18\mathcal{V}^{2/3} \). Requiring a volume of \( \mathcal{V} \sim 10^{14} \) to generate TeV soft terms, a bulk brane will generate a gauge coupling

\[ \frac{4\pi}{g^2} = \tau_b \sim 0.18 \times \mathcal{V}^{2/3} \sim 4 \times 10^8, \]
corresponding to $g \sim 2 \times 10^{-4}$. This produces a force characterised by exceptionally weak coupling. This geometric picture is illustrated in figure 2.

We therefore see that the existence of a hyper-weak gauge boson is a general feature of LARGE volume models. The phenomenology will depend on the mass scale of such a hyper-weak boson and how it mixes with the Standard Model photon. If the hyper-weak boson is entirely decoupled from the Standard Model then it will play little role. However we note that in phenomenological models of branes at singularities, such as those developed in [20], there often exist flavour D7s, which wrap bulk cycles but directly intersect the local branes carrying the Standard Model gauge group.

Let us enumerate possible mass scales for the hyper-weak gauge boson. If the hyper-weak U(1) is coupled to the quark sector, then the condensate $\langle \bar{q}q \rangle \sim \Lambda_{\text{QCD}}^3$ will spontaneously break the U(1), leading to

$$m_{Z'} \sim g\Lambda_{\text{QCD}} \sim 10^{-4} \times 100 \text{ MeV} = \mathcal{O}(10) \text{ keV}. \quad (4.4)$$
If the breaking arises from the MSSM down-type Higgs, then assuming $\tan \beta \sim 10$ the expected mass scale is

$$m_{Z'} \sim 10^{-4} \times 20 \text{ GeV} \sim 1 \text{ MeV}. \quad (4.5)$$

Finally, if $U(1)_{\text{HW}}$ is broken by the Standard Model Higgs, or the MSSM up-type Higgs, the expected mass scale is

$$m_{Z'} \sim 10^{-4} \times 246 \text{ GeV} \sim \mathcal{O}(10) \text{ MeV}. \quad (4.6)$$

The hyper-weak boson may also be coupled to hidden Standard Model-like sectors from different regions of the bulk, generating $m_{Z'} \sim (10^{-4} \to 10^{-5}) \times v_{\text{hidden}}$. If $v_{\text{hidden}}$ is the weak scale - which is the scale of supersymmetry breakdown and the scale at which soft terms are generated - this again gives a mass for the hyper-weak boson of $\sim 10 \text{ MeV}$.

We now discuss the phenomenology and current experimental bounds on hyper-weak interactions with different boson masses. In general, massless hyper-weak bosons are tightly constrained by fifth force experiments. For MeV scale vector boson, our discussion mainly follow a series of papers by Fayet [39–42]. We will see that low energy precision observables, such as $g_e - 2$ and $g_\mu - 2$ and $e - \nu$ scattering, constrains the coupling of the lepton sector to the hyper-weak $U(1)$. Axion-like decay of the $\Upsilon(b\bar{b})$ and $\psi(c\bar{c})$ mesons provide tight constraints on the coupling of the quarks, whereas atomic parity violation constrain coupling product of the quark and electron. We shall see that many of these bounds are at comparable orders of magnitude to the generic expectation of the size of the hyper-weak coupling in the LARGE volume scenario. We also discuss the possibility of having light dark matter residing on the bulk brane. Finally, the experimental constraints on hyperweak gauge bosons with electroweak or higher scale masses are briefly reviewed. These constrains mainly come from electroweak precision observables and collider searches and as expected are rather mild.

### 4.1 Mixing between the visible sector and the hyper-weak $U(1)$

In explicit string constructions of quasi-realistic models it is very common to have matter sectors living on different branes. There are usually several $U(1)$ gauge fields that have to be properly normalised. Even if different $U(1)$’s can be normalised to be orthogonal, loop corrections will induce mixing between different $U(1)$’s. These will then induce an effective charge to the Standard Model fields with respect to an otherwise hidden $U(1)$. The phenomenology of light weakly coupled gauge bosons has been studied in some detail in [41–46], and depends in an important way on the relative size of the mixings and
mass terms. Consider, for instance, a system of two 4D gauge bosons, \( A_{\mu}^a \), subject to the following Lagrangian

\[
\mathcal{L} = -\frac{1}{4} Z_{ab} A_{\mu}^a A_{\nu}^b - \frac{1}{2} M_{ab}^2 A_{\mu}^a A_{\mu}^b - j_a^\mu A_{\mu}^a, \tag{4.7}
\]

where \( A_{\mu\nu}^a = \partial_\mu A_{\nu}^a - \partial_\nu A_{\mu}^a \) and \( j_a^\mu \) denote the currents to which each gauge boson couples. Positive kinetic energy requires the matrix \( Z_{ab} \) to be positive definite.

The kinetic and mass terms may be diagonalised in the usual way, through the transformation

\[
A_{\mu}^a = N_{ab} A_{\mu}^b = \left( Z^{-1/2} \right)^a_c R_{cb} A_{\mu}^b, \tag{4.8}
\]

where

\[
R = \begin{pmatrix}
\cos \theta & -\sin \theta \\
\sin \theta & \cos \theta
\end{pmatrix}, \tag{4.9}
\]

denotes the two-by-two rotation matrix that diagonalises the symmetric matrix defined by 
\( \hat{M}^2 = Z^{-1/2} M^2 Z^{-1/2} \).

Explicitly, taking without loss of generality

\[
Z = \begin{pmatrix}
1 & \lambda \\
\lambda & 1
\end{pmatrix}
\text{ and } M^2 = \begin{pmatrix}
m_A^2 \\
\mu^2 \\
m_B^2
\end{pmatrix}, \tag{4.10}
\]

we have

\[
Z^{-1/2} = \sec 2\alpha \begin{pmatrix}
\cos \alpha & -\sin \alpha \\
-\sin \alpha & \cos \alpha
\end{pmatrix}, \tag{4.11}
\]

where the angles \( \theta \) and \( \alpha \) are given by

\[
\sin 2\alpha = \lambda \quad \text{and} \quad \tan 2\theta = \frac{2\mu^2 - (m_A^2 + m_B^2)\lambda}{(m_A^2 - m_B^2)\sqrt{1 - \lambda^2}}. \tag{4.12}
\]

The corresponding mass eigenvalues then are

\[
M_{\pm}^2 = \frac{m_A^2 + m_B^2 - 2\mu^2 \pm \Delta}{2(1 - \lambda^2)}, \tag{4.13}
\]

where

\[
\Delta^2 = (m_A^2 - m_B^2)^2 + 4\mu^4 - 4\mu^2 (m_A^2 + m_B^2) + 4\lambda^2 m_A^2 m_B^2. \tag{4.14}
\]

The final Lagrangian is

\[
\mathcal{L} = -\frac{1}{4} A_{\mu\nu}^a A_{\mu\nu}^a - \frac{1}{2} \left( M_{\mp}^2 A_{\mu}^+ A_{\mu}^\mp + M_{\pm}^2 A_{\mu}^+ A_{\mu}^- + N_{ab} j_a^\mu A_{\mu}^b \right), \tag{4.15}
\]

where the combined redefinition is

\[
N = \frac{1}{\sqrt{1 - \lambda^2}} \begin{pmatrix}
\cos(\theta + \alpha) & -\sin(\theta + \alpha) \\
\sin(\theta - \alpha) & \cos(\theta - \alpha)
\end{pmatrix}. \tag{4.16}
\]

Several limiting cases are of particular interest.
• In the absence of kinetic mixing, $\lambda \to 0$, we have $\alpha \to 0$ and so $N \to R$ reduces to the usual rotation, with mixing angle $\tan 2\theta = 2\mu^2 / (m_A^2 - m_B^2)$.

• If $m_A^2 = \mu^2 = 0$, then one of the gauge fields is massless, $M_1^2 = 0$, while the other has mass $M_2^2 = m_B^2 / (1 - \lambda^2)$ and $\sin 2\theta = \sin 2\alpha = \lambda$. In this case only the massive vector acquires a coupling to both currents

$$L_{\text{int}} = j^\mu_{1A} A^-_\mu + \frac{1}{\sqrt{1 - \lambda^2}} (j^\mu_{2A} - \lambda j^\mu_{1A}) A^+_\mu.$$ (4.17)

• If the entire mass matrix vanishes, $M_{ab}^2 = 0$, then we may take $R = 1$ and so $\theta = 0$. In this case $N = Z^{-1/2}$ and so both gauge bosons in general couple to both currents

$$L_{\text{int}} = \cos \alpha \frac{1}{\sqrt{1 - \lambda^2}} (j^\mu_1 A^-_\mu + j^\mu_2 A^+_\mu) - \sin \alpha \frac{1}{\sqrt{1 - \lambda^2}} (j^\mu_1 A^+_\mu + j^\mu_2 A^-_\mu),$$ (4.18)

with $\sin^2 \alpha = \frac{1}{2} \left[ 1 - \frac{1}{\sqrt{1 - \lambda^2}} \right]$ and $\cos^2 \alpha = \frac{1}{2} \left[ 1 + \frac{1}{\sqrt{1 - \lambda^2}} \right]$ (and so $\sin 2\alpha = \lambda$, as above).

These limits show that the nature of the bound on new gauge bosons depends in an important way on whether or not the boson masses play a significant role. Since the photon mass is known not to be larger than $3 \times 10^{-36}$ GeV [47] (see also, however, [48]), only the latter two categories are relevant for photon energies greater than this (or for wavelengths smaller than a few kpc). Within this range, it is eq. (4.18) which applies for photon-mixing applications when the wavelengths involved are much longer than the hyper-weak gauge boson’s Compton wavelength. Otherwise it is eq. (4.17) that applies. The second regime of application of the above formulæ is to mixings between the hyper-weak boson and the Standard Model $Z$ boson (or the hypercharge gauge boson, $B_{\mu}$), as is appropriate for applications to bounds coming from particle accelerators.

4.2 Bounds on light vector bosons

If the hyper-weak bosons are sufficiently light, then their mixings with photons are described at low energies either by eq. (4.17) or eq. (4.18), depending on the size of the hyperweak boson mass compared with the wavelengths present in the experiments. In this case there are two types of bounds: those constraining the existence of a new light boson coupled to ordinary electrically charged particles; and those constraining the couplings of the photon to exotic millicharged particles [49], that start life as particles coupled only to the hyperweak boson. Of course, the millicharge constraints are only relevant if such new particles exist in addition to the low-energy hyperweak gauge boson.

Direct bounds on vector bosons. By far the strongest constraints arise if the couplings of the light gauge boson can violate the principle of equivalence, and if the mass is small enough that the range of the new hyperweak force is macroscopically large. In this case very strong bounds arise, coming from fifth force experiments on Earth [40], and from other...
tests of General Relativity [47, 51]. For example, lunar laser-ranging limits the difference between the acceleration of Earth and Moon towards the Sun to be [47]

\[
\Delta a = (-1.0 \pm 1.4) \times 10^{-13},
\]

and so if the mass of the hyperweak boson, \(U\), satisfies \(m_U < 10^{-18}\) eV, then

\[
\frac{|Q_S(Q_E/M_E - Q_M/M_M)|}{4\pi G M_S} \lesssim 10^{-13},
\]

where \(M_I\) and \(Q_I\) for \(I = E, M, S\), respectively denote the mass and hyperweak charge for the Earth, Moon and Sun. If the hyperweak boson were to couple to lepton number with strength \(g\), for instance, then to good approximation we have

\[
\frac{Q_I}{M_I} = \frac{g N_{eI} (m_n X_I)}{(4\pi G)^2} = \frac{g X_I}{m_n},
\]

where \(N_{eI}\) and \(N_{nI}\) count the total number of electrons and nucleons in object \(I\) and \(m_n\) is the nucleon mass. \(X_I\) here denotes the proton fraction, \(X_I = N_{pI}/N_{nI} = N_{eI}/N_{nI}\), and so, being made mostly of hydrogen, for the Sun \(X_S \approx 1\). In this case the bound on \(g\) is very strong:

\[
g \lesssim 10^{-6} \left(\frac{G m_n^2}{X_E - X_M}\right)^{1/2} \sim \frac{10^{-25}}{\sqrt{|X_E - X_M|}}.
\]

If, however, the hyperweak boson couples only to ordinary matter through its mixing with the photon, then its coupling is strictly proportional to the electric charge, \(Q_{em}\), of the source. In this case the direct bounds on the existence of new light bosons are weaker than above because of the difficulty of directly detecting the presence of the hyperweak-photon in the presence of the huge background presented by ordinary electromagnetic interactions. In the absence of millicharged particles the best direct bounds then come from the failure to observe photon/hyperweak-photon oscillations [44, 52] and from tests of the inverse square form of the Coulomb interaction [53], if the mass of the hyperweak boson, \(U\), lies in the range \(10^{-9}\) eV < \(m_U < 10^5\) eV relevant for experiments on Earth or oscillations between the Earth and the Sun.

The resulting bounds constrain the kinetic mixing \(|\lambda| \lesssim 10^{-8}\) if \(m_U \sim 10^{-6}\) eV, but extend down to \(|\lambda| \lesssim 10^{-13}\) for hyperweak masses of order \(m_U \sim 10^2\) eV. The bounds deteriorate to \(|\lambda| \lesssim 10^{-5}\) once \(m_U \lesssim 10^{-9}\) eV or \(m_U \gtrsim 10^5\) eV. If \(\lambda\) is generated by loops of 4D fermions of mass \(m\) carrying both a unit of ordinary and hyperweak charges, then

\[
\lambda \sim [egV/(4\pi)^2] \log(m/\mu),
\]

where \(e\) and \(gV\) are the strengths of the electromagnetic and vector hyperweak couplings and \(\mu\) is an appropriate renormalization point.\(^{11}\) In this case a bound like \(|\lambda| \lesssim 10^{-8}\) would correspond to \(gV \lesssim 10^{-6}\).

**Bounds on millicharged particles.** Another strong class of bounds exists if the hyperweak gauge boson is effectively massless on the scale of the experiments of interest, and there also are other exotic light particles that initially couple only to the hyper-weak boson. In this case eq. (4.18) implies that these new exotic particles acquire miniscule couplings to the ordinary photon [43, 44, 45]. The absence of evidence for such particles in

\(^{11}\)See ref. [43] for calculations of \(\lambda\) in more complicated stringy and higher-dimensional frameworks.
Table 1: Approximate bounds on coupling of standard model fermions to a MeV scale hyperweak vector boson $U$. The superscript $V(A)$ denotes vector(axial) couplings. If no superscript is present, the coupling correspond to $\sqrt{(g_V^2 + (g_A)^2)}$. The subscript labels the particle to which the bounds should be applied. The mass $m_U$ is in units of MeV.

<table>
<thead>
<tr>
<th>Coupling</th>
<th>Bound</th>
<th>Experimental measurement</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g_V^e$</td>
<td>$10^{-4}m_U$</td>
<td>$g_e - 2$</td>
</tr>
<tr>
<td>$g_A^e$</td>
<td>$5 \times 10^{-5}m_U$</td>
<td>$g_e - 2$</td>
</tr>
<tr>
<td>$g_V^\mu$</td>
<td>$10^{-3}$</td>
<td>$g_\mu - 2$</td>
</tr>
<tr>
<td>$g_A^\mu$</td>
<td>$5 \times 10^{-6}m_U$</td>
<td>$g_\mu - 2$</td>
</tr>
<tr>
<td>$</td>
<td>g_e g_\nu</td>
<td>$</td>
</tr>
<tr>
<td>$g_{c(b)}^V$</td>
<td>$10^{-6}m_U$</td>
<td>$B(\psi(\Upsilon) \rightarrow \gamma + \text{invisible})$</td>
</tr>
<tr>
<td>$</td>
<td>g_A^e g_V^\mu</td>
<td>$</td>
</tr>
</tbody>
</table>

astrophysics and cosmology excludes mixings of order $|\lambda| \lesssim 10^{-14}$ for $m_U \lesssim 10^4$ eV, while Earth-based experiments require $|\lambda| \lesssim 10^{-4}$ for $m_U \lesssim 10^6$ eV. Limits quickly deteriorate for masses much larger than this.

4.3 Massive vector bosons

Since the mixings of massive vector bosons with photons tends not to generate mini-charged particles, the nature of the constraints obtained in this case can be weaker. Since precision electroweak measurements test the form of fermion couplings to the photon and $Z$ boson, they provide limits on the existence of hidden sector vector bosons that mix with these particles.

Depending on the brane model constructed, the hyperweak $U(1)$ can either couple to the lepton or quark sectors only, or to both sectors simultaneously. If the bulk and the SM branes do not intersect at all, the hyperweak $U(1)$ could still couple to the visible sector via the NS-NS $B_2$ and R-R forms. The latter leads to kinetic and mass mixings, and is discussed in detail in [44]. Here we focus on the case when the bulk and the SM branes intersect.

For a vector boson $U$ of $O(10)$MeV, a number of experimental bounds exist on various couplings of the SM fermions to this ‘hidden’ sector boson. A collection of such bounds adapted from a series of papers by Fayet is displayed in table 1. In general the vector and axial coupling bounds are different, and their scaling with the $U$ boson mass $m_U$ depends on the scale of the process involved. For instance, for the calculation of the anomalous magnetic moment of a fermion $f$, both the vector and axial contributions can be cast into the form

$$\delta a^V(A)_f \sim \left( g^{V(A)} \frac{m_f}{m_U} \right)^2 L^{V(A)} \left( \frac{m_f^2}{m_U^2} \right),$$

where $L^{V(A)}$ are loop functions. The axial coupling $g^A$ scales as $m_U$ due to the enhancement of order $m_f^2/m_U^2$ from the longitudinal part of the $U$ propagator, and is reflected by the fact that $L^A$ can be approximated by a constant. For the vector part, when $m_U \gg m_f$,}

\footnote{For our present purpose, it is not necessary to detail the $O(1)$ coefficients, as we are more interested in the order of magnitude estimate.}
\[ L^V \sim \text{const.}, \text{ hence again we have } \frac{g^V}{m_U} \sim \text{const.} \]

On the other hand, for \[ m_U \ll m_f \], the situation is analogous to QED contributions to \( \delta a \), and we have instead \( \left( \frac{m_f}{m_V} \right)^2 L^V \sim \text{const.} \).

It should be noted that this set of bounds may be considered as applying separately to leptons or quarks, or to both. Which of these couplings are relevant is a model dependent question. However it is interesting to observe that the bounds are comparable to the estimated hyperweak coupling in the LARGE volume scenario with \( m_U \sim \mathcal{O}(10)\text{MeV}. \)

For a vector boson of \( \mathcal{O}(10) \text{ keV} \) discussed before (4.4), the axial coupling bounds are still valid, implying very stringent bounds on the axial couplings of \( \mathcal{O}(10^{-7}) \) or below. The vector coupling bound for the muon remains the same, whereas the one for the electron is reduced to \( \mathcal{O}(10^{-5}) \). It thus appear unlikely that a keV scale vector boson would be compatible with the LARGE volume models under consideration.

The existence of a hidden brane with a weakly coupled \( U(1) \) also leads to the natural possibility of having dark matter (\( \chi \)) residing in the hidden sector. An interesting scenario of having light dark matter (LDM) of \( \mathcal{O}(10) \text{ MeV} \) was discussed in [40, 54, 55]. In these works, the couplings of the LDM and those of the SM particles to the extra \( U(1) \) are unrelated. However in the LARGE volume scenario, both type of couplings are expected to be of \( \mathcal{O}(10^{-4}) \). Using the result that the total LDM annihilation cross section at freeze out should be roughly equal 4 to 5 pb, a relation obtained in [40] implies in general

\[ |f_\chi| |f_{SM}| \sim 10^{-6} \frac{|m_U^2 - 4m_\chi^2|}{m_\chi(1.8\text{MeV})}, \quad (4.23) \]

where \( f_\chi \) and \( f_{SM} \) are generic couplings of the LDM and SM particles to the hyperweak \( U(1) \). We see that the couplings involved in the hyperweak sector is close to the ball park leading to efficient LDM annihilation into SM particles.

For a heavy vector boson with mass \( m_V \) of the electroweak scale or above, limits can be obtained from electroweak precision observables and collider \( Z' \) searches. Constraints from electroweak precision observables generally limit \( m_V \) to be at least a few hundred GeV, and upper bound on kinetic mixing of \( \mathcal{O}(10^{-3}) \) [57]. There is essentially no constraints on a massive vector boson with mass above a TeV [58]. The agreement between SM predictions and LEP-II measurements on \( e^+e^- \rightarrow f \bar{f} \) implies either \( m_V > 209\text{GeV} \), and/or that its coupling to \( e \) and \( f \) be smaller than \( \mathcal{O}(10^{-2}) \). Tevatron searches have also set limits on \( Z' \) couplings to u- and d- quarks [59], with coupling limits again of \( \mathcal{O}(10^{-2}) \) which are relaxed approximately logarithmically with \( Z' \) mass.

5. Conclusions and outlook

In this note we have analysed general aspects of continuous global symmetries (abelian and non-abelian) in string theory and their implementation in local models within the context of LARGE volume string compactifications. Assuming that a Standard Model-like theory is engineered through an appropriate local combination of branes, we have

\[ \text{An exception is the bound from atomic parity violation which constrains the quark and electron coupling product. This could have implications on model building.} \]
identified two interesting phenomenological features. The first is that the field theory couplings and interactions are expected to be determined by approximate continuous global flavour symmetries, which are inherited from the approximate isometries of the local metric. While consistent with precise statements on global symmetries in string theory [1], this is a counterexample to the folk theorem that continuous global symmetries are not relevant to effective field theories derived from string compactifications. A particularly attractive feature is that it may allow both the fermion mass and weak hierarchies to be simultaneously generated by the LARGE volume. The natural expansion parameter for the breaking of the flavour symmetry is the inverse radius of the compact space, \( l_s/R_b \sim 0.01 \). While on the surface such a parameter is very attractive for explaining the fermion mass hierarchy, more work is necessary to carefully determine the correct powers of this parameter that will enter the expansion in a realistic model.

The second interesting feature of such models is the possible existence of new hyper-weakly coupled gauge groups under which Standard Model matter is charged. Such groups would have \( \alpha_{\text{bulk}} \sim 10^{-9} \) and are associated with branes that wrap bulk cycles but have local intersections with the Standard Model branes. If broken by bifundamental matter acquiring a weak-scale vev - for example the Higgs fields - the gauge bosons of such groups would acquire masses of \( m_{Z'} \sim 10 \text{MeV} \) through the Higgs mechanism. While difficult to discover at high-energy accelerators, the existence of such light, weakly-coupled gauge bosons may be accessible to low-energy precision experiments.

Let us finish with some general remarks regarding cosmological implications. The nonexistence of exact global symmetries has important implications for several cosmological scenarios. In particular there are (non) topological defects that require a global symmetry to be present - for example textures, global monopoles, global strings or semi-local strings [60]. If all symmetries are gauged, we may be inclined to conclude that these defects should not be present as a general prediction of string theory. However, based on the logic of this paper we may wonder whether approximate low-energy global symmetries may be sufficient for these defects to play a cosmological role.

1. Global strings and monopoles. Cosmic strings and monopoles can arise from the breaking of global or local symmetries and the topological conditions for their existence are the same for both global and local symmetries. However the physics is very different. For instance the force among global monopoles/strings is independent of the distance and they annihilate more efficiently than the local ones, and the presence of a Goldstone mode provides a channel for radiation that is not available in the local case. Even though exact global monopoles and strings may not be allowed in string theory, we can still think about approximate global defects and estimate the effect these would have in cosmology. As described in the paper we expect the would be Goldstone modes to be bulk modes with mass at the KK scale, which should be contrasted with [61]. While detailed physical implications of this scenario are beyond the scope of this article, it is clear that the quality of the approximate symmetry will play an important role.

2. Semi-local strings. Semi local strings are formed when a product \( G_{\text{global}} \times G'_{\text{local}} \) is
broken to $H_{\text{global}}$ and the vacuum manifold is simply connected. In the simplest case $G_{\text{global}} = SU(2)$ and $G_{\text{local}} = U(1)_{\text{local}}$ and $H_{\text{global}} = U(1)_{\text{global}}$. As the full local $U(1)$ is broken cosmic strings are expected [62]. Again, semi-local strings may exist due to the approximate global symmetries, with stability properties determined by the amount of breaking. For a recent discussion on semi local strings and their potential role in string theoretical models see [3].

3. **Textures.** In the standard classification of topological defects, if the vacuum manifold $\mathcal{M}$ is such that the third homotopy group is non-trivial $\pi_3(\mathcal{M}) \neq I$ textures are induced, corresponding to a non trivial winding of the field once the points at infinity in our 3D space are identified (turning it into an $S^3$). In contrast to other defects the scalar field is always in the vacuum manifold and therefore only gradients of the scalar field contribute to the energy. These would be naturally cancelled by a non-trivial configuration for the gauge field in the covariant derivative and therefore mostly global textures have been considered for cosmological implications. By Derrick’s theorem it is clear that a texture is unstable to collapse, but in an expanding universe it stretches until it comes inside the horizon when it starts to shrink. As for other defects, the gravitational field of textures can distort the isotropy of the CMB. Their effects have been recently studied in [64] in order to explain the cold spot observed in the CMB. Again, the absence of global symmetries in string theory would indicate that global textures are not possible. But an approximate global symmetry originating from a gauge symmetry may still allow the formation of early universe textures. For instance for a light gauge field $A_\mu$ with mass $m_A$, a local texture of size $L \ll 1/m_A$ will behave as a global texture.

Overall, it is interesting that both Abelian and non-Abelian continuous global symmetries can be present in string theory, albeit in an approximate way. In the examples we have studied the presence of the approximate global symmetry is tied to extra-dimensional geometries that naturally give rise to hierarchies. Such approximate global symmetries can have important phenomenological and cosmological implications. It will be worthwhile both to further investigate their implications and to study explicit realisations in semi-realistic local D-brane models, and we hope to return to this topic in future.

**Acknowledgments**

We are grateful for discussions with B. Allanach, R. Brustein, B. Dolan, N. Dorey, J. Garriga, J. Gauntlett, J. Polchinski, G. Ross, E. Sezgin, P. Shellard, N. Turok and M. Williams. CB is supported in part by research funds from the Natural Sciences and Engineering Research Council (NSERC) of Canada, the Killam Foundation and McMaster University. Research at the Perimeter Institute is supported in part by the Government of Canada through NSERC and by the Province of Ontario through MRI. He thanks the Centre for Theoretical Cosmology (CTC) at Cambridge University for hospitality while part of this work was done. JC is funded by Trinity College, Cambridge. He also thanks the University of Texas at Austin for hospitality while part of this work was carried out.
and was supported in part by NSF Grant PHY-0455649. CHK is supported by a Hutchison Whampoa Dorothy Hodgkin Postgraduate Award. LYH thanks the Gates Cambridge Trust and ORSAS UK for financial support. AM is supported by STFC. FQ is supported by STFC and a Royal Society Wolfson Award. He acknowledges the Mitchell family and the organizers of the Cook’s Branch meeting 2008 for hospitality.

References


[36] For reviews see G. Ross, TASI lectures, Boulder Colorado U.S.A. (2000);
H. Fritzsch and Z.-Z. Xing, Mass and flavor mixing schemes of quarks and leptons, Prog.
D.J.H. Chung et al., The soft supersymmetry-breaking Lagrangian: theory and applications,

[37] For early references see for instance A.I. Sanda, A problem for theories with spontaneous
T. Inami and C.S. Lim, Effects of superheavy quarks and leptons in low-energy weak processes
65 (1981) 1772];
T.W. Appelquist, D. Karabali and L.C.R. Wijewardhana, Chiral hierarchies and the flavor
(1990) 213.

[38] A. Ceresole, G. Dall’Agata and R. D’Auria, KK spectroscopy of type IIB supergravity on
AdS_5 x T_11, JHEP 11 (1999) 009 hep-th/9907216;
O. Aharony, Y.E. Antebi and M. Berkooz, Open string moduli in KKLT compactifications,
L. Kofman and P. Yi, Reheating the universe after string theory inflation, Phys. Rev. D 72
(2005) 106001 hep-th/0507257;
M.K. Benna, A. Dymarsky, I.R. Klebanov and A. Solovyov, On normal modes of a warped
B.V. Harling and A. Hebecker, Sequestered dark matter, JHEP 05 (2008) 031
arXiv:0801.4015;
J.F. Dufaux, L. Kofman and M. Peloso, Dangerous angular KK/glueball relics in string

[39] P. Fayet, Effects of the spin 1 partner of the Goldstino (gravitino) on neutral current


[41] P. Fayet, Constraints on light dark matter and U bosons, from psi, Y, K^+, pi^0, eta and etaprime

[42] P. Fayet, U-boson production in e^+e^- annihilations, psi and Y decays and light dark matter,

[43] S.A. Abel and B.W. Schofield, Brane-antibrane kinetic mixing, millicharged particles and

[44] S.A. Abel, J. Jaeckel, V.V. Khoze and A. Ringwald, Illuminating the hidden sector of string
theory by shining light through a magnetic field, hep-ph/0608246.
S.A. Abel, M.D.Goodsell, J. Jaeckel, V.V. Khoze and A. Ringwald, Kinetic mixing of the
photon with hidden U(1)s in string phenomenology, arXiv:0803.1449.


