ON THE DESIGN OF DIELECTRIC-LOADED
RECTANGULAR DEFLECTORS†

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The deflecting properties of dielectric-loaded rectangular deflectors excited in the dominant LSM$_{10}$ mode are theoretically investigated and the essential results for $S$-band deflectors are summarized graphically. It is concluded that due to its long attenuation length a rectangular waveguide loaded symmetrically with beryllium oxide slabs is ideally suited to Blewett-type rf beam separators. This approach represents a realistic method of producing usable beams of elementary particles for counter experiments at the Brookhaven Alternating-Gradient Synchrotron.

1. INTRODUCTION

A group at Argonne National Laboratory$^{(1)}$ recently suggested the use of dielectric-loaded rectangular waveguides as deflectors in Blewett-type$^{(2)}$ rf beam separators. This type of structure leaves considerable latitude for satisfying a variety of design requirements and any attempt to find a universal solution is doomed to failure. It seems, instead, more profitable to summarize the essential information in a few graphs permitting the instant design of the deflector suited to each particular application. The mathematical background underlying this approach and the application to the design of $S$-band deflectors are the subject of this note.

Insertion of a dielectric slab along the narrow side of a rectangular waveguide reduces the phase-velocity but leaves the transverse character (i.e. $E_z = 0$) of the dominant TE$_{10}$ mode unchanged, whereas insertion along the broad side leads to a hybrid character (i.e. $E_z \neq 0$, $H_z \neq 0$) of the dominant mode, which is now called LSM$_{10}$. The transverse deflecting properties of this mode were recognized and described by Chang, Dawson, and Kustom.$^{(1)}$ They also pointed out that operation in higher order modes would result in a larger aperture at a given frequency. To achieve non-degenerate operation in higher modes would, however, require elaborate mode suppressors. It seems doubtful that the increased losses and mechanical complexity of a deflecting structure with mode suppressors can be justified by the gain in aperture, in particular since the deflector must be placed in a strong magnetic focusing channel.

† Work performed under the auspices of the U.S. Atomic Energy Commission.

![FIG. 1. Geometry of asymmetrical dielectric-loaded rectangular deflector.](image)
2. THEORY

2.1. The asymmetrical structure

The propagation of electromagnetic waves in rectangular waveguides partially filled with dielectric material was first investigated by Pincherle,\(^{(3)}\) and has since become a textbook problem. Nevertheless, it may be useful to repeat here those results which are directly needed for the analysis of the structure shown in Fig. 1. The dispersion relation \(k = k(k_c)\) for propagating \(LSM_{mn}\) modes \((H_y \equiv 0; \text{TE}_{mn} \text{ at cutoff})\) is given by\(^{(1)}\)

\[
\epsilon k^2_y \tan h k^2_y d - k^2_y \tan h k^2_y (b - d) = 0 \tag{1}
\]

and for \(LSE_{mn}\) modes \((E_y \equiv 0, \text{TM}_{mn} \text{ at cutoff})\) by\(^{(1)}\)

\[
(k^2_y)^{-1} \tan h k^2_y d + (k^2_y)^{-1} \tan h k^2_y (b - d) = 0 \tag{2}
\]

with

\[
k_x = m\pi/a \tag{3}
\]

\[
(k^2_y)^2 = -(k^2 - k^2_x - k^2_z) \tag{4}
\]

\[
(k^2_y)^2 = \epsilon k^2 - k^2_x - k^2_z. \tag{5}
\]

Our first problem is to determine the geometry of a structure which is capable of supporting an \(LSM_{10}\) mode with phase velocity \(v_p = c\) at prescribed operating frequency \(\omega = kc\), aperture \(d\), and relative dielectric constant \(\epsilon\). The dimension of the broad side \(a\) is obtained from

\[
\epsilon \tan h \frac{\pi d}{a} - \kappa \tan h \frac{\pi(\kappa d - d)}{a} = 0 \tag{6}
\]

with \(\kappa^2 = (\epsilon - 1)(ka/\pi)^2 - 1\) and \(b = \frac{1}{2}a\). At small apertures, that is \(d \to 0\), Eq. (6) can be replaced by

\[
ka \approx \pi(\epsilon - 1)^{-1/2}(1 + \frac{1}{2}\epsilon d/b). \tag{7}
\]

The cutoff frequencies of the three lowest modes \(LSM_{10}, LSM_{20}, LSM_{01}\) are obtained from Eq. (1) by imposing \(k_z = 0\). For small apertures we find in first order approximation,

\[
k_C(\text{LSM}_{10}) \approx \frac{\pi}{a\sqrt{\epsilon}} \left[1 + \frac{1}{2}(\epsilon - 1)\frac{d^2}{b}\right] \tag{8}
\]

\[
k_C(\text{LSM}_{20}) \approx 2k_C(\text{LSM}_{10}) \tag{9}
\]

\[
k_C(\text{LSM}_{01}) \approx \frac{\pi}{b\sqrt{\epsilon}} \tag{10}
\]

At small apertures \(k_C(\text{LSM}_{01}) \leq k_C(\text{LSM}_{20})\); however, at larger \(d\) the inequality is reversed.

The field configuration of the \(LSM_{10}\) mode in the interesting case of \(v_p = c\) can be described by the following equations, in which the time dependent factor \(e^{i\omega t}\) is omitted and natural units \((\mu_0 = c = 1)\) are used. One can write

in region I:

\[
E^I = \begin{bmatrix}
\sinh k_x y \cos k_x x
\hline
1 + (k/k_x)^2 \cosh k_x y \sin k_x x
\end{bmatrix} e^{-jkz} \tag{11}
\]

\[
H^I = \begin{bmatrix}
0
\hline
j(k/k_x) \cosh k_x y \sin k_x x
\end{bmatrix} e^{-jkz} \tag{12}
\]

and in region II:

\[
E^I = C_s \begin{bmatrix}
\sin k_{11}^{II}(b - y) \cos k_x x
\hline
[1 + (k/k_x)^2] (k_x/k_{11}^{II}) \cos k_{11}^{II}(b - y) - i(k/k_x) \sin k_x x
\end{bmatrix} e^{-jkz} \tag{13}
\]

\[
H^I = C_c \begin{bmatrix}
0
\hline
j(k/k_x) \cos k_{11}^{II}(b - y) \cosh k_x y \sin k_x x
\end{bmatrix} e^{-jkz} \tag{14}
\]

with \(k_x = \pi/a, (k^2_y)^2 = (\epsilon - 1)k^2 - k^2_x\), and

\[
C_s = \sinh k_x d \sinh k_{11}^{II}(b - d) \tag{15}
\]

\[
C_c = \cosh k_x d \cosh k_{11}^{II}(b - d). \tag{16}
\]

Note that Eq. (1) imposes the relationship \(C_c/C_s = \epsilon k_x/k_{11}^{II}\).

Calculation of the transverse shunt impedance \(R = E_y^I/\partial P, \text{attenuation constant } 2\alpha = \partial_x P/P, \text{and group velocity } v_g = P/W\) requires the determination of the equivalent deflecting field \(E_0\), stored energy per unit length \(W\), propagating power \(P\), and losses per unit length \(\partial_x P\). The losses are composed of dielectric and wall losses, and it is convenient to write

\[k\partial_x P = C_D \tan \delta + C_W r_s\]

where \(\tan \delta\) is the dielectric loss tangent and \(r_s\) the surface impedance divided by the wave impedance of free space \(Z_0 = c\mu_0\). Simple integration yields

(1) the propagating power \(P = P^I + P^II\) with

\[
k^2 P^I = \frac{1}{4}\pi(k/k_x)^4 [1 + (k/k_x)^2] S_{C1} \tag{17}
\]

\[
k^2 P^II = \frac{1}{4}\pi(k/k_x)^2 (k/k_{11}^{II})^2 [1 + (k/k_x)^2] S_{C1} C_c \tag{18}
\]

(2) the stored energy \(W = W^I + W^II + W^I + W^II\) with

\[
k^2 W^I = \frac{1}{4}\pi(k/k_x)^2 [1 + (k/k_x)^2] [S_{C1} + [1 + (k/k_x)^2] S_{C1}] \tag{19}
\]

\[
k^2 W^II = \frac{1}{4}\pi(k/k_x)^4 [1 + (k/k_x)^2] S_{C1} \tag{20}
\]
\[ k^2 W_{\text{per}}^{11} = \frac{1}{2} \pi \epsilon (k/k_y)(k/k_y)[1 + (k/k_x)^2] \cdot \left( S_{\text{II}} + [1 + (k/k_x)^2](k_y/S_{\text{CT}}) C_{\text{II}}^2 \right) \]  

(21)

\[ k^2 W_{\text{m1}}^{11} = \frac{1}{8} \pi (k/k_y)^3 [1 + (k/k_x)^2] S_{\text{CT}} C_{\text{m1}}^2 \]  

(22)

The power loss coefficients

\[ C_D = k^2 W_{\text{per}}^{11} \]  

(23)

\[ C_W = \frac{1}{2} k a (k/k_y)^2 [1 + (k/k_x)^2] (1 + C_{\text{II}}^2) + (k/k_y)^3 (k/k_x) S_{\text{CT}} + (k/k_y) S_{\text{CT}} C_{\text{II}} C_{\text{II}}. \]  

(24)

In the preceding expressions the following shorthand symbols are being used:

\[ S_{\text{III}} = k x f_0 \sinh k x y \, dy = \frac{1}{2} \sinh 2k x d - \frac{1}{2} k x d \]  

(25)

\[ S_{\text{CT}} = k x f_0 \cosh k x y \, dy = \frac{1}{2} \sinh 2k x d + \frac{1}{2} k x d \]  

(26)

\[ S_{\text{III}} = k_y f_0 \sin^2 k y (b - y) \, dy = \frac{1}{2} k_y^2 (b - d) - \frac{1}{2} \sin 2k y (b - d) \]  

(27)

\[ S_{\text{CT}} = k_y f_0 \cos^2 k y (b - y) \, dy = \frac{1}{2} k_y^2 (b - d) + \frac{1}{2} \sin 2k y (b - d). \]  

(28)

It should be pointed out here that the accuracy of numerical results can be checked to some extent by calculating \((W_{\text{per}}^{11} + W_{\text{m1}}^{11} - W_{\text{per}}^{11} - W_{\text{m1}}^{11}) / W\) which must be 0.

The equivalent deflecting field on a synchronous extreme relativistic particle is directed mainly in the y-direction and is given by

\[ E_y = jk^{-1} \partial_y E_z = \cos k x y \sin k x x. \]  

(29)

There also exists a deflecting field in the x-direction

\[ E_x = \sinh k x y \cos k x x. \]  

(30)

In contrast to dielectric-loaded circular deflectors, the deflecting field here varies over the aperture.

We define for the asymmetrical structure \( E_0 = E(x = \frac{1}{2} a, \, y = \frac{1}{2} d) = \cosh \frac{1}{2} k x d \) and, somewhat arbitrarily, the field variation in the deflecting plane as

\[ A_y = (\cosh k x d - 1) / E_0. \]  

(31)

For small apertures \( A_y \approx \frac{1}{2} (\pi d / a)^2 \). To keep \( A_y < 15 \) per cent, it is necessary to make \( d \leq \frac{1}{2} a \). It would be possible to use only a fraction of an actually larger aperture, but it will be shown below that this approach leads in general to a reduced shunt impedance. The preferred solution is to use a symmetrical structure, which has essentially twice the aperture in the y-direction for a given field variation. In the x-direction about \( \frac{1}{2} a \) is a useful aperture for both structures.

2.2. The symmetrical structure

The field distribution of the LSM\(_{10}^{\text{m}}\) mode in a symmetrical structure (Fig. 2) is unaffected by the insertion of a metallic boundary along the symmetry plane, which produces two separate asymmetrical structures with \( b = \frac{1}{4} a \). Consequently most results obtained for the asymmetrical structure are also valid for the symmetrical deflector. In particular the dispersion relations for LSM\(_{10}^{\text{m}}\) and LSM\(_{20}^{\text{m}}\) modes are given by Eq. (1) with \( b = \frac{1}{4} a \). However, the cutoff frequency of LSM\(_{01}^{\text{m}}\) in the symmetrical structure must be found from

\[ \tan k_y d = \sqrt{\epsilon} \cot \frac{\sqrt{\epsilon} k_y (b - d)}{\epsilon}. \]  

(32)

The equivalent deflecting field on the axis of the symmetrical structure is \( E_0 = 1 \) and the definition of \( A_y \) remains unchanged. Finally, the expressions for the figures of merit \((R, \, \alpha, \, v_g)\) of the symmetrical structure are easily found by taking into account that

\[ (P)_{\text{SYM}} = 2(P)_{\text{ASYM}}, \]

\[ (W)_{\text{SYM}} = 2(W)_{\text{ASYM}}, \]

\[ (C_D)_{\text{SYM}} = 2(C_D)_{\text{ASYM}}, \]

and

\[ (C_W)_{\text{SYM}} = \frac{1}{2} k a (k/k_y)^2 [1 + (k/k_x)^2] C_{\text{II}}^2 + 2(k/k_y)^3 (k/k_x) S_{\text{CT}} + (k/k_y) S_{\text{CT}} C_{\text{II}} C_{\text{II}}. \]  

(33)

3. DESIGN CONSIDERATIONS

The general formulae of the preceding section have been evaluated numerically for the frequency
$f = 2.855$ GHz ($\lambda = 10.5$ cm) at which existing rf beam separators operate.\textsuperscript{(6)} At this frequency $r_s = 0.37 \times 10^{-4}$ for copper and $\tan \delta \approx 10^{-4}$ for selected dielectric materials independent of $\epsilon$. The structure constant $C_W$ being smaller than $C_D$ (typically $C_D = 0.1$ to 0.2 $C_W$), a small variation in the actual $\tan \delta$ will not substantially affect the conclusions reached.

The shunt impedance as function of dielectric constant with the geometrical aperture $2d$ as a parameter is plotted in Fig. 3. It may be seen, that at fixed $d$ the maximum shunt impedance is obtained by choosing $\epsilon \approx 6$; beryllium oxide appears, therefore, to be the preferred dielectric material. But since commercial beryllium oxide\textsuperscript{(6)} has a $\tan \delta \approx 4$ to $5 \times 10^{-4}$ whereas aluminum oxide or fused silica have a $\tan \delta \leq 10^{-4}$, their use would actually lead to higher values of the shunt impedance. However, the exceptionally good heat conductivity of beryllium oxide practically imposes its choice in long-pulse applications.

After selection of the dielectric material one may choose the geometrical aperture by considering the required attenuation length. For maximum efficiency one would make the attenuation length equal to the deflector length, but it may occasionally be desirable to go to larger values of $\alpha^{-1}$ despite the associated reduction in shunt impedance.

Using Eq. (6), the geometrical dimensions of symmetrical structures with $a = 4b$ were determined as function of aperture $2d$ and dielectric constant $\epsilon$, and the results are plotted in Fig. 4. Single-mode propagation is assured as long as the cutoff frequency of the $\text{LSM}_{20}$ mode is above the operating frequency at which $v_p \approx c$. The boundary imposed by the requirement for single-mode propagation is depicted in Fig. 4, together with the more restrictive limitations due to the permissible field variation in the separation plane. It follows in the practically important case of $\epsilon \approx 6$, that the
useful aperture ($A_y \approx 15$ per cent) is about 2.5 cm \times 2.5 cm, which may be smaller than the geometrical aperture required to achieve the prescribed attenuation length.

The above discussions show that dielectric-loaded rectangular deflectors have several adjustable parameters which make this type of structure useful in various applications. But due to its long attenuation length the dielectric-loaded reflector seems to be ideally suited to Blewett-type rf beam separators, which have now become a realistic method of producing usable beams for counter experiments.\(^{(7)}\)

ACKNOWLEDGEMENTS

The author would like to thank Drs. J. P. Blewett and D. M. Lazarus for helpful comments on the manuscript.

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Received 3 March 1970