ORGANISATION EUROPÉENNE POUR LA RECHERCHE NUCLÉAIRE
CERN EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH

1962 EASTER SCHOOL FOR PHYSICISTS

Using the Nuclear Emulsion Technique in Conjunction with the CERN Proton Synchrotron and Synchro-Cyclotron

held at St. Cergue, April 8-18, 1962

PROCEEDINGS
edited by

N. Doble
W.O. Lock

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document.
The 1962 Easter School for Emulsion Physicists covered a wide range of topics. In preparing these Proceedings we have rearranged the order of the talks to some extent in order to give a more logical presentation of the subject matter. We are grateful to the speakers who kindly provided us with the manuscripts of their talks. Almost all of the students (who are listed in Appendix B) acted as Scientific Secretaries for one or more sessions which has enabled us to report the substance of the discussions following each talk. We thank them for their hard work.

We must also thank the following people who contributed to the success of the School and to the production of these Proceedings: Mr. E. Bissa, Mr. A. Cyvoct and Mr. W.A. Roberts for the technical arrangements at St. Cergue; Mr. A. Bondi and the Scientific Information Service for the diagrams; our colleagues in the Emulsion Group for assistance with the proof reading; and Miss S. Fland, Miss S. Greenstreet and Mrs. K. Wakley for the patient typing of the many stencils.

We apologise for the delay in the appearance of these Proceedings, which has been due to a combination of circumstances beyond our control.

M. Doble
W. O. Lock

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* * *
INTRODUCTORY TALK

K. Gottstein,
Max Planck Institut für Physik, Munich.

I. THE PURPOSE OF THE SCHOOL

This is to be a "School for Physicists using the Nuclear Emulsion Technique in Conjunction with the CERN Proton Synchrotron and Synchro-cyclotron". Why has it been found desirable to set up such a school? Is it not true that the physicists using the nuclear emulsion technique have plenty of opportunities to learn all about it at their home laboratories? In a sense this is certainly true. Many of you come from laboratories that have done very valuable and famous work using the emulsion technique, and there is no better place to learn the emulsion technique than in those laboratories. Nevertheless, we think that there is a genuine need for a school of the kind attempted here. If one wants to experiment nowadays with nuclear emulsions at a large accelerator in a way which really results in revealing a new piece of physical information, then there are many things one has to know in addition to the recipes of how to process the emulsions and how to make reliable measurements in them.

To start with, the emulsion technique has no longer the outstanding position it had several years ago when many of the most important discoveries of those days were made by it. In the meantime the bubble chamber has been invented and developed into an extremely powerful tool and the electronic techniques have been much refined. The large accelerators have come into operation. New devices like spark chambers, luminescent chambers, semiconductor detectors are being applied, or are being tested for application, in high-energy nuclear physics experiments. Many experiments which could only have been done by the emulsion technique a decade ago can now be carried out faster and more conveniently by one of the newer techniques,
whereas for some other experiments the emulsion is still the most suitable tool today. The emulsion worker who plans another emulsion experiment must therefore know enough of these newer techniques, and of the results obtained and obtainable by them, to be sure that he is not just attempting to do something that has already been done, or is being done, or could be done, in another and more efficient way. Of course, the mere fact that some problem has been attacked, is being attacked, or could be attacked by another technique does not mean that it is taboo to the emulsionist. If something has already been done then one may judge whether the result is clear and unambiguous, or whether it should rather be repeated, and, if so, by which technique. If something is being done, then there may be some room for doubting whether it will be finished, and lead to a clear-cut result, and it may take some knowledge not only of the experimental technique applied, but also of the experimenters involved, to form a personal opinion as to the wisdom of starting a competitive experiment using emulsions. Finally, if an experiment could be done better by, say, a hydrogen bubble chamber but no bubble chamber group plans to do it in the near future for lack of time or interest, then there might be a case for the emulsion worker to go ahead. He may find something interesting and may stimulate the appetite of the bubble chamber people who will then finish the job. But in general the emulsion worker should concentrate on those experiments which can only be done by emulsions because here his contribution becomes most valuable. Again, in order to find these experiments he must know not only his technique but also the possibilities and the limitations of the others.

So this is one justification for our school in which we shall try to show what the emulsion technique really can do, and what one should better leave to the bubble chambers, counters, spark chambers, etc.

A second object of the school is to study the particular aspects of experimenting with emulsions near a large accelerator. Again this is something which it is difficult to do far away from an accelerator. There are many technical problems to be overcome before
the emulsion can be exposed at the accelerator, problems that do not exist in cosmic ray work. They will become evident in the lectures on the CERN accelerators and beam transport systems, on exposure techniques, and on some typical experiments.

Another field to be covered by this school are some recent advances of the emulsion technique itself. Apparatus like pulsed magnets for exposing emulsions in fields of more than 100,000 gauss have a size and complexity that make it difficult to transport them over long distances. It would also be uneconomical if each laboratory, for example, set up its own condenser bank. Therefore, large apparatus of this nature will only be constructed at a few places, preferentially near the accelerator. Physicists at the other laboratories should nevertheless know about them, and use them as far as possible.

This should be sufficient to sketch what some people would like to call the "philosophy" of the school but what—with more respect towards philosophy—could also be called its programme. In addition, however, the programme will also include lectures on the general foundations and methods of the emulsion technique like the photographic process, the types of emulsions available, their handling and processing, the scanning and measuring methods. We shall now look at this programme in a little more detail.

Let us consider the first mentioned point first. Let us compare the special features of the emulsion, track chamber, and electronic techniques and discuss the fields in which each has particular value.

II. COMPARISON OF THE PRESENT TECHNIQUES OF HIGH-ENERGY NUCLEAR PHYSICS

We shall not consider here those techniques which are still in the state of being tested or developed, and shall thus limit ourselves to the scintillation and Čerenkov counters, to the expansion
and diffusion cloud chambers, to bubble chambers, spark chambers, and, of course, nuclear emulsions.

Each of these tools has specific properties. Therefore it is sometimes this, sometimes that technique which is better, or only applicable under the conditions of a given experiment.

Scintillation counters have a relatively high resolution in time, i.e. a short "dead" time, and are therefore of great value in all investigations which aim at precise measurements of flight and lifetimes, involve coincidences of several counters, and require large statistics. On the other hand, pure counter experiments often do not give much information on the details of the reactions studied. Frequently it is impossible to say whether one particle, or several particles simultaneously, have traversed the counter, whether secondary reactions took place inside the counter, and which were the direction of flight, the mass, charge, and velocity of the particle.

The situation is better in this respect with the Čerenkov counter. The Čerenkov counter is also well suited for use with fast coincidence circuits, and can be combined with scintillation counters. In fact, the antiproton was discovered by such a combined arrangement of scintillation and Čerenkov counters\(^1\). Also at CERN combinations of Čerenkov and scintillation counters have been extensively used\(^2\). The Čerenkov counter has the great advantage of a precise velocity resolution (\(\Delta \beta \approx 0.001\)) in addition to a high counting efficiency. It can also be made to have angular discriminating properties. Of course, a Čerenkov counter is much more complicated to make and to run than a simple scintillation counter.

The other detectors mentioned have one thing in common that the counters lack: they make the tracks of charged particles directly visible. This, however, is more or less the only thing they have in common. In almost all other respects they have vastly different characteristics.
The expansion cloud chamber may have large dimensions, can be triggered by counters and exposed in a magnetic field, but has a considerably longer "dead" time than any of the other types of chambers.

Compared to the expansion cloud chamber, the diffusion cloud chamber has the advantage that it can be operated with hydrogen under high pressure. The pair production of neutral V-particles in π-p reactions has been demonstrated in this manner. One disadvantage of the diffusion cloud chamber is that the thickness of its sensitive layer is rather limited.

As was just said, cloud chambers can be triggered by counters so that only events of a special type are registered. This possibility does not yet exist for bubble chambers although there has recently appeared some hope that by the use of ultrasonic waves the problem of triggering bubble chambers may be solved. This disadvantage of the bubble chamber is, however, balanced by an array of important advantages. The bubble chamber has a repetition rate much higher than that of the cloud chamber, may have dimensions comparable to those of the largest cloud chambers, can also be exposed in magnetic fields and, most important, may be filled with liquid hydrogen.

On the other hand, a hydrogen bubble chamber is a device of great technical complexity and therefore expensive to build. It is also expensive to run, due to the continuous hydrogen consumption, and to the safety problem which requires constant supervision and alertness from a large crew. Much more economic in every respect is the spark chamber which has recently attracted much attention as a most welcome addition to the arsenal of detectors available for elementary particle experiments. It has clearing times smaller than 1 μsec. It allows tracks to be located with an accuracy of about 1 mm. It is easy to build, inexpensive, and can be triggered. For the latter reason and for the fact that one can put $10^6$ particles/sec through the chamber it should be ideal for looking at rare and complicated events. At CERN several spark chamber experiments are
being planned to investigate the polarization of the recoil proton
from \( \pi^- + p \rightarrow \pi^- + p \), the leptonic decay modes of \( K^+ \), the interactions
of neutrinos, the relative \( \Sigma - A \) parity, and other problems. Spark
chambers can be put into a magnetic field, and may even be combined
with a Čerenkov counter and a bubble chamber, thereby making possible
the identification of particles of a given mass among the tracks in
the bubble chamber\(^4\).

The nuclear emulsion technique permits microscopic track
measurements of high precision and reproducibility and therefore the
determination of ranges and specific ionizations with an accuracy un-
attainable by any of the other techniques. As an example one might
recall that the range of a muon from the decay at rest of a pion is
0.6 mm in nuclear emulsion and 11 mm in the liquid hydrogen of a
bubble chamber. The mean diameter of a developed silver grain in
G5 emulsion is about \( 5 \times 10^{-2} \) mm, of a bubble in the liquid hydrogen
chamber about 0.2 mm. The ratio (track length/unit of range measure-
ment) is in this case about 1000 for the emulsion, but only about 50
for the hydrogen bubble chamber. Ranges and distances of the order
of one micron which are quite unresolvable by other means are still
easily measurable in nuclear emulsion.

If the outcome of the experiment depends less on the ac-
curacy of measurements in individual reactions, and more on the speed
of finding correlated events separated by distances of the order of
1 cm or more (like the production and decay of neutral particles) then
the chambers become superior to the emulsions. The diameter of the
field of view in the microscope is only fractions of a millimetre,
whereas in the photographs from a large cloud or bubble chamber one can
at one glance survey an area with a diameter of the order of 1 metre.
Cloud and bubble chambers therefore allow investigations which are al-
most impossible in emulsions due to the difficulty of spatial cor-
relation.

Scanning for events with a rather small cross-section which
involves following great lengths of track is for the same reasons
less time-consuming in track chambers than in emulsions even though
the density of the absorbing material in the former may be smaller.
One will therefore prefer track chambers to emulsions also in
experiments of this type, unless measurements of extreme precision
have to be performed on the events found.

Table I summarizes some of the more important features
of the various techniques. From this table one gets a rough idea
of the advantages and disadvantages of each of the techniques men-
tioned. If there is a particular physics problem to be attacked,
the right technique to do the job should be carefully selected.
The professional emulsion worker is particularly exposed to the
temptation not to observe this rule. This is because his technique
allows him in principle to do almost any experiment that the other
techniques can do if one disregards the time and effort required to
do it. Every reaction in which charged particles are involved is
beautifully and clearly visible in all its details, and the only
problem is to find enough of these reactions and make the necessary
measurements within a reasonable time. The emulsion contains about
as much hydrogen per cc as a hydrogen bubble chamber; there is only
the problem of identifying the events with this free hydrogen. Again
this identification takes time and effort but can often be done.
But, as we said in the beginning, whenever an emulsion physicist
thinks: "This is an experiment I can do", his immediate further thoughts
should be: "Is the emulsion technique superior to the other techniques
here? Is it at least likely to come up with a good result within
reasonable time? Can I therefore justify the effort required to do
the experiment? Or should I rather use another technique for this
experiment, or do another experiment for which emulsions really are
the best solution?"

As examples we shall now mention a few experiments in the
cases of which the emulsion technique was indeed considered to be a
most suitable, if not the only suitable, one.
III. EXAMPLES OF EMULSION EXPERIMENTS

1. The interactions of 1.5 GeV/c K\(^-\) mesons

This is an experiment proposed to be carried out by several groups in Europe, India, and the United States. Its purpose is the study of the production of hyperfragments, if possible also of Λ\(^0\)-Λ\(^0\) and of Ξ hyperfragments, of the production and absorption of Ξ hyperons, of small angle elastic scattering of K mesons, and of related problems. It is a typical emulsion experiment in that the observations proposed cannot be made as well by any other technique because of the short hyperfragment and Ξ ranges and small scattering angles to be measured.

2. The magnetic moment of Λ\(^0\) and Ξ\(^+\) hyperons

The groups of Bristol, CERN, Lausanne, and Rome plan an experiment by which they want to measure the magnetic moment of the Λ\(^0\) and Ξ\(^+\) hyperons. The experiment is based on the known facts that the Λ\(^0\) and Ξ\(^+\) hyperons produced by pions are polarized with respect to the production plane, and that the decay pions from the hyperon decays show an angular asymmetry with respect to the polarization vector. In a strong magnetic field the polarization vector will precess about the field direction by an angle which depends on the magnitude of the magnetic moment. Detailed quantitative calculations of this effect have been published, e.g. by N. Schmitz\(^5\). In order to obtain precession angles of a measurable order of magnitude, magnetic fields of more than 100,000 gauss are required. Nevertheless, large statistics will still be necessary. Why, then, is this an emulsion experiment? The bubble chamber technique would, in principle, be better suited for the detection and angular measurement of the hyperon decays, but there do not yet exist in a working condition bubble chambers that can operate in or near magnetic fields of that strength, although such chambers are being developed by, for example, Dr. Bergmann at Munich. Magnet coils for the exposure of emulsions in pulsed, very high magnetic fields do exist, however.
For this technical reason the experiment can probably be carried out in emulsions earlier than in bubble chambers. Quite recently, however, the magnetic moment of the $A^0$ has been measured in a spark chamber experiment at Brookhaven.

3. "Burning spot" experiment

This experiment, which was proposed by the Bern Group, takes its name from the fact that a very well-collimated pencil beam of high intensity will be directed on a small spot in one corner of the emulsion stack, or on an external target of high Z material just outside the stack. The intensity will be so high that the emulsions would be black ("burnt") in the beam region where a very large number ($\sim 10^6$) of interactions would take place. In these interactions a considerable number of baryons and antibaryons would be produced, some of which would leave the region of the "burning spot" and would produce tracks in the clear parts of the emulsions. The whole arrangement will be placed into a high field, pulsed magnet so that the tracks of positively and negatively charged baryons would be separated. If one scans at a distance of $\sim 2$ cm from the burning spot or target one hopes to be able to pick up easily the tracks of negatively charged baryons and antibaryons, the interactions of which can be studied.

The emulsion technique is suitable here because it allows the observation and identification of tracks close to a highly irradiated target. The experiment requires, however, a high degree of collimation. It is proposed to use a lead collimator, 2 m long, with an aperture of $5 \times 5 \text{ mm}^2$.

4. Search for Dirac monopoles

This experiment was carried out recently at CERN. If magnetic monopoles exist, they may be formed by proton-nucleon collisions in a target in which they then get bound. By strong, pulsed magnetic fields they should be liberated again from the target. They should
ionize heavily and therefore leave very characteristic tracks in nuclear emulsion. The experiment, which was carried out in various ways, consisted essentially in the irradiation by protons of different kinds of targets which were placed in strong, pulsed magnetic fields, either during the irradiation or afterwards. If any monopoles had been liberated some would have been guided by the accelerating fields into the nuclear emulsions which were placed, in the last version of the experiment, at a distance of a few metres from the machine. So far, none have been found. The upper limit for the cross-section is of the order of $10^{-40}$ or $10^{-39}$ cm$^2$.

The emulsion technique is excellently suitable here because of its simplicity, its power of integrating over time, and its distinctive power for very heavily ionizing tracks, but the experiment can, and has, also been done with counters.

IV. CONCLUSIONS

We have shown only a few examples of CERN emulsion experiments from which some of the main advantages of this technique become discernible. Of course we could have mentioned many more examples of emulsion experiments carried out or planned in various places in the world. A good one would have been the determination of the $\pi^0$ lifetime in which flight paths of a fraction of a micron have to be measured, a feat directly possible only with emulsions, although indirect ways also exist. [See talk by H. Heckmann in Part VIII.2

We summarize again: the high spatial resolution, the discriminating power (i.e. the possibility to make direct measurements on the tracks), and the relative ease of handling the emulsions are the principle assets of the emulsion technique. Whenever these factors become decisive the application of the emulsion technique is indicated. But one should, of course, never forget that there are usually several ways to the solution of a problem in physics, and
that one cannot, therefore, divide up the questions of physics according to techniques which might be competent for them. There is no limit to ingenuity, and the advent of a good new idea can completely change the experimental situation.

At the end of the introduction I should like to say just a few words on some of the technical aspects which the emulsion physicist has now to face and which will be treated in greater detail in this school. I spoke before of the relative ease of the emulsion technique. This is true if one compares the mere preparation and exposure of an emulsion block with, for example, the construction and operation of a bubble chamber. But this situation is changing. The construction and use of pulsed magnets is already requiring more complete technical work from the emulsion physicist, and as far as the design, testing and monitoring of beams go, the emulsionist will now have to share fully the burden of the work required from the track chamber and electronics physicist if he does not want to remain in the position of perpetual parasiting on other people's beams. Therefore this school will stress particularly the problems of beam optics and particle separation. I only want to remind you here that, for example, the new K beam for the North Hall will consist of seven quadrupole lenses and three bending magnets. The design, alignment, and adjustment of a beam of this type is, of course, a major task that requires several months of work by a group of physicists.

I cannot go into any details here in this short introductory talk. All the details will be given to you in the sessions that follow. It was my intention only to recommend to your particular attention some points of the programme which cover aspects of modern emulsion work about which we, as emulsion physicists, perhaps still think too little. It was not necessary for me here to stress the other parts of the programme which deal with the more conventional aspects of our technique, aspects the importance of which has always been obvious. Without the knowledge of the special types of nuclear
emulsion available, without the know-how of processing and measuring, no emulsion experiment is possible.

At the beginning of this school and as a member of its Organising Committee, I should like to thank CERN and all of its members who made this School possible. I also thank all those colleagues who have agreed to lecture here and thus to contribute to the success of the school. They will now introduce you into the fields in which they are experts and which are so important for fruitful work with the emulsion technique.

* * *

REFERENCES


4) See, for example, Spark chamber symposium, Rev.Sci.Instr. 32, 480 (1961).


* * *
**DISCUSSION**

**Lock** : The magnetic moment of $\Lambda^0$ has been measured by a spark chamber experiment at Brookhaven, but the error was large; the quoted value is $-1.5 \pm 0.5$ nuclear magnetons. Therefore another experiment is planned on this machine in order to reduce the error.

**Richter** : I have one question about the comparison of spark chamber and bubble chamber. You have given here a resolution time of $10^{-6}$ sec for a spark chamber. Now I think it is rather difficult to use this resolution time for the spark chamber and to compare these two resolution times. I have the feeling it would be better to say a bubble chamber is seeing every track coming along within about 1 msec, while a spark chamber sees every track within $10^{-6}$ sec. But to make this track visible you cannot take this resolution time of $10^{-6}$ for the spark chamber. You have, I think, of the order of 50 msec to get separate tracks. The question is, how do you define this resolution time for those two chambers?

**Gottstein** : I think that the definition of the resolution time is the time by which you can distinguish that two events were not simultaneous. In a spark chamber you can do it with an accuracy of $10^{-6}$ sec because it takes about that long to clear the gas again before the next spark; whereas in a bubble chamber—unless you use information from counters—you know only that the event happened while you took the pictures, and a picture can be taken in intervals of ~1 sec. What I had in mind when I wrote $10^{-6}$ sec for the spark chamber was a comparison with the time resolution of the counters. I agree that the registration interval given for the bubble chamber
Gottstein (cont.): is of a somewhat different nature and therefore not really comparable.

Teucher: I think what you have written is too favourable for the spark chamber because actually as far as I know you can not take a second picture for, let us say, 10 or 20 msec in a spark chamber. The sensitive time is of the order of 1 μsec. But if you want to compare how much data you can take I think you have to take actually the recovery time, and all these things, such as power supply and so on, go in and as far as I know this is at the moment of the order of 10 - 20 msec, so even if you have a pulse length of 100 msec which is about the best you can get at the proton machine you can take something of the order of 10 pictures per pulse with the spark chamber but not more than that. So the data collection is very good because with the bubble chamber you can take just one picture and with the spark chamber you can take ten. But if you go to electron accelerators where you have extremely short pulse lengths, then it remains at one picture per pulse for the spark chamber.

Gottstein: The figure of $10^{-6}$ is significant when you have to distinguish one track from the next one. It is quite true that you cannot take another photograph after $10^{-6}$ sec, but only after this much longer recovery time. But if you want to make time measurements, then you should use this figure of $10^{-6}$.

Nikolić: Speaking about the exposure of emulsions by the 1.5 GeV/c K$^-$ beam, you mentioned as a typical emulsion experiment the investigation on hyperfragments, and especially that of the hyperfragments with two bound Λs produced by Σ$^-$.
Nicolić (cont.): Then you also mentioned a general research on $\Xi^-$ particles to be a typical emulsion experiment. Can you substantiate the latter opinion?

Gottstein: I just reported what the people who want to do this experiment wrote specifically in their proposal. They want to study the production of hyperfragments and of $\Xi^-$. The investigation of hyperfragments is a typical emulsion experiment. I have not calculated the kinematics for the $\Xi$ particles produced in such reactions so I do not know at the moment what ranges they could have. In any case, the emulsion technique would allow rather precise measurements on the $\Xi$ tracks also, if such are found.

Nikolić: Emulsion represent, for instance, a particularly suitable technique for the study of production of short-lived charged particles, especially of $\Sigma^-$. Another advantage of this technique is the fact that it allows one to work with a very high particle intensity. For example, emulsions are and will be used in determining the pion flux in the CERN neutrino experiment.

* * *

3348/MF/kw
<table>
<thead>
<tr>
<th>Detector</th>
<th>Registration interval</th>
<th>Direct track visibility</th>
<th>Surveyable track length</th>
<th>Advantages</th>
<th>Disadvantages</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scintillation counter</td>
<td>( &lt; 10^{-9} ) sec</td>
<td>no</td>
<td>–</td>
<td>High time resolution.</td>
<td>Tracks not visible.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Large statistics attainable.</td>
<td>Poor space resolution.</td>
</tr>
<tr>
<td>Čerenkov counter</td>
<td>( &lt; 10^{-9} ) sec</td>
<td>no</td>
<td>–</td>
<td>High velocity resolution in addition to above.</td>
<td>As above.</td>
</tr>
<tr>
<td>Expansion cloud chamber</td>
<td>( 1 - 10 ) min (^1))</td>
<td>yes</td>
<td>( \sim 1 ) m</td>
<td>Large dimensions. Sensitive volume not limited as in diffusion chamber.</td>
<td>Low density (^2).</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Triggerable by counter.</td>
<td>Long dead-time.</td>
</tr>
<tr>
<td>Diffusion cloud chamber</td>
<td>( \sim 10 ) sec</td>
<td>yes</td>
<td>( \sim 1 ) m</td>
<td>Large dimensions. 20 at ( \text{H}_2 ) possible.</td>
<td>Shallow sensitive volume.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Triggerable by counters.</td>
<td>Density still low (^2).</td>
</tr>
<tr>
<td>Bubble chamber</td>
<td>( \sim 1 ) sec</td>
<td>yes</td>
<td>( \sim 1 ) m</td>
<td>Large dimensions. Uniform substance of high density.</td>
<td>Not yet triggerable by counters.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Short life of bubble nuclei (( \sim 1 ) msec).</td>
<td></td>
</tr>
<tr>
<td>Spark chamber</td>
<td>( \sim 10^{-6} ) sec</td>
<td>&quot;yes&quot;</td>
<td>( \sim 1 ) m</td>
<td>Large dimensions. High density. Simple. Triggerable by counters.</td>
<td>Tracks invisible inside plates.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Limited spatial resolution.</td>
</tr>
<tr>
<td>Nuclear emulsion</td>
<td>–</td>
<td>yes</td>
<td>( \sim 0.5 ) mm</td>
<td>Precision measurements.</td>
<td>Complex composition.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>High spatial resolution.</td>
<td>Small field of view.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>No time resolution.</td>
</tr>
</tbody>
</table>

\(^1\) 5 to 10 sec possible with supercompression.

\(^2\) Density of hydrogen gas: \(0.09 \) g/litre. Density of liquid hydrogen: \(59 \) g/litre.
THE CERN PROTON SYNCHROTRON AND ITS TARGETS

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The CERN PS machine accelerates protons up to a momentum between 10 and 28 GeV/c with a repetition rate of 1 and 5 sec, respectively. At top energy there are about $2 \times 10^{11}$ protons accelerated per cycle and steered onto a target.

I. PS MACHINE

The general layout of the PS is given in Fig. 1. The protons, which are produced by the ion source, are accelerated up to 50 MeV along the three tanks of the linear accelerator. At this energy they are injected into the PS ring. There the protons are kept on the orbit of 200 m diameter by the magnetic field of the 100 magnet units. The field of those is rising in about 1 sec from 14.7 gauss to 10, 12, or 14 kgauss, depending on the final momentum which is requested. Synchronized with the rise of the main magnetic guiding field, the protons are accelerated by the 16 cavities, which are placed in long straight sections each five magnet units apart. This leaves four long straight sections free for other purposes such as injection and beam extraction. During acceleration the beam consists of 20 bunches. The velocity of the protons is about $\frac{1}{3}$ of the velocity of light at injection, and approaches roughly the velocity of light at the end of acceleration. In the range from 10 to 28 GeV/c each of the 20 bunches is about 10 nsec long in time and has a distance of 100 nsec in time to the next one. The revolution time is 2 µsec.

The vacuum chamber of the machine has a clearance of 7 cm vertically and 14.8 cm horizontally. The beam uses the whole cross-
section at injection and has a diameter of about 7 mm at the end of acceleration. It is kept in radial position by phase lock and radial control of the acceleration servo system. The radial position changes some 10 mm during acceleration and has a reproducibility of 2 mm from cycle to cycle.

Due to misalignments of the magnet units of the order of 2 mm and to equipment (lenses, magnets, shielding) for external beams, which are placed close to the main magnets, the closed orbit differs from the theoretical orbit up to ±10 mm in the horizontal plane and up to ±3 mm in the vertical plane. Furthermore, there are differences in radial beam position at straight sections between two focusing and two defocusing magnet ends.

The effective clearance of the machine for radial beam position is about 6 cm. Beam steering in this plane can be done by mistuning the radial feedback in the acceleration servo. Beam steering in the vertical plane is only possible with vertical d.c. magnets, which produce an orbit shape roughly looking like six sine-waves.

II. EXPERIMENTAL AREAS AND TARGET POSITIONS

Two experimental areas (South and North Hall) are in use at the moment. A third one (East area) will come into use at the end of 1962.

Figure 2 shows a layout of beams for South and North Hall, following the targets in straight sections 1, 4, and 7. For internal irradiations (without external beams) targets can be put in other places as well.

The practical situation in the target area, consisting of outgoing beams with lenses, magnets, separators, and shielding, is shown in Fig. 3. There are three different beams, coming along from target 1, which is at the end of the fourth magnet in this picture. In order to get secondary beams at small production angles in the
target, the standard target positions are the pump manifolds in the downstream end of each magnet unit (Fig. 4), between the overhanging coils. Therefore the targets are normally in the magnet stray field which at the actual target position is about 1/3 of the field in the centre of the magnet.

Figure 5 shows a target unit placed in a piece of the vacuum chamber. The hole in front presents the chamber cross-section. Normally the vacuum pump is connected on the right-hand side.

III. TARGET MECHANISM AND TARGET HEADS

Each target unit consists of two supporting rods, which can be moved radially by remote control. The target heads turn 90° around horizontal axes (or 180° for the fast target at the top of Fig. 5, which cuts through the beam. At this operation the beam stays at its place during target operation, while for all other target operation the target stays at its place and the beam moves). The main part of the target operation is to bring the target into its up position and to push the beam onto it.

The target heads normally used are shown in Figs. 6 and 7. The foil targets are used to make stacks of foils for internal irradiation as well. The block targets are used to make short bursts, as long as not too refined optics are required in the external beam. For consuming only some 10% of the beam for a short burst, the fingers of the fast target can be flipped through the primary proton beam.

For external beams, which required a small source, the point source targets of Fig. 7 are used. They are placed in such a way that they point in the direction of the secondary beam.

The long burst targets have a small mass to get as many traversals as possible. The disc is placed on top to get rid of modulation in the burst shape. The short burst targets are available
in three different dimensions in order to cover a certain range between two incompatible requests: source size and burst length as small as possible.

The burst shapes which can be produced with these target heads are given in Fig. 8. The modulation in the long burst is due to ripple of 600 and 50 cycles/sec in the main magnetic field during the flat top. To improve this situation a filter will be installed in a couple of months. To produce the long burst, the beam is de-bunched right at the beginning of the flat top and spirals slowly out onto the target by having a slight slope in the magnetic field during the time of the flat top.

A short burst is produced by steering the beam with rf against the target. The burst length depends on sweeping speed and mass of the target. The first one can be increased by excitation of betatron oscillations with the rf knock-out. This makes a sharp front edge of the short burst; however, it cannot affect the back tail, which is determined by the target mass only.

IV. MACHINE CYCLE, PROGRAMMING, AND SHARING

There are a number of standard machine cycles available, for example with 2 sec repetition: 200 msec flat top and 19.2 GeV/c top energy for long bursts, or 20 msec flat top and 21.9 GeV/c for short bursts. Figure 9 shows the standard timing signals which are distributed with each cycle.

Facilities for programmed operation giving only some pulses (e.g., every 2nd, 3rd ... 90th pulse, or 2 out of 5, and other combinations) to a special target in the machine are provided, and it is usual to have several experimental teams working together. There is also the possibility of intensity sharing on two long or two short bursts which are produced simultaneously (Fig. 10). Intensity can be adjusted by deformation of the closed orbit, so that the beam has
different distances from the two targets at the beginning of the burst. This is done with horizontal kicker magnets similar to the dumping in Fig. 12.

V. BEAM BEHAVIOUR DURING TARGET OPERATION

Before intercepting a target, the beam has a diameter of about 7 mm. Protons which are hitting the target produce either a nuclear interaction, or they are diffraction or Coulomb scattered. Scattering blows up the beam, as shown in Fig. 11, where a thin Al foil and Au foil are pushed into the beam. Because of this effect, the protons are not only lost by interactions in the target, but a rather big fraction is lost on the chamber wall all around the machine. Other protons make interactions in the support of the actual target head.

In order to keep this background from the chamber wall and the target support small, a special dumping technique has been developed. The closed orbit is horizontally deformed by energizing d.c. kicker magnets as shown in Fig. 12. This procedure makes parts of the chamber wall, which are far away from the target area, dumping places. The effect of this procedure can be seen in Fig. 13 where the irradiation of a gold foil is shown without and with this dumping technique.

* * *
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DISCUSSION

Key : What will be the loss of protons interacting at the target by using the dumping technique?

Richter : I can answer this question by giving some more information about the example given at the end of the lecture (Fig. 13). By the dumping technique, the number of interactions in the aluminium frame is diminished by a factor of 2 only. Taking an irradiation time twice as long, the ratio between wanted and unwanted interactions is improved by a factor of 20.

Nikolić : What is your definition of target efficiency and what is the influence of Coulomb scattering of protons on this efficiency?

Richter : The measurements of target efficiency have been made up to now with a large foil target only. This foil of 5 and 10 μ aluminium with a size of 6 × 8 cm² covers almost completely the vacuum chamber aperture. The time for moving it into position is only a very short fraction of the total burst length. For this target J.A. Geibel calculated from the actual burst shape a target efficiency

\[ \eta = \frac{\text{protons absorbed and diffraction scattered}}{\text{total number of accelerated protons}} \approx 66\% . \]

This is done by assuming that the initial slope of the secondary burst has no loss of protons due to Coulomb scattering. Further measurements with the same target with observation of the protons which are lost on the chamber wall due to Coulomb scattering gave target efficiencies of somewhat less than 60%. Since this type of target should be more efficient than the targets
Richter (cont.) : which are normally used, this efficiency seems to be the upper limit which one can expect under very favourable conditions (target in centre of aperture; mass of target support negligible compared with mass of actual target-head; no orbit deformation by sharing).

Doble : Could you say a little more about the choice of target material, particularly the advantages and disadvantages of light and heavy materials?

Richter : The advantage of beryllium is that the Coulomb scattering is rather small, that means that the efficiency from this point of view should be the highest possible. The disadvantage is that the interaction length is rather long. If you want to have a long burst you employ a heavy material which is not what is required to have refined optics. Targets normally used in the PS machine are at the moment made of beryllium. Only when we have supply difficulties with this type of target are aluminium targets used. Copper targets have been used previously (neutrino experiment). In conclusion, if you want a long burst you use beryllium targets or a light material with mass number near to it, and only for special purposes heavy materials are used.

Dahl-Jensen : If the machine conditions are such that the flat top time is 200 ms at ~ 22.7 GeV/c, and if for some reason this time falls to ~ 50 ms, will the top peak energy then be changed?

Richter : This is simply due to a fault of the machine and what one is going to do depends on the special machine fault.
Teucher: Could you say something about the present intensity of circulating protons and about the possibilities at CERN to improve it? What is the maximum intensity which the PS actually can reach?

Richter: For the moment the maximum intensity is $2.8 \cdot 10^{11}$ protons/pulse. In normal operating conditions the intensity is $1.5 \cdot 10^{11}$ protons/pulse. Improvements for obtaining a higher intensity are now being carried out on the linear accelerator. Next year the value of $3.5 \cdot 10^{11}$ protons/pulse may be reached.

* * *
Fig. 1.  General plan of the CERN PS

Fig. 2.  Layout of beams in the North and South experimental halls
Fig. 3. Experimental set-up in the target area with beam aligned from target 1, which is at the back of the picture.

Fig. 4. A PS magnet unit
Fig. 5. Standard targets

above: mounted in pump manifold; target heads are seen through the clearance of the vacuum chamber; pulling magnets on the right; motor-drive for change of radial position on the left.

below: the actual targets enlarged; long burst foil target on the bottom rod; short burst ‘fast target on the top rod.
LONG BURST

Material: Be, 1 or 10 or 20 mm, d = 3 or 6 mm.
axis for flipping

**Material:** Be or Al.
**h** = **w** = 2 x 3 mm$^2$
or 3 x 4 mm$^2$
or 5 x 5 mm$^2$
axis for flipping

The disc serves to reduce the magnet ripple structure in the burst. The amount of secondaries coming from the disc should be less than 1/10 of those coming from the 1mm rod.

A long burst from this target can be obtained with beam spiraling towards inside. Large modulation (500 c/s from magnet ripple) cannot be avoided.

**STANDARD POINT SOURCE TARGETS**

**Fig. 7.** Details of targets to give "short" and "long" particle bursts

LONG BURST    SHORT BURST

**Fig. 8.** Details of the short and long particle bursts
Fig. 9. FS magnet cycle and timing signals

Fig. 10. Beam sharing with two targets working simultaneously
Radioautographs of irradiation at 24 GeV

Source and intensity distribution

aluminium-foil 10μ thick

gold-foil 7μ thick

Fig. 11. Radiographs of the irradiation of thin foils of 6 × 8 cm² area
Fig. 12. Closed orbit deformation to dump scattered protons

Normal irradiation

7 μ gold foil on 1mm al. frame

Irradiation with dumping

Black spot describes area for

50%  85%  95% of interactions made in the target

Fig. 13. The effect of dumping
METHODS TO CALCULATE BEAM TRANSPORT SYSTEMS

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I. INTRODUCTION

In most experiments with accelerators it is necessary to use well-collimated beams of secondary particles with a small momentum spread. Focusing is obtained with quadrupoles and momentum analysis with bending magnets. We shall give here some equations and some general considerations that were found useful in designing secondary beams for the CERN PS. In all these beams the angles between the particle trajectories and the axis of the beam transport system are of the order of $10^{-2}$ radians, so that the tangents and sines can be taken as equal to the angles themselves with high accuracy. This paraxial approximation enormously simplifies the mathematics.

Surveys of beam transport techniques have been given by Chamberlain\(^1\), and Luckey\(^2\). The basic theory of alternating gradient focusing is due to Courant and Snyder\(^3\). Several of the subjects that we shall discuss are treated in an elementary way in the recent book by Livingood\(^4\). Extensive articles about quadrupoles have been published by Septier\(^5\). The characteristics of the bending magnets and quadrupoles\(^6,^7\) used at the CERN PS have been measured by Langeseth et al. A computer programme to trace trajectories of secondary particles in the CERN PS stray field has been written by Kuiper et al.\(^8\). A collection of formulae to calculate beam transport systems has also been given by Auberson\(^9\).
II. EQUATIONS OF MOTION IN A QUADRUPOLE FIELD

A schematic cross-section of a quadrupole magnet is shown in Fig. 1. The profile of each pole is a rectangular hyperbola with the equation

\[ xy = \frac{n^2}{2} \quad (1) \]

where \( R \) is the radius of the inscribed circle. The axis of the quadrupole is the \( z \)-axis.

The magnetic field distribution can be derived from the magnetic scalar potential

\[ -V = G \ xy \quad (2) \]

so that the components of the field are

\[ B_x = -\frac{\partial V}{\partial x} = G y \]
\[ B_y = -\frac{\partial V}{\partial y} = G x. \quad (3) \]

We note also that

\[ \frac{\partial B_x}{\partial y} = \frac{\partial B_y}{\partial x} = G. \quad (4) \]

From these equations we see that a quadrupole is completely described by giving its gradient \( G \). The maximum value of \( G \) that can be realized in a practical quadrupole is limited by saturation of the steel to about

\[ G_{\text{max}} \approx \frac{1.1}{R} \text{ Wb/m}^3. \quad (5) \]

We shall use MKS units, expressing distances in m, magnetic fields in \( \text{Wb/m}^2 \) (= 10^4 gauss), and gradients in \( \text{Wb/m}^3 \) (= 10^2 gauss/cm).

The equations of motion of a particle with charge \( e \) and relativistic mass \( m \), travelling in the \( z \)-direction with constant velocity
\[ v_z = v \text{ are} \]
\[
\frac{d}{dt} \left( m \frac{dx}{dt} \right) = F_x = -e v B_y = -e G v x
\]  
(6)
\[
\frac{d}{dt} \left( m \frac{dy}{dt} \right) = F_y = +e v B_x = +e G v y.
\]

The direction of the forces is also indicated in Fig. 1, which shows that \( F_x \) is everywhere directed towards the plane \( y = 0 \), while \( F_y \) is always directed away from the plane \( x = 0 \). Therefore the quadrupole shown in Fig. 1 is horizontally focusing and vertically defocusing. The roles of the two planes can be exchanged by reversing the polarity of the quadrupole.

The magnetic forces are always perpendicular to the direction of motion so that \( m \) and therefore the \( z \)-component of the momentum \( p = mv \) are constant. Eliminating \( t \) with the relation \( z = vt \) we can rewrite Eq. (6) as
\[
\frac{d^2 x}{dz^2} + \frac{eG}{p} x = 0
\]  
(7)
\[
\frac{d^2 y}{dz^2} - \frac{eG}{p} y = 0.
\]

The quantity \( p/e \) is called the magnetic rigidity and is usually indicated by the symbol \( B_\rho \). A particle with magnetic rigidity of 1 Wb/m has a momentum of 0.300 GeV/c.

III. THICK LENS EQUATIONS FOR A QUADRUPOLE

If the length \( L \) of the quadrupole is so small that the change of \( x \) and \( y \) inside the quadrupole can be neglected, Eq. (7) can be integrated directly and becomes
\[
\Delta \left( \frac{dx}{dz} \right) = - \frac{eL}{B_\rho} x
\]  
(8)
\[
\Delta \left( \frac{dy}{dz} \right) = + \frac{eL}{B_\rho} y.
\]
In this approximation the quadrupole behaves as a thin lens with focal length

$$f_0 = \frac{B_0}{3L}$$  \hspace{1cm} (9)

that is focusing in one plane and defocusing in the other plane. Although this formula is very useful for a first estimate, it is not sufficient for accurate work. We shall therefore derive the exact solution of Eq. (7) and prove that the quadrupole can be represented by a thick lens with different focal distances and principal planes for the motion in the focusing and defocusing planes, respectively.

Let us introduce a constant

$$K^2 = \frac{G}{B_0}.$$  \hspace{1cm} (10)

Taking $z = 0$ at the entrance of the quadrupole we can write the solution of Eq. (7) as

$$x = x_0 \cos K z + \frac{x'_0}{K} \sin K z$$

$$y = y_0 \cosh K z + \frac{y'_0}{K} \sinh K z$$  \hspace{1cm} (11)

where $x_0$, $x'_0$, $y_0$, and $y'_0$ are the initial values of $x$, $dx/dz$, $y$, and $dy/dz$.

By differentiation of Eq. (11) we find

$$x' = -K x_0 \sin K z + x'_0 \cos K z$$

$$y' = K y_0 \sinh K z + y'_0 \cosh K z.$$  \hspace{1cm} (12)

Assume now that a particle is emitted from a point $P$ located off the $z$-axis in the plane $z = z_P < 0$ as shown in Fig. 2. In the field-free region the particle travels in a straight line and will therefore enter the quadrupole with the initial conditions

$$x_0 = x_P - x'_P z_P, \quad x'_0 = x'_P$$

$$y_0 = y_P - y'_P z_P, \quad y'_0 = y'_P.$$  \hspace{1cm} (13)
By substituting Eq. (13) into Eq. (11) and (12), where we also put $z = L$, we get the position and angles of the trajectory at the exit of the quadrupole. From there on the particle travels again in a straight line given by

$$
\begin{aligned}
    x &= \left[ \cos KL - K(z-L) \sin KL \right] x' \\
        &+ \left[ \frac{1}{K} \sin KL + (z-L-z_p') \cos KL + Kz_p'(z-L) \sin KL \right] x'_p \\
\end{aligned}
$$

$$
\begin{aligned}
    y &= \left[ \cosh KL + K(z-L) \sinh KL \right] y' \\
        &+ \left[ \frac{1}{K} \sinh KL + (z-L-z_p') \cosh KL - Kz_p'(z-L) \sinh KL \right] y'_p. \\
\end{aligned}
$$

(14)

It is always possible to find a point $P'_x$ so that for $z = z(P'_x)$ the coefficient of $x'_p$ vanishes. In the plane $z = z(P'_x)$ all particles emitted from point $P$ pass through the same point so that this is the image plane of the plane $z = z_p$ for the horizontal motion. There is also an image plane $z = z(P'_y)$ for the vertical motion, but always $z(P'_x) \neq z(P'_y)$, except when $K = 0$, which is a trivial case.

It is shown in geometrical optics that a system with the imaging properties as given by Eq. (14) can be described completely by giving its principal planes and focal length. These can be found most easily by considering an incident ray with $x'_p = y'_p = 0$. In the image principal plane for the horizontal motion $x = x_p$, so that the coefficient of $x_p$ in Eq. (14) must be unity. This gives for the position of the image principal plane

$$
    z(P'_x) = L + \frac{\cos KL - 1}{K \sin KL}.
$$

(15)

This ray crosses the $z$-axis at the image focal point

$$
    z(P'_x) = L + \frac{\cos KL}{K \sin KL}.
$$

(16)
and the focal length is
\[ f_z = \frac{1}{K \sin KL}. \]  
(17)

The corresponding equations for the vertical plane are
\[ z(H_y') = L - \frac{\cosh KL - 1}{K \sinh KL} \]  
(18)
and
\[ z(F_y') = L - \frac{\cosh KL}{K \sinh KL} \]  
(19)
and
\[ f_y = -\frac{1}{L} \cosh KL. \]  
(20)

In practical cases KL << 1 so that the trigonometric and hyperbolic functions can be developed in a power series, retaining only the first term. Using also Eqs. (9) and (10) we then find for the focusing plane
\[ z(H_x') = \frac{L}{2} - \frac{L^2}{24 f_0} \]  
(21)

\[ f_x = f_0 + \frac{L}{6} \]  
(22)

and for the defocusing plane
\[ z(H_y') = \frac{L}{2} + \frac{L^2}{24 f_0} \]  
(23)

\[ f_y = f_0 - \frac{L}{6}. \]  
(24)

Note that the middle of the quadrupole is at \( z = \frac{L}{2} \).

By considering a trajectory which is parallel to the z-axis at the exit of the quadrupole, we can calculate the position of the object principal planes \( H_x \) and \( H_y \), and the object focal points \( F_x \) and \( F_y \). One then finds that \( H_x \) and \( H_x' \) lie symmetrically with respect to the middle of the quadrupole, while the same is true for \( F_x \) and \( F_x' \),
etc. This result is obvious since the whole quadrupole is symmetrical around its middle.

Equations (21) to (24) are sufficiently accurate for all practical purposes. We see from Eq. (22) that in the focusing plane a quadrupole is weaker than would follow from the thin lens approximation. Inspection of Fig. 2 shows that the actual trajectory is closer to the axis and therefore in a weaker field than would follow from the thin lens approximation. The reverse is true in the defocusing plane.

IV. DOUBLETS AND TRIPLETS

A single quadrupole makes a real image in only one plane, whereas for a beam transport channel focusing in both the x and y plane is necessary. This condition can, for example, be met with a doublet consisting of two quadrupoles with approximately equal strength but opposite polarity. The beam envelope in the x and y plane of a doublet is shown in Fig. 3. Particles are emitted from a point P on the axis, and must be brought to an image P_x' in the horizontal plane and P_y' in the vertical plane. P_x' and P_y' can be different. The projection of the particle motion on the x-plane has a larger excursion from the x-axis in the second quadrupole, which is focusing in the x-plane, than in the first one which is defocusing in the x-plane. Therefore, the focusing effect of the second quadrupole is stronger than the defocusing effect of the first quadrupole, and the difference of these two effects gives the nett focusing action of the doublet in the x-plane. The same argument is valid for the y-plane.

It follows from Eqs. (22) and (24) that in the focusing plane a quadrupole is weaker, while in the defocusing plane it is stronger than would follow from the thin lens approximation. Since the nett focusing action of a doublet depends on the difference of the effects of the two quadrupoles, a doublet is considerably weaker than would
follow from the thin lens approximation. The same is true for other combinations of quadrupoles.

The task of the beam designer is now to find the gradients $G_1$ and $G_2$ in the two quadrupoles (which have equal length $L$) that give images $P_x'$ and $P_y'$ for a given object $P$. Even in this very simple case an explicit solution for $G_1$ and $G_2$ can only be derived in the thin lens approximation of Eq. (9) which can give quite large errors. When the proper equations (21) to (24) are used this approach becomes hopeless.

The only way out is to assume certain values of $G_1$ and $G_2$ and to trace two rays emitted from $P$, one in the $x$- and one in the $y$-plane. From the distances at which these rays pass $P_x'$ and $P_y'$ one estimates how much $G_1$ and $G_2$ should be changed and repeats the ray tracing with the modified values of $G_1$ and $G_2$. This procedure is repeated until the rays pass $P_x'$ and $P_y'$ at a small enough distance. A Mercury computer programme that works on this basis and traces rays through complete beam transport systems with several intermediate images has been written by Van der Meer\(^1\). In practice one would first design a beam with a desk computer in the thin lens approximation and then use the computer programme to obtain the accurate values of the quadrupole strengths.

Some general comments can be made about a doublet. Even if $P_x'$ coincides with $P_y'$, the values of $G_1$ and $G_2$ are generally different. If $l < l'$, then $G_1 > G_2$, and vice versa. Only when $l = l'$ we have $G_1 = G_2$, as can readily be seen from the symmetry of the system in that case.

Inspection of Fig. 3 shows that the angular magnification in the plane $DF$ is larger than in the plane $FD$. It is well known from geometrical optics that the transverse magnification in the plane $FD$ is usually at least twice as large as in the plane $DF$. There is no objection against this and in many cases one can make good use of it. The difference in magnification can be enhanced by increasing $s$. Note
that by doing so one also decreases the maximum angles $x'_{\text{max}}$ and $y'_{\text{max}}$
for which particles emitted from $P$ will pass through the doublet.

The alternative arrangement is a triplet, consisting of a central quadrupole of length $L$ and a quadrupole with length $\frac{1}{2} L$ on each side. Figure 4 shows the beam envelopes. In this case the magnification in both planes are rather equal. It is customary to have the same gradient in the two outer quadrupoles, which will in general be different from the gradient in the central quadrupole, but this is not at all necessary. Goldberg et al.\textsuperscript{11} have studied a beam with a triplet with different gradients in all three quadrupoles. This allows them to vary the magnifications in the course of an experiment while keeping $P_x$, $P'_x$, and $P_y'$ fixed.

In the literature\textsuperscript{1},\textsuperscript{12} one finds graphs for the focusing properties of doublets and triplets. However, these assume that $P_x'$ and $P_y'$ coincide, the triplet is symmetrically excited and choose some specific values of $s$. Therefore they are of very limited use.

V. MATRIX FORMULATION

Let us consider any optical system in which the displacement $x_1$ and slope $x_1'$ of a particle trajectory at the exit are related to the corresponding values $x_0$ and $x_0'$ at the entrance by the linear equations

$$x_1 = a_{11}x_0 + a_{12}x_0'$$
$$x'_1 = a_{21}x_0 + a_{22}x_0'$$

Using matrix notation we can write Eq. (25) as

$$\begin{pmatrix} x_1 \\ x'_1 \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} x_0 \\ x_0' \end{pmatrix}.$$  \hspace{1cm} (26)

Assume now that this system is followed by a second optical system
that transforms the vector \((x_1, x_1')\) into the vector

\[
\begin{pmatrix}
 x_2 \\
 x_2'
\end{pmatrix} =
\begin{pmatrix}
 b_{11} & b_{12} \\
 b_{21} & b_{22}
\end{pmatrix}
\begin{pmatrix}
 x_1 \\
 x_1'
\end{pmatrix}.
\] (27)

The relation between \((x_2, x_2')\) and \((x_0, x_0')\) is then

\[
\begin{pmatrix}
 x_2 \\
 x_2'
\end{pmatrix} =
\begin{pmatrix}
 b_{11} & b_{12} \\
 b_{21} & b_{22}
\end{pmatrix}
\begin{pmatrix}
 a_{11} & a_{12} \\
 a_{21} & a_{22}
\end{pmatrix}
\begin{pmatrix}
 x_0 \\
 x_0'
\end{pmatrix}.
\] (28)

where the matrices \(A\) and \(B\) are multiplied by the well-known rules for matrix multiplication. In this way we can trace a ray through any number of optical elements by simply multiplying the corresponding matrices. Note that the order of the matrices in this product is the inverse of the order in which the particles traverse the corresponding optical elements. This order must be carefully observed, since in general for matrices \(A\) and \(B\)

\[
A B \neq B A.
\] (29)

If \(A\) is the transfer matrix for particles passing from left to right through some optical system, the matrix for particles passing from right to left is \(A^{-1}\), defined by the relation

\[
A A^{-1} = 1.
\] (30)

We shall show in Section VII that the determinant of all matrices representing combinations of quadrupoles and drift spaces is unity, that is

\[
a_{11}a_{22} - a_{12}a_{21} = 1.
\] (31)

It is readily verified therefore that

\[
A^{-1} = \begin{pmatrix}
 a_{12} & -a_{11} \\
 -a_{21} & a_{22}
\end{pmatrix}.
\] (32)

Equation (31) provides a convenient method for detecting errors in calculations involving matrix multiplications.
The matrix formulation is especially useful for the calculation of beam transport systems whose focusing properties are a periodic function of $z$, since simple rules for the multiplication of identical matrices exist. Its main application is therefore in the theory of betatron oscillations in particle accelerators.

The multiplication of matrices is a rather laborious work. To find the transfer matrix of a long beam transport system it is often easier to trace one ray with $x_0 = 1$, $x_0' = 0$ to find $a_{11}$ and $a_{21}$, and then to trace a second ray with $x_0 = 0$, $x_0' = 1$ to find $a_{12}$ and $a_{22}$. Even so, matrices can often be very helpful in studying beams. Suppose, for example, that one wants to find the effect of varying a particular quadrupole somewhere halfway in a long beam. Having traced the two rays mentioned above we can write the matrix for the total beam as the product of three matrices, corresponding to the part of the beam before the variable quadrupole, the variable quadrupole itself, and the part after the variable quadrupole. In this way the change in the total transfer matrix due to the variable quadrupole is readily calculated.

The transfer matrix for a thin focusing lens with focal length $f_0$ is

$$
\begin{pmatrix}
1 & 0 \\
-\frac{1}{f_0} & 1
\end{pmatrix}
$$

(33)

and for a drift space of length $s$ it is

$$
\begin{pmatrix}
1 & s \\
0 & 1
\end{pmatrix}
$$

(34)

From Eqs. (11) and (12) we see that the exact matrices for the focusing and defocusing plane of a quadrupole are
\[
\begin{pmatrix}
\cos KL & \frac{\sin KL}{K} \\
-K \sin KL & \cos KL
\end{pmatrix}
\]

and
\[
\begin{pmatrix}
\cosh KL & \frac{\sinh KL}{K} \\
K \sinh KL & \cosh KL
\end{pmatrix}
\]

VI. ABERATIONS

The focusing properties of a quadrupole depend on the momentum of the particles. This effect, called chromatic aberration, limits the momentum band that can be accepted if a given unsharpness of the image is not to be exceeded.

Let us consider now a beam transport system, shown in Fig. 5, that images P in P' for particles with momentum \( p_0 \). The transfer matrices from P to some intermediate point R, and from R to P', are \( B \) and \( A \), respectively. The transfer matrix \( M \) from P to P' is then
\[
M = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} = \begin{pmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{pmatrix}.
\]

Since \( P' \) is the image of \( P \)
\[
a_{11}b_{12} + a_{12}b_{22} = 0
\]
and the magnification is
\[
\frac{x'_L}{x_0} = \frac{x'_F}{x'_1} = a_{11}b_{11} + a_{12}b_{21}.
\]

The trajectory of a particle with \( p = p_0 + \Delta p \) is the same as that of a particle with \( p = p_0 \) but with all gradients changed by an amount
\[
\Delta g = -g \frac{\Delta p}{p}
\]
We now assume first that only in a short piece of quadrupole, located at \( R \), the gradient is changed by this amount \( \Delta G \). The modified transfer matrix \( M' \) is then

\[
M' = \begin{pmatrix}
a_{11} & a_{12} \\
a_{21} & a_{22}
\end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\frac{\Delta G dz}{B \rho} & 1 \end{pmatrix} \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}.
\tag{40}
\]

Using Eq. 37 we find

\[
M_{12}' = -\frac{a_{12}b_{12} \Delta G dz}{B \rho}.
\tag{41}
\]

Therefore a trajectory starting from \( P \) with \( x_0 = 0 \), \( x_0' = a_0 \) passes point \( P' \) at a distance

\[
x_1 = -\frac{a_0 a_{12} b_{12} \Delta G dz}{B \rho}
\tag{42}
\]

and appears to come from an object at \( P \) located at a distance

\[
\delta = \frac{-a_0 a_{12} b_{12} \Delta G dz}{(a_{11} b_{11} + a_{12} b_{21}) B \rho}
\tag{43}
\]

from the axis.

It is clear that

\[
x_R = b_{12} a_0.
\tag{44}
\]

Moreover, using Eqs. (32) and (38) we find that

\[
x_R = -a_{12} a_1 = \frac{-a_{12} a_0}{a_{11} b_{11} + a_{12} b_{21}}.
\tag{45}
\]

Combining now Eqs. (39), (43), (44) and (45) we get

\[
\delta = \frac{-1}{a_0} x_R^2 \frac{G}{B \rho} \frac{\Delta \rho}{\rho} dz.
\tag{46}
\]

The total apparent displacement of the object at \( P \) is found by integrating Eq. (46) from \( P \) to \( P' \). If particles are emitted from \( P \) in the angular interval \(-a_0 < x_0' < +a_0\) the total apparent increase of the object size for a particle with momentum \( p_0 + \Delta p \) is
\[ d = \frac{2}{\alpha_0} \frac{\Delta p}{p} \int_0^P x^2 \frac{G}{B_p} \, dz. \]  
\[ \text{(47)} \]

In this equation \( G \) is, of course, a function of \( z \). Note that a particle with \( p = p_0 - \Delta p \) has the same \( d \).

Using Eq. (7) we can rewrite Eq. (47) as

\[ d = -\frac{2}{\alpha_0} \frac{\Delta p}{p} \int_0^P x \, x'' \, dz \]  
\[ \text{(48)} \]

and by partial integration we obtain

\[ d = \frac{2}{\alpha_0} \frac{\Delta p}{p} \int_0^P (x')^2 \, dz. \]  
\[ \text{(49)} \]

The length of the projection on the \( x \)-plane of a particle trajectory between \( P \) and \( P' \) is

\[ L_t = \int_0^P \frac{d s}{\sqrt{1 + (x')^2}} \, dz = \int_0^P dz + \frac{1}{2} \int_0^P (x')^2 \, dz. \]  
\[ \text{(50)} \]

Therefore \( L_t \) exceeds the distance \( z_{P'} - z_P \), measured along the \( z \)-axis by an amount

\[ \Delta L = \frac{1}{2} \int_0^{P'} (x')^2 \, dz \]  
\[ \text{(51)} \]

and we can write Eq. (49) as

\[ d = \frac{4}{\alpha_0} \frac{\Delta p}{p} \Delta L. \]  
\[ \text{(52)} \]

From the latter equation we see that it is not possible to make with quadrupoles a beam without chromatic aberration, since any trajectory is always longer than the direct distance \( z_{P'} - z_P \). The smallest \( \Delta L \) is obtained with trajectories that are as short as possible.
and everywhere make small angles with the z-axis. This argument can be used to judge qualitatively the relative merits of different quadrupole systems. To find the chromatic aberration in practice it is usually quicker to trace a ray with momentum $p_0 + \Delta p$ through it, rather than to use Eq. (49) or Eq. (52).

Van der Meer has shown\(^1\) that by placing a sextupole at a place where the horizontal images of different momenta are separated due to the dispersion of a bending magnet one can correct the chromatic aberration in a vertical plane. Experiments on this carried out at CERN have not yet been conclusive, but Ticho\(^1\) has made a separated K-beam with $\Delta p/p = \pm 3\%$ by non-linear shimming of some bending magnets.

Up to now we have assumed perfect quadrupole fields, but in reality there are appreciable deviations from this ideal case, due to the finite width of the pole, saturation of the steel, and end effects. If it is known how $G$ depends on $x$ and $z$, these non-linear aberrations can be expressed by a similar formula as was derived for the chromatic aberration. In principle one could correct for it with a suitable non-linear lens, but in practice it is simpler to use only the central part of the quadrupole. In the CERN quadrupoles, non-linear aberrations can be neglected, even for high quality separated beams, if the distance of the trajectory from the axis never exceeds 0.7 $R$.

VII. PHASE SPACE REPRESENTATION

In this section we shall apply some of the results of statistical mechanics\(^1\) to beam transport problems. We first remark that Eq. (6) can be derived from a Hamiltonian

$$H = \frac{1}{2} \left( e G v_x^2 - e G v_y^2 + \frac{p^2}{m} + \frac{p_y^2}{m} \right) \tag{53}$$

where $p_x = m v_x$ and $p_y = m v_y$, by means of the canonical equations of motion. We must allow $G$ to be a function of time, since the particle may pass through several quadrupoles with different gradients.
The particle motion in the x and y plane is independent and therefore we shall consider only the motion in the x-plane that is given by

$$H_x = \frac{1}{2} \left( e G v_x^2 + \frac{p_x^2}{m} \right).$$

(54)

As a particle passes through a quadrupole the values of $p_x$ and $x$ change continuously. Using a co-ordinate system with axes $x$ and $p_x$ we can plot this sequence of pairs of values $(x, p_x)$. If $G$ is constant the point $(x, p_x)$ representing the particle in phase space describes a curve given by the equation $H_x = \text{const}$. The general proof of this statement is given in statistical mechanics, but for this simple case we can verify it by multiplying both sides of Eq. (6) by $dx/dt$ and integrating.

For $G > 0$ the phase space trajectories are ellipses. This situation is shown in Fig. 6. In a defocusing plane we have $G < 0$ and the phase space trajectories are hyperbolae.

Let us now follow the ensemble of representative points in phase space of all particles emitted by a source and passing through a beam transport system. At each point along the axis of the system this ensemble of points lies inside a closed curve, whose shape varies continuously since the points representing the particles move in phase space. Now, Liouville's theorem says that the area enclosed by this boundary curve is constant for all systems of particles whose motion can be derived from a Hamiltonian as given by Eq. (54) where $G$ may depend on time. Again we refer to statistical mechanics for a proof of this theorem.

When the particle beam is limited by several diaphragms the boundary curve in phase space is a complicated polygon, but in many cases it can be approximated by an ellipse. Such an ellipse is transformed into a new ellipse in a transformation as given by Eqs. (26) and (27). It is readily verified that the area of the new ellipse is equal to the area of the original ellipse, multiplied by
the determinant of the transfer matrix, which therefore must be unity in order to satisfy Liouville's theorem. Successive phase space ellipses in a focusing quadrupole are shown in Fig. 7.

It is usually more convenient to use \( x' = \frac{dx}{dz} \) instead of \( p_x \) since the former has a direct geometrical meaning. As long as angles are small and the particle is not accelerated, these two variables are equivalent.

VIII. BENDING MAGNETS

Practically all beams contain bending magnets for momentum analysis. The transfer matrix for a bending magnet with arbitrary angles between the end faces and the beam has been given by Edwards and Rose \(^{[16]}\). We shall discuss here a much simpler arrangement that is practically the only one used in high-energy beams. It is shown in Fig. 8. The end faces of the bending magnet are perpendicular to the axis and the beam passes symmetrically through it. In the horizontal plane such a magnet behaves merely as a drift space with length \( L \).

Due to the edge focusing effect, each edge has on the vertical motion the effect of a thin focusing lens with focal distance

\[
 f_e = \frac{\rho}{\tan \frac{1}{2} \varphi} 
\]

(55)

where \( \rho \) is the radius of curvature of the particle orbit inside the magnet. In high-energy beams \( f_e \) lies in the range \( 10^2 \) to \( 10^3 \) m, while \( L \) might be 2 m. Therefore the effect of the two edges can be represented by a single thin focusing lens with focal length

\[
 f = \frac{\rho}{\varphi} - \frac{L}{\varphi^2} 
\]

(56)

at the centre of the magnet.
Assume now that we have a bending magnet producing a given deflection, and we ask ourselves how it could be used together with the doublet shown in Fig. 3 so that the dispersion of the beam is largest. The total deflection angle for particles with momentum \( p_0 \) is \( \varphi \), and for particles with momentum \( p_0 + \Delta p \) is \( \varphi - \Delta \varphi \). To compare the positions A and C we draw the phase space ellipses in these points. One can show that the area of each ellipse is

\[
S = \pi \Delta x' (\Delta x')
\]

where the symbols are explained in Fig. 9, and according to Liouville's theorem

\[
\left( \pi \Delta x' \right)_A = \left( \pi \Delta x' \right)_C.
\]

The solid ellipses in Fig. 9 are the phase space ellipses for \( p = p_0 \), and the ones drawn in interrupted lines are for \( p = p_0 + \Delta p \) after the bending magnet. At point C we have \( \Delta \varphi > 2 \Delta x' \) so that the two ellipses are completely separated. Therefore also the images at \( p_x' \) for \( p_0 \) and \( p_0 + \Delta p \) are completely separated. At point A we have \( \Delta \varphi < 2 \Delta x' \) so that the ellipses and therefore also the images at \( p_x' \) partially overlap. From this discussion we see that the dispersion obtained with a given \( \varphi \) is proportional to the width of the beam in the bending magnet, which therefore should be made as large as possible. Note that the beam cross-section in point C fits nicely in the bending magnet since normally the width of the poles is several times larger than the height of the gap.

The dispersion obtained by placing the bending magnet in between the two quadrupoles is larger than at A, but in general somewhat smaller than at C. This arrangement can be interesting if a very different magnification in the x and y plane, and therefore a large s, is used.

Looking at Fig. 4 we see that a large \( \Delta \) in the bending magnet can be obtained by splitting the triplet at its plane of symmetry and inserting the bending magnet there. One has then two doublets,
the first of which makes a parallel beam, while the second focuses
the beam down to an image. This situation is shown in Fig. 10.
Such a system, with $P_2'$ and $P_4'$ coinciding, and with $\mathbf{L} = \mathbf{L}'$, so that
it has unit magnification, has been discussed by Courant\(^{17}\) and
Penner\(^{18}\). The disadvantage is that now four quadrupoles are re-
quired instead of two. However, if one wants to make high inten-
sity beams in the 25 GeV range the quadrupoles become so large that
even if the single doublet of Fig. 3 is used one would in general
compose each quadrupole of two separate quadrupoles of length $\frac{1}{2} L$.

The arguments given above also apply for the dispersion
obtained with electrostatic separators. Since especially in this
case one wants to obtain the maximum possible dispersion, the set-up
of Fig. 10 is frequently used. It is also possible to use one tri-
plet to make the beam parallel inside the separator and then a second
triplet to bring it to an image again\(^{19}\), instead of the two doublets
shown in Fig. 10.

A beam with two successive deflections is shown in Fig. 11.
Particles emitted from $P$ are brought to an intermediate image at $P_2'$
by $Q_1Q_2$ and a final image at $P_2''$ by $Q_3Q_4$. Momentum selection is ob-
tained with $BM_1$ and a slit at $P_2'$. We assume that a second deflec-
tion after $Q_3Q_4$ is necessary, for example, to remove background pro-
duced in the edges of the slit at $P_2'$ or to fit the beam inside the
available experimental area. Let the deflections produced by $BM_1$
and $BM_2$ at $P_2'$ and $P_2''$ be $\theta_1$ and $\theta_2$, respectively. We take the
trajectory of a particle emitted from $P$ with $x_0 = 0$, $x_0' = 0$ and
$p = p_0$ as the $z$-axis. A particle emitted from $P$ with $x_0 = 0$ and
with $p = p_0 + \Delta p$ will be deflected less and passes $P_2'$ at a distance

$$x(P_2') = \frac{\Delta p}{p} \theta_1.$$  \hspace{1cm} (59)

Under the influence of the doublet $Q_3Q_4$ that images $P_2'$ in $P_2''$ with
magnification $m_2 < 0$ the particle passes $P_2''$ at a distance from the
axis
\[ x(P_x) = (m_2d_1 + d_2) \frac{\Delta p}{p} \]  

(60)

To obtain the sharpest possible image at \( P_x \) the images for different momenta should coincide. The condition for this is

\[ \frac{d_2}{d_1} = -m_2 \]  

(61)

that is, the two bending magnets should deflect in the same direction and the ratio of their deflections must be equal to the magnification of the second doublet. For systems with more than two deflections similar relations can be derived.

I should like to express my thanks to S. van der Meer for many stimulating discussions about beam design.

* * *

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3348/HP/KN
Fig. 1: Components of field and force in a quadrupole.

Fig. 2: Principal planes and focal points of quadrupole.
Fig. 3: Beam Envelope in Doublet.

Fig. 4: Beam Envelope in Triplet.

Fig. 5: Calculation of Chromatic Aberration.
Fig. 6: Particle motion and corresponding phase space plot in the focusing plane.

Fig. 7: Successive phase space ellipses in focusing plane.

Fig. 8: Beam passing symmetrically through a bending magnet.
Fig. 9: Separation of phase space ellipses by bending magnet.

Fig. 10: Momentum analysis with bending magnet in between two doublets.

Fig. 11: Beam with two successive deflections.
A SIMPLE EXAMPLE IN BEAM OPTICS

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I. INTRODUCTION

In order to illustrate the theory developed in the previous lecture, we will consider in detail a beam of the type shown in Fig. 11 of Dr. de Raad's paper. In such a beam with an intermediate image in the horizontal plane, it is possible to make a good momentum selection, and also to focus all particles within the desired momentum band in a sharp image. Such requirements are especially typical of emulsion experiments with a pulsed magnet.

II. CALCULATION OF LENS STRENGTHS AND
TRAJECTORIES: FIRST STAGE OF BEAM

We will neglect altogether the effects of the fringing field of the FS.

Let us assume that the position at which we want to focus the beam is 36 metres from the FS target, and that a convenient position for the intermediate focus is at 18 metres. Typical dimensions for the first stage, illustrated in Fig. 1, are

\[
\begin{align*}
\text{target to } Q_1 & : 6 \text{ metres} \\
Q_1 \text{ to } Q_2 & : 2 \text{ "} \\
Q_2 \text{ to BM} & : 5 \text{ "} \\
\text{BM to focus} & : 5 \text{ "}
\end{align*}
\]

Length of bending magnet: 2m; Bending angle: \(15^\circ = 262\) mR.

In practice, these dimensions are determined by the geometry of the machine itself, by the maximum strengths of lenses and bending
magnets for the momentum of the beam, and by the presence of other apparatus and shielding.

We will use only the simplest approximation\textsuperscript{*), that a quadrupole lens of length $L$ with field gradient $G$ behaves as a thin lens with a focal length

$$ f_0 = \frac{B_0}{G L} \tag{1} $$

for particles with a magnetic rigidity $B_0$; and that a bending magnet (of length $L$ and bending angle $\varphi$) behaves as a thin lens in the vertical plane, of focal length

$$ f = \frac{L}{\varphi} \tag{2} $$

We now use

$$ \frac{1}{u} + \frac{1}{v} = \frac{1}{f} \tag{3} $$

and find, with the notation of Fig. 1, for $Q_1$ focusing in the vertical plane,

$$ Q_0 I_0' = 11.0 \text{ metres} \quad \text{in the vertical plane} $$

$$ Q_1 I_1' = 4.485 \text{ metres} $$

$$ Q_1 I_1 = 1.965 \text{ metres} \quad \text{in the horizontal plane} $$

$$ f_{Q_1} = 2.925 \text{ metres} \quad f_{Q_2} = 2.840 \text{ metres}. $$

(The image in the vertical plane is not necessary: the beam might just as well be parallel in this plane.)

If the lenses available can give these focal lengths for the momentum chosen, we can continue with the design.

The ray diagram of Fig. 1 can be drawn from these figures, and from it one can obtain the angular magnification, and hence the linear magnification at the image. We find

<table>
<thead>
<tr>
<th></th>
<th>Vertical plane</th>
<th>Horizontal plane</th>
</tr>
</thead>
<tbody>
<tr>
<td>angular magnification</td>
<td>0.34</td>
<td>1.22</td>
</tr>
<tr>
<td>linear magnification</td>
<td>2.94</td>
<td>0.82</td>
</tr>
</tbody>
</table>

\textsuperscript{*} In two appendices added since the lecture was given, computer calculations of the same beam are discussed.
It can also be seen that the solid angle accepted by the beam is limited in the vertical plane by \( Q_1 \) and in the horizontal plane by \( Q_2 \). For the standard PS quadrupoles, the useful working aperture, if good optical qualities are desired, has a radius of approximately 7 cm. We can therefore see that the solid angle accepted by the beam is

\[
\text{approximately } \left( 2 \times \frac{7}{600} \right) \times \left( 2 \times \frac{3}{600} \right) = 2.72 \times 10^{-4} \text{ steradians.}
\]

The dispersion is easy to calculate: the bending magnet follows the two lenses, and so we do not have to think about them. The total bending angle is 262 mrad, and at 5 m the change in deflection for a 1% change in the momentum of the particles will be of the order of 0.262 \( \times \) 0.01 \( \times \) 5000 mm = 13.1 mm. Thus (neglecting aberrations) if our object is 5 mm wide and the magnification is 0.8 in the horizontal plane, the momentum band we can select by putting a slit at this image will have a "sharpness" of 4/13.1 % of \( p \).

III. THE SECOND STAGE OF THE BEAM

We are now in a position to calculate the second stage of the beam. Generally one has rather more freedom in the choice of layout the further away one is from the machine. In order to avoid any more algebra we will consider the various ways in which the arrangement used in the first stage of the beam can be re-employed. These are illustrated in Fig. 2. We will neglect any small changes in the focusing caused by re-positioning the bending magnet with respect to the lenses.

a) Placed in the same order as before. Fig. 2a)

Since the angular spread of the beam in the horizontal plane at the exit of the first stage is greater than at the entrance, the horizontal aperture will now be limited at the second lens in the second stage of the beam. The overall linear magnification in the vertical plane will now be \( \times 8.6 \), giving a large image in this plane.
b) Placed in the same order as before, but with the signs of the lenses interchanged. Fig. 2b)

The trajectories in the horizontal and vertical planes are now interchanged. This is satisfactory from the point of view of acceptance. The linear magnification will be the same in both planes, \( \times 2.41 \), which is larger than is necessary. In order to have zero dispersion at the final image

\[
\frac{d_2}{d_1} = \frac{\phi_2 L_2}{\phi_1 L_1} = 2.94 .
\]

\( d_1, d_2 \) are the deflections produced by the two bendings, \( \phi_1, \phi_2 \) are the bending angles, and \( L_1, L_2 \) are the distances from the centres of the bending magnets to the foci. If \( L_1 = L_2, \phi_2 = 45^\circ \) which is unrealistic.

\[\text{c) Placed in reverse order, but with the bending magnet displaced. Fig. 2c)}\]

If we neglect the small focusing effect of the bending magnet, we see that the acceptance is the same as for the first stage of the beam (for particles of the correct momentum), and that the over-all magnification in both planes is unity. The bending angle required to give zero dispersion at the final image is \( 23^\circ \), taking \( L_2 = 4 \) metres as a convenient position for the magnet.

In order to see what happens to the off-momentum components of the beam in the second stage, it is only necessary to consider them as coming from displaced objects at the intermediate focus. We can see that in each of these three cases, the acceptance in the horizontal plane for off-momentum particles will be severely limited at the second pair of lenses, where the dispersion will be of the order of 3.9 cm per \( 1\% \Delta p/p \). A field lens which makes an image of the bending magnet aperture at the second pair of lenses can be used to reduce this effect. If a single lens is used, the gain in acceptance in the horizontal plane will be offset by a loss in the vertical plane. A doublet or triplet field lens can be used to give focusing in both planes.
IV. PHASE SPACE

For a more detailed knowledge of the momentum selection of the beam, its acceptance in momentum and angle, and the size of the final image, it is necessary to use phase-space diagrams. We will consider only the horizontal plane, and first the situation for particles of the correct momentum. Phase space diagrams at various points in the system are shown in Figs. 3a) to 3f). x and x' are the distances from the axis and the angle with the axis of the particle trajectory. In order to construct them it is only necessary to have the information contained in Fig. 1.

Figure 3a) is the object phase space. Particles emitted with positive x' from one edge of the target are indicated by the heavy line. Fig. 3b) shows the effect of 6 metres drift space. The x' remain unchanged, but the change in x increases linearly with x'. Q1 has the effect shown in Fig. 3c): x remains unchanged, but the change in x' is proportional to x. (cf., the transfer matrices 33 and 34 in Dr. de Raad's talk). Two metres drift space then results in Fig. 3d). Figure 3e) shows the effect of Q2. Ten metres further drift space gives the image phase space shown in Fig. 3f).

In Section II above, we decided to set a limit at the lenses of |x| ≤ 7 cm. This limit is shown as the broken line at |x| = 7 cm in Fig. 3d) and 3e). It has been transferred by inspection to both the object and image through all the intermediate diagrams. Any other limiting aperture can be treated in the same way.

V. CHROMATIC ABERRATION AND DISPERSION

Since f is proportional to p for all beam elements, the position of the images for the off-momentum particles can be calculated by changing the focal lengths of all the lenses accordingly. Thus, for the first stage of the beam in the horizontal plane, one finds that particles whose momentum is 5% lower than the correct momentum will be
focused 1.26 m before the image point I for the correct momentum. The phase space diagram for these particles at I will therefore be that of particles 1.26 m beyond their image point, at which image point the magnification is some 13% lower. This situation is illustrated in Fig. 4.

The effect of dispersion is to displace the diagram of Fig. 4 both in $x$ and $x'$. For $-5\% \Delta p/p$, $x$ is displaced by $-6.55$ cm and $x'$ by $-13.1$ mR. (With this choice of sign, the bending angle has a $-$ sign.) The combined effect of chromatic aberration and dispersion is shown in Fig. 5. Phase space diagrams for the off-momentum components can be drawn for any point in the system, once the appropriate ray-diagrams have been constructed. Then it is possible to see in more detail, for example, what is the effect of dispersion on the transmission of the second half of our beam.

VI. FINITE LENGTH APERTURES

If the momentum slit we place at the intermediate image had no length, we could take all particles with $|x| \leq W/2$ as being transmitted, where $W$ is the width of the slit. However, we need slits of 50 cm to 2 metres length for high-energy beams. The limits set by a slit of length $L$ and width $W$ are:

$$\frac{W}{2} - \frac{L}{2} |x'| = |x|,$$

and these limits give the lozenge-shaped boundary lines in Fig. 5, which have been drawn for $W = 14$ cm and $L = 1$ m. These boundary lines can easily be referred back to the object phase space, and forward to the final image phase space. It can be seen that in the case of the $-5\% \Delta p/p$ component, the particles transmitted will be mostly those emitted from the target with negative $x'$, and in the case of the $+5\% \Delta p/p$ component, those emitted from the $+x$ side of the target will be transmitted.
It is clear that one cannot specify the $\Delta \Omega$ and $\Delta p$ of a beam separately, but that one must perform an integration over $\Delta \Omega, \Delta p$ and the width of the target. To the accuracy required for beam studies, this can be done graphically using the phase space diagrams.

VII. ESTIMATION OF IMAGE SIZES

Once the limits set by the various beam elements on the different momentum components have been transferred to the final image phase space, a fairly accurate idea of the intensity distribution across the image can be obtained. More important, it can easily be seen where the most serious limits on the transmission or on the width of the image are, and where improvements can most easily be made. In practice one would study many variants in the design before adopting a final solution which would be a compromise between many factors.

VIII. WEDGE ABSORBERS

The spread of momenta at the final image can be reduced by placing a wedge-shaped absorber at the intermediate image, where particles of different momenta are separated in space, so as to place the greatest thickness of material in the path of the particles of the highest momentum. All the particles in the beam can then be brought to rest in a well-defined region of detector. This type of arrangement has been used successfully in the Murray beam at Berkeley, where the momentum bite in the first stage of the beam was $\pm 1\% \Delta p/p$, and in the second stage, $\pm 1/4\%$. A loss of about $15\%$ of the intensity was observed due to multiple Coulomb scattering in the wedge. Thus, an increase of approximately $70\%$ was obtained in the flux of particles in the desired momentum band at the final image.
IX. SOURCES OF UNSHARPNESS IN IMAGES

In separated beams, and in emulsion experiments (e.g. with a pulsed magnet) where it is desired to place stacks close to the direct beam, it is very important that the image be sharp. Aberrations due to lens defects are avoided by reducing the aperture of the lenses until they are negligible. Chromatic aberrations have been discussed already: the only way to improve the sharpness in this respect is to reduce $\Delta p$ or $\Delta \Omega$. We are left with the effects of scattering from the edges of collimators or from the material in the path of the direct beam.

Collimators and defining slits should obviously be placed as far from the final image as possible, so that scattered particles and secondaries produced in the slits are removed from the beam by the momentum selection of subsequent elements in the beam.

The r.m.s. angular deviation for a particle of momentum $p$ traversing a thickness $L$ of material or radiation length $x_0$ is

$$\sqrt{\langle \Theta^2 \rangle_{av}} = \frac{21}{(p\beta)} \sqrt{\frac{L}{x_0}}$$

For a beam of 10 GeV/c particles traversing 30 metres of air (radiation length 303 m at room temperature)

$$\sqrt{\langle \Theta^2 \rangle_{av}} = 0.63 \text{ mrad}.$$ 

The r.m.s. deflection in position projected on our horizontal or vertical planes is given by

$$x_{r.m.s.} = \Theta_{r.m.s.} \frac{L}{\sqrt{6}}$$

$$= 7.8 \text{ mm in this case.}$$

3348/3P/kw
The situation in a beam with lenses is rather complicated. However, it is clear that at 10 GeV/c the unsharpness due to multiple scattering in air cannot be neglected. For 1 GeV/c particles, the situation is ten times worse.

For high energy (≥ 10 GeV/c) beams, it is usually sufficient to replace the air by helium at atmospheric pressure contained in polyethylene bags. The scattering in helium is approximately four times less than in air for the same path length. The end windows of the bags (≈ 3 x 10^{-4} radiation lengths each) can be placed near the target and at images in the beam, so that scattering in the windows will contribute very little to the image width. For beams of momentum much less than 10 GeV/c it is necessary to use vacuum if good image quality is desired. Again, windows (≈ 10^{-3} radiation lengths) should be placed near the image points.
Appendix 1

BEAM TRAJECTORY CALCULATIONS

E. Malamud

Track Chamber Division, CERN.

A section has been added to the original van der Meer Mercury programme (No. 660) which facilitates writing data tapes and which prints out the results in a form adequate for many calculations. The original programme, together with the new sections, is Mercury programme No. 707-P1.

Restrictions

Programme No. 707 contains the following conditions which could be modified in the programme if desired.

1. The first two lenses (doublet) between given focal points will be optimized. These focal points can be real, virtual, or at infinity. Infinity is a large distance such as 1000 metres. A symmetrical triplet optimization routine exists in the van der Meer programme; however, it is not presently used.

2. Lens strengths expressed in (metres)$^{-1}$ can be read-in where desired.

3. Bending angles are given in degrees: $1^\circ < \varphi < 51.5^\circ$ ($51.5^\circ = 0.9$ radians). If bendings are not all in the same sense, indicate signs.

4. Each point where there is a beam element or where information is desired on the trajectories is given a number. Points at infinity must also be numbered. The maximum number of points is 25.
Data Tape

<table>
<thead>
<tr>
<th>A</th>
<th>1 1 -1</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>-1 1</td>
</tr>
<tr>
<td>C</td>
<td>-1 1 1</td>
</tr>
</tbody>
</table>

Part A has three columns of numbers: the first column has the spacing between points measured in metres; the second column has bending angles in degrees and the "sign" of quadrupoles, e.g. $1 = \text{vertically focusing}, -1 = \text{horizontally focusing};$ the third column has the effective length of elements measured in metres. Where there is neither magnet nor lens, write 0 in the second and third columns.

Part B has two columns: the first is the number of the beam point at which it is desired to indicate a lens strength; the second is the lens strength in (metres)$^{-1}$.

Part C has groups of four numbers which indicate the starting and stopping points for the doublet optimization routine: vertical object, vertical image, horizontal object, horizontal image.

The terminating lines after sections A, B, and C must always be present even if nothing is put in sections B and C.

Example of a data tape

\begin{verbatim}
\begin{tabular}{llll}
\hline
<&2>&<&2>&<&2>&<2>
| & 0 & 5 & 1 & 1.5 &  \\
\hline
\end{tabular}
\end{verbatim}
It is desired that the first quadrupole be vertically focusing and the others alternate in sign. Also the vertical rays must be parallel between quadrupoles 2 and 3, horizontal rays focused at slit No. 1, and both planes focused at slit No. 2. Assume that the effective length of all the elements is 1.1 metres. When writing the data tape always use . for the decimal, never a comma. First the points in the beam are numbered:

<table>
<thead>
<tr>
<th></th>
<th>Distance (from preceding point)</th>
<th>Angles and signs</th>
<th>Effective lengths</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Target</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>Q1</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>M1</td>
<td>2</td>
<td>30</td>
</tr>
<tr>
<td>4</td>
<td>Q2</td>
<td>2</td>
<td>-1</td>
</tr>
<tr>
<td>5</td>
<td>Dummy points. In the output</td>
<td>1000</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>these can be ignored.</td>
<td>-2000</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td></td>
<td>1000</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>Q3</td>
<td>6</td>
<td>1</td>
</tr>
<tr>
<td>9</td>
<td>slit No. 1.</td>
<td>1.5</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>Q4</td>
<td>0.5</td>
<td>-1</td>
</tr>
<tr>
<td>11</td>
<td>Q5</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>12</td>
<td>slit No. 2.</td>
<td>1.5</td>
<td>0</td>
</tr>
</tbody>
</table>

In section B is fed in a value calculated for Q3 (point 8), e.g. if one thinks Q3 should have a strength of $1.5 \text{ m.}^{-1}$ one writes 8 \ 1.667.

In section C are given focal points which will optimize Q1Q2 and Q4Q5: 1,...,5 (to infinity) 1,...,9, and 9,..., 12 9 12. This could be omitted if these lens strengths were already known in which case their values would be put in by section B. Point 6 is not needed but is put in to illustrate how to write minus infinity.
The complete data tape is then:

```
  0  0  0
  2  1  1.1
  2 30  1.1
  2 -1  1.1
 1000  0  0
-2000  0  0
 1000  0  0
  6  1  1.1
  1.5  0  0
  0.5 -1  1.1
  1  1  1.1
  1.5  0  0
  1  1  -1
  8 .667
-1  1
  1  5  1  9
  9 12  9 12
-1  1  1  1
```

**Results**

In the first table are printed the point number in the beam, the accumulated distance in metres from the beginning, the strength of lenses expressed in metres$^{-1}$. This is either a strength calculated by the optimization programme, a strength fed in on the original data tape, or a strength for a bending magnet (vertical focusing action only) computed from the given angle and effective length by the formula $\frac{\theta^2}{L}$. The next column contains a 1 for vertical focusing quadrupoles, a -1 for horizontally focusing, and for bending magnets the angle, now expressed in radians. Then follows the distance from the beam point preceding and the effective length in metres. If the optimization programme has no solution, the computer prints out "no convergence".
The next two tables are the trajectories. The strength is repeated again and then follow four columns which are the four elements of the trajectory matrix: displacement and angle for a unit angle at the beginning, displacement and angle for a unit displacement at the beginning.

Following the trajectories for the normal momentum particle are values for particles with a difference in momentum of 5%, 2% and 1%. These are computed with dispersion turned off, i.e. taking into account only chromatic aberration. Only two elements of the trajectory matrix are shown: the displacement for a unit angle at the beginning, and the displacement for a unit displacement at the beginning. These displacements are increments which must be added to the normal momentum values to get the trajectory for an off-momentum particle.

Finally values of dispersion, expressed in metres, are printed out for 5%, 2% and 1% momentum change. These are calculated assuming lenses are achromatic.

E. Malamud
Appendix 2

W. Toner

Nuclear Physics Division, CERN.

To complete the illustration, the beam has been calculated using the computer programme described in Appendix 1. The positions and signs of the lenses were specified, together with the bending angles and the positions of the images. The programme was run through for several values of the second bending angle around the one given by the hand calculations. Tables Ia) and Ib) are the data input and results of the calculation in the case for which the dispersion at the final images was zero. The programme takes account of the first two terms in the power series expansion for the trigonometric and hyperbolic functions, but the strength printed in the output is the "conventional" strength, $\frac{C_0}{E_0}$. Table II shows the comparison between the hand calculations and the computer results. It can be seen that although the hand calculations will give a good idea of the qualitative performance of the beam, computer calculations (or iterative hand calculations) are necessary for detail.

The print-out gives the trajectories in the form of the four matrix elements of the trajectory as it leaves each position [cf. de Raad, Section V, Eqs. (25), (26) and (27)]. These are displacement per unit angle ($a_{12}$), angle per unit angle ($a_{22}$), displacement per unit displacement ($a_{11}$), and angle per unit displacement ($a_{21}$). The phase-space diagrams can easily be constructed, as the matrix elements are just the new co-ordinates of points on the axes of the original phase-space ellipses (or parallelograms as used in Fig. 3). Limits set by lens apertures and slits can either be transferred graphically by inspection, or algebraically. For the off-momentum components, the chromatic aberration and dispersion must be taken into account in doing this.
The values shown for chromatic aberration and dispersion are for \(-1\% \Delta p/p\). The computer takes the bending angles as positive, unless otherwise instructed; the dispersion has therefore a +ve sign.
### Table 1a)

**Data Input**

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### Beam optics K-beam new programme

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### Vertical orbits

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### Horizontal orbits

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<th>per unit displacement</th>
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**Chromatic aberration**

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</table>

**Chromatic dispersion**

0.010

<p>| 1        | 0.000                  |
| 2        | 0.000                  |
| 3        | 0.000                  |
| 4        | 0.000                  |
| 5        | 0.013                  |
| 6        | 0.039                  |
| 7        | 0.015                  |
| 8        | 0.004                  |
| 9        | 0.000                  |</p>
<table>
<thead>
<tr>
<th>( f_c )</th>
<th>( Q_1 )</th>
<th>( Q_2 )</th>
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<th>Hand Calculation</th>
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<tbody>
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<td></td>
<td>2.13 m</td>
<td>2.37 m</td>
<td>2.93 m</td>
<td>2.84 m</td>
</tr>
<tr>
<td>Angular magnification at intermediate image</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Horizontal plane</td>
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<td>Vertical plane</td>
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<td>Bending angle for zero dispersion</td>
<td>25°</td>
<td></td>
<td>23°</td>
<td></td>
</tr>
</tbody>
</table>
Fig. 1 Trajectories in the first stage of the beam:

Fig. 2a

Fig. 2b

Fig. 2c

Some possible layouts for the complete beam
Fig. 3 Phase space diagrams for $\Delta p=0$ horizontal plane
--- Limit set by $r_0 = 7\text{cm}$

Fig. 4
Chromatic aberration

Fig. 5 Chromatic aberration and dispersion
{} Limit set by 1m long, 14cm wide, slit at image
SOME PROPERTIES OF BEAMS WITH ELECTROSTATIC SEPARATORS

S. van der Meer
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I. PRINCIPLE OF THE ELECTROSTATIC SEPARATOR

The curvature of the trajectories of charged particles in a magnetic field depends on their momentum. Particles with different rest mass, but the same momentum, will follow the same trajectories.

Static magnetic fields alone are therefore not sufficient for separating different types of particles. It is possible, however, to use electrostatic fields for this purpose. First, a momentum analysis is made with magnets, so that all particles in the beam have about the same momentum. Then an electric field is used for mass analysis. The particles are passed between two oppositely charged plates. Since they have the same momentum, the heavier particles will move more slowly and therefore stay longer in the field between the plates. The force exerted by the field does not depend on the speed of the particle (as it does in magnetic fields), so that the heavier particles will be given a greater transverse momentum. They will therefore be deflected more.

In practice, high-energy particles are not strongly deflected in electrostatic fields. The maximum electric field strength that can be produced at present over a distance of 10 cm in vacuum is of the order of 80 kV/cm. Relativistic particles will be deflected by such fields as they would be by magnetic fields of ~ 250 gauss. Therefore, electrostatic separators for high-energy purposes are usually large and are mostly used near the limit of electrical breakdown.

The force on a particle with charge e in an electric field (field strength E) is

\[ F = eE. \] (1)
We suppose that the particle enters the separator at right angles to the field with a velocity \( v \). It will then traverse the separator (length \( L \)) in a time

\[
T = \frac{L}{v} = \frac{L}{\beta c} .
\]  

(2)

Therefore the transverse momentum after the separator will be

\[
P_L = \int_0^T F \, dt = \frac{eE}{\beta c} \cdot
\]  

(3)

The total deflection angle produced by the separator is then

\[
\alpha = \frac{P_L}{p} = \frac{eE \, L}{p \beta c}
\]  

(4)

if \( p \) is the forward momentum of the particle, which is not changed by the field.

If we express \( p \) in \( eV/c \), this reduces to

\[
\alpha = \frac{E \, L}{p \beta} \quad \left( E \text{ in } V/m, \quad L \text{ in } m, \quad \alpha \text{ in rad} \right).
\]  

(5)

If two different types of particles are present, the difference in deflection will be (if \( p \) is the same for both)

\[
\Delta \alpha = \frac{E \, L}{p} \cdot \Delta \left( \frac{1}{\beta} \right).
\]  

(6)

It may be seen that \( \Delta \alpha \) is much smaller than \( \alpha \) for relativistic particles (\( \beta \approx 1 \)). Evidently this means that for a good separation the field strength must be kept very well constant, and that the field must be very homogeneous.

Equation (6) may be written as follows for highly relativistic particles (see, for example, Ref. 1):
\[ \Delta \alpha = \frac{5L}{2p^3} \Delta (m^2) \]  

(7)

if \( m_0 \) is the rest mass.

It may be seen from this that the difficulty of separation increases very rapidly with the particle momentum. At present, anti-protons and pions up to 4 GeV/c have been separated (at CERN). It is believed to be possible to go up to 8 or 10 GeV/c with electrostatic separators. Above this region, radio-frequency separators (see Ref. 2) will probably be more interesting. For K-\( \pi \) separation the limit is lower.

The deflection \( \alpha \) of the "wanted particles" by the electric field is mostly compensated by a properly adjusted magnetic field. The latter is often superimposed on the electric field. Sometimes a bending magnet is also placed before and after the separator. Of course, magnets do not influence the separation \( \Delta \alpha \); their purpose is to keep the general direction of the beam straight and independent of the exact particle momentum.

II. SOME GENERAL PROPERTIES OF SEPARATED BEAMS

In Fig. 1a) a simple arrangement for separating particles is shown. \( S \) is a small source of particles with about the same momentum (it is supposed that momentum analysis has been performed before \( S \)). \( Q_1 \) and \( Q_2 \) are quadrupoles. It is assumed that a magnetic field is superimposed on the electric field in the separator such that the wanted particles are undeflected and arrive at \( A_1 \). The unwanted particles are then deflected by the amount \( \Delta \alpha \) and arrive at \( A_2 \). A slit (usually called "mass slit") passes the wanted particles only. The source is imaged on the mass slit by \( Q_1 \) and \( Q_2 \).

The mass separation is usually done in the vertical plane, the momentum analysis horizontally. This has the advantage that they do not interfere with each other.
Usually the wanted particles are less numerous than the unwanted ones (e.g. at 3 GeV/c there are 100 times more pions than antiprotons in a typical PS beam). For obtaining good separation it is therefore necessary that the separation distance $S$ [see Fig. 1a)] is several times greater than the image size at the mass slit. The image size is not a very well-defined quantity; usually the intensity distribution is more or less gaussian with long tails.

In the first instance it might be expected that the image size is equal to the source size multiplied by the magnification of the system. To this is added, however, contributions from chromatic aberration, non-linearities and fluctuations of the field in lenses, bending magnets and separator, effects due to scattering of particles, etc. A careful estimation of all these contributions can be made for each case. If the image size thus found is called $I$, experience shows that the "separation factor"

$$S = \frac{S}{I}$$

must be at least about 2.5 or 3. Of course, the relative abundance of wanted and unwanted particles will influence this value.

It is clear that the effect of the separation angle $\Delta \alpha$ will be greatest if the inherent angular spread $\alpha$, between the particles passing through the centre point of the separator, is smallest. This is illustrated in Fig. 2a), which shows the ellipse*) in phase space, inside which all particles are situated. The dashed ellipse is for the unwanted particles which are deflected by an amount $\Delta \alpha$. Both ellipses refer to the situation at the separator. Vertical displacement is called $y$, slope $y'$.

In the part of the beam after the separator the ellipses may be transformed either simply by means of a drift space as shown in Fig. 1a), or by a more complicated combination of quadrupoles.

*) In principle this figure should be a polygon rather than an ellipse, depending on the collimators used in the beam; in practice the difference is not great.
and drift spaces. At the mass slit, finally, they may be as shown in Fig. 2b). They stand upright, since there is an image at the mass slit. The image width and the separation $\delta$ are shown. The displacement of the two ellipses in the $y'$ direction is always relatively small if $\delta$ is small.

By considering how the line ABCD, connecting the centres of the two ellipses, is transformed between separator and mass slit, and how the ratio $AB/AD$ is always conserved, it may be understood that in the first (and, in practice, rather good) approximation the separation factor is given by

$$ S = \frac{\delta}{I} \approx \frac{A_2}{a_1}. $$ (9)

Any aberrations between the separator and mass slit must either be neglected, or transformed backwards to the separator and included in $a_1$ as well as in $I$.

Clearly $a_1$ should be kept as small as possible in order to obtain a good separation factor. This implies (since the area of the ellipses is constant throughout the system) that the beam height $d$ in the separator should be a maximum, i.e. that the distance between the separator plates should be filled with particles. In particular, it means that the beam envelope in the separator should not be converging as in Fig. 1a), but rather parallel as in Fig. 1b).

If this procedure is followed, i.e., if the beam height in the separator is equal to the electrode spacing, it is possible to give a simple quantitative relation between the separation factor, the acceptance of the system, and the total voltage on the separator.

If the target height is $t$ and the accepted angle at the target $a_0$, then the acceptance is $a_0t$. According to Liouville's theorem

$$ a_0 t = a_1 d. $$ (10)
Using Eqs. (7) and (9) we have

\[
a_0 t = \frac{EL}{2p^3} \Delta(m_0^2) \cdot \frac{d}{S}.
\]  

(11)

Since we have supposed that the beam height \(d\) is equal to the electrode spacing, the product \(Ed\) is equal to the voltage \(V\) between the separator plates, so that

\[
a_0 t = \frac{L \Delta(m_0^2)}{2p^3} \cdot \frac{V}{S}.
\]  

(12)

The target height \(t\) cannot be decreased below a certain minimum value without losing efficiency. Assuming \(t\) to be given, we may see that the angular acceptance \(a_0\) is proportional to the voltage between the separator plates and the length of the separator.

In addition to a certain separation factor, we also require a certain absolute spatial separation at the mass slit, in order to avoid contamination with unwanted particles scattered by the slit material. This requirement may sometimes make it necessary to accept the less ideal arrangement of Fig. 1a) instead of that of Fig. 1b).

III. DESCRIPTION OF A SEPARATED ANTIPOTON BEAM

The initial design of the "fast antiproton" beam, used at the CERN PS, was described in Ref. 3) around the middle of 1960. Since that time some changes have been made. The final layout is shown in Fig. 3 (it is the beam at the bottom of the drawing).

The first bending magnet in the beam \(M_a\) provides the momentum analysis, in combination with the "momentum slit" (the black rectangle following \(M_a\)). The quadrupoles \(Q_{13}\) and \(Q_{10}\) make an image of the target on the momentum slit in the horizontal plane.
The horizontally focusing quadrupole $Q_3$ makes a second intermediate image just before the magnet $M_4$. Finally, this is again imaged, on the mass slit at the end of the beam (before $M_7$), by $Q_{12}$ and $Q_4$. Vertically, $Q_{13}$ and $Q_{14}$ make an image of the target in the centre of $Q_3$. This is again imaged on the mass slit by $Q_{13}$ and $Q_1$. The bending magnets $M_6$, $M_9$, $M_{13}$ and $M_{12}$ were necessary to bring the beam to the bubble chamber (marked HBC 31, i.e. the Saclay 31 cm H$_2$ bubble chamber).

Following $Q_1$, the separator tank can be seen, with on each side the compensating magnets $M_4$ and $M_4$. The latter two provide the vertical deflection that cancels the electrostatic deflection for the wanted particles.

The magnet $M_7$ behind the mass slit removes low momentum background particles (particles produced in the material of the mass slit and muons from $\pi-\mu$ decay). Quadrupole $Q_2$ is vertically defocusing and therefore spreads out the beam vertically before it enters the bubble chamber. This is desirable in order to make scanning of the bubble chamber pictures easier.

Finally, the magnet $M_{C3}$ just before the bubble chamber deflects the particles vertically so that they pass about through the centre of the chamber, despite the strong magnetic field of the chamber itself.

It may immediately be seen from Fig. 3 that space requirements have seriously limited the design. The thick screening wall (between $Q_3$ and $M_{10}$) with its pillars, as well as the other existing beams, necessitated the detour and the use of the four bending magnets $M_{10}$, $M_9$, $M_{13}$ and $M_{12}$. The angle of omission at the target is $12.25^\circ$, which is uncomfortably high from the point of view of intensity. It was necessary in order to leave space for other beams. Again, the place of the bubble chamber was planned before the beam, with the consequence that for use with K's the beam is rather long.
The use of doublets instead of triplets followed partly from space considerations (for \( Q_3 \) and \( Q_{10} \)), partly from the requirement of small magnification (about 1.5\( \times \)) in the vertical plane, necessary for obtaining a small image at the mass slit. The horizontal magnification is much greater (about 50\( \times \)).

A consequence of the high magnification in the horizontal plane is that the image at the mass slit is a horizontal line rather than a spot. The dimensions of the mass slit are about 7 mm vertically and 150 mm horizontally.

Also because of the high horizontal magnification, great care had to be taken in order to ensure that the momentum dispersion caused by the bending magnets is zero at the mass slit. Figure 4 illustrates this. By having two intermediate horizontal images as described above, the dispersion of the first bending magnet is compensated by the other four. This condition determines the position of the second intermediate image.

In the vertical plane only one intermediate image is used. Each additional intermediate image would increase the chromatic aberration. One is necessary for the following reason. A vacuum pipe is used throughout the beam in order to avoid air scattering. Near \( Q_3 \) a thin mylar window separates the good synchrotron vacuum from the bad separator vacuum\(^*\). Scattering in this window does not increase the image size, since it is near to the intermediate vertical image.

Since there are no lenses between the separator and the mass slit, the situation corresponds to the case of Fig. 1a) rather than Fig. 1b). This is not ideal, but the difference is small because of the small ratio of separator length to drift space. The vertical width of the beam inside the separator is nearly constant and the electrode spacing corresponds to it (although it is slightly greater in order to avoid background from particles scattered by the electrodes). The voltage between the plates is 84.0 kV for 14 cm

\(^*\) It must be bad, because at a pressure of \( 10^{-4} \) Hg a higher voltage can be used than at, for example, \( 10^{-6} \) mm.
distance, and the length of the electrodes is 9 m. The beam was used to separate antiprotons up to 4 GeV/c.

In order to prevent particles hitting vacuum chamber walls, poles of lenses, etc., the envelope of the beam is limited by lead collimators. These are placed between the target and the first lens. A further collimator for the vertical plane is situated inside M3, where the vertical beam envelope is wider, so that relatively less background is caused by the particles scattered by the collimator.

An additional slit is situated inside Q3, around the intermediate vertical image of the target. This slit has a vertical width of about 1 cm, so that the image is well within its aperture. Much of the background around the image is absorbed, however,

A sextupole lens was installed near the momentum slit in order to compensate the chromatic aberration of the quadrupoles in the vertical plane, as described in Ref. 3). The principle of this device is as follows.

The field inside a sextupole lens (Fig. 5) may be described with the scalar magnetic potential

\[ V = Cr^5 \sin 3\varphi = C(3x^2y - y^3) \]

in which C is a proportionality factor. The contribution to the focusing strength in the vertical plane, given by the sextupole, is proportional to

\[ \frac{\delta B_x}{\delta y} = \frac{8^2V}{8x\delta y} = 6Cx. \]

Since near the momentum slit the particle momentum is also proportional to x, it is possible by introducing a sextupole field of correct strength and polarity to compensate the first-order chromatic aberration of the quadrupoles.

Actually, the sextupole was found to give no measurable improvement in image sharpness. This was probably due to misalignment
of the momentum slit, which decreased the accepted momentum band by nearly a factor 2, as was found from analysis of the bubble chamber photographs.

The solid angle accepted at the target is about 60 \( \mu \) steradians, the accepted momentum band \( \pm 0.3\% \). Under the most favourable circumstances, the height of the image at the mass slit was found to be 1.7 mm between points of half-intensity. This was measured at 3 GeV/c momentum, and with a target consisting of a 1 mm thick cylindrical rod, a few centimetres long, pointing in the beam direction.

The intensity was down to 0.2\% at a distance of 4 mm from each side of the image centre. The separation between \( \pi^- \) and \( \bar{p} \) was 35 mm at 3 GeV/c and 15 mm at 4 GeV/c.

With a slow burst on the target, from \( 2.5 \times 10^{11} \) protons at 19 GeV/c circulating in the PS, there were 50 antiprotons behind the mass slit with 10\% background (mostly muons). With short burst operation, as required for the bubble chamber, the target efficiency was smaller and the intensity dropped by a factor 3.

* * *

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3) Achromatic beam for particle separator, S. van der Meer, CERN 60-22.
DISCUSSION

Harmsen : How did you start to build up the beam? I think you did not start from the values you have planned for the layout of the beam. How many quadrupoles and lenses did you use?

Van der Meer : That beam is actually a beam which we started with. We had plenty of time to think about this beam, because the separator was not ready and its building was delayed. In fact I think it is better if you start on paper and calculate by trial and error the necessary values. Many people have written papers on the theory of beam optimization, but in practice there are many difficulties such as shortage of apparatus and the layout of the hall. So it is a massive subject, and you need to co-operate with other people. But if you have a good experiment, it is usually worth while to wait and to take a lot of trouble collecting apparatus and finding the space. Of course, the actual mounting is done by the machine group, but anyone who proposes an experiment must be prepared to do the first designs himself and show that he understands the problem.

*   *   *

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Fig.1 Simple arrangements for separating particles with beam a) converging
  b) parallel through separator.

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Fig. 1a

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Fig. 1b

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Fig.2 Phase space ellipses (inside which all particles are situated)
  a) at the separator
  b) at the mass slit.
**Fig. 4**  Beam optics for separated antiproton beam

**Fig. 5**  Magnetic field lines inside a sextupole lens.
BEAMS OF PARTICLES OF FINITE LIFETIME

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I. INTRODUCTION

The preceding lectures have shown us how to design a beam, or, given an existing beam, how to calculate its performance. To complete the discussion of beam transport we must add one theoretical limitation that has not been mentioned until now. Our particles have been assumed to live forever. Thus they can travel as far as needed. Our beam is limited only by the quantity of beam transport equipment available and the size of the experimental hall. Now suppose that we want a beam of K mesons or \( \pi \) mesons, or any other particle with a finite life. Consider the K meson since, at present, beams of K mesons are of great interest:

Mean life : \( \tau = 1.2 \times 10^{-8} \) sec

Mean free path for decay : \( L = \beta c \gamma \tau \)

For example, for K mesons of momentum \( p = 1 \) GeV/c 
\( \beta = 0.9, \ \gamma = 2.25, \ \ L = 8.5 \text{ m} \)

This means that in each 8.5 metres of length from the target in the PS to the emulsion stack we lose intensity by a factor of 1/e. This has two serious consequences:

i) The absolute flux may diminish beyond the point where a given experiment is feasible.

ii) The percentage of unwanted particles such as pions which decay less rapidly, or protons which do not decay at all, will increase. This means that the rejection ratio required in the separation stages is higher. In practice this means either increasing the length of the separator or decreasing the image size. In either case intensity is lost.
So, our theoretical designs must be modified. We can no longer always have the ideal system where we use one length to make a momentum selection, another length for forming a good image before the separator, parallel beam in the separator, enough length to make the desired magnifications, etc. We must use the available length to perform as many functions simultaneously as possible. To some extent one can judge the "efficiency" of a beam or more likely the ingenuity of its designers, by how completely the space between target and emulsion stack is filled with beam transport elements. The Berkeley K-meson beams are excellent examples.

II. SOME WAYS OF SHORTENING A BEAM

1. The fringing field of the machine

This is usually a nuisance, but it can also be used to make a first momentum selection. This, however, fixes the momentum of the beam or rather makes it completely dependent on the machine peak energy. Also it is no longer possible to have both signs of particles.

2. Edge focusing in bending magnets

The momentum-selecting bending magnet can be used to focus vertically as has already been pointed out. If a parallel-faced magnet is placed symmetrically on the beam it has no horizontal focusing action and has a vertical focusing action

\[ f \sim \frac{L}{\alpha^2} \]

where L is the length of the magnet and \( \alpha \) is the bending angle.

For a 1-metre magnet and \( \alpha = 40^\circ \), \( f \sim 2 \) metres. This is a useful focal length. The magnet can be combined with a horizontally focusing quadrupole to take the place of a doublet.
In a very refined beam design, the small higher order horizontal effects can be useful. In this case the angle of the magnet axis with respect to the beam axis is a free parameter.

If a bending magnet is used to focus vertically, or for that matter to bend through large angles, it will probably be necessary to provide shimming. In the CERN bending magnets shimming has been done to make \( J^6 B d^6 \) a constant along any straight line through the magnet. Since the magnets are usually used with high-momentum particles, and consequently small angles, this is quite good. However, for very curved trajectories through the bending magnets, additional shimming is needed (see Fig. 1).

3. **Parallel beam in the separator**

This can be given up to gain some length at the expense of the efficiency of separation. This is illustrated in Figs. 2a), 2b), and 2c). Here \( d \) is the object size, \( I \) is the image size, \( \alpha \) is the separation angle referred to a point in the middle of the separator, and \( s/I \) is the relative separation.

**III. SOME PRACTICAL EXAMPLES**

Now keeping in mind that there are certain things we can do to shorten a beam, let us see how in a couple of practical examples a maximum length is imposed.

1. It is desired to stop 5 \( K^- \) per picture in a hydrogen bubble chamber. For simplicity assume

\[
K^- \text{ flux at } 15^\circ = 3 \times 10^{-3} /\text{proton GeV/c ster \cdot } \frac{P_K}{1 \text{ GeV/c}}. 
\]

The number of stopping \( K^- \)'s will be

\[
3 \times 10^{-3} \Delta \Omega \Delta p N \cdot \frac{P_K}{1 \text{ GeV/c}} \cdot e^{-L/\lambda} D. 
\]
N is the number of protons in the FS per pulse that are effective in making interactions in the target. For the discussion we can take $10^{11}$ protons times a target efficiency of 70%. L is the length of the beam and $\lambda$ is the decay length. As $p_K$ becomes higher we gain in two ways: i) there is more flux from the target according to the oversimplified spectrum above; ii) there is less loss from decay. But we also lose in two ways: i) it becomes harder to separate; ii) we lose more in the factor $D$ which is the loss due to interaction and multiple scattering in a degrader in which the momentum is reduced from $p_K$ to almost stopping. Thus there is an optimum momentum for each length, which is illustrated in Fig. 3.

If we now require 5 $K^+$s per picture, then it is clear that for a given length of beam, a certain minimum $\Delta\Omega\Delta p$ solid angle momentum acceptance is needed. This gives us a boundary condition shown by the continuous line in Fig. 4.

If a certain momentum is chosen, then another boundary condition can be placed on the curve. The dotted line is the minimum length beam in which particles of the chosen momentum can be separated. Beams in the cross-hatched region will "work".

2. An existing beam in the North Hall ($k_t$) was examined to see if it could be used for a stopping $K$ experiment in emulsions. The arguments are very similar to the above.

A combination of decay in flight and interaction in a copper degrader while coming to rest gives the set of curves shown in Fig. 5.

The number of $K^+$s coming from the target for a given solid angle and momentum band will look as shown in Fig. 6.

The result of folding these two figures together is shown in Fig. 7.

From these curves the best operating conditions can be chosen. There are some additional factors:

i) the maximum momentum is limited by the separator;
ii) the momentum band is limited by the separator as well as by the requirement that the straggling of the stopping particles in the emulsion stack is not too large;

iii) the thickness of the degrader is also limited by the requirement that straggling and multiple scattering be kept small.

* * *

PRACTICAL ASPECTS

In this last section on beam transport we will discuss briefly some practical problems.

1. Characteristics of beam transport equipment

Some of the features of quadrupoles, bending magnets, and separators in common use at the PS are listed in the table. Most beams you will work with are constructed as a combination of these elements\(^6,7,8\).

2. Trajectory calculations

These are done on a digital or analogue computer. The digital computer is, of course, more accurate but often takes more time before results are ready for use. The analogue computer must be used carefully if values of quadrupole currents are to be taken as absolute. But it is a useful device, for it traces directly on a sheet of paper the beam trajectories. One application is to trace a family of beam trajectories for different momenta in order to see where is the best position to place counters for optimizing different elements.

It is the experience in many beams that values of quadrupole currents found in calculations are within 1% of the best values found empirically. However, if two quadrupoles are placed very
close together, as in the old antiproton beam, the fringing fields cancel and the values of the lenses may be altered by as much as 5%.

3. Magnetic measurements

Philosophies differ as to their necessity. They are long and tedious but can save valuable machine time. A wire under tension T and carrying a current i will follow the trajectory of a particle of momentum p, p being given directly by T/i. This method is the best way to calibrate the momentum of a beam. It is also the best way to check the linearity of the lenses and spot any mistakes in alignment. In general, the lenses in use have been shimmed so that the focal length is the same for particles passing through at any distance from the axis. However, when lenses are used near saturation and with their full aperture they should be checked either with a floating wire or a long thin flip coil that integrates B•d/2.

The momentum band in a beam is determined in the first approximation by calculating the dispersion at the momentum slit. This can be checked with the floating wire. In practice the solid angle acceptance is a function of momentum, and the actual momentum band must be found by calculations in phase space as shown by Toner.

4. Physical approach to the PS

...designing a beam one has some choice of emission angle and target position. This has already been discussed but we summarize here the various factors involved.

i) Emission spectrum as a function of angle.

ii) Effect of fringing field on particles of different moments and different sign. In general the target position in the PS must be changed for each sign and each momentum, as well as the peak energy of the machine. But these changes cannot be too large. They can be reduced by increasing emission angle which may reduce flux, or moving the target out into the straight section which may reduce solid angle due to geometrical limitations.
iii) Geometrical limitations: posts, other beams, walls, and the dimensions of the straight section. A bending magnet placed close to the FS will interfere with the operation of the FS, and magnetic shielding will be needed.

SETTING UP

No discussion intended to familiarize potential users of the FS with problems of beam transport can be complete without some mention of the practical problems involved. In most cases emulsion physicists do not design the beams they use. And it is infrequent that they have the opportunity to make any important modifications or, indeed, the time in which to do so. Therefore the most common situation is the following: You are an emulsion physicist and have come hundreds of kilometres to Geneva with your precious emulsions tucked under your arm. You have fought for your experiment for months, defended it in dozens of committee meetings, won a hard battle with the intricacies of machine scheduling, and now finally your great day has come. You are given the FS for some period of time—say, 5 shifts (30 hours) or maybe even a week. Now what do you do? You soon realize a fundamental truth. An hour or two lost during a bubble chamber run or a counter experiment is a far smaller percentage of their total running time than an hour or two lost during an emulsion exposure. The hours fly by quickly and one sees one's experiment slipping through one's fingers. The two years of scanning that will follow the exposure of course represent a great deal more work than the exposure. But the few hours during the exposure can seal the fate or seriously compromise the conditions for the part of the experiment that follows. Those of you who have worked at other accelerators know this problem. Those of you who have not as yet done an experiment at an accelerator, where the clock is always ominously present, can only learn these problems from the experience itself.

Let us be more specific now and discuss some of the practical aspects.
1. Magnet polarities and currents

These are checked before the run starts. Polarities are checked with one of the little hand gadgets available. All slits and openings in shielding walls are checked to see that they are open and of the right dimensions.

2. Counters

Counters are an indispensable part of running. They serve three purposes:

i) **Optimization of elements.** One always must optimize the radial position of the target in the PS and the settings of the bending magnets. This takes between 1 and 2 hours. One hopes that the quadrupoles have been optimized by someone else, for this is a very lengthy procedure.

ii) **Determination of profiles.** This is particularly important in emulsion exposures. A small scintillation crystal (e.g. 2x8 mm) can be used to determine a beam profile with a resolution of the order of 1 mm. The whole counter is mounted on a carriage which can be displaced \( \frac{1}{2} \) mm at a time in either horizontal or vertical direction from the control room. A complete profile can be done in 20 minutes if the work is done efficiently. It helps to have graphs prepared beforehand and to arrange the scalars so they turn off automatically after a preset count is reached in the monitor counters. This avoids much slide rule work.

iii) **Monitoring.** First of all a monitor is needed that looks directly at the target and is used to indicate that the target is operating properly and is used to normalize counts in the beam. This counter should be independent of the peak energy of the PS or the target radial position. It is sometimes arranged to look at \( \gamma \) rays coming from the target at a large angle, such as 90°. Then another counter or group of counters must be set up in the beam to indicate that the beam is operating correctly: magnets, lenses...
and separator at correct values, vacuum system operating, and all apertures open. This system is usually the same that is used to optimize the beam and measure the profiles. One thing must be watched carefully — if the beam is set up with a long burst (100 ms) and then is run on the short burst (1 ms) for a pulse magnet experiment, it must be verified that the monitor and the counters in the beam are not losing counts due to pile-up. Of course, an emulsion physicist will have the most faith in a test plate. Some time during the run an intercalibration is made between test plates and counters.

3. Profiles

These can also be determined by:

i) polaroid camera — can take less than five minutes;

ii) x-ray film — can take about 20 minutes and has better resolution than the polaroid camera;

iii) test plates — takes about 1-2 hours. This is useful when one is interested in the "tail" of the profile (background) which is hard to determine with counters.

* * *

REFERENCES

6) Quadrupoles. CERN PS/MM 31 (1957).
7) "Citron" lenses. CERN Yellow Report 61-12.
8) Bending magnets. FS/Int EA 60-14.
Discussion

Key : Would you explain the wire technique which you mentioned.

Malamud : If you take a wire which is under tension $T$ and has a current $i$ flowing through it, then when placed in the system of magnets, it will assume the trajectory of a particle of momentum "$p"$, where "$p" is directly proportional to $(T/i)$. This can be used in many ways but basically it is a very accurate method of following the path of a particle through the system to the emulsion stack. If it is performed carefully, this method will give an error of less than $1\%$. One can also use it to trace rays through a quadrupole, and to check that rays far from the axis are focused at the same point as those close to the axis. On the other hand, if you have a system of magnets each of which is all right in itself, you can use the wire technique, if you want to know what happens when they are lined up and the current switched on. There are some disadvantages in using it; if the wire passes through a focal point it becomes unstable, so one has to trace the system in between such points.

Harmsen : How long a beam can you check using the wire method, and how long a wire can you use?

de Raad : The longest wire I remember as having used was about 40 metres long, and for that we used wire of the highest tensile strength that we could find. So we used wire of $\frac{1}{16}$ mm diameter of beryllium-copper that has a tensile strength of 50 Kg/mm$^2$. Other people I think use constantan wire, which is also quite strong, but I am not certain on this point.
Harmsen : Is this method with the wire more accurate than computing the trajectories using the matrix formula?

Malamud : The formula does not tell you where the magnets are on the floor, and in the matrix you do not have the momentum of the particles.

Harmsen : Is the counter method better than the wire for optimizing the beam?

Malamud : Both are useful for different reasons. For a momentum calibration you use the wire method. To optimize the quadrupoles, I think the counter method is best.

Van der Meer : The wire measurements can be done when the machine is not running, while the counters need machine time. At the energies of the PS the wire measurements are not better for the quadrupoles than the magnetic measurements but is worth while to use the wire method with a bending magnet.

Malamud : The only advantage in using the wire method with quadrupoles is to see if they are lined up properly and that the coils are working correctly.
Table I

Some Miscellaneous Facts Concerning Magnets

Basic formula: \( H = 1.26 \frac{n}{l} \)
- \( n \) = number of turns
- \( i \) = current in amperes
- \( H \) = field in gauss
- \( l \) = gap in cm.

Wire measurements: \( p = 2.34 \frac{T}{i} \)
- \( p \) = momentum in MeV/c
- \( T \) = tension in wire, in grams
- \( i \) = current in amperes.

Some figures for estimating the costs of large electromagnets:

Let \( V \) = volume of magnetic field measured in litres.
Then for 1 litre of field one has:
- for an \( H \)-type bending magnet exclusive of design costs:
  - 1,500 SF for magnet
  - 100 SF for magnet support;
- for a standard-type quadrupole:
  - 1,500 SF for magnet
  - 125 SF for magnet support.

Power requirements and weights:

- for 1 litre of field in an \( H \)-type bending magnet:
  - 250 kg and 2 kW of power;
- for 1 litre of field in a standard-type quadrupole:
  - 100 kg and 5 kW of power.

Some figures on standard CERN magnets:

Bending magnets (\( H \)-type):
- Dimensions: gaps, 11, 14, 17, 20 cm (normal is 14 cm);
  - gap width 52 cm or 42 cm with tapered poles;
  - field with 14x52 cm gap, 18,000 g (18,000 gauss).
- Quadrupoles: 20 cm in diameter,
  - maximum gradient: 1100 gauss/cm (11 WH/m²).
Some figures on standard CERN magnets (cont.)

Resistance of all magnets 0.225 ohm.

Power and current (at maximum field or gradient):
- 2-metre long bending magnets and quadrupoles - 830 amp. 155 kW;
- 1-metre long bending magnets and quadrupoles - 675 amp. 100 kW.

Weights: bending, 2 metre, 29 tons; 1 metre, 15 tons;
- quadrupole, 2 metre, 9 tons; 1 metre, 5 tons.

Bending magnets: 14 cm gap
- 1-metre magnet max field 1.64 Wb/m² max. bending power 1.88 Wb/m²
- 2-metre magnet " " 1.74 " " " " 3.63 " "

Quadrupoles: 20 cm aperture
- 1 metre long
- 2 metres long
0.4 m effective length ("Citron" type).

Separators:
- Ramm: 9 metres
- Cresti: 3 metres

maximum field is about 50 kV/cm at 10 cm spacing.

Regulation of magnet currents and separator voltages is about 1/1000.

* * *
Fig 1 Trajectory for particles of low momentum passing through bending magnet.

Fig 2 Beam length and separation for beam

a) converging
\[ \text{Sep} = \frac{\pi L}{2} \]
\[ \text{S/I} = \frac{\pi L}{2d} \]

b) diverging
\[ \text{Sep} = \frac{\pi L}{2} \]
\[ \text{S/I} = \frac{\pi L}{2d} \]

c) parallel through separator

Fig 3 Relative number of stopping \( k \) particles as a function of beam momentum for different lengths of beam.
Fig. 4 Solid angle-momentum acceptance needed with different lengths of beam.

Fig. 5 Attenuation-beam momentum curves for k particles brought to rest (k, beam)

Fig. 6 Flux of k particles coming from the target as a function of beam momentum for a given solid angle and momentum band (k, beam)

Fig. 7 Relative number of stopped k's (flux x attenuation) as a function of beam momentum for different lengths (k, beam)
GENERAL PROBLEMS OF EMULSION IRRADIATIONS

J.C. Combe

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This talk will deal with the general problems connected with exposing emulsion stacks, and the way in which they are generally solved. The following talk by Mr. Roberts will describe a few practical technical devices for making such exposures.

I. DEFINITION OF THE PATH;
BEAM CHARACTERISTICS

Let us suppose that the beam has already been designed and that its main components (quadrupole lenses and bending magnets) are in position. Its path is then known, and by means of reference marks (on the lenses or magnets, or on the ground) a specialized group from the NFS Division plan the exact position on the site of all the additional equipment required (counters, tables for holding the emulsion stacks, etc.).

However, the position of the beam path may still vary; slight changes of position can be obtained by varying the current intensity in the lenses and bending magnets. The position in which the beam reaches maximum intensity and the corresponding parameters of the optics (intensity of current in the lenses and bending magnets) can be very accurately determined by means of mobile counters having a few centimetres of motion in two suitably chosen directions. (This subject is dealt with in the chapter entitled "Monitoring").

Once a beam path has thus been fixed, an accurate study of the absolute value of the intensity, of the angular distribution and the geographical distribution of the particles can be made by means
of emulsions, which are chosen to be thin (between 25 and 100 μm) for speed of processing and scanning.

**Intensity measurements** are made on plates exposed either horizontally or perpendicularly to the beam. Plates 25 μm thick are adequate for horizontal exposures. The tracks can easily be counted up to intensities of $10^6$ particles/cm$^2$; they can be used for ionization measurements, angular measurements, and for detecting interactions. The error on the thickness, which may exceed an average of 15%, prevents very accurate measurement of beam intensity. In this particular case it is preferable to make a perpendicular irradiation (namely, beam perpendicular to the surface of the emulsion). Error in the plate thickness then has no effect. However, in this case, for good track spotting, the thickness should be at least 100 μm. A 25 μm plate is developed and ready for scanning an hour after irradiation; a 100 μm plate may take three or four hours; this is often a great drawback.

The measurement of absolute intensity, even when done on horizontally irradiated plates, can be used to calibrate the counting rate of the scintillators placed in the beam. It is always possible, although often inexplicably difficult, for a counter to give the correct number of particles. Comparison with the value measured in the emulsion, which serves as a reference, can eliminate this trouble.

The angular distribution of the particles in the vertical and horizontal planes is an excellent means of assessing the quality of the beam. As a rule there is a peak with low angular dispersion accompanied by a background of varying spread. The peak represents the "pure" part of the beam (particles whose nature and momentum are clearly determined). A very low angular dispersion inside this peak is often an essential requirement for certain experiments. For instance, the scattered proton beam (Born or C2 beam) used naturally (without quadrupole lenses) has a very low angular dispersion (of the order of 1 milliradian). This characteristic feature makes it
extremely valuable for all experiments calling for a parallel incident beam (for instance, experiments on scattering at small angles).

The background accompanying the peak is produced by stray particles of varied origin (collimators, lenses, magnets, decay of unstable particles, etc.). A rough study of its nature (ionization, angle, mean free path for interaction) may help to determine its origin and reduce its importance.

The presence of \( \gamma \)-rays is very quickly shown by the presence of pairs of electrons. The number of neutrons can be estimated from the intensity of the stars which they produce; the size distribution of the stars gives an idea of the neutron energy distribution. Weak intensity of electrons (a few per cent) is more difficult to observe. To detect \( \mu \) mesons one should use an absorber sufficiently thick (at least 80 cm of Pb) to destroy all the other particles. The number of \( \mu \) mesons can be determined from counting in two photographic plates, one placed before and one after the absorber. Contamination by a low percentage of \( \mu \) mesons can be easily measured.

The geographical distribution of the particles inside the beam, and the dimensions of the beam, are important data for obtaining uniform intensity of irradiation in a big emulsion stack. Beam dimensions may go from cross-sections of the order of 1 cm\(^2\) up to 100 cm\(^2\) and over, according to whether the beam is more or less focused, and according to the size of the collimators defining its shape.

When it is wished to know quickly the position and dimensions of a beam, x-ray films are used (results in 10 minutes, if there is a developing installation, however elementary, nearby) or Polaroid film (results in 30 seconds without any installation). X-ray films or Polaroid films need to receive an intensity of about \( 10^7 \) particles per cm\(^2\) in order to produce a suitable image. A Polaroid film accompanied by an assembly of fine scintillators producing light will give an image with a smaller number of particles (about \( 10^6 \) per cm\(^2\)). (A more detailed description of this instrument is given in Section IV.2. by Mr. Roberts.)
II. POSITIONING OF THE STACK

When the position and characteristics of the beam are known the stack has to be placed so as to receive the particles correctly.

The first important point is the angle at which the particles enter. In the majority of cases one tries to have the particles enter parallel to the surface of the emulsion. Since the beams, especially at some distance from the target and in the absence of strong focusing, are completely horizontal, the technical problem of positioning the stack is limited to constructing a table with a perfectly horizontal surface. Mr. Roberts will describe this type of table with the additions which he has made to give it increased flexibility and make it of more general use: viz. a choice of three positions in relation to the beam (horizontal, vertical, and perpendicular), revolving mounting-plate, variable height and remote control etc. With such a table the horizontal position can be defined to within 1 milliradian. Perpendicular exposures are usually required for studies of scattering at small angles in emulsions, or for distortion measurements. It is necessary for the angular dispersion of the beam to be small (less than 5 or even 2 milliradians).

A second important point is the shape and dimensions of the beam cross-section and the geographical distribution of the particles, which is generally not uniform. The stack must then be moved at regular intervals so as to obtain uniform exposure of different parts.

III. SAFETY OF THE STACK

When the exposure takes place in a separated particle beam, measures should be taken to prevent unwanted particles, from entering the stack as a result of the faulty operation of certain beam transport components. Double safety measures are applied in this case. Firstly the target is automatically stopped as soon as variations of a few per cent appear either in the intensity of the magnet and lens power
supply or in the difference in potential between the plates of the separator. At the same time the table supporting the stack is lowered (in about 1 second) and the emulsions are taken out of the beam. When everything returns to normal the table automatically rises again to its original position. This special table will be described by Mr. Roberts.

IV. DETERMINATION OF CERTAIN CHARACTERISTIC VALUES CONCERNING THE STACK

1. Dimensions of the stack

There is a limitation on the thickness of the stack connected with the difficulty of stacking; 200 pellicules, 600 µ in thickness, namely 9 to 12 cm in height, is about the maximum number that can be well stacked (see Section IV.3 for further details). The width of the stack depends both on the dimensions of the beam and the nature of the events which it is intended to study in the emulsions; 10 to 15 cm is a common dimension. The length depends even more on the nature of the studies which are to be carried out. To illustrate the kind of considerations which may influence its choice, suppose one wishes to study the production of trident events by 17 GeV/c π mesons, i.e. π → 3π. For the purpose of eliminating the primary electrons, which may also give tridents, these electrons must be recognized by multiple scattering measurements. Thus 10 cm of primary track is necessary. In order to discover the events the tracks should be followed individually. Again 10 cm of track is necessary to work at an optimum speed. Finally, as soon as an interaction is found the secondaries should be followed in turn over 10 cm to distinguish by multiple scattering measurements the π → 3π events from the π → π + 2e events. In this way the required length of the stack is established, in this case 30 cm, which is somewhat above the average usually chosen. Moreover, in this study similar conditions governed the choice of the width as 15 cm.
2. **Intensity of the exposure**

When the studies have to be carried out by following individual tracks at minimum ionization, it is preferable to choose a low intensity (about $10^3$ particles/cm²). Scanning is much easier and faster and the total length of the tracks is always sufficient to give a satisfactory number of events. For example, an intensity of $3 \times 10^3$ particles/cm², equivalent to two tracks per 100 μm field in a 600 μm thick pellicle, corresponds to a total length of about two kilometres of track in 100 pellicles (average utilizable length of each track: 10 cm, and width of stack 10 to 15 cm). In this medium-sized stack there will therefore be 6000 interactions of all kinds and it will be possible statistically to study a phenomenon with a cross-section of the order of 0.2 - 0.3 mb. Moreover, even this low intensity will represent the work of 5 or 6 microscopists for six months, and the price of the stack, which will be of the order of SF 5,000 (SF 4,000 per litre), will not prohibit the exposure of a second similar stack if more events are needed.

When area scanning is employed in search of events with tracks which are not at ionization minimum, the intensity may be much higher (between $5 \times 10^4$ and $10^5$ particles per cm²). In perpendicular exposures and in the case where the type of study calls for high intensity, one can go as high as $10^6$ particles per cm². A special, less sensitive, type of emulsion, or normal emulsions specially developed, can stand much higher intensities (e.g. in the monopole experiment and experiments on high-energy fission).

* * *

**DISCUSSION**

Holthuizen : I would like to raise a question concerning the horizontal characteristics of the beam. I understand that for a normal beam the angle with the horizontal plane is less than 0.2 mrad. However, if a separator is used,
Holthuizen (cont.)

a deflection in the vertical direction occurs which
is corrected by means of two magnets, one before the
separator and one after it. What is the angle with
the horizontal plane in this case?

Harmsen

I think that this is a question of the last quadrupoles.
One can arrange to have the value of 0.2 mrad as with
other beams.

Teucher

What is the maximum time that the emulsion can tolerate
in the neighbourhood of the machine, before becoming
blackened?

Combe

That depends, of course, on many factors, such as posi-
tion with regard to the ring and the number of beam
apertures which are open. However, two extreme cases
might be of interest. 1) Inside the ring the relevant
time would be approximately five minutes. 2) Outside
the ring, in the South Hall, with all apertures closed,
one finds practically nothing but muons, and it would
take, say, 5 to 10 hours to obtain a density of \(10^6\) or
\(10^5\) particles per cm\(^2\). Between these extremes one has
a vast range of possibilities.

Teucher

The \(k_1\) beam was particularly bad in this respect, and
it was necessary to introduce a special shielding in
order to make the exposure feasible.

Combe

Here one has something just between the two extreme
cases particularly with regard to the Hall. The North
Hall is "open" to the ring as opposed to the South Hall
where one has a solid concrete shielding wall. The \(k_1\)
beam was very typical. One has pions which are stop-
ping in very high intensity somewhere around the stack,
and the exposure is very long (approx. 24 hrs) with a
Combe (cont.) : very small number of particles to be collected. That means that any background produced by particles that are $10^4$ times more abundant can give the same blackening.

Dahl-Jensen : The value for the muon flux in the South Hall (approx. $10^5$/cm² in 10 hrs) seems to me an extreme overestimate; our experience would suggest less than $10^2$/cm² in 30 hours.

Combe : This may be a question of solid angle. The conditions inside the ring may have been different. If there is a good deal of iron inside the ring, the shielding thickness could be increased by a factor of two. When I made the relevant exposures there was practically only the shielding wall. I agree that the flux could be less by the order of magnitude as you suggest.

Dahl-Jensen : With regard to your remarks concerning an angular spread of up to several degrees in a focused beam, I think that it would be fair to compare the number quoted for the angular spread of the unfocused $\alpha$ beam (0.2 mrad) with the angular spread of the focused beam, namely 0.5–0.6 mrad.

Combe : I cannot take the case of every beam, but, of course, one can focus much more with a low-energy beam.

Nikolić : I would like to stress that Dr. Combe's remarks concerning background are intended merely to serve as illustrations, since, of course, one cannot give a general value of the background in the South Area or North Hall because so many factors are involved that one finds a variation of 1 or 2 orders of magnitude. One must consider each individual case, especially because situations continually change (as, for instance, the shielding, geometry, and so on).
TECHNICAL PROBLEMS OF EMULSION IRRADIATIONS

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I. STACK EXPOSURES

The type of equipment necessary to make emulsion exposures to beams from the PS has gradually become more complex, as the experiments themselves have evolved. Our first needs were very simple, in fact just a table of suitable dimensions, adjustable in height, with the addition of a three-footed stand on top for levelling purposes. This, together with the use of an accurate spirit level giving a precision of \( \frac{1}{30} \) of a milliradian, was quite adequate for simple horizontal exposures, and has been used as a basis for more complicated arrangements, as is shown in Fig. 1. On top of this assembly we have built a turntable, graduated in degrees, with a vernier scale giving a precision of \( \frac{1}{2} \) minute. In addition there is a cross-slide with a vertical support, fixed to the turntable. This vertical support has been milled at 90°, with respect to the cross-slide, to a high degree of accuracy. For angular exposures in the vertical plane, we have made a simple device that consists of two flat plates hinged together at one end. It has a quadrant graduated in degrees fixed to the bottom plate and a vernier scale fixed to the top plate, giving a precision of \( \frac{1}{10} \) of a degree.

With this apparatus one can obtain accurate exposures, in the horizontal, vertical, and perpendicular planes, plus any angular exposure needed. Here I should like to define our use of the terms horizontal, vertical and perpendicular, as in fact one error in exposure was made due to a misunderstanding of these terms.
The horizontal plane gives no trouble: the difficulty arises in the difference between vertical and perpendicular. We use the following definitions:

**Vertical** - emulsion plane in line with the beam, but turned through 90° with respect to the horizontal;

**Perpendicular** - emulsion plane at 90° to the beam.

In the early days of emulsion irradiations to beams from the FS we were obliged to use many tables. The reason for this was the large number of stacks exposed, together with the variety of stack holders used, requiring different table heights to ensure the correct exposure of individual stacks. Recently, large collaborative stacks have been used which cut down the amount of preparatory work and save a considerable amount of machine time. The collaborating laboratories also have the possibility of exchanging plates, which is especially important when following-through is necessary.

To irradiate these large stacks, where in some cases several shots into the same stack are necessary, an automatic movement in the vertical direction is needed. Figure 2 shows a tripod carrying a motorized central column with an attached platform. This central column has a keyway accurately cut along its length into which is fitted an adjustable key. In this way movement about the axis of the column is avoided, and height adjustment can be made without fear of displacement of the stack in the Y direction. To facilitate the operation of this table from a distance, there is a simple device which registers the revolutions of the gear-box drive on an electromechanical counter, which is connected to the table by a length of cable. Thus, changes in height of the table can be measured in either direction to a precision of 0.5 mm. In addition, stop-start and reversing switches have been included on the control box carrying the counter, so that all operations can be performed from outside the exposure area.

The apparatus so far described is what we use in most cases, but this does not mean that these are the only methods of making exposures. For example, the group at Bern use a different method of
lining up a perpendicular exposure. They mount their stack on a platform which has three levelling screws, and they fix to the emulsion holder a mirror which is accurately held in the same plane as the emulsions. They place a pocket torch at beam height, pointing in the beam direction, about ten metres away from the stack. The eye is placed in line with the torch; when one sees the reflection of the torch bulb, the stack is lined up. This type of alignment is not, of course, always possible, due to the complexity of some beam layouts.

When exposures to the K beam were proposed, a new set of problems presented themselves. This beam yielded of the order of 10-15 K\(^{-}\) particles per pulse; consequently exposures of upwards of twelve hours were envisaged. Any major change in the separator conditions, or in the current in any of the many magnets and quadrupoles, could mean that pions would enter the stack. To avoid this possibility we designed a special table, shown in Fig. 3. This table has a movement of 30 cm in the vertical direction, which is operated pneumatically, enabling the stack, which is fixed to the table, to be lowered quickly out of the beam. To control this table movement a device was made which also controls the target of the FS. This device consists of a simple electrical circuit with several relays, which accepts an "off" signal from the control panels of the bending magnets, quadrupoles, and separator. In the event of large fall-off or complete cut of current to any one element, the device gives an alarm signal, stops the FS target operating, and indicates which element has failed. At the same time the table can be lowered. In this way, while the stack is out of the beam the fault can be rectified, then the target put into operation and the table raised to its original position. All these operations can be carried out from the counting room by remote control.

The table height relation to beam centre can be adjusted manually according to the size of stack exposed. The stacks are mounted on a platform independent of the table. This enables various types of stack holders to be mounted in the laboratory and slid into position as required. The stack holders used in the K\(^{-}\) exposures had two slots milled in the base at 90\(^{\circ}\) to the beam direction. This allows movement
for lining up the stacks in the horizontal plane; the screws which traverse the slots are then tightened and the stack is held firmly in position during the exposure.

This special table can also be used as a beam stopper, possibly in connection with pulsed magnet exposures. It could be used in the following way. Due to the charging rate of the condenser bank it is desirable to accept, say, every fourth pulse of the PS. The table could be placed in front of a collimator, with a suitable piece of lead on it, large enough to cover the hole in the collimator. The table could then be operated in coincidence with the repetition rate of the condenser bank. In this way it would not be necessary to stop the PS target; when other groups are using the machine this mode of operation would ensure the most efficient use of the accelerator.

II. LINING UP

As Dr. Combe has already pointed out, there is a competent team of surveyors at CERN, and, when given the necessary fiducial points by the designers of a particular beam, they are able to line up any ancillary apparatus to optical accuracy. All our levelling tables have a line traced through the centre, and for most straightforward experiments these tables are aligned with this line as the beam centre in the horizontal plane. The table is then adjusted to the required height, and lastly the top platform is levelled with the aid of the three adjustable feet and an accurate spirit level. In the case of a vertical exposure, when the turntable is used, the vertical support is lined up in the centre of the turntable, with the vernier scale set at 0°. Then the cross slide can be moved the required amount to place the stack in position.
III. BEAM SHAPE AND POSITION

After the mechanical alignment is completed, the problem of the determination of the shape and actual position of the particle beam arises. In working with high intensity extracted beams, such as the extracted proton beam from the SC, this presents no problem, as both X-ray and polaroid films give good results with an exposure of the order of $10^7$ particles per cm$^2$. With the scattered-out beams from the FS an exposure of this order would take much too long, as the intensity per pulse is usually only of the order of $10^3 - 10^4$ particles per cm$^2$ in the most favourable conditions.

In setting up the $K^-$ beam (K beam) we solved this problem by using a device developed by the Argonne group, headed by Prof. A. Roberts. This device consists of a Polaroid Land film-holder attached to a box containing a large number of 5 mm diameter rods, 10 cm long, of scintillation material. Each rod is shielded from the others by a tube of thin-walled aluminium foil. In the film-holder the polaroid negative is exposed in the beam direction to the end of these rods. With this apparatus a reduction in exposure time can be obtained of about a factor 10 compared with that required for a polaroid film exposed alone. We are making what we hope will be a rather more refined type of beam intensifier, using the Argonne model as a basis (see Fig.4). In the Argonne apparatus the rods do not actually touch the film, resulting in a certain amount of diffusion in the image obtained. We are using a 2 mm diameter rods of only 3 cm in length, and the whole scintillation chamber will be spring-loaded against the film. In this manner better image definition should be obtained. If this apparatus is successful it will help a great deal to speed the setting-up of beams for emulsion exposures.
DISCUSSION

Göing: : There is a great deal of heavy material around the stack during exposure (supports and so on). Has the background produced by scattering in this material been measured?

Roberts : Heavy material would cause scattering; however, we use "tufnol" for all our stack holders. This material is strong but not very dense. In fact, we have never had any noticeable trouble of this nature.

Dahl-Jensen : I would like to know the dead-time of the dropping-table.

Roberts : The table was designed to drop a total distance of the order of 30 cm in 800 milliseconds. However, since a target stopper is used, the main function of the dropping-table is to take the stack out of the beam while any adjustments are being made. Certainly a stack could not be removed more quickly than the target.

Pahl : With regard to vertical and horizontal exposures, could you give an example where one has an advantage over the other?

Nikolić : When, for example, the angular distribution of particles under investigation in the vertical and horizontal planes may be quite different.

Teucher : I think that the original consideration was a simple technical factor. The only problem was that the available beam had a rectangular cross-section, thus it was more economical to arrange the stack with the plane of the emulsion parallel to the long side of the rectangle.
Combe: An additional reason for using the vertical exposure was concerned with dipping tracks. For example, if one wants dipping tracks of three degrees, it is much better to have a vertical exposure where the plane of the emulsion can be rotated about a horizontal axis.

Gottstein: Can the motor which moves the exposure table in the vertical direction be operated from outside, during the exposure, so that the beam could be distributed more uniformly throughout the stack?

Roberts: This could be arranged, at least in principle.

Heckmann: I would like to mention a couple of techniques which we have found useful in the exposure of emulsion stacks at Berkeley. The first is the use of the common pocket electroscope to monitor the general background radiation in the region of the emulsion stack; as a very general rule an emulsion stack that receives a radiation dose of 20 mr has accumulated enough particle tracks, whether primary beam or background particles, to seem further exposure unwise. Monitoring radiation of dosimeters is a particularly simple way of checking the efficiency of shielding against background radiation in new experimental arrangements. Dosimeters also serve to check the levels of background during a long exposure. Any significant fluctuations may indicate a malfunction in the machine or beam transport system.

Another use of the dosimeter that we have found valuable is as a beam monitor. In cases where the exposure is not very critical, a dosimeter placed directly in the beam, together with the emulsion stack, can record to an accuracy of about 2 the integrated flux of particles. If I recall correctly, a reading of 10 mr corresponds closely to $3 \times 10^8$ minimum ionizing particles per cm$^2$. \( \text{NP/sb} \)
Roberts: Actually, this dosimeter system was used in Levi-Setti's $K^-$ exposure (March 1962).

Heckmann: A second technique, which we used to evaluate a beam of stopping $K^-$ mesons, involved a stack of, say, 10 to 20 by 200 microns thick emulsions sandwiched between outdated emulsions to form a stack 1" thick. In particular, I am thinking of a time when the $K^-$ beam was completely new, the geometry was untried, and, to make matters worse, we had to begin exposing about 20 various stacks in about 24 hours, after the beam was set up. After the standard method for checking beam flux and contours with plates, we exposed the stack of 200 micron pellicles. The stack was developed immediately after exposure. By cutting corners we were examining the test stack in about 5 hours. Several hours later we had information on the absolute flux of $K^-$ mesons, their momentum resolution, dispersion of the beam, and the amount of background. The beam proved to be satisfactory in all respects and the series of exposures was completed on schedule.

What intensity is required for polaroid exposures? About $10^6$ per cm$^2$?

Roberts: The Argonne apparatus gave us an advantage of the order of a factor of 10 immediately, and I think that the intensity was, in fact, of the order of $10^5$ to $10^6$ per cm$^2$. I can give no details about the new apparatus until it is finished, but I hope to get an advantage of at least the same order of magnitude.

*   *   *

$348/№/sb$
Fig. 1.  Simple exposure table

Fig. 2.  Exposure table with motorised vertical movement.
Fig. 3. "Dropping" table

Fig. 4. Polaroid beam camera
USE OF TARGET IN SECONDARY PARTICLE BEAMS

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I. INTRODUCTION

The study of the interactions of elementary particles between each other is a very important problem of high-energy physics. One experimental method of carrying out such a study is to bombard a target containing some of these elementary particles with a beam consisting of others. The characteristics of the particles produced (nature, angle, energy ...) are studied by recording them in photographic emulsions, with or without the assistance of a high magnetic field (100-200 kilogauss). The incident beam may be formed (with varying degrees of difficulty, it is true) of a certain number of known elementary particles (protons, antiprotons, neutrons, π, μ and K mesons, electrons, neutrons, and γ rays). The target can only contain nucleons (neutrons or protons). The proton, in its purest form, is supplied by hydrogen gas or liquid hydrogen. A less pure form, which can often be used and is easier to handle, consists of hydrogen compounds such as the polyethylenes: (CH₂)n.

The neutron does not exist in a free state. The simplest nuclear structure in which it is found is deuterium, whose use, like that of hydrogen, involves certain technical difficulties.

It may be thought that all nuclei, since they contain nucleons, could provide targets which would generally be easy to construct. Unfortunately, these nucleons are bound together by nuclear forces which deform the interaction between the incident particle and the nucleon which it is wished to study. However, from certain high-energy studies it appears that this deformation is not entirely prohibitive, especially in light nuclei (Li, Be, C).
This point is illustrated by the experiment to measure the magnetic moment of the \( \Lambda^0 \) where, in particular, an attempt is made to produce \( \Lambda^0 \) at a given angle (18°) by the reaction \( \pi^- + p \rightarrow \Lambda^0 + K^0 \) (see description of the experiment in Section VI).

If hydrogen is used, the \( \Lambda^0 \) has maximum polarization (which is of interest for the purposes of the experiment) and has a well-defined energy for a given angle of production, which makes it easier to identify; its magnetic moment can also be measured with greater accuracy. Unfortunately, the use of liquid hydrogen in the presence of the coils for the pulsed magnetic field, necessary in this experiment, is an extremely delicate question.

A hydrogen compound \((\text{CH}_2)_n\) was chosen as a target. The polarization of the \( \Lambda^0 \) produced in the reactions between the \( \pi^- \) and the nucleons of the carbon nucleus is less, but is still adequate.

The Fermi motion of the nucleons means that the energy of the \( \Lambda^0 \) emitted at a given angle is not constant. But this drawback does not spoil the experiment; on the contrary, the technical difficulties of setting up are greatly reduced.

Moreover, the polyethylene contains a higher concentration of hydrogen than liquid hydrogen itself; the polyethylene target can be smaller for the same amount of hydrogen and, for this particular experiment, makes for better over-all geometry.

It may sometimes happen that even heavier nuclei are used, particularly when it is wished to produce only sample particles (hyperons, \( \gamma \) rays, heavy fragments ...), or when it is wished to study phenomena which are no longer interactions with nucleons but with nuclei. This, moreover, is a field in which there is great interest at present.
II. A FEW REMARKS CONCERNING CONDITIONS
UNDER WHICH TARGETS ARE UTILIZED

When the study of a problem touches upon the particles produced by a target placed in a beam, the question of efficiency, which is linked with the machine time required, constitutes one of the most important practical points. In general, an emulsion experiment takes a maximum of one week of machine time; this time depends finally on its scientific value. A week corresponds to about $1.7 \times 10^5$ pulses at the rate of one pulse every two seconds.

Efficiency is bound up with many factors, some of which are mentioned below.

1. Beam intensity

The most intense external beams set up at CERN (1 to 15 GeV/$c$, 8 to 27 GeV protons, neutrons) provide about $10^4 - 10^5$ particles per cm$^2$ per pulse. (Examples of beams which have been constructed at CERN:

- $12$ GeV/$c$ $\pi^-$, intensity $1.4 \times 10^4$ cm$^{-2}$ per $10^{11}$ protons;
- dimensions $3 \times 6$ cm$^2$.

Protons in the neighbourhood of 20 GeV/$c$: $\sim 3 \times 10^4$ protons per cm$^2$;
- dimensions $1.5 \times 7$ cm$^2$.)

The nature of the particle, the energy and angle of production, its mean life, the pulse length, the definition in momentum $\Delta p/p$, and the optical system used, are all factors affecting beam intensity. The minimum intensity acceptable for a beam intended for bombarding a target is $10^4 - 10^5$ particles per cm$^2$. Certain beams will soon be improved as, for instance, the $a_2 (\pi^+)$ beam, in which it is expected to obtain $10^7$ particles per pulse in the neighbourhood of 1.5 to 2 GeV/$c$.

Proton beams reaching $10^{14}$ particles per pulse will also soon be extracted, and this high intensity may change the field in which they are used in practice.
High intensities are generally obtained with optics producing intense focusing. The dimensions of the beam are then small (of the order of 1 cm²) but the particles have a relatively large angular dispersion (more than 1°) which is a disadvantage for certain purposes (for the study of scattering at small angles, for instance).

2. Purity of the beam

This is with respect to the nature of the particles, their angular dispersion, and their energy definition.

Until now, beams of rare particles (separated beams of K⁻, p, ...) could not be produced with both the purity and the intensity necessary for providing primary beams useful for bombarding targets. This intensity is below 50 particles per pulse. However, a beam with an intensity of $10^7 \pi^-$ per pulse will contain about $10^5 \bar{p}$ and can form an excellent low-energy antiproton beam (in the neighbourhood of 1 GeV/c).

$\pi^-$ and proton beams are generally contaminated by electrons, $\gamma$ rays and neutrons, which can be fairly easily eliminated (converter and magnetic field); the $\pi^-$ beams may also contain muons, which are more difficult to eliminate.

The $\pi^+$ are naturally produced and transported with an appreciable number of protons. In order to be able to use them alone, separation has to be carried out which slightly reduced the intensity.

For instance, in the experiment for measuring the magnetic moment of the $\Sigma^+$, the $\Sigma^+$ are produced by the interaction

$$\pi^+ + p \ [\text{actually} \ (\text{CH}_2)_n] \rightarrow \Sigma^+ + K^+. $$

The presence of protons among the $\pi^+$ in no way harms the reaction, but increases the background. It seems possible to obtain a separated $\pi^+$ beam giving more than $2 \times 10^4 \pi$ mesons per pulse on the target.

NB Separated beams are necessary for track chambers and, particularly, emulsion techniques.

Counter techniques make the separation by selecting in the un-separated beam and retaining, by electronic means, only those
particles which concern them, and which are identified in
general by choosing those which have given m and v values
(momentum and velocity), thus determining m, and the nature
of the particle.

3. Angular dispersion

The angular dispersion can be made small enough (of the
order of a few milliradians) if a system of optics which is strongly
focusing is avoided, and if a detecting position at some distance
from the last lens is adopted. In both cases the intensity per unit
area is reduced. The same difficulty with regard to intensity ap-
ppears when, in order to obtain a more parallel beam, one has to use
a collimator which stops particles with a trajectory which diverges
too much relative to the optical axis. The collimator is moreover
a rather unsatisfactory means (but there are no others) when it is
wished to produce a narrow beam which will just cover a small-sized
target, less than 1 cm², for instance. The stray scattering which
occurs at the output edges of the collimator acts as a source of
particles, which accumulate in a comparable way to those to be pro-
duced in the target and mask their properties. A certain number of
precautions then have to be taken, which cost a great deal in instal-
lation and intensity (for example, analysing magnetic field, long dis-
tance between the target and the detector, and further focusing).

4. Energy definition

The energy definition (more accurately the momentum p) is a
quality which can be highly developed by an appropriate study of the
optical system focusing the particles, but unfortunately always at the
cost of the intensity. The Δp/p bands normally accepted vary according
to the experiments; usually they are 1 to 5 per cent.

5. Volume of the target

When one has an idea of the value of the cross-section of
the phenomenon studied, for a beam of given intensity, the optimum
volume of the target for suitable efficiency can be calculated. When
the volume has been determined, the relation between the three dimen-
sions will be chosen as a function of certain criteria which vary ac-
cording to the experiment (natural dimensions of the beam, solid angle
between target and detector, angles at which the particles produced
are recorded). Other considerations may have some influence and lead
to modification of the volume of the target.

A target, for instance, modifies the particles of a beam and
those which are produced, and the thicker it is the more marked is
this effect. Additional angular dispersion by multiple scattering
occurs, and at the same time absorption causes a spread of the energy
spectrum. Moreover, secondary reactions take place, and the particles
produced by these reactions mix with those produced by the phenomenon
under study. It is often necessary to carry out magnetic analysis of
the particles coming out of the target.

For example, in liquid hydrogen targets, especially when
they are small, the thickness of the walls of the receptacle may play
an all-important part by producing stray radiation which is unwanted,
and difficult to eliminate in certain cases.

6. Distance between the target and the detector

This parameter is of great importance for efficiency. If
the detector is near to the target, the solid angle is large and more
particles are recorded (the effect is proportional to the square of
the distance). Unfortunately, in order to preserve good angular reso-
lution in detecting particles produced, a compromise has to be made
between the distance and the volume of the target. There are also
cases where the distance cannot be reduced as desired; sometimes, for
instance, a magnetic field has to be used (namely, several metres) to
drive away stray particles. Then, the use of a pulse magnetic field
producing very high fields (100 to 200 kilogauss) can be of valuable
assistance by keeping this distance very small. This point is partic-
ularly important when the particles produced are very unstable (A°, Σ+ ...).
This question of efficiency is also obviously bound up with the cross-section of the reaction which it is wished to study. Thus, when measuring the magnetic moment of the $\Sigma^+$, it is hoped to produce 400 $\Sigma^+$ in the emulsion stack in 10,000 pulses of the PS, with a 1 GeV/c $\pi^+$ beam containing some $10^4$ particles per pulse. The cross-section of $\Sigma^+$ production is taken as equal to 0.26 mb and the over-all geometry is very favourable to the collection of a high number of $\Sigma^+$ (very short distance between target and detector: an average of 2 cm). (See description of this experiment in Section VI.)

III. HYDROGEN TARGETS

Some details of targets containing hydrogen [(CH$_2$)$_n$, hydrogen gas, liquid hydrogen] will be given.

There are no special difficulties involved in the preparation and use of most targets made of other elements. However, certain precautions sometimes have to be taken, for instance with beryllium, when care has to be taken not to breathe in the dust while machining, because it is toxic.

Whenever possible, solid targets are used. There are no particular drawbacks attaching to the liquid state, except that the walls of the container detract from the purity of the target. The gaseous state has the disadvantage of low density. This drawback can be lessened by greatly compressing the gas (sometimes several hundreds of atmospheres). Densities superior even to those of liquids can thus be reached, but the walls become too thick and produce too much stray radiation.

Hydrogen may be used in all three states. However, hydrogen in liquid form is clearly preferable in general, in spite of certain difficulties which can be overcome fairly easily nowadays.

Hydrogen can be used as a target in a very simple way, in the form of polyethylene (CH$_2$)$_n$. Many experiments have been carried out
in this way. The results obtained with pure carbon are subtracted from those obtained with (CH$_2$)$_n$, and the difference gives the contribution of the hydrogen.

The method is obviously long because the experiment has to be performed twice (once with CH$_2$ and again with C). Statistical accuracy also suffers, especially when the stray events due to the carbon are more numerous than the events produced in the hydrogen.

Hydrogen has also been used in the form of a gas at a pressure of 40 atmospheres $^1$ for studying the photoproduction of $\pi^0$ from protons. The detecting emulsions were inside the hydrogen were inside the hydrogen gas, which posed the technical problem of preventing the reducing action of this gas on Ag Br. The solution consisted of cooling to -60$^\circ$C.

Pressures of 150 atm were reached; the density was then 0.035 g/cm$^3$ (half that of liquid hydrogen). The whole was kept at the temperature of liquid nitrogen. In such a target the wall thickness (1.2 g/cm$^2$, as compared to the hydrogen content 1.5 g/cm$^2$) complicated the experiment considerably; the particles emanating from the gas had to be separated from those produced in the walls.

Many gas targets have been constructed and used$^2-7$). All these targets achieved similar performances: pressure $\sim$ 150 atm, d $\sim$ 0.03 g/cm$^3$, at 77$^\circ$K.

**Note**: A technique which is more characteristic of nuclear emulsions consists of incorporating the hydrogen in the emulsion.

- Tubes (outside diameter 150 $\mu$, inside diameter 100 $\mu$) filled with gas at high pressure (600 atm) and incorporated in the emulsion$^8$.

- Thin wires (20-50 $\mu$) of (CH$_2$)$_n$ incorporated in the emulsions, in which the interactions due to the carbon are separated from those due to hydrogen.

It also seems possible to immerse photographic emulsions directly in liquid hydrogen, with a few precautions but without too much trouble$^9,10$). Simple and efficient target-detector assemblies can thus be made.
The field of liquid hydrogen targets will not be considered in greater detail, and mention will be made of the important technical problems which arise. These problems are the result of fundamental characteristics of liquid hydrogen.

In the first place hydrogen can only be kept liquid at a very low temperature (20°K). This, therefore, gives rise to successive difficulties for its production and storage, and then for transferring it from the dewar to the target and keeping it in the target without too high an evaporation rate.

In the second place, hydrogen combines very easily with the oxygen in the air. The reaction, which is exothermic, easily becomes explosive. It is therefore necessary as a general rule to avoid mixing hydrogen with air, especially in an enclosed space and where there is a considerable concentration of hydrogen. All targets should, therefore, be surrounded by an efficient ventilation system; its construction should be sturdy, and the causes of destruction (e.g. overpressure) should be avoided or attenuated and catalysts of the reaction (sparks, flames ...) eliminated.

The safety regulations for the use of hydrogen at CERN are strict and mean that a new target whose characteristics have not been tested has to undergo many weeks of inspection and testing before being approved and brought into use.

A copy of these safety regulations can be obtained from the CERN Safety Officer. A certain number of indications on this subject can be found in the literature.

1. A few physical properties

<table>
<thead>
<tr>
<th></th>
<th>Boiling point (^{\circ})K</th>
<th>Density (g/cm^3)</th>
<th>Steam-generating heat (\text{cal/mole})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\text{H}_2)</td>
<td>20</td>
<td>0.070</td>
<td>214</td>
</tr>
<tr>
<td>(\text{D}_2)</td>
<td>23</td>
<td>0.161</td>
<td>302.3</td>
</tr>
<tr>
<td>(\text{He})</td>
<td>4.2</td>
<td>0.125</td>
<td>20</td>
</tr>
<tr>
<td>(\text{N}_2)</td>
<td>77</td>
<td>0.81</td>
<td>3,000</td>
</tr>
</tbody>
</table>
2. **Styrofoam targets**

The liquid hydrogen is put in styrofoam containers. These targets were in use a few years ago \(^{16-18}\). They are gradually being discarded, and as far as is known have never been used at CERN.

Nevertheless styrofoam has good qualities. It has excellent heat insulation properties, and it is light \((0.03 \text{ g/cm}^3)\) and cheap. It can be stuck together with araldite. Containers with double walls, between which circulated the cold hydrogen gas produced by the evaporation of the liquid, were used to reduce losses, which were still of the order of 1 litre per hour \((\frac{1}{4} \text{ litre under the best conditions})\).

3. **Metal targets using a vacuum as insulator**

Figure 1 shows a diagram of a hydrogen target and its filling system \(^{19}\). These targets, when well constructed, are definitely more stable, have a longer useful life, and are easier to handle.

They consist essentially of an appendix A (Fig. 1), of variable size, which contains the hydrogen through which the beam will pass. This appendix is as well insulated as possible from the outside atmosphere. It is permanently connected to a liquid hydrogen reservoir, which ensures that there is a constant quantity of liquid in the appendix and prevents the liquid from boiling. The reservoir itself is insulated from the outside atmosphere by a vacuum, and at the same time is surrounded by liquid nitrogen. The appendices are generally interchangeable and can be used with the same reservoir.

The gas produced by the boiling of the hydrogen is evacuated through \(V_1\) into the outer air at some distance from the experimental hall. A number of papers \(^{19-26}\) have been published on the construction and utilization of targets of this kind.

A brief survey will now be made of the most important technical problems related to the construction and use of these targets and the various solutions which have been adopted.
In the first place, during production and storage there is the phenomenon of the conversion of the ortho type molecule into the para type.

The heat produced by this conversion necessarily causes the liquid to evaporate. In a normal mixture of 75% ortho - 25% para, loss of liquid due to this phenomenon is about 1.3% per hour. When the mixture reaches the proportion of 50% ortho the loss is only 0.5%. Therefore catalysing converters have to be used during the production of the liquid hydrogen to obtain mixtures which are richer in para and will then be converted more slowly\textsuperscript{27}).

Next, the target has to be filled. This operation calls for a transfer line from the dewar to the target itself, which should be well designed and well constructed if the liquid hydrogen is not to evaporate before arriving in the target. This line should also be as well insulated as possible from the atmosphere (vacuum, no direct contact between metal parts and atmosphere).

The filling is performed according to certain important rules which will not be given here. Losses during filling are about the same as the daily losses when the target is in operation.

The target should be robust and absolutely reliable and it should be designed to avoid heat losses as far as possible.

These losses are due to radiation (which varies as \(T^4\)) and to heat conduction through the target structure (which varies as \(T\)). High heat losses lead to added expense and the need for greater attention. The increased amount of bubbles in the liquid may produce variations in density; security problems will be more serious (because of the greater amount of \(H_2\) gas to be evacuated). Good targets have losses of between 0.05 and 0.10 litres per hour\textsuperscript{21}). If the hydrogen has already been converted, losses can be as low as 0.1 litre per day.

As a rule the reservoir is surrounded by liquid nitrogen (Fig. 1) which can reduce losses by a factor of the order of 80. The appendix (hydrogen cell) is insulated by the vacuum and, in addition, in certain more recently constructed targets, is wrapped in a layer of superinsulating...
material consisting of a large number of reflecting layers. At CERN, where there are several targets constructed on this principle, the hydrogen cell is enclosed in 200\( \mu \) thick aluminized mylar foil. This system eliminates the liquid nitrogen envelope.

The target is usually precooled with liquid nitrogen before being filled with hydrogen. The very large target, 3 metres long and with a capacity of 100 litres, used by Dr. Von Dardel's group, takes five hours to precool and consumes 150 litres of liquid nitrogen. Where the targets are superinsulated they can be directly and much more quickly filled with liquid hydrogen.

The use of superinsulated cells is preferable from a security point of view. It appears that any sudden considerable increase in temperature due to an accidental cause (such as the destruction of the vacuum) need not be catastrophic. The whole structure has sufficient inertia for the heat to be transmitted slowly and for the hydrogen to evaporate at such a rate that the gas can be easily evacuated.

The thermal conductivity can be reduced by a suitable choice of material for the structure of the target (in connection with the thermal conductivity of various materials, see a list of references given in Handbuch der Physik).

There are also many advantages to be gained from the heat exchange between the cold gas which is evacuated and the part of the structure which is in contact with it\(^{29}\).

The vacuum in its turn is constantly used to improve the insulation of the parts in which there is liquid hydrogen. From the heat point of view, a vacuum of \(10^{-4}\) mm of mercury is adequate. The hydrogen cell itself acts as a vacuum pump: air entering through a small leak will immediately be frozen.

From a technical construction point of view, to reduce difficulties, rigid connections between the hot and cold parts should be avoided. Excellent insulating joints can be made with styrofoam, polystyrene or teflon\(^{30}\).
The material which is the most generally suitable for constructing targets is stainless steel. It is strong, a poor heat conductor, and it can be welded. It is available in the form of tubing or foil (up to 25μ thick) which can be used to make the hydrogen cells and the windows. For the latter, mylar is almost as good.

Mylar tends to be used increasingly at CERN. A target entirely made of mylar (hydrogen cell and envelope) was used for an experiment. In many cases mylar has replaced stainless steel for the windows to advantage: for example, a 1-metre long target which represents 7 g/cm² of liquid hydrogen has windows of 0.7 g/cm² stainless steel on the cell and 1.6 g/cm² aluminium on the envelope, making 2.3 g/cm² in all. If these windows were made of mylar they would represent only 0.5 g/cm² and 0.3 g/cm² respectively, namely 0.8 g/cm². Moreover, mylar behaves very well at low temperatures.

4. Deuterium targets

Liquid deuterium is produced directly from the gas in apparatus which is very similar in principle to a hydrogen target. Liquid hydrogen\(^{31}\) or liquid helium\(^{33}\) is used for liquefying it.

The deuterium gas can be recovered after the liquid has been used.

IV. CONCLUSIONS

A general review has been made of the problems posed by the construction and use of targets.

In practice, when a liquid hydrogen target is used at CERN many of these problems are solved with the help and advice of those responsible at CERN.

It is even easier to solve these problems if one of the many existing targets which have already operated can be used.

The principle and the characteristics of these targets are continually changing. It is difficult to give a really up-to-date
picture. There is a whole range of targets with lengths (Fig. 1) from 20 cm (3-4 litres volume) to 300 cm (100 litres volume). Every time it is planned to use a liquid hydrogen target at CERN it is necessary to contact those responsible at CERN, well in advance, in order to obtain from them the necessary information.

* * *

REFERENCES


14) CPS User's Handbook, Chapter X.
DISCUSSION I

Harmsen: How is an external π beam obtained?

Combe: By placing a target in the internal beam, producing π's which are then focused along the beam channel.

Bertin: Do the intensities given relate to 1 cm² or to the whole beam?

Combe: These values should be regarded as rough estimates; they concern the whole beam.

Potenza: What is the maximum intensity of high-energy particles which can be tolerated in the emulsion?

Combe: The emulsion is an extremely adaptable detector; from $10^2$ to $10^5$ particles/cm² the counting can be done in G5; up to $10^8$ particles/cm² the stars produced can be counted in K0 and K1; from $10^8$ to $10^{11}$ the fission of uranium is counted.

Potenza: What is contamination and how is it detected?

Combe: The contamination of a beam depends on a large number of factors about which one cannot generalize. There may be a γ background detected by pairs of electrons, or there may be neutrons found from the stars without primaries (their multiplicity also makes it possible to have a rough idea of their energy spectrum). For instance, in order to estimate μ contamination, the flux before and after a 100 cm Pb absorber is compared.
DISCUSSION II

Harmsen : Do solid and gas targets have to be cooled?

Combe : No.

Dahl-Jensen : It is possible to obtain hydrogen in the solid state?

Teucher : It is not technically possible; too low a temperature would have to be reached and maintained by means of the circulation of liquid He; this has not so far been attempted.

Heckmann : Some years ago Prof. Alvarez proposed to drop pellets of solid hydrogen through the beam in the Bevatron. The proposal was never actually put into practice, but apart from some obvious difficulties it seems to offer a means of obtaining a pure, well-defined hydrogen target suitable for many emulsion experiments.

* * *

334.8/NP/amg
Fig. 1 Schematic diagram of liquid hydrogen target and filling system

(From Rev. of Sci. Instr. 22, 1006, 1951.)
THE PROPERTIES OF PHOTOGRAPHIC EMULSIONS

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I. CHARACTERISTICS OF PHOTOGRAPHIC EMULSIONS

Ordinary photographic emulsions consist of gelatine in which a very large number of silver-halide crystals are embedded. The silver-halide mostly used is silver-bromide with the addition of up to 5% silver-iodide. Silver-bromide crystals have cubic structure of the same type as sodium chloride, the distance between an ion and its nearest neighbour being $2.88 \times 10^{-8}$ cm. Depending on the way the silver-bromide is precipitated, the crystals grow one of the spatial structures belonging to the cubic system; the faces most commonly present in commercial emulsions are octahedral. The small quantities of silver-iodide will dissolve in the bromide without changing the crystal arrangement, apart from slightly increasing the spacing of the ions in the lattice.

The crystal size in the photographic emulsions ranges from \( \sim 0.03 \mu m \) in Lippman emulsion to \( \sim 20 \mu m \) in some high-speed optical emulsions.

Nuclear emulsions differ from ordinary optical emulsions by a higher silver-bromide content, a smaller average crystal diameter, and a very narrow spread of the crystal diameters, as shown in Fig. 1. Extensive use is made of nuclear emulsions as thick pellicles, i.e. emulsions without support of acetate or glass.

The differences between nuclear emulsions and ordinary optical emulsions are illustrated in Table 1 below.

The atomic composition of a nuclear emulsion is an important "parameter", especially when calculating the stopping power of the emulsion. The atomic composition of a given nuclear emulsion depends
Table 1

<table>
<thead>
<tr>
<th></th>
<th>Optical emulsion</th>
<th>Nuclear emulsion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ag Br/gelatine % by weight</td>
<td>50/50</td>
<td>83/17</td>
</tr>
<tr>
<td>&quot; &quot; &quot; % by volume</td>
<td>15/85</td>
<td>50/50</td>
</tr>
<tr>
<td>Average crystal diameter</td>
<td>0.5 - 3</td>
<td>0.07 - 0.3</td>
</tr>
<tr>
<td>Emulsion thickness</td>
<td>~ 10 µm</td>
<td>400 - 600 µm</td>
</tr>
</tbody>
</table>

on the absolute water content and is subject also to changes introduced during manufacturing. Ilford has reported the average composition of 40 batches of G5 emulsion in equilibrium with air at 58% RH. In the next table is given this mean composition together with the composition of G5 emulsions of densities (D in g/cm³) different from the mean density of the 40 batches.

Table 2

|                      | 40 batches
|----------------------| D = 3.828 ± 0.035 | at D in g/cm³                      |
| Ag                   | 1.817 ± 0.029    | 0.7169 × D - 0.9271 ± 0.0047       |
| Br                   | 1.338 ± 0.020    | 0.528 × D - 0.6831 ± 0.0035       |
| I                    | 0.0120 ± 0.0002  | 0.0047 × D - 0.0061 ± 0.0025      |
| C                    | 0.277 ± 0.006    | 0.6871 - 0.1072 × D ± 0.0036      |
| H                    | 0.0534 ± 0.0012  | 0.1283 - 0.0196 × D ± 0.0006      |
| O                    | 0.249 ± 0.005    | 0.5613 - 0.0815 × D ± 0.0029      |
| N                    | 0.074 ± 0.002    | 0.2182 - 0.0378 × D ± 0.0011      |
| S                    | 0.0072 ± 0.0002  | 0.0214 - 0.0037 × D ± 0.0001      |

(The limits given are at two standard deviations.)
The density in a tiny volume is expected to fluctuate from the average density of the pellicle as a whole. However, from straggling measurements on \(\mu\)-ranges from \(\pi - \mu\) decays these fluctuations can be estimated to be not larger than 1%.

Some times it may be desirable to bring an emulsion into equilibrium (within 1%) with air of another relative humidity than 58%. The time required to obtain this equilibrium depends strongly on the desired humidity and on the temperature. Zero humidity (vacuum) equilibrium is achieved in a month only, 50% RH requires about a fortnight, 75% RH a few days, and 85% RH less than a day to reach equilibrium at room temperature. At lower temperatures much more time is needed.

The gain or loss in weight and volume of 1 cc of emulsion at 58% RH brought to equilibrium with air at other humidities can be read from the following table.

<table>
<thead>
<tr>
<th>% RH</th>
<th>mg/cc</th>
<th>(\times 10^{-3})/cc</th>
<th>mg/g</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>-69</td>
<td>-58</td>
<td>-18</td>
</tr>
<tr>
<td>32</td>
<td>-45</td>
<td>-38</td>
<td>-11</td>
</tr>
<tr>
<td>58</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>72</td>
<td>+43</td>
<td>+36</td>
<td>+11</td>
</tr>
<tr>
<td>84</td>
<td>+112</td>
<td>+95</td>
<td>+30</td>
</tr>
</tbody>
</table>

(These values are measured at "room temperature" - at 25° they are approximately doubled.)
From this table it follows that 1 g of water taken up in the emulsion occupies ~ 0.84 cc, and also that the dimensions of a plate or pellicle will change with humidity. Plates kept under dry conditions (low temperature and/or low humidity) tend to strip off from the glass or to break the glass-backing, depending on the thickness of the coating. Pellicles kept dry turn brittle unless extra plastifier has been introduced in the emulsion before pouring.

Water-soaked emulsions start melting at 35°C, whereas dry emulsions stand temperatures up to ~ 60°C (95° has been reported). If emulsions are to be kept at very low temperatures, care should be taken to get rid of "free" water (not chemically bound to gelatine) before lowering the temperature. Otherwise, ice crystals may be formed and give rise to unwanted distortions [and perhaps accelerated fading if the emulsions were kept at the low temperature during exposure (see section C)].

For a water-soaked emulsion it is somewhat different, because there you have no water in gelatine but gelatine in water, and the whole mixture will freeze as one medium.

Distributed pressures up to 800 atm do not affect the emulsions, but higher pressures may give a blackening after processing due to mechanical deformations in the silver bromide crystals. A sharp pencil will give pressures of about 2000 kg/cm² during writing on the emulsion – this pressure exposes the emulsion locally.

If you plot as usual, the pressure on some emulsion against the compression in % you will find that it behaves elastically up to a compression of 1%, which corresponds to a pressure of about 50 kg/cm². If you increase the pressure, then you will have 2% compression already at 60 kg/cm², implying that the emulsion had started to flow.

It is often desired to have an emulsion in contact with some material or to "load" an emulsion with some substance. It should be noted, therefore, that the recording properties of a photographic emulsion
are affected by:

- oxidizing or reducing substances;
- acid (or alkaline) substances;
- hygroscopic substances.

Example: Metals less noble than silver (iron, copper, lead, brass, aluminia, etc.), will reduce electrochemically the silver and thus cause an unwanted blackening

\[ \text{Me} + \text{AgBr} \rightarrow \text{Ag} + \text{MeBr}. \]

It has frequently been observed that an iron or brass bolt in contact with the emulsion "eats away" a large part of the emulsion.

The chemical action of the various substances decreases with decreasing temperature. So emulsion in contact with hydrogen gas (at some hundred atmospheres) at room temperature partly will be reduced at the surface, whereas an emulsion in liquid hydrogen is not damaged.

The gelatine used in photographic emulsion belongs to the group of natural proteins. It is generally made from selected clippings of calf hide, ear and cheek, or from pigskin and bone. The main action (but by no means the only) of gelatine in photographic emulsions is to keep the silver halide crystals well dispersed and, as far as possible, to prevent clumping of the crystals. Certain, less understood, properties of gelatine make it possible to prepare emulsions with very much better photographic properties than emulsions containing any other dispersing medium.

Gelatine is built up from a number of amino acids - one gelatine molecule consists of 350-500 amino acid units, the molecular weight being ~ 40,000. Gelatine molecules contain both \(-\text{NH}_2\) and \(-\text{COOH}\) groups at the ends and on side chains. This means that gelatine possesses acidic as well as basic properties. The mechanical and chemical properties depend, therefore, on the $\text{pH}$ of the surrounding medium.
At some $P_H$ the number of $-\text{NH}_3^+$ and $\text{COO}^-$ groups are equal—or, in other words, the molecule as a whole has no net charge. This $P_H$ is called the isoelectric point and lies for gelatine around 4.8. At this $P_H$ the swelling is minimum and the gelatine has the highest chemical stability. Gelatine shows hysteresis effects—the properties under some external conditions depend on the pre-history.

II. THE MECHANISM OF EXPOSURES

When photographic material receives a heavy exposure free silver and halogen will be produced. The general reaction is

$$\text{Br} + \text{energy} \rightarrow \text{Br}^* + e$$

$$\text{Ag}^+ + e \rightarrow \text{Ag}.$$  

The quantity of energy absorbed in the silver halide crystal required to make the liberated silver detectable is far above the energy absorbed during a "normal" exposure where only a latent image is formed. Provided effective methods to remove the liberated halogen are used, direct proportionality has been established between the energy absorbed and the amount of silver liberated. When these high-exposure data are extrapolated to weaker exposures the straight line passes through the origin. From this it may be assumed that free silver is formed also in the region of normal exposure. X-ray diffraction pattern analysis on exposed emulsions shows likewise the existence of metallic silver in the latent image region.

How is the energy absorbed by the silver-halide crystal, and how are the silver nuclei formed which make the crystal developable? According to the widely accepted theory of Guerney and Mott, the photo-chemical process occurs in two steps—a motion of electrons, followed by a motion of ions. By absorption of light quanta or by energy dissipation by a penetrating charged particle, an electron will be lifted from its usual energy level in the crystal lattice to the conductance
band where it can move around freely. In the crystal there exists a large number of imperfections—deviations from the ideal crystal lattice or impurities of silver or silver-sulphide. Here an electron can be trapped and thus form a negatively charged speck. Between the ions of the lattice exists a number of interstitial silver ions; they are the carriers of the photo-current in a silver-halide crystal measured in complete darkness. These interstitial silver ions are attracted by the negative specks and, on their arrival, are discharged and form a neutral silver atom. This silver atom is now an effective trap for the next electron. The positive hole formed when the electron was released moves to the surface, where a bromine atom escapes.

Most of the known photographic effects can be interpreted by this theory, but the weak point is that the velocity of the positive hole movement must be much less than the velocity of the interstitial silver ions. Otherwise, the negative specks will be discharged by the positive holes before a silver ion arrives at the speck. The experimental evidence, however, is that the mobility of the holes is several orders of magnitude greater than that of the silver ions (no mass transport is involved in this process, only a repeated shift of charges). It must be required, therefore, that by some mechanism the positive holes are more readily trapped than the electrons.

Most of the bromine atoms liberated will be taken up by the surrounding gelatine. Suggested reactions are

\[
\begin{align*}
\text{Br}_2 + \text{H}_2\text{O} & \rightarrow \text{H} \text{OBr} + \text{HBr} \\
\text{H}O\text{Br} + \text{gelatine} & \rightarrow \text{gel-Br} + \text{OH}^- \\
\text{Br}^* + \text{gelatine} & \rightarrow \text{gel-Br} \\
\end{align*}
\]

where \text{gel-Br} indicates an undefined bromination product of gelatine.
The bromine could migrate to the latent image centres also and rehalogenate the silver ion, or it could recapture the trapped photoelectron and again become a halide ion. At very heavy exposures the gelatine is not able to accept the large quantities of bromine; rehalogenation will take place to a high degree, reducing the photographic effect of the exposure. When less sensitive nuclear emulsions are exposed to large fluxes of particles (> $10^4$/cm$^2$) the net result can be that no latent image is present after exposure. This effect is known as solarization.

Another aspect of interest in the work with nuclear emulsions is the recording properties at very low temperatures. From the Guerney-Mott theory it must be expected that only liberation and trapping of electrons take place - the interstitial silver ions are practically immobile at temperatures below -200°C. At these temperatures another process may occur; electrons could be trapped in "shallow" specks, where they will remain until they are released easily by thermal excitation when the temperature is subsequently raised.

When the emulsion is heated (prior to processing) the second stage in the Guerney-Mott mechanism takes place - the interstitial silver ions move to the negative specks. The latent image silver is much more dispersed at low temperature exposure than at room temperature, the reason being that when an electron is trapped the lack of ion mobility will prevent discharge of the trapped electron by an interstitial silver ion. The probability of trapping two or more electrons in the same speck is small, as the already trapped electron will repel the next one approaching the speck. The electrons trapped in shallow traps may help in building up larger latent image nuclei when they are released by thermal energy at temperatures where the mobility of the interstitial ions is sufficient to complete latent image formation.

The recording properties of nuclear emulsions at low temperatures differ from type to type. Ilford G5 is reported to yield 75% of the usual sensitivity when exposed at liquid hydrogen temperature.
Ilford C2, on the contrary, has been reported to lose its registering power at low temperatures. The Russian NIKFI-R cannot record relativistic particles at liquid hydrogen temperature unless modifications are applied to the emulsion, such as exclusion of iodide or introduction of the hypersensitizing TEA. The lack of low temperature sensitivity of NIKFI-R has been explained by the loss of energy through fluorescence is a marked effect at low temperatures in silver bromide crystals containing iodide. The intensity and wave-length band of the fluorescence is a complex function also of the crystal size and of the degree of perfection in the crystal lattice.

III. THE MECHANISM OF FADING

The latent image is subject to changes between the time of exposure and the time of processing. Here distinction can be made between two processes, a physical and a chemical, both of which tend to decrease the number of developable crystals, the effect is usually termed fading. The physical fading is a thermal ejection of an electron from a latent image silver speck. In this way a silver ion is formed which may migrate away from the silver speck. There is experimental evidence that more than a critical number of silver atoms is required in a speck to make the crystal developable. The loss of a single silver atom can thus turn a developable crystal into a non-developable one. Consequently, specks having just the critical number of silver atoms (which almost certainly is less than 10-15 and probably is about 2-3 only) are very sensitive to this type of fading. It has been shown (Fig. 2) that the processed sensitivity decreases rapidly during the first days after exposure; the reason may be that a considerable number of crystals possess a speck of just the critical size and are thus easily faded by this thermal mechanism. According to this hypothesis emulsions exposed at low temperatures should be more sensitive to fading because of the high degree of dispersion
of the latent image silver. The rate of physical fading differs very little at normal temperatures (0 - 25°C) - reported values indicate less than $7 \times 10^{-4}$ reduction per day in the processed sensitivity. To prevent thermal fading, the exposed emulsions have to be kept at very low temperatures (liquid nitrogen).

Alboy and Faraggi have suggested that the chemical fading follows the reaction

$$2 \text{Ag} + \text{O}_2 + 2\text{H}_2\text{O} \rightarrow 2 \text{Ag}^+ + 4 \text{OH}^-.$$

Alboy and Faraggi (Fig. 3) and many others have shown that the rate of fading at constant temperature increases exponentially with humidity and that fading is accelerated in pure oxygen gas but decelerated in inert gases. However, it must be kept in mind that the above-mentioned chemical reaction, although temperature-sensitive (around a doubling of the reaction speed for a 10 degrees rise in temperature), is very much more sensitive to the absolute water concentration. When the emulsion is in equilibrium with moist air this absolute concentration grows exponentially with temperature (at constant relative humidity).

It has been shown (Barron and Wolfendale, Fig. 5) that, if access of water to an emulsion is prevented by adequate sealing, the rate of fading (interpreted mostly as physical fading) is as low as 0.06% per day at 25°C over three months. The storage times for a 50% reduction in processed sensitivity were found to be

$$520 \pm 50 \text{ days at 25°C and}$$
$$1040 \pm 200 \text{ days at 5°C.}$$

Figure 4 (Leide) shows the dangerous fading regions (shaded) at various humidities and temperatures. It also confirms that fading is a function of the absolute water content rather than of temperature.
A surprising conclusion from the data is that the widely used storing of emulsions in refrigerators where the humidity is high results in some fading, whereas fading is lower when emulsions are stored at 20° and 50% RH!

Because of the smaller average number and size of the latent image specks in emulsions with small crystals, fading is expected to be greater than in emulsions with larger crystals; the crystals in the small crystal emulsion may be more delicate with respect to fading.

IV. HYPERSENSITIZATION OF NUCLEAR EMULSIONS

In the last years a strong interest has been attached to the study of nuclear emulsions with considerably increased sensitivity. An old, well-known treatment - hypersensitization by TEA (triethanolamine) - has been applied and has led to excellent results. The increase in sensitivity by bathing an emulsion in a 2.5% buffered (pH = 9.5) aqueous solution for about 40 minutes amounts to 1.7 \times the sensitivity of the same emulsion without hypersensitization. The hypersensitized emulsions seem to be stable for a few months.

The hypersensitization mechanism has been explained (Idanov et al.) as a silver-catalyzed reduction of silver-bromide by the TEA, thus building up subcritical silver specks. During exposure these will more readily grow large enough to make the crystal developable than in the case of non-hypersensitization.

Ilford has recently started the production of hypersensitized emulsions - still on an experimental basis. Their sensitivity is nearly twice that of non-hypersensitized emulsions. If the emulsions are kept dry, fading is low over a few months. In the first Ilford samples the degree of hypersensitization showed a gradient from air surface of the emulsion.
V. TYPES OF NUCLEAR EMULSIONS

1. Ilford Emulsions

Most of the nuclear emulsions used today are supplied by Ilford. Ilford has succeeded in manufacturing nuclear emulsions with rather constant properties, as far as sensitivity and atomic composition are concerned. Ilford supplies three types of emulsions which record tracks of all particles at all velocities, namely G5, K5 and L4. The processed sensitivity obtainable for all three, ranges from ~ 20 - ~ 30 blobs/100 μ; the difference in the three types lies only in the silver-bromide crystal diameters - ~ 0.27 μm, 0.20 μm and 0.14 μm respectively. Under equal conditions fading is reported to increase in the order G5-K5-L4. The discrimination power for tracks of slow particles and also the accuracy in measuring angles and very short ranges increase in the same order. In addition, the visibility of tracks is improved because a general background will be less marked when the grains are smaller.

K2 and L2 are less sensitized emulsions, recording protons to about 80 MeV, corresponding to a β of 0.4 for a singly charged particle.

K1 is still less sensitized and records protons up to 7 MeV (β = 0.12).

K0 is the least sensitive emulsion, used mainly in fission studies.

Ilford emulsions are in general available as plates, pel- licles, and in gel. form. They are also available diluted with gelatine, loaded with lithium or boron, and with extra plasticizer for use in vacuum or under dry conditions.

2. Eastman Kodak emulsions

Kodak nuclear emulsions are rarely used in high-energy work. According to Kodak, their relativistic particle sensitive emulsion
is considered an experimental product, not subject to Kodak's regular standards of uniformity". Kodak nuclear emulsions are not available as pellicles and are mostly used for autoradiography.

Types available are NTB 3, NTB 2 (protons up to 300 MeV), NTB (protons up to 50 MeV), and NTA (protons up to 20 MeV).

3. Gevaert Emulsions

Gevaert has recently succeeded in producing a minimum ionization sensitive emulsion called NUC 7.15, whose processed sensitivity is as high as 50 blobs/100 μ. Tests have shown that fading in 100 μ test plates is negligible after a month's storage. The diameter of the silver-bromide crystals is ~ 0.15 μm, comparable with those of Ilford L4 (0.14 μm). This emulsion thus possesses a very high sensitivity combined with a high discrimination power due to the small grains. In sensitivity it is comparable to the famous Russian NIKFI-R but has about half the crystal diameter of the latter.

Gevaert NUC 3.07 is a less sensitive emulsion meant for autoradiography. The crystals are exceedingly small (~ 0.07 μm) and the emulsion is able to record particles with β's up to 0.5.

4. Agfa Emulsions

A minimum sensitive emulsion is hoped to be put on the market some time this year. The crystal size distribution will be extremely sharp - the grains will be cubic and the emulsion will contain 82-84% silver bromide.

5. Russian and Japanese Emulsions

For the sake of completeness two of the most outstanding nuclear emulsions from other countries should be mentioned, namely the Russian NIKFI-R and the Japanese FUJI BT-7A. Both of them are
minimum sensitive, the crystal diameters being 0.28 μm and 0.27 μm, respectively. Nikfit-R records relativistic particles with blob densities around 50/100 μ. In case of hypersensitization, Nikfit-R yields up to 100 blobs/100 μ.

The properties of the above-mentioned nuclear emulsions are compiled in Table 4.

VI. LOADING OF NUCLEAR EMULSIONS

Nuclear emulsions can be loaded with practically all elements. In this presentation, the technique of loading will be described in two groups: heterogeneous (the loading phase is visible after processing), and homogeneous (the loading phase is so dispersed in the emulsion that visual inspection is not possible).

1. Heterogeneous loading

Nearly all insoluble compounds can be filled into an emulsion in the form of grains with diameters larger than 1 μ. Loadings of this kind are easily mixed with a melted emulsion prior to pouring. As discussed in section I, care should be taken that the loading does not react chemically with the emulsion. Typical loadings of this category are minerals (SiO₂, diamond, graphite, TiO₂, etc.), insoluble salts (PbSo₄, CoWo₄, WO₃, etc.), glass, plastics, protected metals, etc.

Loading by wires and tubes is often preferred as the scan for events is facilitated when it can be done along the wire. Another advantage is that thin tubes can be filled with a reactive substance. Wire thicknesses from 1-50 μm have been used. The most important problem in wire loading is to reduce the distortions around the wire, the region where one usually wants to make accurate measurements. Good results have been obtained in attempts to restore the original thickness of the emulsion after processing by resins of various kinds.
Flakes, foils and powders can be placed between pellicles in a stack provided care is taken so that the material does not damage the emulsion. Likewise, pure or loaded gelatine layers can be poured on or between emulsion layers (sandwich emulsions).

**Homogeneous loading.** After soaking emulsions in solutions of various soluble compounds these compounds will be present in the emulsion after subsequent drying. Better control of the quantity is obtained by adding a known volume of the solution of a known concentration to a known volume of melted emulsion. In this way, Ilford prepares loaded emulsions containing 16 mg Li/cm³ (as Li₂SO₄), 23 mg B/cm³ (as Na₂B₄O₇), or 270 mg Bi/cm³ (as Na BiO₃).

The following list may serve as a guide for the quantities of various substances which can be loaded into an emulsion by simple impregnation or addition.

<table>
<thead>
<tr>
<th>Substance</th>
<th>Percentage</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>H₂O</td>
<td>200%</td>
<td>(with development immediately after exposure)</td>
</tr>
<tr>
<td>D₂O</td>
<td>10%</td>
<td></td>
</tr>
<tr>
<td>Li₂SO₄</td>
<td>5%</td>
<td></td>
</tr>
<tr>
<td>Li₂B₄O₇</td>
<td>10%</td>
<td></td>
</tr>
<tr>
<td>Na₂B₄O₇</td>
<td>10%</td>
<td></td>
</tr>
<tr>
<td>NaN₃</td>
<td>3%</td>
<td></td>
</tr>
<tr>
<td>Th(NO₃)₄</td>
<td>0.3%</td>
<td></td>
</tr>
<tr>
<td>UO₂(CH₃COO)₂</td>
<td>2%</td>
<td></td>
</tr>
</tbody>
</table>

The list can be extended to include almost all soluble compounds—provided adequate care is taken to prevent chemical destruction of the emulsion (by buffering or by complexing). Strong complexing agents (e.g., versenates) enable us to load emulsions with large quantities of heavy elements such as UO₂, Th, Bi, etc. (de Carvalho).
It is possible to add up to 50% of insoluble compounds as colloidal particles to an emulsion, but this procedure requires rather sophisticated methods. For example, PbSO₄ and CoWO₄ can be precipitated together with silver-bromide during the manufacture of the emulsion, the loading consisting of submicroscopic particles protected by gelatine.

* * *

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II. THE MECHANISM OF EXPOSURES


III. THE MECHANISM OF PADING


IV. HYPERSENSITIZATION OF NUCLEAR EMULSIONS

Faraggi and Gerin, Hypersensibilization des Emulsions Nucléaire Ilford par la Triettanolamine, Saclay 1959.

V. TYPES OF NUCLEAR EMULSIONS

Booklets and private letters from the various manufacturers.

VI. LOADING OF NUCLEAR EMULSIONS

de Carvalho and da Silva, Notas de Fisica 4, 12 (1958).
### Table 4

<table>
<thead>
<tr>
<th>Manufact.</th>
<th>ILFORD</th>
<th>KODAK</th>
<th>GEVAERT</th>
<th>NIKFI</th>
<th>FUJI</th>
</tr>
</thead>
<tbody>
<tr>
<td>type</td>
<td>G5</td>
<td>K5</td>
<td>L4</td>
<td>K2</td>
<td>L2</td>
</tr>
<tr>
<td>diam. μm</td>
<td>0.27</td>
<td>0.20</td>
<td>0.14</td>
<td>0.20</td>
<td>0.14</td>
</tr>
<tr>
<td>highest β record.</td>
<td>1</td>
<td>0.40</td>
<td>0.12</td>
<td>0.10</td>
<td>1</td>
</tr>
<tr>
<td>P</td>
<td>T MeV</td>
<td>all</td>
<td>80</td>
<td>7</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>R mm.</td>
<td>all</td>
<td>20</td>
<td>0.3</td>
<td>0.2</td>
</tr>
<tr>
<td>K</td>
<td>T MeV</td>
<td>all</td>
<td>4.0</td>
<td>3.5</td>
<td>2.5</td>
</tr>
<tr>
<td></td>
<td>R mm.</td>
<td>all</td>
<td>10</td>
<td>0.15</td>
<td>0.08</td>
</tr>
<tr>
<td>π</td>
<td>T MeV</td>
<td>all</td>
<td>13</td>
<td>1</td>
<td>0.7</td>
</tr>
<tr>
<td></td>
<td>R mm.</td>
<td>all</td>
<td>3.5</td>
<td>0.04</td>
<td>0.02</td>
</tr>
<tr>
<td>e</td>
<td>T KeV</td>
<td>all</td>
<td>4.0</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>α</td>
<td>T MeV</td>
<td>all</td>
<td>all</td>
<td>140</td>
<td>110</td>
</tr>
<tr>
<td></td>
<td>R mm.</td>
<td>all</td>
<td>all</td>
<td>5</td>
<td>3</td>
</tr>
</tbody>
</table>

Particles of higher charges are always recorded.
DISCUSSION

Combe : Now there are some emulsions recording particles up to $\beta = 0.40$ (K2 Ilford), some up to $\beta = 0.65$ (Kodak). How difficult would it be to produce emulsions recording up to a $\beta$ between 0.65 and 1? Such a property could be used in experiments where the wanted particles have a $\beta$ between 0.65 and 1, and are produced with a lot of unwanted particles with $\beta = 1$.

Dahl-Jensen : Dr. Vanderhaeghe has described an excellent method of mixing two types of emulsion of the same grain size but different sensitivity to get an emulsion with a sensitivity which is about the mean of the two. You can also hypersensitize a less sensitive emulsion to obtain the desired sensitivity.

E. Fletcher : Does the treatment of the emulsions before exposure (e.g. the temperature at which they are kept) affect their sensitivity?

Dahl-Jensen : Barron and Wolfendale again exposed some of the emulsions, which had been stored both at 5° and 25° C for 500 days, and found no decrease of sensitivity for either temperature.

Heckman : Could you tell me why processed L4 emulsions often have large, amber-coloured crystals? These crystals are difficult to remove and seriously affect particle tracks which enter them.

Teucher : It can be prevented by refixing; L emulsions should always be refixed.

334.8/NF/amg
Heckman: I wonder if you could account for the varying degree of success one has when one tries to hypersensitize emulsions. In particular, I know that emulsions have been useful several months after they were sensitized, yet I have experienced cases where the rise in the single-grain background has made the emulsion useless in about 24 hours.

Dahl-Jensen: If you use too high concentrations of TEA you will expose your plates by the hypersensitization. My only experience is with two stacks of K5 emulsion which could be kept for about three months before the hypersensitization disappeared. It is reported by Faraggi and Garin (see bibliography) that you can reproduce the hypersensitization by a certain procedure, getting stability for more than six weeks.

Key: You mentioned that the proportionality between the energy absorbed and the amount of silver liberated was established at very high exposures and then extrapolated to normal exposure. Why cannot you just measure the points?

Dahl-Jensen: Because these points correspond to about $10^5$ silver atoms per cm$^2$ and you need about $10^6$ atoms to make a chemical analysis. However, you can measure such small quantities by counting the $\gamma$ rays from Ag after irradiation in a reactor.

Teucher: Could you say something about attempts to manufacture emulsions without gelatine?

Dahl-Jensen: Black and white films for cinemas are very often made with polyvinylalcohol instead of gelatine, but you can only use a very thin layer because it is very difficult
Dahl-Jensen : to get your developer into it. Attempts have been made to replace partly the gelatine of nuclear emulsions by polyvinylalcohol but for the same reason it has been very difficult to get a uniform development.

Zakrzewski : How does the emulsion change depending on the inhomogenities you introduce?

Dahl-Jensen : We have found that with, for example, diamond powder (about 8 μm in diameter) you can tolerate 50 mg/100 cm² emulsion (200 μm thick). A track passing through the centre of a grain or above can easily be seen, whereas a track hitting the bottom part of the grain can be examined later on after reversing the plate. The distortion of a steep track near the grain is less than 1-2 μm, which is not much compared with the distortion of the plate as a whole.

Zakrzewski : Which grain size can be used when you are to study short prongs from interactions inside a grain?

Dahl-Jensen : This is a question of compromise because you must exclude events in an outer layer of the grain where you cannot be sure whether the reaction took place in the grain or just outside. If you use smaller grains your effective target volume will decrease rapidly; if you use larger grains you will miss some of the short prongs. We have estimated that 6-12 μm grains will give you minimum distortion and maximum information.

Bovet : I would like to add a remark on the question of heterogeneous loading of emulsions. Several heavy metals can be introduced directly into the emulsion in the form of small spheres. We have succeeded in
Bovet (cont.): producing spherical blobs of gold with a diameter of 3 μm. These blobs are easily distinguishable from the silver ones after development; such a loading, amounting to 4 mg/cm², does not seriously alter the transparency or optical properties of a layer of 200-600 μ emulsion.

* * *
Fig. 1. a) b) The size-frequence curve of an Ilford K5 emulsion (crystal diameters in μm)

Fig. 2. (Debeauvais-Wack et al. Comptes Rendus 253 p. 2518 (1961))
G. Leide, Latent image Fading in Ilford G-5 emulsions

Fig. 3. (Alboy and Faraggi, I. Phys. Rad. 10 p.105 (1949))

Fig. 4. (Leide, Arkiv f. Fysik 11 p. 344 (1957))

Fig. 5. (Barron and Wolfendale, Brit. I. Appl. Phys. 8 p. 298 (1957))
THE ORDERING OF EMULSIONS

W.O. Lock
Nuclear Physics Division, CERN.

In this talk we shall discuss the ordering of emulsion, and for simplicity will only talk about orders placed with Ilford Ltd. At the moment Fujii, Gaevert, Kodak and Nikkî emulsions are not commonly used in Europe.

The first point to stress is that if your order is for some exposure at CERN, then it is most important that you try to get the maximum information to Ilford. If, for example, you order emulsions without specifying that they are needed for experiment B29 in conjunction with Rome and Bari, then Ilford do not know that they should give the same priority to that particular batch of emulsions as they give to the order which comes from Bari and the order that comes from Rome. Of course if the emulsions are only to be sent to you, to be used in your own laboratories in conjunction with radioactive sources, then this first remark does not apply. On the other hand if these are for any experiment at CERN, it is much better if Ilford know exactly what experiment they are required for and whether you are associated with some other laboratories or not. In fact, if the number of exposures to be made during any one week at CERN is more than about three or four, then Ilford prefer the order to come from CERN, on behalf of the laboratories concerned. Otherwise the orders will come in at different times and it is very difficult for them to make a schedule.

When you give information to Ilford, it is better not to send letters attached to the order but to put the information actually on the order. Further, it is a great help if you send a copy of your order, stamped for 'Information Only', to Mr. Ehrlich.
who is in charge of the production of emulsions. In this way he knows several days in advance what he is going to have to do, as it takes a certain time for the orders to reach him from the order department.

The next topic is the transport of the emulsions to you. In general it is best to have them sent by airmail, and that applies not only to the emulsions but to the glasses. If you ask for your emulsion to be sent by airmail but do not specify anything about the glasses, you may have to wait some time for the latter just when you need them. For example, a parcel sent by rail takes about three weeks to travel from London to Geneva. Then it is very desirable that you should obtain the emulsions as soon as they arrive at your local airport. We usually specify on our order that Ilford should send us a telegram with the flight number and airway bill number of the package as soon as it is despatched. In this way we can have all the customs documents ready waiting at the airport as soon as the aeroplane comes in and we can obtain the emulsions within half an hour of the aeroplane landing.

If you order emulsions to be sent to CERN for an experiment, it helps us considerably if you tell us, as well as Ilford, exactly what you are doing. Otherwise we may have emulsions turning up in CERN, sent on behalf of, let us say Berne, and we will have no idea why they have come. Here if you ask Ilford to send emulsions to CERN, please send us a copy of your letter or order so that we know what to expect. Further, it is advisable to check, before you order the emulsions, that your experiment is definitely on the schedule of the PS or the SC (as the case may be), otherwise you may waste emulsions. If you are worried at all about the packing of the emulsions, and you want them sent in dry ice, for example, then it is preferable for you to provide your own packing and your own box. It is certainly

*) Some consignments sent by air to Geneva during 1962 have arrived showing heavy radioactive fogging, rendering the emulsions unusable. Therefore, for important orders, the safest thing to do is to collect your emulsions personally from Ilford and travel back to your laboratory by train.
appreciated by Ilford if you provide your own box for special jobs.

All these things may sound trivial but with the amount of material that comes to CERN and the number of orders that Ilford receive, attention to these small points makes things go very much more smoothly.
THE HANDLING OF EMULSIONS

M.A. Roberts
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I would like to go back about two years in time. At this stage, when large collaboration stacks were envisaged, Professor Herz of Milan suggested the examination of each pellicle at Ilfords before the total stack was packed and sent to its destination. Dr. Dilworth and I went to Ilford to check the first stack of this kind.

By some confusion in the exchange of information with Ilfords, the stack we had come to check was in fact already packed. This proved to be extremely fortunate, because the people who had packed the stack were obliged to unpack it. At this time Ilfords had the habit of cutting their emulsions in pairs with the tissue paper already between them. In the unpacking they saw for the first time the tremendous difficulty there was in separating the tissue paper from the emulsion. Consequently the emulsions are now cut to size one at a time. Those of you who have been in this work for more than two years will probably have noticed that the emulsions now arrive with the tissue paper quite loose between them.

Another point which also amazed the packers was the amount of foreign matter that was stuck between, and in the, the emulsions. In fact, of this stack of some 150 emulsions we were obliged to reject about 20%. This was largely due to the fact that the stack had already been packed and the foreign matter (e.g. silvers of glass, pieces of emulsions, etc.) had been really pressed into the emulsion. In a stack inspected recently (March 1962) less than 5% of the pellicles had to be rejected.

For the cutting of emulsions Ilfords have a simple guillotine. They have recently sharpened this device but unfortunately the emulsions always have some deformation at the edges and this is very noticeable when one makes a large stack. In a stack of 250 pellicles of 600μ
thickness, 20 cm long and 15 cm wide, that I made up recently, there
was a gap at the centre of some 2 mm, as illustrated in Fig. 1. It
is evident that a stack exposed horizontally in this condition would
not be of much use, if one required the primary particles to pass
through the length of the emulsion. I have suggested that a new cutting
machine be made from stainless steel with a spring-loaded bar to hold
the emulsion during the cutting operation (see Fig. 2). The spring-
loaded bar would be of the order of 3 mm wide, so that any resultant
pressure marking of the edge of the emulsion would be kept to a minimum.
In most experiments measurements are not made in the region close to the
edge of the emulsion.

When the emulsions arrive at the laboratory, each pellicle is
re-examined during the stacking operation, and one sometimes still
finds one or two emulsions that are not perfect. These are placed on
top of the stack to act as a pressure buffer during tightening down.

Cleanliness is most important at this stage; the darkroom
should be kept perfectly clean, like an operating theatre in a hospital.

The choice of stack holder is a problem. It depends on the
experiment and exposure envisaged: whether the emulsions are to be
horizontal, vertical, perpendicular, water soaked, loaded, or for
use in a pulsed magnet. Let us consider first the type of stack holder
used to expose emulsions horizontally.

For example, in the case of an experiment to investigate
spurious scattering one requires the tracks to traverse the complete
length of the emulsion. Therefore the emulsions have to be horizontal
to within about 0.03 mrad. Figure 3 shows the type of holder we use
in CERN for such exposures. It can also be used in certain cases for
the kind of vertical exposures discussed yesterday. It is composed
of a tufnol base 300 x 300 x 30 mm, a tufnol top piece 30 mm wide and
about 3 mm all round smaller than the emulsions, plus four rather
robust clamps to hold the emulsions in place. On the tufnol base we
place a piece of black plexiglas 5 mm thick, cut to the stack size.
This allows the milling cutter to pass the last emulsion without
damaging the tufnol base and in this way the apparatus may be used many
times.
The reason for making the apparatus in this form is because one needs some reference for gridding for most stacks. In addition, due to the distortion introduced at the edges by the cutting operation, one is obliged to mill all around the stack. Therefore we stack the emulsions in as neat a manner as possible on the plexiglas base, place the top piece of tufnol in position and tighten the clamps. The amount of pressure applied can be measured with a simple mechanical lever attached to the clamping screws, which are made long enough for this purpose. A known weight is then applied and the pressure on the emulsions can be calculated exactly.

The pressure applied to a stack of emulsions can be quite high without causing damage. A pressure that is quite reasonable is of the order of 300-350 g/cm².

Now the stack is ready for the milling operation. The whole idea of this apparatus is that one can mill and expose without disturbing the stack. In this way the milled edges of the stack give an excellent reference line for gridding. Further, by milling away the distorted edges, one has done the best one can to make the emulsions flat in the horizontal plane.

The technique of milling the stack is quite important. We have had a special cutter made which is 20 cm long and 5 cm in diameter (see Fig. 3). A length of this magnitude can be tolerated because the emulsion is a rather soft material. This cutter has a very fast helix angle so that it slices rather than shears. The shearing action tends to weld together the edges of the emulsion, but the slicing action of this type of cutter gives a smooth finish, and the emulsions separate quite easily after exposure. The recommended speed of a cutter of this type and diameter for milling emulsions, is of the order of 125-150 turns per minute, and the feed should be of the order of 50 mm per minute. During the whole of the milling operation compressed air should be applied to the cutter to clear away the cut emulsion and to keep the cutter cool. To avoid contaminating the emulsions with any oily matter it is better to filter the air supply.
This milling operation can be quite safely carried out in daylight. Immediately the edge of the emulsion becomes black it provides its own protection and the blackening penetrates to less than 0.5 mm. When one has a large stack (e.g. 200-250 600μ emulsions) it is necessary to divide the stack in half, mill each separately, put the two halves together and remill the complete stack. The reason for this is to avoid light entering due to the gap that can occur in large stacks, caused by edge distortion as is illustrated in Fig. 1.

Figure 4 shows a much smaller stack, in exactly the same fixture; this gives some idea of the flexibility of this type of holder. One can, in fact, hold up to 250 x 600μ emulsions in it. For very small emulsions, say 10 cm x 3 or 4 cm, only two of the clamps may be used. With this holder one has the possibility of stacking emulsions of all sizes up to a maximum of 30 cm long by 15 cm wide.

The only specialized types of stack holders that I shall discuss here are those required for pulsed magnet work (see Fig. 5). The pulsed coils constructed in CERN by our workshop have a central tube giving a working diameter of about 6.5 cm. The problem was to hold the emulsions in the centre of this coil. We have therefore developed the two holders seen in Fig. 5. The right-hand holder, which is made completely of plastic, was constructed for straight-forward vertical exposures. The emulsions are held between two pieces of black plexiglas cut to the required shape, and the whole stack is held together by scotch tape. The stack is then inserted into the circular holder and is ready for exposure. The left-hand holder was designed for the Σ⁺ experiment and is, in fact, two stacks in the same holder. The two stacks are at ± 10° with respect to the beam. The side of the emulsions facing the beam was milled to present a radius of 3 cm to the target. This holder is held together by stainless steel screws and no movement of it was noticed in magnetic fields of up to 200 kgauss in magnitude.

Many other types of stack holders have been made for specialized purposes. For example, several groups (e.g. University College, London) have made some very elegant pieces of apparatus to hold water-soaked emulsions.
After the exposure comes the problem of how to keep the emulsions until it is convenient to do the development, etc. Dr. E. Dahl-Jensen remarked in his talk that it is not very good to keep emulsions at low temperatures after exposure. This is not true, however, if one seals them in polyethylene before putting them in the refrigerator, so that the relative humidity remains of the order of 50-60%.

Perhaps the most delicate part of the whole operation is the demounting, marking, thickness and/or density measurements, and the gridding of the emulsions. For this work we employ a team of four or five people, each with a specific job, which avoids confusion and mistakes.

Demounting

If the milling technique outlined above is used, the best approach is to take a stainless-steel knife, and run it up the corner of the stack. In this way the emulsions will then separate from each other quite easily.

Marking

Figure 6 shows the manner in which most stacks developed in CERN are marked. All the writing, which is done with a hard lead pencil, is on the bottom edge of the emulsion; this avoids resting the hand on the emulsion while writing.

Thickness and Density measurements

Due to the accurate milled edges, the size of the stack can be easily measured and the volume calculated, provided that a sensitive balance is used for weighing the emulsions. Thickness measurements can be made in various ways. Figure 7 shows the apparatus currently used at CERN. The micrometer clock gives a precision of 1 micron, enabling accurate shrinkage factor determinations to be made.
Gridding

Figure 7 also shows the apparatus used at CERN for gridding. The box-like structure is about 2 metres high at the back and 1 metre in front. An ordinary microscope bulb is placed centrally at the top; at the bottom two mirrors are fixed at 90° to one another, resulting in nearly parallel light illuminating the plexiglas top of the front of the assembly. The grid negative is fixed on to this plexiglas top. On the right-hand side of the box is a time switch giving a choice of exposure time of 1-30 seconds. On the front panel there is a micro-ammeter which is operated by a selenium cell giving a measure of light intensity.

Figure 8 shows the square used for positioning the emulsions during gridding, with three-pin location that can be changed according to the emulsion size. Also shown is an enlarged section of the Berkeley-type grid that is normally used at CERN. Many other types of grid exist and can be used with our apparatus.

After the development of the emulsions, to facilitate the quick positioning of plates on the microscope it is desirable to cut the edges of the glass with reference to the grid. This can be done, with the aid of a simple glass-cutting machine, provided that the print of the grid on the bottom of the emulsion can be seen clearly with a hand microscope.

Transport of undeveloped emulsions

After exposure, emulsions which will not be processed at CERN have to be transported as quickly as possible to their destination for development. As I have already mentioned, providing that the emulsion stack is sealed in polyethylene it can be kept at a low temperature, e.g. -5° to -2°, to avoid fading and the accumulation of extra background. Therefore we pack the stack in specially made strong wooden boxes, with a layer of at least 3 cm of styrofoam all around the inside, and construct appropriate compartments of styrofoam to hold the stack, plus sufficient dry ice to keep it cool during transit. This arrangement is illustrated in Fig. 9.
Suitable labels are then attached to the box indicating that it must be kept away from radioactive materials and in a cool place. A telegram is sent advising the recipient of the airway bill number, flight number and the date of arrival.

Whenever possible it is desirable to come to some agreement on procedure with your nearest Customs officials, so that the passage through the Customs of undeveloped emulsions is subject to the minimum of delay.

*   *   *

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DISCUSSION

Sichirrollo: I should like to emphasize that the thickness measurement is useful for measuring the shrinkage factor, for example for precise angular measurements, and you can see where the measurement has been made as the friction causes a dot on the emulsion surface.

Roberts: Yes, the point would make a dot on the emulsion surface, which is easy to find afterwards, but we have rather a large foot on the CERN micrometer clock, so I emphasized that the position must be measured very accurately and written down.

Doble: Would you like to comment on the fact that we are interested in the average thickness and the average shrinkage factor? What are the objections, therefore, to a parallel plate condenser consisting of stainless-steel plates connected to a bridge circuit and moving one plate relative to the other until the bridge is balanced? This should be highly reproducible and would give an average thickness over the emulsion placed between the plates.

Nikolić: I am interested in the manner in which Ilfords produce the large emulsions. Do they make them one by one, or do they cut up a large sheet?

Lock: I shall answer Doble's point first. I made such a piece of apparatus fifteen years ago for a different purpose. I am not sure of the accuracy one could achieve, perhaps to 1 micron. However, even if you made thickness measurements at two or three standard points, and determined the shrinkage factor at these standard points, the assumption made afterwards would be that the
Loch (cont.)

: shrinkage factor is constant over the whole emulsion. If you are interested in a particular event at a particular event at a particular point, I do not think that there is any difference between assuming an average thickness, or from a measurement at another point assuming an average shrinkage factor.

Doble:

: I agree with this. The idea was to eliminate the danger of pressing too hard and compressing the emulsion at that point, or not pressing hard enough, when the inclusion of air pockets would lead to too high a value for the thickness.

Loch:

: What we do is to remove the spring, and just use the weight of the dial micrometer pointer to measure the thickness. In this way we obtain enough pressure without including air gaps, or denting the emulsion.

Key:

: We combine the density method and this method. We weigh the whole stack and then each pellicle, and then on every tenth pellicle throughout the stack we make thickness measurements at ten or twelve points. This gives a good idea of the variation throughout the stack and in each pellicle. Combined with the density measurement we feel this probably gives quite a good value.

Roberts:

: It is probably true to say that the shrinkage factor is reasonably constant over any individual emulsion. In answer to Nikolić, I really am not qualified to comment. It probably depends on the size ordered. Perhaps they produce one metre square of emulsion, pour it onto glass, and cut it afterwards.

Nikolić:

: This is the usual assumption, but recent information suggests that they are produced one by one. I feel this is important for precision measurements.
Dahl-Jensen : We are interested in this for our next exposure. I asked Ilfords, and they said that for 4" by 4" pellicles, they produce a 9" by 9" sheet, then put away the surplus, and cut it into four.

Roberts : I will check on this next time I am at Ilfords. You mentioned pouring onto the glass, and this reminded me of our examination in London of the first stack. We did, in fact, find convex bumps as well as concave indentations due to foreign matter. These bumps were due to chips in the glass, and I believe that Ilfords immediately changed all their pieces of glass.

Allen : For the recent K⁻ exposure, we examined the pellicles and rejected 16 out of 360, which is about 4%. We were quite critical, and some pellicles could have been used if absolutely necessary. Ilfords always manufacture more emulsion than is actually ordered, but are not prepared for a rejection of more than 5%.

Roberts : Yes, after the first examination when we rejected about 20%, Ilfords have been prepared to accept a rejection of up to 5%.

Gottstein : What is the present standard of uniformity in emulsion thickness at Ilfords? A few years ago fluctuations of as much as 10% occurred in a given stack.

Roberts : This is still so, even in one pellicle, but it varies from stack to stack and probably depends on the levelling of the glass plate onto which they pour the emulsion.

Dahl-Jensen : Do you measure the thickness at Ilfords when you check the plates?
Roberts : No, we do not have time.

Dahl-Jensen : We have had pellicles which varied from 64.0μ to 570μ down one edge. How can one avoid this? If you had examined this, it would have been rejected.

Roberts : If we used this criteria, we would probably throw out up to 50%. The Milan stack of 130 pellicles dipped by about one millimetre at one edge.

Harmsen : How many milimetres are lost when the stack is milled?

Roberts : I normally mill off at least three milimetres from all sides. Another point: with a big stack it is advisable after each cut to take up the slack gently on the screws.

Evans : How do you mill the edge of the stack which is by the screw rods?

Roberts : I just mill the front edge until completely flat, and then take it into the darkroom, and turn the stack round and mill the other three edges.

* * *
Fig 1
Schematic diagram of clamped emulsion stack showing effects of edge thickening.

Fig 2
Schematic diagram of proposed device for cutting emulsion.
Fig. 3. Photograph of clamped emulsion stack with special milling cutter.

Fig. 4. Photograph of small emulsion stack in special holder.
Fig. 5. Photograph of two types of stack holders for exposure in pulsed magnets.

Fig. 6
Diagram illustrating the CERN Emulsion Group's method of marking plates and pellicles.

In the case of several stacks being exposed to the same beam.

Type of Plate No.
emulsion
Fig. 7. Photograph of CERN Emulsion Group's gridding and thickness measuring apparatus.

Fig. 8. Schematic diagram of locating device for gridding emulsions plus a section of Berkeley type grid.

3cm space for styrofoam cover
Plate compartment
3cm thick styrofoam
Dry ice compartments

Fig. 9. Schematic diagram of typical packing arrangement for transport of exposed emulsions.
THE PROCESSING OF NUCLEAR EMULSIONS

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Manuscript not received.
DISCUSSION ON THE PROCESSING OF NUCLEAR EMULSIONS

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K. Gottstein (Chairman)
Max-Planck Institut für Physik, Munich

H. Heckman
Lawrence Radiation Laboratory, Berkeley, California

M.W. Teucher
Physikalisches Staatsinstitut, Hamburg

G. Vanderhaeghe
Nuclear Physics Division, CERN

Gottstein: In his talk Prof. Teucher showed us several ways in which one can process nuclear emulsions and obtain good results. He admitted that his survey was rather subjective, and we have here sitting by my side four experts on processing who all treat their emulsions in a different manner and who all get their satisfactory results. Now we have seen this morning that this might not be so very surprising. There are some few hundred parameters in the whole processing process which one can vary and nobody really knows what happens if one varies these parameters. What usually happens is that in a particular laboratory people try some method which they think might work and if it works then they stick to it, and they are very careful not to change anything because one never knows what happens when one changes

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the procedure. On the other hand the technique is rather lengthy and tedious and sometimes one suspects that there are stages which one can modify in a manner that makes the whole thing shorter and easier. For instance Dr. Dahl-Jensen has processed emulsions completely cutting out the soaking time, introducing the emulsion immediately in the cold developer, and he obtained very good emulsions. He also did away with the stop-bath and it worked. Now of course, if one has a good stack exposed at the CERN-machine, one does not dare to deviate from the well-tested route because one might risk to lose the stack. Therefore very few people have really invested much time into investigating the procedure of processing itself. Therefore, I think, it is very useful if we now ask the experts who are here, why do they process as they do and why don't they do it in a different manner. Particularly I should be very grateful if they would tell us something about their failures because that is something from which one can learn most. Now I will hand over this discussion to the experts and let us start with the rolling-on process. May I ask Dr. Vanderhaeghe what he thinks about the rolling-on procedure which Prof. Teucher described this morning?

Vanderhaeghe: I would like to repeat first what Prof. Teucher said this morning that if there are a few points of disagreement between practices of different laboratories, at least on this side of the Atlantic all routines have the same common root so that their differences are generally not very important. Now, about rolling-on, I think that the over-all procedure described this morning by Prof. Teucher is very near to what we are used to doing at CERN and there is only one point of disagreement, which is quite important.
Vanderhaeghe (cont.)

: It is the way in which you prepare the gelatine. We think that it is much better to heat the gelatine up to about 40° to 50° (it does not matter so much), and then cool it down as quickly as possible. I understand very well the argument of Prof. Teucher that it might be better to cool it down slowly in order to have a very uniform solution but, on the other hand, it is known that keeping gelatine at 50° for too long a time breaks the gelatine chains which weakens its sticking properties. One remark about the time of immersion in the gelatine: if your pellicles are a bit too hard, you have to keep them for more than ten seconds in the gelatine in order to have them soft enough to put them easily on the glass.

Gottstein

: I noted that there is one more difference, at least between the CERN-procedure and what Prof. Teucher described, namely that you use a sponge and really clean the emulsion and the glass rather carefully, don't you?

Teucher

: Whereas Vanderhaeghe just immerses the emulsion, is that true?

Vanderhaeghe

: It is true that we wipe off very gently the gelatine left on the emulsion and we clean the glass.

Teucher

: We only clean the glass, we never touch the emulsion. Let me mention one other point which seems to me rather important during the rolling. Sometimes people want to really see what they are doing and they use the yellow safe-lights in a way which is extremely dangerous. I think it has been proved by now that part of the black deposit at the top of the emulsion is due to yellow safe-light. Do not forget that the Ilford
Teucher (cont.): Safe-lights should be used for indirect lighting and with not more than 15 watt behind one filter. So I would really like to recommend some caution when using the safe-lights; you cannot remove the deposits at the bottom of the emulsion.

Dahl-Jensen: First, a few words of warning. In the early stages of processing it was believed that if you want to get a very good glue, you should boil your gelatine solution for some time to make it feel really sticky. It is true that it is a better glue, but it is not gelatine and later on the pellicles tend to strip off the glass again. Therefore I can confirm what Vanderhaeghe said, that you should be very careful in melting your gelatine. He quoted the figure 40°C; we actually use only 38°C. Another thing is that the wetting agent you put into your sticking solution could be dangerous. You cannot use every wetting agent you find, but you should choose one of those meant for photography because some sulpho-compounds can act as a sensitizer or desensitizer for the emulsion and you can therefore get a very undesirable blackening of the surface. Another thing we have forgotten to mention is that the glass-plates prepared by Ilford for sticking onto are rather sensitive. It is recommended to keep them in the Ilford package in a cold place to prevent bacteria from eating the gelatine, because if you form islands on these glass-plates where there is no gelatine, your pellicles will not stick and you will automatically have some bubbles created. Our method of sticking down is somewhat different from what Teucher described. We use the method of painting, i.e., we have a thick rubber mat on which we place the glass which has been cleaned in a dry state using a brush; we clean the pellicle, too, with a dry brush, then we paint the gelatine onto the glass plate. We stick the emulsion first along one line.
Dahl-Jensen (cont.): and then by hand we press it down into contact with the glass. Using a plastic foil (of 50 - 100 μ thickness) placed on top of the emulsion, it is then gently rolled from the middle and outwards, as Teucher described. Using this method you will not get a wet air-surface of the pellicle when it is being rolled on the glass, and so you will reduce the danger of surface fading due to wetting, whilst you will still have fading at the bottom.

Gottstein: Do you do this painting of gelatine before you put the treated glass into water?

Dahl-Jensen: We paint the dry prepared glass-plate from Ilford; thus we do not have to wet it and put it under the water.

Heckman: At Berkeley we do it just about the opposite way, and perhaps it started a couple of years ago when we decided that any immersion of emulsion in water is generally bad for the eradication of your surfaces. So I found perhaps we could make a dry mount, and by dry I mean that the only liquid which would enter into the mounting would be pure ethyl-alcohol. So the idea was to wet the emulsion in alcohol. The length of time for which the emulsion stays in the alcohol is not critical, and we would then mount it on the dry glass. This has been partially successful. There was some sticking, but not good enough. So we then decided we would add water to our alcohol solution, and now we have settled down to an alcohol solution between 65 and 75% in which we immerse our emulsions. This is just to actually wet the surfaces, and if you pull out your emulsion and look at it and shake it, you can see it dry in about 30 seconds. The glass-plate, however, is what we immerse in a gelatine solution. I do not know details of our gelatine solution, but I think it is very similar to the one that Teucher mentioned. Our solution
in which we immerse the glass is formular, diluted by a factor of 2 by water; so it just has the strength of the solution. Our technique of mounting is the following. First we inspect the glass to make sure it is free from dirt; we also wash it with a solution of a detergent (Persolol, or something of this order), and we can inspect each pellicle. We do not wash very much the emulsion per se, but it is somewhat cleansed when we immerse it in the alcohol solution. So we have a vessel of alcohol, which is 65 or 75%, into which we can immerse our emulsion; hence we have an assembly line. One person throws an emulsion in and leaves the emulsion there for 3-5 minutes, and it does not make any difference. So if you are in the process of rolling-on and you break a glass or something drops, or something goes wrong and you have emulsion in this liquid, you do not have to panic. Then we have another vessel which is full of our 50% gelatine solution, and we immerse our glass in that. Then we have beside these a foam-rubber pad. Now we are ready to mount. We take a piece which we call parafilm— it is similar to your polyethylene, though it is a little bit elastic, but it is a waterproof membrane. We put that over the foam-rubber. We take the pellicle out and lay it on the foam-rubber pad, and then we put one of the leading edges of the emulsion (rolling the emulsion out) on the pad and we put a bead of gelatine. You may call this bead of gelatine a 100% strength gelatine solution. As soon as it hits the alcohol on the emulsion it coagulates. Now we take the plate; we are looking through the plate all the time, and you can see how it is going on. Then you put the plate at an angle on the leading edge and it forms a wave—you can see it contact. Now I demonstrate: your emulsion is here, and then you can see by putting your weight on
Heckman (cont.) : like this (it is rolled on) the wave going on, and then if anything is trapped you can see it, then you can back up and turn it; and then the last thing you do is to put your finger underneath and press the emulsion a little bit so that all the excess gelatine is just oozed right out the edge. Then we put a weight on for 30 seconds, take it off and roll it gently to remove the excess gelatine from underneath the pellicle and the surface is dry as soon as you turn it over and strip off the parafilm which you put on the foam-rubber in the first place. We start processing actually two hours after we have done this. But it is normally delayed. So, in a sense, I think that our techniques are similar. We use the same materials, but I think that we have just reversed what we soak and what we do not soak.

Vanderhaeghe : I am a little surprised about this 50% gelatine solution. Is it 50% by weight?

Heckman : You might say it is a solution containing $7\frac{1}{2}$ g per 100 cc of water, instead of the 15.

Dahl-Jensen : We were asked to tell about failures in the processing. We had, around Christmas time, three weeks of processing with 12 batches of emulsion. The last three batches consisted of 80 pellicles belonging to the E19 experiment together with five or six pellicles belonging to a spurious scattering exposure, and 40 NIKFI emulsions belonging to Bombay. The first batch to be rolled-on were the 80 pellicles, then the five or six pellicles belonging to another emulsion batch from Ilford which were mounted on other glasses than the 80 pellicles, and finally the 40 NIKFI emulsions. In the fix-bath later on, we found the 80 pellicles did not show any bubbles, but that the five
Dahl-Jensen: or six spurious scattering plates showed around 100 bubbles per plate. Of course, we looked very carefully at the glass-plates we had left from the sticking down and tried to see if we could not find any failures in them. But the gelatine coating from Ilford is very thin and it was very difficult to detect if there were any islands in it. The only difference we can suspect was that these five or six glass-plates were not perfect, because all the premises in this processing were the same for three different kinds of emulsions, and only these five or six plates showed a very large number of bubbles.

Teucher: We have not said anything so far about the rolling-on of NIKFI emulsions, and so I will take the liberty of quoting one paper by the Berlin Group. They have done quite some work on it, and one thing which is very much different from the preparation of the Ilford emulsions for the mounting is that with NIKFI emulsions it is not absolutely clear just from looking at them which side should be mounted to the glass. In Ilford it is very clear; you have one shiny side which should be mounted, and this you have to use. For the NIKFI emulsions, the side you want to glue to the glass has first to be cleaned. You have to rub away something between 3 or 4 μ, using an alcohol-water solution of about 25 or 30%. If you do not do this, you will have a black deposit at the bottom surface which cannot be removed later, and you cannot look through this under the microscope. The other procedure is not completely different, although the solution used is somewhat different: but if you want to look into it, I think all this has been published in "Nuclear Instruments" in great detail.
Gottstein : I should be interested to know if anyone can tell us anything more about the correlation between numbers of bubbles appearing and the procedure applied. Sometimes it is, as you know, a nuisance that one has a lot of bubbles. Dahl-Jensen has just told us of an example, but maybe some of the other experts know a way of avoiding bubbles. Can anything be said about this?

Vanderhaeghe : Certainly cleanliness is very important. I can say that for the last two years we have avoided trouble with bubbling just by taking care, putting on gloves, and not touching either the back of the emulsion or the glass. But unfortunately this is not the only cause of blistering. Recently we had exactly the same kind of trouble as Dahl-Jensen; we had no time to investigate it carefully, but most probably it came from faults in the coating on the glass.

Gottstein : Teucher told us this morning not to use gloves, but maybe it depends a little bit on the hands one has.

Vanderhaeghe : No, I agree perfectly that inside the gelatine solution you do not need gloves; but for all other operations such as marking, gridding, and so on, i.e. every dry operation, it is much better to wear gloves.

Bull : Is there a minimum time that should elapse between rolling-on and the beginning of the processing? Such a suggestion was made at Bristol some time ago.

Teucher : We always did the mounting in the afternoon, and usually we waited more than 24 hours. But once we had to do it the next day, and also sometimes we had to wait, let us say five or six days, until we could really start processing. In my experience I found no correlation whatsoever between bubbles and this time.
Vanderhaeghe: I agree with Teucher. Nevertheless, a long time ago when this was suggested by Prowse in Bristol that it was better to wait for a certain time after mounting, we did some tests in Brussels and we agreed with this conclusion. But nowadays, with our present procedure, it is no longer necessary, and usually we do the mounting in the afternoons and process next morning without any more trouble. Sometimes we started processing immediately after the rolling-on, and it was still all right. This is, of course, better, because you reduce the fading.

Heckman: Our standard technique is also waiting until the next morning after rolling-on in one afternoon, so typically eight to ten hours. When we roll-on, in principle our emulsion is dry; at least the surface is dry and the interface between the glass and the emulsion is wet with gelatine. After we roll-on and the surface dries, we notice that there are islands on the emulsion which swell relatively to anything else, and they start swelling just as if the rolling-on process did not squeeze all the gelatine up. There were pockets of moisture still there, which may lead one to suspect that adhesion at that point is not as good as at some place else, and we felt we should wait until the moisture has been able to withdraw from that into the emulsion itself and somewhat dries. So there are areas which appeared to us to be probably not as well stuck as they could be, owing to islands of moisture which are not squeezed up during the rolling process. So I would hesitate to say that we ever tried to correlate areas such as these with any bubbles that we observed. In general, it happens on every plate without exception that there are four or five areas which have bubbles with swelling, preferential swelling, and our bubble count has varied from a very few (less than 1%) to the complete catastrophe.
Dahl-Jensen : I can mention that our best processings have been those where we have started the processing immediately after sticking down. By best, I mean no, or extremely few, bubbles and a high sensitivity. One may expect that the sensitivity decreases with time after the rolling-on. If you then process immediately after the sticking down, you should expect a higher processed sensitivity, and there are hints that this is really so.

Teucher : I would like to comment on Heckman's experience. We had the same experience of this local swelling of the emulsion when we had to process the big stack in Chicago. This, I think, usually occurs about half-an-hour after the mounting. Suddenly you can see really such a swelling of sometimes $\frac{1}{2}$ cm$^2$, sometimes even more than that; but one should not be too anxious because in our case it always disappeared about the next morning. It gradually disappeared and we could not find at this point that bubbles occurred later. So if you ever will notice it, just forget it.

Finney : After sticking down the emulsion and before the processing starts, are the plates kept under pressure?

Heckman : We mount the glass on to the emulsion; we weight it very lightly just to keep the edges from folding away essentially for maybe one minute, then we roll and then we stack. We stack each pellicle as we do it until we get maybe three or four inches of glass and pellicles altogether. We might put a little weight on it, but there is no direct intent to weight them for any length of time. But I think, in fact, they are under weight for about 20 minutes during the process, but as soon as we get to stack them each glass and pellicle would peel off the parafilm and dry it.
Gottstein : Is this stacking dangerous, or do you have some poly-
ethylene in between it?

Heckman : From the initial moment of the rolling-on we put the
parafilm on the pellicle and we leave it on, so that
no gelatine sees the top surface of the emulsion and
makes it a very clean product. We leave that para-
film on for about a total of half-an-hour before we
peel it off to dry it.

Bovet : After having heard such complicated processes for
sticking the emulsions on glass, I would like to men-
tion that it is possible to stick the emulsion quite
easily by immersion in cold water, say at 5°C.
Dr. Manfredini in Rome has done so several years ago.

Gottstein : Perhaps each member of the panel would describe briefly
what kind of processing machine they use.

Heckman : We are the wet-hot stage protagonists, and in Berkeley
we are very tight for space. In fact, the emulsion
group there has been in operation for about 12 years,
and we started up the emulsion work in a dark room which
was perhaps 9 by 12 feet in area. When we first
started up we were using one by three inch emulsions,
and this was plenty of room. Then all of us came to
cubic feet of emulsion, and we had to process them still
in this 9 by 12 foot room. So we had the problem of how
we were going to handle such large volumes of emulsion
using a dry-hot stage. So the idea was to try a wet-hot
stage. The first results were quite encouraging, and
so we continued to do it this way. We would merely cool,
presoak, standard presoak, standard developer at 5° for
2 1/2 hours, and then we would withdraw about 50% of the
: developer and mix it with hot water so that the resulting solution would be a 50% strength developer at 22°C. Then we dump the rest of our cold developer from the emulsions and rapidly run in the warm developer; we leave it for an hour or so at 22°C and then use standard stop and fix. In this way we think we alleviated two problems. First, we could do a very good size stack in one small 50 gallon tank. In fact, we have now a tank which is designed to handle up to about 400 sheets, each a foot square. These pellicles are racked up so that the glasses are separated only about 1/8 inch from each other, so we have a rack which is a solid layer of emulsions, and by having complete immersion all the time we do not have the problem of contact with the air which would tend to oxidize and eradic the surface, which was in the early days one of our main concerns. Another advantage of this particular technique is that a great deal of the work has been considerably lessened. In the processing of a large stack the mounting, gridding, density measurements, etc., are a major task and require everybody in the group to take part. This is a one-day effort. After the plates have been rolled-on we put them into the rack, put them into this big tank, and start processing. This is a one-man job only. Our technician runs through the process all by himself, and he has nothing else to do but to make sure that the solutions are mixed, flowing in, flowing out. From the time the pellicles are racked-up initially and dried and placed in the tank they are not touched or moved, except for eliminating air bubbles between the glass, until they are withdrawn from the tank and dried. We might check bubble formation, but on a 200 or 300 pellicle stack it is practically impossible to check every pellicle. So we do nothing about bubbles, we just let them exist and
Heckman (cont.): let them dry out. We do not puncture them, and we find that the emulsion in the middle of the bubble is perfectly usable except for the crater that goes around each one. We have never lost a pellicle because a bubble grew to any considerable size; we had them perhaps up to 1 to 1.5 inch in diameter which is not uncommon.

Gottstein: Does Copenhagen also use the wet-hot stage?

Dahl-Jensen: Yes. Our processing plant has been described in the Suppl. Nuovo Cimento 15, 211 (1960) and therefore I shall not go into much detail. But the difference between our equipment and yours is not very large. It consists of separating the processing in two steps. After the development and stop bath we transfer the racks from the stainless steel tanks to a fixing and washing tank where they stay until we start the alcohol drying for which we have a third set of tanks. Our tanks have an area of 50 by 50 cm² and are 15 cm deep, in which one can place a rack having 11 slots where 11 plates can be put in. For safety we normally use only ten plates and put a glass plate at the top. We have heating from outside by coils which have been soldered on. We have two of these tanks for the actual development. These are operated in such a way that we start in one of them and, after one hour, we start the other one, and then the procedure is such that our pre-cooler and our thermostats always will be ready for the next operation. When we have completed two batches then we transfer the racks to the fixing equipment. If the silver deposits on the surface of the pellicles are too heavy, we remove them in most cases by gentle rubbing with medical cotton-wool between the stop and fixing stages, but not always. Then these two racks are transferred to the fixing equipment.
Dahl-Jensen (cont.): and so on. I think these are the only differences between Berkeley's method and ours.

Vanderhaeghe: I would like to ask a question about the wet-hot stage. Does the Amidol concentration in the solution used during the hot stage depend on the thickness of the emulsion or not? Some people use just the developer without Amidol, some others put half of the Amidol concentration, but I suppose this concentration has something to do with the thickness.

Dahl-Jensen: Is there any dependence between the thickness of the pellicle and the Amidol concentration in the hot developer? I do not know if there is any variation in the effect of a given Amidol concentration in the hot developer on pellicles of different thickness, because we only have experience with 600 μ and 400 μ emulsions. I forgot to say that our second developer (the hot developer) follows a Brussels recipe in which the Amidol has been halved and potassium-bromide doubled, to prevent excessive development on the surface of the pellicle. The reason why we have kept the total concentration of salts more or less equal is that if you have a different concentration then you will have an ion concentration shock giving a sudden change in the swelling. Therefore we are very careful to keep this ion concentration constant.

Vanderhaeghe: I would like to raise a few points which are slightly different from what Teucher described this morning. First, two remarks about the preparation of the developer. Usually we start by dissolving the sodium-sulphite. It seems that this helps a little to dissolve the boric acid. We put the Amidol in this solution after having cooled it down. Because Amidol oxyzides so easily, it is much
Vanderhaeghe (cont.)

better to cool down the solution to about the right
temperature, i.e. 5° or 6°C, and then add Amidol which
still dissolves very easily.

Another point is the temperature of the presoaking.
Usually we start with the pellicles at room temperature;
we put them on the slab and we drop the temperature down
to about 5°C before adding the cold soaking water. Of
course, we know that this means that the soaking time will
be longer, but we prefer that just to avoid the exces-
sive and too sudden swelling. However, there is some
evidence -- I do not want to go into details now -- that
anything you do before the hot stage has not much
influence on distortion and spurious scattering. It
had been suggested some time ago to do the soaking even
slower, starting from an alcohol solution and then in-
roducing water very slowly. It has been tried, I
think, in Copenhagen and no improvement has been found.

Now a few words about the apparatus. We have two large
slabs of about 2 sqm, and they are in stainless steel.
The main reasons for the choice of this material are
mechanical strength and low cost. I do not think that
it is really so important to achieve a very good heat con-
duction, as we all agree that everything has to go very
slowly, so stainless steel is good enough. It is probably
more important to have a very flat surface.

I should like to comment about the stop bath. Here,
then, is a serious difference in our routine with respect
to others. This is really something important because
it has something to do with corrosion. I will first
give the recipe: we add some sodium-sulphite in the
stop bath and correlatively we increase the amount of
Vanderhaeghe: acetic acid to reach the reasonable pH value of about 4.8. We are convinced that the presence of the sulphite in the stop bath is really very important because the stop bath has, in principle, a double role: firstly, to stop the development, and that is achieved by dropping the pH and the temperature; secondly, to destroy and wash out the developer. But usually a part of the developer remains in the emulsion. If you do not take care, the remaining Amidol will be oxidized and this is definitely a cause of corrosion, not the normal corrosion of the silver by hypo, but some more complicated process which has been called "catastrophic corrosion". I recall here an accident we had in Brussels, using the usual stop bath without sulphite. We were trying to improve the stop bath, having in mind to wash out the Amidol. We changed the stop baths three times, and we had a fantastic corrosion! Fortunately we kept the solutions. The three stop baths were of quite different colour: the first one was still colourless; the second one was slightly pink; and the third one was completely red. I remember we discovered this with Bonetti, and we understood then that, trying to wash out the developer from the emulsion we removed most of the sulphite, whereas a fraction of the Amidol remained absorbed by the emulsion. Of course, this Amidol was easily oxidized and we had corrosion. So I insist very much on this point.

Gottstein: Thank you. Shall we discuss this last point first, perhaps? Not all people use stop baths with sodium-sulphite, and not all people get this terrible corrosion.

Heckman: What is your recipe?
Vanderhaeghe: It is about 12 g/litre of sodium-sulphite, and about 1% of acetic acid, just to get a pH value of about 4.8.

Toucher: I just want to add something about the acetic acid concentration. I was asked this morning why I put only one in one thousand, that means 0.1%. This 0.1% I took from Occhialini's last formula for the handling of K5 emulsions, but I must admit that in our old routine, ten years ago, we started by 2% and gradually it went down to about 0.5 to 0.1%. I do not think that this concentration is extremely important because most of the stopping effect comes from the cooling-down, I would say.

Dahl-Jensen: When you are buying sodium-sulphite it is not supplied 100% pure by most manufacturers. You may have only 85% of real sodium-sulphite, the remaining 15% being oxidized to sodium-sulphate. Therefore, if you want to have a very constant sulphite concentration, you should make a chemical analysis of your sulphite and it usually comes out around 85%.

Gottstein: How serious is it to leave the stop bath out altogether? I mean, the pH change is also brought about by the fixer and the stopping is done by lowering the temperature, so I am wondering what will happen if you leave it out completely.

Dahl-Jensen: We have tried to avoid the stop bath and the argument is very simple. You have a developer of pH about 6.4, and you then introduce a stop bath in which one-half per cent acetic acid will give you about 3.8 pH. Most laboratories are using fix baths having a pH of about 4.7. As Gottstein told us, the development is stopped
Dahl-Jensen (cont.): by lowering the temperature: so why introduce this acid stop bath when we are going to an acid fix bath immediately afterwards? We have tried to omit the stop bath several times for small batches of plates, especially spurious scattering plates, and we have found that it has no effect either on the quality of the processing or on the spurious scattering.

Vanderhaeghe: We also tried the same thing, and the result has been the same as regards the spurious scattering. But the plates were slightly less transparent, and this again means that the stop bath has a washing effect.

Gottstein: But did you get this really with two plates from the same batch, everything being done in the same way except for the difference of the stop bath?

Vanderhaeghe: Yes.

Gottstein: Are there any more comments on the whole developing procedure up to the stop bath, inclusive?

Finney: I would just like to make one comment on the pH value of the developer. In Durham we found when trying to develop emulsion at 15°C, using a developer with boric acid, that the quality of the development depended critically on the pH value of the developer. We also found, by accident, that the distillation unit was producing distilled water with a low pH, and the subsequent developer using this water did not, in fact, produce development. Using a buffer solution to control pH we found that, using ordinary tap water, we could get very good developments. This would indicate that one would need to be careful when using ion exchange water of low pH if one was not using
Finney (cont.) : a developer which did not contain boric acid, otherwise the pH of the developer may vary sufficiently to stop development.

Dahl-Jensen : The question of ion exchange water has been raised, and in my opinion it has no influence on the developer because the ion-capacity in distilled water is very, very inferior to the ion-capacity of the chemicals you are putting in.

Another point I should mention is that if you rely on ion exchange water for the presoak, then it can easily have a very low pH which will completely remove the latent image, whereas if you add chemicals which are supposed to buffer the developer, the initial water you use can be extremely acid without affecting the pH of the subsequent solution.

Gottstein : Now we might go to the next step, which is the fixing.

Vanderhaeghe : About the fixing, I have first one question to Teucher. He did not give the pH of the fixing bath this morning. What is your routine with pH? I suppose it is about 6 or 6.5?

Teucher : Yes.

Vanderhaeghe : We have the tendency to use a somewhat lower pH in order to get more transparent plates and to avoid refixing. But we know that this is also dangerous because the lower the pH, the more easily the corrosion occurs. So we usually make a compromise and have a pH of 5 to 5.5. It is not only a question of transparency and clarity of the emulsion, but also of the reduction of the swelling.
Vanderhaeghe (cont.): The isoelectric point being about 4.8, it is better to keep up as near as possible to this value in order to reduce the swelling. As we fix at a temperature of about 10°C (we cannot go lower for technical reasons), we have to be very careful about swelling.

Now I am coming to a point of disagreement. Teucher recommended this morning to fix at a temperature as low as possible. I think that it is a bit dangerous. As Dahl-Jensen said, this might be a cause of choping. If you go too low, around 0°C, some crystals could be formed inside the emulsion. So I do not recommend a very low temperature. I think the best is probably around 6°C.

Teucher: Of course it was not my intention to recommend fixing at -1°C or something like that. I did not mean anything lower than 5°C, and I have never tried to do it below 5°C. So the meaning of my "as low as possible" was, if you can do it, do it at 5°C. I have developed stacks at 8°C and 9°C, and this works sufficiently well, too.

Vanderhaeghe: I forgot something about the silver-bromide in the fixing bath. I think it is a good recommendation to start with some silver-bromide although, according to our present experience, it is not absolutely essential, especially if you start without agitation for a few hours; you will then have enough silver in the fixing bath to avoid the surface corrosion. Only one detail about the preparation of the silver-bromide: you have to wash it to remove the KNO₃.

One word about sodium-sulphate in the hypo to reduce the swelling. It can be used in case of blistering or some catastrophe of this kind, but it does not help very much.
Vanderhaeghe: Recently, we have had a very bad experience with blistering and I added some alcohol to the fixing bath, just to shrink the emulsion a little bit, and this worked very well. You can put 10-20% of alcohol; it helps very quickly and very well, without any trouble.

Teucher: There are several possibilities for building a fixing apparatus. You always need some circulation of your hypo which is rather important, otherwise the old hypo will just collect on the surface of your emulsion and this is pretty bad.

Heckman: Much has been said about agitation throughout the whole process. Vanderhaeghe said that leaving hypo in the stationary state for the first few minutes would be helpful to raise the silver content, then to employ agitation. We have always started agitation almost immediately without necessarily adding any silver bromide.

Teucher: I do think that if you apply a dry-hot stage you need agitation only in the fixing bath, and you do not need any agitation in all the other steps. This might probably be different for a wet-hot stage, because in order to get uniformity of the temperature during the hot stage, you probably need some agitation. As we heard from Dahl-Jensen, he has his pumps in his tank. For the wet development we are now planning to introduce agitation, too, in a somewhat different manner. But I think for the fixing it is absolutely essential whatever you try to do. That means you need a rather gentle stream of hypo, and there are several way how to do it.*

*) Editorial Note: The description of specific processing installations and equipment (e.g. fixing tanks) has been omitted from this report of the discussion.

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Gottstein: I should like to ask Professor Teucher a question. This morning you said that one should avoid changing the fixing bath during fixing. So one should start with enough fixing solution. So what do you consider enough fixing solution for, say, 1 litre of emulsion? Obviously you do not want to have your hypo too much saturated with silver at the end.

Teucher: During all our last experiences we never went above 12 g of silver per litre of solution, and the amount of hypo was adjusted to this figure. This included the 2 g initially put into the hypo. But, of course, this is a matter of taste; other people consider even 10 g dangerous because of bubble formation, and in the old days even 17 or 18 g were allowed.

Vanderhaeghe: We tried to keep below 10 g per litre of silver at the end. That means that sometimes, when we have many pellicles in the same tank, we have to replace a part of the hypo before the end of the fixing.

Dahl-Jensen: We have built an apparatus in Copenhagen [J.Sci.Instr. 37, 360 (1960)] with which we can keep the silver concentration in the fixing baths constant, so that when the silver concentration goes too high a magnetic valve opens and fresh hypo is added to the tanks, and when the lower limit is reached the valve will close again. The reason why we built it was the following: it was believed that corrosion was prevented by keeping the silver concentration up. We believe that if we keep the silver concentration at about 3 g per litre throughout the fixation, we should be safe. At least we have had no catastrophic experience.
Dahl-Jensen (cont.): We have been using sodium-sulphate in the washing stage, and I do not understand why you want to put it in the fixing stage where you already have a very high ion concentration. If you add sulphate to your washing water, you can prevent the emulsion from swelling at this stage, but at some stage you must wash out the sodium-sulphate and then the thickness goes up again to almost the same extent; so in the last year we have left out the sodium-sulphate from the wash-water.

Another point is that Heckman mentioned that they go up to 95% alcohol in the drying. Teucher said that it should not be necessary to go so high. What we have found is that about 75% alcohol will have no further drying effect on the emulsion. So if you go higher than that you are only trying to make the emulsion thinner than required, and you will have a hardening of the surface as Teucher mentioned. Therefore it must be recommended that the final drying bath should not exceed about 80% alcohol.

Gottstein: Thank you. Any more comments?

Vanderhaeghe: I have one suggestion to avoid large swelling during the washing. It has been recommended to add sodium-sulphate at this stage to reduce the bubbling effect, because the sulphate has a slight tanning effect, but it does not help very much. I think alcohol is better. Another way to achieve this very easily is just to add a few per cent of acetic acid. If you keep the pH low, you do not get this raise. I am not sure that we have to wash necessarily with pure water at the end.

Dahl-Jensen: Another possibility must be to add a compound you want to have in your emulsions after they have been processed,
Dahl-Jensen: namely glycerine. It should have the same effect and you do not have to remove it afterwards.

Gottstein: I should like to ask a question. Is really the smaller space which you need for a wet-hot stage the main consideration, or perhaps the only consideration, or do you also believe that you really get better results? May I ask the CERN people. Do you plan in the near future to replace your machine by a wet-hot stage?

Vanderhaeghe: I do not think that we are going to spend a lot of money to transform completely our processing plant in the near future. We can use our plant as it is for the wet-hot stage, and it goes very well. We do not have the facility for agitation, but that does not seem to be really essential. The first attempt, on a large scale, has been made last month by Levi-Setti rather successfully. So, for the moment we have no special reason to change the plant completely. I must say that, for the moment, I am not at all expert in the wet-hot stage, but I do not see any strong argument against it.

Teucher: I would like to say that even the big Chicago dry-hot stage plant was used a few months ago for a wet-hot stage. So most of the dry-hot stage plants can be used for wet-hot stages if someone likes to do it. For more than 10 years I was a strong believer in dry-hot stages, but now for the sole reason of not having enough space, I have to build a wet-hot stage. The problem of spurious scattering and dry- and wet-hot stage is extremely complex, as we know from the Copenhagen and Lausanne conferences.
Gottstein: So, am I correct in summarizing that so far the result of the wet-hot stage is not better and not worse than the result of the dry-hot stage as far as the emulsions go? Does everybody agree with that?

Dahl-Jensen: Our wet-hot stage of course has some disadvantages as well. One is that in most of the processings we also get a silver deposit on the emulsion surfaces that has to be wiped off afterwards. Another thing is that when you study a pellicle after processing, you will find a large number of very tiny grains which have highest concentration in the top and lower in the bottom. It does not affect measurements and visibility in the plates, because they are so extremely small that you have to look for quite a while to see them, but it is not perfect. However, I should like to ask the dry-hot plate people about the gradient in the emulsions, because Cochialini's argument for introducing the dry-hot stage was to avoid a gradient. I have been measuring in a few dry-hot stage pellicles, and I have found a gradient in all of them. Of course, in our own processing we also have a gradient, but it is generally much less than 5%, which is the maximum drop we have found at the surface and bottom of a pellicle. Dry-hot stage pellicles that I have seen show gradients appreciably greater than this.

Vanderhaeghe: At least for what we call a good processing——and I think we have done a few of them in recent years——we essentially have had only some 50% fading over the first 20 µ both sides of the emulsion. It is fading, it is not a gradient.

Teucher: What Dahl-Jensen calls the gradient is, in fact, the fading at the top and the bottom. I must say I really do not
Teucher (cont.)

: think that it is as bad as this curve. It could be the case if you are extremely careless, but if you are careful it does not happen like that. So from this point of view I do not think that I would make a strong argument for building a wet-hot stage.

Heckman

: I have a question, not necessarily on the uniformity of development, but I should say that our experience has been quite good as far as the variation of sensitivity or development of the emulsion is concerned. Our experience in dry-hot stage is very small, and so we have really no comparison in our own laboratory on the relative merits of the uniformity of development. On the question of cleaning your emulsions by rubbing, Teucher mentioned that you could wipe off the silver deposits after the stop bath, when it is quite easy to rub off. The other alternative is to wait until the processing is completed and you have gone to your alcohol and drying, and then we will remove the silver. I do not know if there is any real evidence which way is the better. I can only cite one example. Some years ago when we did the collaborative experiment on antiprotons, one of the techniques we used to measure multiple scattering of very steep particles was a method which we call 'surface angle' which was essentially measuring the angle that a particle makes when it enters into the surface of each emulsion. As it dives deeply into the emulsion, each cell length was, in fact, one emulsion plate. There were several stack exposed, but processed in different fashions and particularly rubbed off in different fashions. We were getting quite good results with our noise. We had about a tenth of a degree noise on measuring the entrance angle to each plate. The plates which Birge and Perkins worked on had, I think, perhaps twice or three times that noise, and one of
Heckman (cont.): the suggestions was that this was perhaps a result of
different treatment of rubbing. We waited to the very
end of the process, finishing off at 95% alcohol. The
surface hardened considerably, so that rubbing was probably
less damaging to the very surface than when you rub it
even in the stop bath, which is the method they used. They
cleaned it off in the stop bath; we cleaned it off as the
last thing, and there was a considerable difference in the
distortion of the angle at the entrance of the top surface.
This has led us to believe that it is perhaps better to
rub the silver deposit off at the very end of the process,
when the emulsion is really hardened and no serious dis-
tortion can be introduced.

Fletcher: Is there any real reason for alcohol drying as opposed to
air drying, or is it just a matter of convenience?

Vanderhaeghe: Certainly alcohol drying is much simpler in many respects.
First, you do not have such a serious edge effect which
occurs at the beginning of the air drying. Secondly, it
goes much faster, and thirdly, as mentioned by Teucher,
you avoid automatically the trouble with bacteria which
occurs sometimes with air-drying. According to me, these
are all very good reasons. I do not think that anybody
has really investigated completely the problem of dis-
tortion, but it is certainly not much better with air-drying
than with alcohol.

Pelosi: What is the correlation between the various kinds of pro-
cessing you mentioned and general distortion?

Teucher: I do not think that this can be answered in a few sen-
tences because there are, of course, many ways in which
one can introduce these distortions. Certainly you can
Teucher (cont.): introduce distortion in the hot stage if you go to too high a temperature. You can introduce distortion if you do not keep your plates in a horizontal position but a vertical direction during fixing. You can introduce distortions by changing the pH too rapidly or if you are careless during the drying stage, especially in the air-drying, if you kept islands of water on your plate, and so on. There are so many reasons that it is really very difficult to give a short answer.

Winzeler: If one adds alcohol to the hypo, would it not be worth while to add it right at the beginning of the fixing, because in the case that bacteria are one reason for bubble formation, this would lower the possibility of bubble formation?

Vanderhaeghe: It has nothing to do with bacteria. I never trusted this hypothesis of bacteria making bubbles in the hypo, just because one of the best killers of bacteria is silver. Ionic silver is a strong antiseptic. I do not recommend putting alcohol in the fixing solution every time because it is quite expensive.

*   *   *
**MICROSCOPES**

G. Vanderhaeghe  
Nuclear Physics Division, CERN

I. INTRODUCTION

The purpose of this lecture is only to give a brief survey of the generally available and commonly used equipment for nuclear emulsion microscopy, adding some comments and recommendations dictated by the practice.

No constructional details of microscopes will be given and very little will be said about "home made" prototypes of microscopes or "gadgets" devised to improve the performances of standard equipment, although some of them are of very great help. Those interested should refer directly to original descriptions.

As general references, I should like to recommend two publications.


J.C. Hodges, Microscope Equipment for Nuclear Emulsion Analysis; UCRL 9089 (1960).

II. OPTICS

Requirements are more stringent than for biological microscopy; some of them are quite in opposite directions. They are: high contrast, high resolution, small depth of focus, flatness of field, large field and long working distance.

---

*This is a shortened version of the talk given by Dr. Vanderhaeghe.*

3348/NP/smg
It is difficult to fulfil all of them together because some are contradictory and there are a limited number of parameters to play with. However, they are not always needed at the same time.

Figure 2 gives a schematic layout of the compound microscope.

\[ \beta = \frac{l}{D} \text{ and } \beta' = \frac{l'}{f_e' f_0' f_e} = \frac{l_T}{f_e' f_0' f_e}, \]

where \( l \) = actual length of a given object
\( D \) = least distance of distinct vision
\( l' \) = size of the intermediate image given by the objective
\( f_e' \) = eyepiece focal length
\( f_0' \) = objective focal length
\( T \) = optical tube length.

The over-all magnification is given by:

\[ G = \frac{\beta'}{\beta} = \frac{-}\frac{l_T}{\frac{1}{f_e' f_0' f_e}} = \frac{T}{f_e} \times \frac{D}{f_e} = M \times N, \]

with \( M \) = objective or primary magnification,
\( N \) = eyepiece or secondary magnification.

Objectives

The most important parameter is the numerical aperture

\[ NA = n \sin \epsilon \quad \text{(see Fig. 1)} \]

where \( n \) = refractive index of the medium in contact with the front lens,
\( \epsilon \) = semi-angle of aperture of the cone of rays received by the objective.

If \( \epsilon = 30^\circ \), \( NA = 1.0 \times 0.5 = 0.5 \), for a dry objective (Fig. 1a) and \( NA = 1.5 \times 0.5 = 0.75 \), for an oil-immersion objective (Fig. 1b). In practice, for a dry objective \( NA < 1.0 \) whereas for an oil-immersion objective \( NA \leq 1.4 \).
The resolving power is defined as

\[ \delta = \frac{0.5\lambda}{NA} \]

for \( \lambda = 0.5 \, \mu m \),

\[ \delta = \frac{0.25 \, \mu m}{NA} . \]

Thus, for good resolution, one needs large NA.

For two points at a distance \( \delta \) from each other and with \( D = 25 \, cm \), one has

\[ \beta = \frac{0.25 \times 10^{-4} \, cm}{NA \times 25 \, cm} = \frac{10^{-6}}{NA} . \]

The minimum angle of resolution of the eye being about \( \beta' = 0.3 \, mrad \), the minimum magnification required to see separately these two points under the microscope is therefore

\[ G = 300 \, NA , \]

giving

\[ \beta' = G\beta = 3.10^{-4} . \]

In practice, \( 500 \, NA \leq G \leq 1000 \, NA \), with

\[ 5 \leq N \leq 25 \quad \text{and} \quad 20 \, NA \leq N \leq 100 \, NA . \]

\[ T = 160 \, mm \times 1.5 \, (\text{correcting lens}) = 240 \, mm , \text{ so that} \]

\[ \frac{2.4 \, mm}{NA} \leq f_o \leq \frac{12 \, mm}{NA} . \]

Thus, large NA means short focal length which means greater technical difficulty.

On the other hand, working distance and depth of focus are both inversely proportional to NA. (Note that standard objectives for biological microscopy are designed for great depth of focus.)

Three types of objectives are commonly used for nuclear emulsion work:
i) Achromat \((N < 50)\);

ii) Fluorite \((N \geq 50)\), contains one fluorite lens;

iii) Apochromat \((N \geq 50)\), contains more than one fluorite lens.

Aplanatic objectives are more complicated. They provide a flatter field but have a very short working distance.

The best solution (adopted so far only by Zeiss) seems to be Monochromat because, having not to correct for chromatic aberration, one can adjust parameters to other requirements (e.g. depth of focus and flatness of field).

Table 1 gives a list of recommended objectives (not exhaustive).

**Eyepieces**

There are two well-known basic types: Huyghens or negative (Fig. 3a) and Ramsden or positive (Fig. 3b).

Corrected eyepieces contain three or more lenses. The most commonly used are:

- Hyperplan (negative) \(5 \leq N < 25\);
- Periplan (negative) \(5 \leq N < 25\);
- Kellner (positive) \(N \geq 15\);
- Orthoscopic (positive) \(N \geq 15\).

Compensating eyepieces (to be used with apochromatic objectives for chromatic correction).

Large field eyepieces (positive for good focusing of reticule, negative for flatter field).
### Examples of recommended combinations

<table>
<thead>
<tr>
<th>Purpose</th>
<th>G</th>
<th>Objective (M)</th>
<th>Eyepiece (M)</th>
</tr>
</thead>
<tbody>
<tr>
<td>alignment, rough location</td>
<td>100 x</td>
<td>Leitz 10 x</td>
<td>Periplan 10 x</td>
</tr>
<tr>
<td>rapid scanning, grid reading</td>
<td>200 - 300 x</td>
<td>Koristka 26 x</td>
<td>10 x G.F.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Cooke 20 x</td>
<td>10 x G.F.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Leitz 22 x</td>
<td>10 x Periplan or G.F.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Koristka 30 x</td>
<td>10 x Periplan or G.F.</td>
</tr>
<tr>
<td>general scanning</td>
<td>500 - 800 x</td>
<td>Leitz 53 x</td>
<td>Kellner 15 x or G.F.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Koristka 55 x</td>
<td>Kellner 15 x or G.F.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Cooke 45 x</td>
<td>Kellner 15 x or G.F.</td>
</tr>
<tr>
<td>critical scanning, following minimum tracks,</td>
<td>900 - 1200 x</td>
<td>Leitz 53 x</td>
<td>20 x G.F.</td>
</tr>
<tr>
<td>scattering, blob counting</td>
<td></td>
<td>Wild 50 x</td>
<td>20 x G.F.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Cooke 45 x</td>
<td>20 x G.F.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Zeiss 50 x</td>
<td>-</td>
</tr>
<tr>
<td>grain counting, gap measurements, accurate</td>
<td>1500 - 3000 x</td>
<td>Leitz 100 x</td>
<td>20 x or 25 x G.F.</td>
</tr>
<tr>
<td>dip measurements</td>
<td></td>
<td>Zeiss 50 x 2</td>
<td>-</td>
</tr>
</tbody>
</table>

### Light sources and condensers

They are of two types: critical (Fig. 4a) and Köhler (Fig. 4b).

The first needs a very uniform light source over an area large enough to illuminate properly the object. In the second, an intermediate image of the light source is made at the position of the lamp-condenser, in front of which a ground glass is placed (sometimes it is the lens itself which is ground). This system is the most commonly used for nuclear emulsion work; of this type is the well-known Berek, two-diaphragm condenser. The top lens (cap) can be removed for illumination of a wider field or interchanged. For example, Leitz make two special caps in addition to the standard one:
Code name  |  N.A.  |  Use
---|---|---
ORZEL  | 0.65  | for thicknesses of glass + emulsion from 1.2 to 4.8 mm
GRAFG  | 1.40  | for high resolution in depth. (oil-imm.)

Another possibility, adopted by Koristka and Zeiss, is to use an objective of low magnification but large N.A., e.g. Koristka 30× or Leitz 22×.

Filters

For normal work: glass or gelatine filters. For accurate work or use with monochromat objective: interferometric filters.

Measuring accessories

Reticules and graticules

Diamond scratch - good but diffraction effects.
Photographic - sometimes too opaque.
Photographic - made with a needle on photographic emulsion, which is then developed - difficult but very good.

Clausen micrometer

Plane parallel glass rotating about an axis perpendicular to the optical axis; allows lateral shift of image of ~0.01 μm; inconvenient: total displacement < 20 μm.

Made by: Koristka (Pochtrolino), Leitz (Stodiek), Zeiss-Jena.

Special eyepieces

Filar micrometer - many types; care about back-lash.
Movable and adjustable slit - very useful for microphotography.
Ehrlich eyepiece - adjustable square-field diaphragm.
Centring pinhole.
III. MECHANICS

General purpose microscopes

There are many good microscopes for scanning and simple measurements. Among the best available in western countries are the Leitz Ortholux, the Cooke 4000 and the Koristka R4. Some special pieces can be added to them to widen their capabilities, e.g. large stages, rotating stages, micrometric gauges, etc.

Measuring microscopes

The main requirements are: low noise in X and Z movements, long travel in X, a range of pre-set cell lengths on lead screw, convenient and accurate rotating device for alignment. An additional facility of great help is, of course, digitizing of Y co-ordinates and automatic computing of differences of second and higher order.

Three different principles have been applied successfully: spring-supported stage (Koristka HS2 and Zeiss-Jena); optical glass leading a floating objective (Leitz Stodieck and Russian HBO-8M); stage sliding on a straight rod (Koristka R4, American Brower).

The stage noise of these microscopes is generally lower than 0.02 μm.

Special all-purpose stage

Mr. M.A. Roberts, of CERN, has designed a new large stage, which seems to be very versatile. I would like to ask him to give a brief description of it now.
LARGE MICROSCOPE STAGE
(N.A. Roberts)

For some time the need has been apparent for larger and more precise stages for microscopes used for the study of nuclear emulsions. The average microscope suitable for this work has a stage of some \(15 \times 15\) cm, with a movement in the X direction of only 5-7.5 cm and in the Y direction of about 4 cm. The only exception to my knowledge is the Koritska \(R4\), which has an X movement of 10 cm. Nearly all laboratories working with nuclear emulsions already have all the microscopes they need. Therefore I have designed a self-contained stage to fit a Cooke \(M4000\) microscope. With very little modification it could be adapted for use with other microscopes such as the Leitz Ortholux. It will give an X movement of 20 cm, a Y movement of 5 cm, a rotary movement to the top stage of \(8^\circ\) either side of centre, plus a hydraulic coarse and fine focus for the Z movement. The Z movement can be adjusted to be accurately parallel to the optical axis of the microscope.

Due to the length of the X movement, and the 1 mm pitch of the screw controlling this motion, an electric motor will be built into the system, facilitating the movement from one part of the plate to another in the X direction. It is hoped that the X movement, by virtue of its semi-kinematic design, will give a motion of such precision that scattering measurements will be possible over the full 20 cm movement.

The first tests of this instrument should take place in the autumn of this year.
DISCUSSION

Bovet: One word about reticules to put into the eyepieces. For special measurements it is very useful to draw a particular reticule. We always made these ourselves with the photographic method using Kodak P651 films which become quite transparent if carefully processed. The opacity of the lines can be made as small as one likes.

Teucher: Have any measurements been made of the stage noise of Zeiss-Jena microscopes? They have been done for small cell-lengths but they do not give any quotation for long cell-lengths. Some indications seem to show that it increases rapidly with cell-length, above a few millimetres.

Vanderhaeghe: Looking at interferometric measurement of the stage noise of a Zeiss microscope which is at present in CERN for demonstration, I do not think that the stage noise could increase so much with cell-size. It is approximately constant with some small variations not exceeding ± 0.02 μ over 50 mm of stage travel.

Dahl-Jensen: The noise of the Zeiss scattering microscope is certainly less than of an HS2. I have just one other point; when you make reticules, you can use thread from spiders. It is extremely thin and very difficult to handle but you can obtain an excellent reticule.

Grote: May I answer Prof. Teucher's question? We have measured the stage noise of three Zeiss microscopes up to a cell-length of 8 mm. We found in all cases that
Grote (cont.): for large cell-lengths the stage noise was less than 50% of the stage noise of Koristka MS2 microscopes, whose values we have found in the literature. The stage noise measured by us included reproducible and irreproducible stage noise, therefore the value can be lowered still more. The results are to be published in the "Jenaer Rundschau".

Zakrzewski: We get very fine hair lines from BALSA cement. They are better than those from spiders webbs. We tried this and got very good results.

* * *
Table 1

Recommended objectives

A. **Dry objectives**

<table>
<thead>
<tr>
<th>Objective</th>
<th>Magnification</th>
<th>NA (numerical aperture)</th>
<th>Working Distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cooke</td>
<td>10 x</td>
<td>0.30</td>
<td></td>
</tr>
<tr>
<td>Leitz</td>
<td>10 x</td>
<td>0.25</td>
<td></td>
</tr>
<tr>
<td>Koristka</td>
<td>12 x</td>
<td>0.30</td>
<td></td>
</tr>
<tr>
<td>Cooke</td>
<td>20 x</td>
<td>0.50 / 1700 μm</td>
<td>working distance</td>
</tr>
<tr>
<td>Leitz</td>
<td>25 x</td>
<td>0.50 / 1000</td>
<td></td>
</tr>
<tr>
<td>Koristka</td>
<td>26 x</td>
<td>0.65 / 1300</td>
<td></td>
</tr>
<tr>
<td>Leitz</td>
<td>45 x</td>
<td>0.65 / 680</td>
<td></td>
</tr>
</tbody>
</table>

B. **Oil-immersion objectives**

<table>
<thead>
<tr>
<th>Objective</th>
<th>Magnification</th>
<th>NA (numerical aperture)</th>
<th>Working Distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Leitz KS</td>
<td>22 x</td>
<td>0.65 / 2200</td>
<td></td>
</tr>
<tr>
<td>Cooke</td>
<td>25 x</td>
<td>0.65 / 2000</td>
<td></td>
</tr>
<tr>
<td>Koristka</td>
<td>30 x</td>
<td>1.00 / 3000</td>
<td></td>
</tr>
<tr>
<td>Koristka</td>
<td>30 x</td>
<td>1.10 / 1000</td>
<td></td>
</tr>
<tr>
<td>Cooke Fl. LWD</td>
<td>45 x</td>
<td>0.85 / 1500</td>
<td></td>
</tr>
<tr>
<td>Wild Fl.</td>
<td>50 x</td>
<td>1.00 / 250</td>
<td></td>
</tr>
<tr>
<td><em>Zeiss Fl.</em></td>
<td>50 x</td>
<td>1.00 / 1900</td>
<td></td>
</tr>
<tr>
<td><em>Zeiss Monochrom. Fl.</em></td>
<td>50 x</td>
<td>1.35 / 600</td>
<td></td>
</tr>
<tr>
<td>Leitz KS Fl.</td>
<td>53 x</td>
<td>0.95 / 1000</td>
<td></td>
</tr>
<tr>
<td>Koristka</td>
<td>55 x</td>
<td>0.90 / 1350</td>
<td></td>
</tr>
<tr>
<td>Koristka</td>
<td>55 x</td>
<td>1.00 / 700</td>
<td></td>
</tr>
<tr>
<td>Leitz KS Apo.</td>
<td>100 x</td>
<td>1.32 / 370</td>
<td></td>
</tr>
<tr>
<td>Koristka Fl.</td>
<td>100 x</td>
<td>1.25 / 530</td>
<td></td>
</tr>
<tr>
<td>Leitz L 591</td>
<td>100 x</td>
<td>1.32 / 700</td>
<td></td>
</tr>
</tbody>
</table>

C. **Water-immersion objective**

<table>
<thead>
<tr>
<th>Objective</th>
<th>Magnification</th>
<th>NA (numerical aperture)</th>
<th>Working Distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Leitz W</td>
<td>50 x</td>
<td>1.00 / 440</td>
<td></td>
</tr>
</tbody>
</table>

* The Zeiss Monochromats do not fit standard microscopes.
MEASUREMENT OF CURVATURE IN A MAGNETIC FIELD

W. M. Gibson
H. H. Wills Physics Laboratory, Bristol University.

I. INTRODUCTION

Many of us are now in a position to be interested in the measurement of momentum and charge by magnetic deflection in emulsion, simply because there is in CERN the apparatus for producing high magnetic fields¹). I shall now try to present the elementary theory in a form ready for use.

II. RADIUS OF CURVATURE

Balance of centrifugal and magnetic forces on a particle gives:

\[ r = \frac{p^2}{eH} \]

\[ 1 \text{ electron-volt} = \frac{e}{300} \text{ erg} \]

therefore

\[ r = \frac{(p^2)_{\text{e.v.}}}{300H} \]

\[ = \frac{(p^2)_{\text{e.v.}}}{6 \times 10^7} \text{, if } H = 200,000 \text{ gauss} \]

where \( r \) is the radius of curvature in \( \text{cms} \), \( p \) is the momentum in c.g.s. units, \( H \) is the electronic charge in c.s.u.

So

\[ r = \frac{(p^2)_{\text{eV}}}{60} \text{ cms} \]

Let us keep to the value 200,000 gauss and discuss what we can do with it. If you wish this lecture may be repeated backwards, rephrasing it as a logical case for choosing 200 kgauss. Higher field

¹)

For the details of this apparatus see VII.1 by L. Hoffman.
will be useful when we can make it reliably, but at present there is
not much overlap between what is useful and what can be made reliably.

III. MEASUREMENT OF CURVATURE

Usually we want to extract the maximum information from a
limited length of track, L(cm). We use the co-ordinate method, at
four points spaced by \( t = \frac{1}{2} L \), where \( \theta_1 \) and \( \theta_2 \) are the relative angles
of the chords joining the first and last pairs of points, having y
co-ordinates \( y_1, y_2 \) and \( y_3, y_4 \), respectively.

Then

\[
\frac{1}{r} = \frac{(\theta_1 - \theta_2)}{2t} = \left(\frac{y_1 - y_2 - y_3 + y_4}{t}\right) \frac{2t}{t} = \frac{D_M}{2t^2},
\]

where

\[ D_M = y_1 - y_2 - y_3 + y_4 \text{ (cm)}; \]

therefore

\[ D_M = \frac{2t^2}{r} = \frac{120t^2}{pc} \text{ with } pc \text{ in MeV} = \frac{13.3 L^2}{pc} \text{ " " " "}. \]

In microns:

\[ D_M = \frac{133 L^2}{pc} \text{ " " " GeV, } L \text{ in cm}. \]

IV. MULTIPLE SCATTERING

This itself provides a means of measuring momentum, but
also limits the accuracy of the magnetic method to an extent we shall
now calculate.

Let us measure the y co-ordinates of four points on the track
of a particle deflected only by scattering. Then the angle between the
extreme chords is
\[ \Theta_s = \frac{D_3}{t}, \]

where \( D_s = y_1 - y_2 - y_3 + y_4 \) (cm).

If the r.m.s. deflection over a length \( l \) is

\[ \phi = K \cdot l^{1/2} \cdot \beta, \]

where \( K \) is the scattering constant as derived below.

Then the r.m.s. angle between tangents at points 2 and 3 is

\[ \phi_2 = \frac{K}{p \beta} \cdot t^{1/2} \]

the r.m.s. angle between chord 1 - 2 and tangent at 2 is

\[ \phi_1 = \frac{1}{\sqrt{2}} \cdot \frac{K}{p \beta} \cdot t^{1/2} \]

and the r.m.s. angle between chord 3 - 4 and tangent at 3 is

\[ \phi_3 = \phi_1. \]

Therefore mean square total deflection is

\[ \Theta_s^2 = \phi_2^2 + \phi_3^2 = \frac{5}{2} \left( \frac{K}{p \beta} \right)^2 \cdot t. \]

Therefore r.m.s.

\[ \Theta_s = \sqrt[4]{\frac{5}{2}} \cdot \frac{K}{p \beta} \cdot t^{1/2}. \]

Therefore r.m.s.

\[ D_s = \sqrt[4]{\frac{5}{2}} \cdot \frac{K}{p \beta} \cdot t^{1/2}. \]

For the scattering constant \( K \) we must take the ordinary value, without cut-off, of

\[ K_{\text{chord}} \approx 33 \text{ MeV/c degrees.} \quad (\text{100} \mu)^{-1/2} \]

\[ = 5.8 \text{ MeV/c radians.} \quad \text{cm}^{-1/2}, \]

with two further factors:

1) \( \sqrt{\frac{\pi}{2}} \) to get r.m.s. values from arithmetic means;

2) \( \sqrt[4]{\frac{\pi}{2}} \) because our formula requires \( K_{\text{tangent}}. \)

\[ \text{Editorial note: For reference see Powell, Fowler and Perkins, The Study of Elementary Particles by the Photographic Method, p.114 (1957).} \]

33k8/NP/smg
The net result is therefore:

\[ K = K_{\text{chord}} \left( \frac{3\pi}{4} \right)^{1/2} \]

and hence r.m.s.

\[ D_s = \left( \frac{5}{3} \right)^{1/2} \left( \frac{2\pi}{4} \right)^{1/2} \left( \frac{L}{2} \right)^{3/2} \]

\[ = \frac{2.20}{p\beta} L^{3/2}, \text{ with } D_s \text{ and } L \text{ in cm, } p\beta \text{ in MeV/c} \]

or:

\[ D_s = \frac{22.0}{p\beta} L^{3/2}, \text{ with } D_s \text{ in } \mu \text{m, } L \text{ in cm, } p\beta \text{ in GeV/c}. \]

V. ACTUAL MAGNITUDES

These are listed in the tables for three different values of L. The momentum p is used as the principal variable, with values of kinetic energy T and \( \beta \) for pion and proton attached.

The absolute magnitude of \( D_M \) is greater than \( D_s \) at the high energies where measurements are difficult. As \( \beta \) and L decrease, \( D_s \) begins to become large enough to make a single measurement of \( D_M \) useless.

However, as long as \( L > 1 \) mm, and range \( > 3 \) mm, \( D_M \) is always comfortably greater than \( D_s \).

VI. TECHNIQUE OF MEASUREMENT

Since \( D_M \) increases as \( L^2 \), one measures at a small number of points as widely placed as possible. In principle three points provide a sufficient measure of curvature \(^a\), but give no check of its regularity. The four-point method which we have discussed provides some check against mistakes and single scatters. A single scatter near the middle of the track can cause a serious undetected error even in the four-point method. The only way to be sure is to make measurements at five or more points, and check that the curvature is as uniform as multiple scattering allows.

---

\(^a\) Editorial note: \( D_M \) (3 points) = \( y_1 - 2y_2 + y_3 = \frac{L^2}{4} = \frac{9}{8} D_M \) (4 points).
VII. CORRECTION FOR DISTORTION

With multiple scattering it is possible to correct for the various types of distortion by assuming that they are random and vary with cell size in a prescribed manner.

When trying to measure a magnetic deflection, one must measure and subtract the deflection due to C-shaped distortion. To do this it is necessary to have a reference line which is known to have been straight at the time of exposure. Such a reference line may be provided by the track of a beam particle of known momentum; or the bottom surface of the emulsion may be assumed to have remained undistorted.

To use such reference lines, one must have a means of projecting them vertically to the level of the track under observation. This may be done by means of vertical tracks from a subsidiary exposure. For these subsidiary, perpendicular, exposures, the normal scattered proton beam from the CERN PS is quite satisfactory. The intensity must be about $5 \times 10^4$ protons per cm$^2$, so that a vertical track can be found within about 25 $\mu$ of each point to be corrected.

In the pellicles already under examination the corrections are commonly about 10 $\mu$, and can be made with an accuracy of about 1 $\mu$. This is why, in discussing the tables, we may consider that a $D_m$ of less than 1 $\mu$ is useless.

In this report I have drawn heavily upon the work of colleagues in Bristol, especially B.D. Jones and W.A. Venus to whom I am especially grateful for correcting errors in the final calculations.
MEASUREMENTS ON 1 cm OF TRACK

<table>
<thead>
<tr>
<th>$P$ ($\text{GeV}/c$)</th>
<th>$D_M$ ($\mu$)</th>
<th>$D_s$ for $\beta = 1$</th>
<th>Pion</th>
<th>Proton</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
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Conclusion: $D_M > 2.5 D_s$ whenever range is adequate.
### Measurements on 2.5 mm of Track

<table>
<thead>
<tr>
<th>$p$ (GeV/c)</th>
<th>$D_m$ ($\mu$)</th>
<th>$D_s$ for $\beta = 1$</th>
<th>Pion</th>
<th>Proton</th>
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<td>T</td>
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<td>100</td>
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<td>28</td>
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**Conclusion:** $D_m > D_s$ whenever range is adequate.
### Measurements on 1 mm of Track

<table>
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<tr>
<th>p (GeV/c)</th>
<th>D_M (μ)</th>
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<th>Proton</th>
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<td>50</td>
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<td>14</td>
<td>9</td>
<td>1.8</td>
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</table>

Conclusions: \( D_M > D_s \) and possibly useful for pions from 100 MeV/c to 1 GeV/c; whilst 1 mm is never much use for protons.
DISCUSSION

Pahl : Are the numbers you have given not dependent on the type of emulsion used?

Gibson : The values of $D_8$ (due to multiple scattering) certainly do depend on the emulsion. So if you use diluted emulsions, you can hope to have a smaller multiple scattering. This may be useful in certain cases, but in other cases the wide spacing between grains, which you have in the dilute emulsions, may give more uncertainty than the gain due to reduction in the scattering constant. As Zakrewski pointed out, distortion is very often worse in diluted emulsions than it is in normal ones.

* * *

;348/NP/smg
AUTOMATION OF MICROSCOPE WORK

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H.H. Wills Physics Laboratory, University of Bristol

GENERALIZATIONS AND ANALOGY

One of the advantages of the emulsion technique is the ease and directness with which we can obtain a mental picture of a nuclear process.

The x and y co-ordinates of each charged particle are presented to our eyes directly, and the z co-ordinate is inferred from the position of a focusing screw: very little training, and no mental effort, is required to obtain a clear three-dimensional picture of a process. The whole mind is free to consider the time-scale and the nuclear interpretation of the tracks seen.

In contrast, bubble chamber photographs give no immediately obvious information about the z co-ordinate. Such information must be obtained from comparison of pairs of stereoscopic photographs. This comparison must include correction for the refractive index of liquid and window; it is therefore normally made with the aid of a computer. Having used a computer to calculate the three-dimensional form of an event from the two stereoscopic photographs, we naturally make further use of the computer, e.g. to calculate curvatures of tracks in the magnetic field, and to work out the kinematics of reactions. This is an extremely powerful technique for carrying out major experiments for which extensive preparation is possible and for meeting situations which are already well understood. But its flexibility in exploratory work is limited by the extent to which men can obtain from the computer, and interpret, information beyond that for which a full programme has already been designed.
So throughout any discussion of how to speed up emulsion work by mechanization, one guiding principle must be: RETAIN THE FLEXIBILITY OF THE TECHNIQUE.

For several years I used to repeat to myself and others this incantation, as a way of closing the discussion. This attitude I now see to have been as limited as that of the man who is discouraged from buying a car by fears of losing the freedom which he enjoys as a pedestrian. We all know, many from experience, that the flexibility of a man's technique is increased by the possession of a car: certainly he cannot make a sharp right-angle turn at high speed, and he may at times have to park a mile from his objective, returning to pedestrian techniques for the last difficult mile, but mechanization does increase his potentiality for useful work.

Now let us return to the microscope. I shall describe the operations which may usefully be mechanized under four headings.

1. **Printing co-ordinates**

This facility may be considered as part of a general-purpose microscope, as a Decca navigator is part of a general-purpose aeroplane. It may be used casually, e.g. for recording the position of events with respect to grid or other reference marks, or systematically for the recording of lengths, directions or profiles of tracks. Instead of accumulating handwritten paper, one collects rolls of printed co-ordinates, with printed identification marks, the eyes always remaining on the eyepieces.

When one knows in advance what will be the principal use of the printed co-ordinates, one can arrange a parallel output: Barkas's digitized microscope feeds an IBM card-punch; cards describing particular types of events may then be used as input for a computer programmed to calculate the kinematics of possible processes.

In Rome and in CERN, the y co-ordinates from a filar eyepiece are fed to a printer which is a standard calculating machine. In CERN this can be set to print only, to subtract for track length or direction, or to calculate and accumulate second differences.
Koristka have now digitized a filar eyepiece, and are starting on x. In Britain, Bryen Brothers have produced a completely digitized microscope, in the first instance for measuring spectra of neutrons from the Electricity Board's reactors. We hope to get one as a general-purpose instrument.

2. Measurement of ionization

Here the ideal probably is the lacunometer of Lausanne: the track is driven at constant speed across a reference line, the observer operates a switch at the beginning and end of every blob. The machine counts the total number of blobs, and records the distribution of blob- and gap-lengths.

This type of machine is a useful thing to have in any laboratory, since it can increase the speed and quality of a measurement which forms an important part of many experiments.

3. Motor-driven stage

This facility may also be considered as part of a microscope intended for use in many types of experiment: it merely permits movement over a wider range of speeds, and more uniform than can be obtained with the fingers. The advantage of a motor becomes most marked in experiments where many long, flat, parallel tracks are being followed in a search for interactions and scatters. Moving the stage irregularly, one stops only when an event is visible in a single field of view but when the stage moves uniformly, a small-angle scatter can be seen as a change in velocity at right-angles to the motion of the stage.

4. Finding events of a pre-determined type

Many machines have been built for specific experiments, or types of experiment. Some retain the human observer, while many replace him by a photomultiplier. Some include fully automatic track-following of the type used in Franckenstein. They make possible
experiments having certain of the features of counter experiments, since they share the power of the electronic technique to reject large amounts of information while recording large numbers of events satisfying pre-selected criteria.

* * *

8/NP/smg
GRAIN DENSITY AND IONIZATION LOSS IN EMULSION

D.H. Perkins
H.H. Wills Physics Laboratory, Bristol University.

The study of the distribution of grains in emulsion tracks is part of a more general problem on fluctuations, and can equally well be applied to tracks in bubble chambers, dead-time of counters and so on.

I. ELEMENTARY THEORY OF IONIZATION LOSS

Consider a particle \((\mathbf{H}, v, ze)^*)\) passing at distance \(b\) from an electron.
Assume the electron is
(i) stationary during the collision,
(ii) free.

By Gauss' theorem,

\[
\int \mathbf{D} \cdot d\mathbf{S} = 4\pi q, \text{ or, with } K = 1
\]

\[
\int_{-\infty}^{\infty} E_1 \cdot \mathbf{n}_b \cdot dx = 4\pi ze,
\]

where the integration is over the cylinder of radius \(b\).

\*)

Here \(H, ze\) denote mass and charge of particle;
\(m, e\) denote mass and charge of electron;
\(v\) denotes velocity of particle;
\(x\) denotes length along direction of motion of particle;
\(t\) denotes time;
\(F\) denotes force;
\(D\) denotes induction due to electric field \(\mathbf{E}\);
\(d\mathbf{S}\) denotes element of surface area;
\(q\) denotes enclosed charge;
\(K\) denotes dielectric constant of detecting medium;
\(E_1\) denotes component of electric field perpendicular to direction of motion of particle;
\(Z, A\) denote mean atomic number and mass number of detecting medium.
Hence, the impulse $\delta p$ received by the electron will be

$$\delta p = \int P \, dt = \int eE_1 \, dt = \frac{e}{v} \int E_1 \, dx = \frac{2ze^2}{bv}$$

This formula is true both relativistically and non-relativistically; when $\gamma$ is large the pulse is sharpened but the area remains unchanged, since $E_1$ is increased by a factor $\gamma = \frac{1}{\sqrt{1-v^2/c^2}}$

whilst the collision time $\tau \sim \frac{b}{v}$ is reduced by a factor $\gamma$. By symmetry, we also see that $\delta p$ is perpendicular to $\vec{v}$.

If $\delta p \ll mc$, the energy transferred to the electron is given by

$$\delta E = \frac{(\delta p)^2}{2m} = \frac{4ze^4}{2mv^2} \cdot \frac{1}{b^2}$$

Let $N = \text{No. of electrons/cc}$. The number in a cylindrical shell of length $\Delta x$, radius $b + b + db$ is $2\pi bdb\Delta x$, so that the energy lost by the particle $N$ in traversing $\Delta x$ is

$$\Delta E = \frac{N \cdot 4ze^4}{mv^2} \cdot \frac{2\pi bdb}{b^2} \cdot \Delta x$$

Hence the total rate of energy loss, from electrons at all distance $b$ is

$$\frac{dE}{dx} = \frac{\Delta E}{\Delta x} = \frac{4\pi Nze^4}{mv^2} \cdot \int db \frac{db}{b} = \frac{2\pi Nze^4}{mv^2} \log_e \left[ \frac{b_{\text{max}}}{b_{\text{min}}} \right]$$

$$= \frac{2\pi Nze^4}{mv^2} \log_e \left[ \frac{\delta E_{\text{max}}}{\delta E_{\text{min}}} \right]$$

for $\delta E \ll nc^2$.
subject to the assumptions (i) and (ii) above. Let us now look at these assumptions more closely.

Assumption (i) (electron stationary)

If we call $\Delta y$ the distance travelled by the electron, velocity $u = \delta p/m$, during the collision time $\tau = 2b/\nu \gamma$, the condition may be stated as

$$\Delta y/b << 1$$

where $\Delta y = ur$. Then

$$\Delta y = \frac{ur}{b} = \frac{\tau \delta p}{mb} = \frac{2 \delta p}{\nu \gamma} = \frac{4z e^2}{\nu \gamma b^2 \gamma} = \frac{4z}{\beta^2 \gamma} \cdot r_e$$

where $r_e = e^2/4\pi \epsilon_0 m_c^2 = \text{classical electron radius}.

Hence, if $b_{\text{min}}$ denotes the minimum value of $b$, we must have

$$\frac{4z r_e}{\beta^2 \gamma b_{\text{min}}} << 1.$$ 

As shown below, the quantum mechanical lower limit on $b$, set by the dimensions of the electron wave-packet, is $b_{\text{min}} = \hbar / \nu \gamma$.

Hence, the condition required is that

$$\frac{4z r_e}{\beta} \cdot \frac{mc}{\hbar} = \frac{4z}{137 \beta} << 1, \text{ where } \chi_c = \frac{\hbar}{mc} = 137 r_e.$$

Thus, for singly-charged particles ($z = 1$), the electron may be considered stationary provided $\beta > 1/137$ (= $\beta$ of K-electron in H-atom). This condition is always satisfied except for extremely slow particles; for the latter, the energy loss would be reduced, but it is in any case complicated by the phenomenon of "capture and loss".

Assumption (ii) (electron free)

The extent to which the electron can be regarded as free, when in fact it is bound in an atom, determines $\delta E_{\text{min}}$, or $b_{\text{max}}$. 

8/NP/smg
The collision time $\tau = 2b/\nu_0$. The Fourier transform of $E_i$ against $t$ gives the distribution in frequencies $\nu$ of the component plane electromagnetic waves falling on the electron. For example, for a gaussian-shaped electric pulse of width $\tau/2$, the distribution in amplitude of the various frequencies $0 < \nu < \nu_{\text{max}}$ is also gaussian, of width $2/\tau$.

Thus, $\nu_{\text{max}} \sim \frac{2h}{\tau} = \frac{h\nu_0}{b} > h\nu_0$

(where $\nu_0 = \text{frequency of bound electron}$), if the electron is to be regarded as free.

Setting $h\nu_0 = \overline{1}$, an effective mean ionization potential, we then have

$$b_{\text{max}} = \frac{h\nu_0}{\overline{1}} \quad (2)$$

In addition, there is a restriction on $b_{\text{min}}$, which comes from the fact that the electric field must not vary appreciably over the dimensions of the electron i.e. $b_{\text{min}} \sim \lambda$, where $\lambda = \text{De Broglie wavelength of the struck electron as seen by the incident particle}$. The momentum of the electron in the frame of the incident particle is $p' \approx mv\gamma$. Hence $\lambda = h/p' = h/mv\gamma$. Thus, quantum-mechanically,

$$b_{\text{min}} \approx \frac{h}{mv\gamma} \quad (3)$$

Classically, there will also be kinematic limitations on $b_{\text{min}}$. Thus, if $M \gg m$, one has

$$\delta E_{\text{max}} = 2mv^2 \quad \text{(for } v^2 \ll c^2, \text{ the electron velocity relative to } M \text{ is reversed on impact).}$$

$$= E \quad \text{(for } v \sim c).$$

One can see from equation (1) that the dependence of the energy loss on log $b$ arises from the assumption that the electron is non-relativistic. If $\delta E \sim c \delta p \quad (u \sim c)$, then $\delta E \propto 1/b$ instead of $\propto 1/b^2$, hence the cross-section tends to zero as $b \to 0$ instead of rising logarithmically. Thus, the classical effective lower limit is given by

$$\left(\delta E_{\text{max}}\right)_{\text{classical}} \sim mc^2.$$
Also $\delta E_{\text{max}} = \frac{2e^2}{b_{\text{min}}^2}$, so that $(b_{\text{min}})_{\text{classical}} \approx \frac{2e r_e}{\beta}$ (4)

This lower limit is of the order of $r_e$, the classical electron radius.

Comparing (3) and (4), one gets, for $v \sim c$,

$$\frac{\left(\frac{b_{\text{min}}}{b_{\text{min}}}ight)_{\text{q.m.}}}{\left(\frac{b_{\text{min}}}{b_{\text{min}}}ight)_{\text{classical}}} \approx \frac{\lambda e}{r_e} \frac{137}{2\gamma}$$

(5)

Thus, if $\gamma \ll 137$, one should use $(b_{\text{min}})_{\text{q.m.}}$, whilst if $\gamma \gg 137$, one must use $(b_{\text{min}})_{\text{classical}}$, or $\delta E_{\text{max}} \sim m c^2$. When $\gamma \sim 1$, one finds $(b_{\text{min}})_{\text{q.m.}} > (b_{\text{min}})_{\text{classical}}$.

Assuming the quantum mechanical limits, one thus obtains from (2) and (3):

$$\frac{b_{\text{max}}}{b_{\text{min}}} = 2\pi \cdot \frac{m v^2}{(1 - \beta^2) I}$$

(6)

This formula is approximate in the sense that we have not carried out properly the Fourier analysis of the electromagnetic pulse in getting $b_{\text{max}}$. When done correctly, the result is

$$\frac{b_{\text{max}}}{b_{\text{min}}} = \frac{2m v^2}{(1 - \beta^2) I}$$

(6a)

Substituting in (1), one finally obtains

$$\frac{dE}{dx} = \frac{4\pi N e^2}{m v^2} \cdot \left[ \log_e \frac{2m v^2}{(1 - \beta^2) I} - \beta^2 \right]$$

(7)

where the extra term $-\beta^2$ comes in the correct treatment (Bethe). This formula is correct provided $\bar{m} \gg m$, i.e. provided the centre of mass is fixed in $\bar{m}$. If $\bar{m} \sim m$, the limits are changed by a factor accounting for reduced masses; also, if the particle $\bar{m}$ is an electron, exchange terms must be added. However, (5) is good enough for most cases of
interest; (see Rossi p. 23 et seq. for the correct formulae).

The main features are:-

i) $\frac{dE}{dx} \propto z^2$ of the incident particle;

ii) $\frac{dE}{dx} \propto \frac{1}{v^2}$, if $v < c$;

iii) $\frac{dE}{dx}$ increases logarithmically as $\beta \to 1$;

iv) $\frac{dE}{dx}$ is independent of $M$, i.e. is a function of $v$ only.

It may be noted again that when $v << c$, so that $nv^2 \sim \bar{v}$, none of the electrons are free, i.e. collisions are adiabatic, and $\delta E_{\text{max}} \rightarrow \delta E_{\text{min}}$ (capture and loss).

If we express $\frac{dE}{dx}$ in MeV/gm rather than MeV/cm, then

$$N = \text{No. of electrons/gm} = \text{No} \cdot \frac{Z}{A}$$

where

$\text{No} = \text{Avogadro's number.}$

$Z/A$ varies only slowly with $Z$, hence $\frac{dE}{dx}$ (per gm) is of the same magnitude for all substances. $(dE/dx)_{\text{min}} \sim 2 \text{ MeV/gm}$, and varies only by a factor two for different absorbers. It may be noted that $b_{\text{max}} \propto \gamma$, and $b_{\text{min}} \propto 1/\gamma$, hence $dE/dx$ increases as $\gamma \to \infty$ because of an increase in the numbers of very close and very distant collisions.

*) Numerically (for $v \sim c$):-

$$4\pi \text{No} \frac{\frac{Z}{A} e^2}{\frac{4}{3} \pi r_e N_0 \frac{Z}{A} m_e^2} = 4\pi r_e N_0 \frac{Z}{A} m_e^2,$$

where $m_e^2 = 0.5 \text{ MeV};$

$N_0 = 6.10^{23}$; $r_e = 2.8 \times 10^{-13}$ cm;

$Z/A \sim \frac{1}{2};$

$$\bar{v} \approx 10Z \text{ eV}$$

therefore

$$\frac{dE}{dx} \approx 0.15 \log e \frac{10^6 \gamma^2}{10Z};$$

hence for $\gamma \sim 4$, $(dE/dx)_{\text{min}} \sim 2 \text{ MeV/gm}$ for $Z = 5$. 

3348/FP/swg
II. LOCAL ENERGY LOSS - POLARIZATION

In emulsions (or bubble-chambers), large values of $\delta E$ mean energetic $\delta$ rays, most of the grains of which do not contribute to the track. Hence, there is a cut-off $\delta E_{\text{max}} \sim 3$ KeV imposed by the fact that tracks result only from local ionization losses. For

$$\delta E = 3\text{KeV}, \quad b_{\text{min}} = r_e \frac{z\sqrt{2}}{(\delta E/mc^2)^{1/2}} = 18r_e \sim 5 \times 10^{-12} \text{ cm}. $$

The relativistic increase in grain density is then due to increase in $b_{\text{max}}$ only.

In deriving the above formulae, we used Gauss' theorem, namely $\int \mathbf{D} \cdot d\mathbf{S} = 4\pi q$. Actually $\mathbf{D} = K\mathbf{E}$ where $K > 1$ for a polarizable medium. Thus, the impulse $\int \mathbf{E} \cdot d\mathbf{t}$ is reduced by a factor $K$ for those collisions where the macroscopic structure of the medium is important. Physically, the moving particle induces a polarization field which opposes the applied field wherever it can. This will be true, roughly, if $b_{\text{max}} \gg$ inter-atomic distance. Now, from (2)

$$b_{\text{max}} \approx \frac{h\gamma}{m} = \frac{h}{mc} \cdot \frac{mc^2 \gamma}{m} $$

for $\gamma > 1$.

If we take $b_{\text{max}} \gg 10^{-6}$ cm ($\approx 100 \times$ interatomic distance), then the condition for effective polarization is that

$$\gamma \approx \frac{1}{m} \left( \frac{mc}{h} \right) \cdot \frac{1}{mc^2} \cdot b_{\text{max}}, \text{ where } b_{\text{max}} \gg 10^{-6} \text{ cm}. $$

With $\bar{I} = 10Z \approx 300 \text{ eV}$ (for emulsion); $mc^2 = 5 \times 10^5 \text{ eV}$;

and

$$\frac{h}{mc} = 3.8 \times 10^{-11} \text{ cm},$$

one obtains $\gamma \geq 15$.

For a gas (e.g. cloud-chamber), the density as compared with emulsion is lower by a factor $10^3 - 10^4$, hence the limiting $\gamma$ is some 10 times higher.
For more details, see for example Sternheimer.

III. RANGE-ENERGY RELATIONS

From the value of \( \frac{dE}{dx} \) the range follows:

\[
R = \int_0^E \frac{dE}{\frac{dE}{dx}}.
\]

Numerical calculation of the R/E relation then gives a value for \( \bar{I} \), which in the case of G5 emulsion gives \( \bar{I} = 330 \text{ eV} \).

IV. THE PROCESS OF GRAIN FORMATION IN EMULSIONS

The energy \( E \) received by an electron in a collision of impact parameter \( b \) is, as we have seen above, (provided \( E << m_c^2 \)):

\[
E = \frac{\frac{4}{3} z^2 e^4}{2 m_e v^2} \cdot \frac{1}{b^2}.
\]

The probability of such a collision is

\[
f(b)db = 2 \pi b \cdot \delta(b) \propto d(1/E).
\]

Therefore \( f(E)dE \propto dE/E^2 \) is the energy spectrum of electrons
(for \( E_{\text{min}} < E < E_{\text{max}} \)).
Assumptions about grain formation:

**Assumption (A)** (Demers, etc.)

a) Crystals are distributed at random – almost correct.

b) All crystals are of the same diameter (and spherical) – nearly correct.

c) All crystals are of the same sensitivity.

d) **Energy loss, \( \epsilon \), is continuous**, (i.e. we neglect fluctuations).

e) All crystals in which the energy loss \( \epsilon > \epsilon_0 \) will be developed.

Let \( J \) eV be the local energy loss per unit length of track (for a constant \( v \)).

If \( d = \) diameter of AgBr crystals \((\approx 0.2 - 0.4 \mu)\), then the path length \( l \) is given by \((l/2)^2 = (d/2)^2 - x^2\).

If \( N = \) number of crystals/cm, then the number of crystals which have centres within distance \( x \) of the trajectory is

\[
n = \pi x^2 \cdot N = \frac{\pi N}{4} (d^2 - l^2) = \frac{\pi N}{4} d^2 \left( 1 - l^2/d^2 \right)/\text{cm}.
\]

If all such grains are to be developed, then \( Jl > \epsilon_0 \), and hence

\[
n_{\text{developed}} = \frac{\pi N d^2}{4} \left( 1 - \frac{\epsilon_0^2}{J^2 d^2} \right) \approx \left( 1 - \frac{K}{J^2} \right)/\text{cm}, \text{ where } Jd > \epsilon_0 \text{ of course}.
\]

[Dotted curves (----) show the variation predicted by the formula, in disagreement with the observed (-----).]

The assumption (b) can be modified by assuming a range in crystal diameters, \( d \). This still does not produce agreement with experiment. We must conclude that \( \epsilon_0 \) is not constant; i.e. there is
a large variation in sensitivity of the crystals. In fact from the experimental data, it is possible to estimate \( f(e_0)de_0 \). The spread of this distribution is much greater than that observed for experiments with light. Thus, the assumption of a continuous, uniform energy loss per unit path length is proved to be incorrect.

Assumption (ii)

The discrepancies above can be resolved in terms of the formation of \( \delta \) rays, i.e. there are large fluctuations in the rate of the energy loss. Let us assume therefore that crystals are made to develop as a result of single acts - the creation of single \( \delta \) rays. The number of single \( \delta \) rays \( \propto z^2 \), i.e. \( n \propto z^2 \), as indeed is found. For example, if more than 1 \( \delta \) ray were required per crystal (say 2), then \( n \) should vary as \( z^4 \), contrary to experiment. The spectrum of \( \delta \) rays may be written as \( \sim C \cdot \delta E/E^2 \). The mean energy of a \( \delta \) ray between limits \( E_2 \) and \( E_1 \) is:

\[
\bar{E} = \int_{E_1}^{E_2} E^{-2} \, dE = \ln \left( \frac{E_2}{E_1} \right), \quad (\text{if } E_2 \gg E_1).
\]

Therefore energy loss (total) \( \sim C \ln E_2/E_1 \).

\( E_{\text{max}} \) (the maximum energy of \( \delta \) ray which is confined to the crystal) is \( \sim 5 \) KeV. Let us assume that all \( \delta \) rays of energy \( \geq E_{\text{max}} \) are effective, and that this spectrum follows the \( 1/E^2 \) law although this law breaks down for \( E < E_{\text{max}} \) (400 eV say).

The proportion of the total energy lost as local ionization is

\[
\frac{1}{\left[ \ln \frac{10^6}{5000} \right]} = \frac{1}{\ln \frac{10^6}{5000}}.
\]
where \( E_{\text{max}} \approx mc^2 \approx 10^6 \text{ eV} \) is the effective upper limit on energy transfer. The total energy loss in AgBr \( \approx 500 \text{ KeV/mm} \); and local energy loss in AgBr

\[
\approx 500 \frac{\ln(5000/E_g)}{\ln(10^6/E_g)} .
\]

The mean energy of \( \delta \)'s in local ionization is

\[
E_g \ln \frac{5000}{E_g} .
\]

Number of grains developed \( \approx 250/\text{mm} \).

Therefore

\[
250 \frac{E_g}{E_g} \ln \frac{5000}{E_g} = \left( \ln \frac{5000}{E_g}/\ln \frac{10^6}{E_g} \right) \cdot 5 \cdot 10^5 .
\]

Therefore

\[
E_g \ln \frac{10^6}{E_g} = 2000; \text{ therefore } E_g \approx 250 \text{ eV}.
\]

Thus, the grains which are developed result from liberation of individual \( \delta \) rays in the range 250–5000 eV. Because the \( 1/E^2 \) law is not valid for \( E < \bar{E} \), the actual \( E_{\text{min}} \) will be lower (say 100 eV).

V. EXPERIMENTAL MEASUREMENTS OF THE IONIZATION

1. Measurements near minimum ionization

For particles of ionization near the minimum value, any method of estimating the ionization may be used; grain counting, blob counting etc. All such parameters vary almost linearly with the ionization in this region.
The variation of, for example, blob-density, with p/mc, for p/mc ≈ 1, is of great importance, particularly now that accelerator beams of high energy are available.

The principal results are as follows:-

i) The blob-density (or grain density) passes through a minimum, and attains a plateau value.

ii) The magnitude of the rise above minimum to "plateau" is of the order of 10-16%, occurring for p/mc ~ 30-100, though there is no general agreement regarding the magnitude of the increase or its variation with energy.

a) Bristol (1952)  b) Washington (1954)  c) G-stack Dublin (1952)

The increase in local ionization results from additional low-momentum transfers to electrons of the medium in "distant" collisions, the spectrum of energy transfers $E'$ being a function of particle velocity (especially below $\bar{V}$). The number of developed grains then depends critically on the sensitivity of the emulsion, and the degree of development which determine $E'$, the effective minimum energy transfer below which grains are not developed. Very careful measurements in well controlled conditions are needed to resolve these problems.
2. **Measurements at high ionization**

a) **Blob density**

It can be expected that if, as suggested above, the formation of grains (developed crystals) in an emulsion is due to the liberation of single $\delta$ rays, the spatial distribution of grains should be Poissonian. Numerous measurements confirm that this is indeed the case. It is strictly true only in so far as the size of the original halide crystals (0.25$\mu$) is small compared with that of the developed grains (0.7$\mu$ diameter).

The number of blobs (unresolvable clusters of grains) is then easily seen to be $n_b = n_g e^{-n_g d}$, where $d$ = grain diameter and $n_g$ is the true number of developed grains per unit length. When $n_g$ is small, $n_b \approx n_g$, but as $n_g d \to 1$ or $> 1$, appreciable "blogging" together of the grains occurs, and thus $n_b \ll n_g$.

$n_b$ passes through a maximum when $n_g d = 1$, and $n_b$ is double-valued, corresponding to ($n_g d > 1$ or $< 1$). Thus blob-density, although a useful parameter for low values of $n_g$, is not applicable to tracks of high specific ionization. It is generally used only for tracks of $n_g d \ll 0.35$.

b) **Grain density**

In this method, each agglomeration is subdivided (mentally) into individual grains. Again saturation occurs for high values of $n_g$, and the counting is very subjective.

c) **Measurements involving gap-length**

Parameters involving the spatial distribution of grains along the track (e.g. number of gaps greater than a certain length; mean gap-length; ratio of numbers of gaps exceeding a certain length) have long been recognized as superior to a simple grain-count, at least for tracks of ionization $g \gg g_{\text{min}}$.

i) **Gap-length distribution**

If the distribution of developed grains is truly Poissonian, the gap-length distribution must be exponential.

The actual distribution was investigated by many groups, unfortunately with divergent results. Let us call $l$ the gap
length, as indicated. For \( l > 0.5\mu \), all experiments agree that
\[ N (> l) \propto \exp (-gl). \]

a) O'Ceallaigh  
b) Fowler-Perkins(F.P.)  
c) Barkas  
d) Cortini et al.  
e) Blau et al

For \( l < 0.5\mu \), some groups find an exponential distribution down to
\( l = 0 \), whilst others find that the curve "bends over" for \( l < 0.4\mu \).
Now, the separation between centres of the halide crystals, for
\( l = 0.4\mu \) is 0.4 + 0.7 \( \approx \) 1.1\( \mu \) (0.7\( \mu \) = developed grain diameter). This
is about 4 times the separation between centres of consecutive crystals
is G5 emulsion (\( \sim 0.25\mu \)). It is therefore very difficult to see why,
if the gap distribution is exponential for \( l > 0.4\mu \), it should not also
be exponential down to \( l \approx 0 \). It seems likely that there are subjective
errors in estimating gaps of
\( l < 0.4\mu \), connected with
diffraction effects. Because
of the uncertainty in the experiments, it is not
practicable to give a full
comparison of experimental
results with different models of
grain formation.

ii) Measurements based on mean gap-length

At present, therefore, we shall assume for simplicity that
\( (N (> l)) \propto \exp -gl \), for all \( l > 0 \). This relation has been confirmed by
F.P. for relativistic particles of \( z = 1 \to z = 12 \) (constituting a range
of 150:1 in ionization). Thus the gap density \( (N > l \) per unit length)
is given by \( H(\ell) = H(0) \exp(-g\ell) = B \exp(-g\ell) \) since \( H(0) = B \), the blob density. F.P. suggested \( g = \frac{1}{\ell} \ln \frac{B}{H} \) as a measure of the ionization. Further it was found empirically that

\[
B = g \exp(-ga)
\]

\[
H = g \exp(-g(\ell + a))
\]

where the parameter \( \alpha \) depends on development, and is of the order of the grain diameter \((\alpha \approx 0.7\mu \approx a)\). Experimentally, F.P. found that as \( z^2 \to \infty \),

\[ g \to 5000 \text{ mm}^{-1} \]

associated with an exponential distribution of gelatine gaps between crystals of mean length = 0.2\( \mu \).

Provided \( g \) is not too large \( g < 6g_{\text{min}} \) \((g/g_{\text{min}}) \) is the same function of ionization, irrespective of small changes \((50\%)\) in development.

VI. STATISTICAL ERRORS

1. Track of restricted length

Given a certain length of track, we may ask what is the optimum value of \( \ell \) for counting gaps. Assume \( N_B \) and \( N_H \) (the number of blobs and the number of gaps > \( \ell \) actually counted, respectively) are independent statistically:
\[ N_H = N_B e^{-g/l} \]
\[ g/l = \ln B/H = \ln \left( \frac{N_B}{N_H} \right) \]
\[ \frac{\sqrt{\Delta g^2}}{g} = \frac{1}{g/l} \left( \frac{1}{\sqrt{N_B}} + \frac{1}{\sqrt{N_H}} \right) \]

\[ = \frac{1}{g/l \sqrt{N_B}} \left( 1 + e^{g/l/2} \right) = X \text{ (say)} \]

Let \( g/l/2 = z \). Then \( X \) is a minimum when \( dx/dz = 0 \). Differentiating, one finds the condition is that \( z = 1 + e^{-2} \).

This gives a minimum for \( x \) when \( z \approx 1.28 \), i.e. \( l \approx 2.56/g \). However, the minimum is very flat, and any values of \( g/l \geq 3 \) are preferable, since the errors in measuring \( l \) are thereby reduced. The above treatment neglects correlations between \( N_B \) and \( N_H \), which are important when \( g/l < 2 \) (\( B/H < 5 \)). When \( g/l > 2 \), the r.m.s. fractional error in \( g \) reduces to
\[ \sigma_g = \frac{\sqrt{\Delta g^2}}{g} = \frac{1}{g/l \sqrt{N_B}} \left( 1 + e^{g/l/2} \right) = \frac{1 + e^{g/l/2}}{g/l \sqrt{N_H}} e^{g/l/2} \approx \frac{1}{N_H \left( \ln B/H \right)} \]

When \( g/l \) is large, this formula become exact.

If all the information in the track were used this would entail determining the length of every gap (O'Ceallaigh's method) - the error would be
\[ \sigma_g = \frac{\sqrt{\Delta g^2}}{g} = 1/N_B \]

(since the distribution is an exponential, the r.m.s. error on the mean gap length = mean gap length/\( \sqrt{N} \)).

With \( g/l = 2 \),
\[ \sigma_g = \left( \frac{1 + e}{2} \right) \frac{1}{\sqrt{N_B}} \approx 1.35 \sqrt{N_B} \]

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with \( g_l = 3 \),

\[
\sigma_g = \frac{1 + e^{1.5}}{3 \sqrt{N_B}} = \frac{1.83}{\sqrt{N_B}};
\]

and for \( g_l = 2.5 \)

\[
\sigma_g = \frac{1 + e^{1.25}}{2.5 \sqrt{N_B}} = \frac{1.8}{\sqrt{N_B}}.
\]

Thus, in the optimum region, the \((B,H)\) method yields an error only a factor 1.8 larger than that obtained when all gaps are counted. Clearly, it is much easier to count blobs, and 15 or 20\% of all gaps, that is, those greater than 3 times the average length.

2. **Track of unrestricted length**

When the statistical accuracy is not limited by the track-length available and the accuracy required is not high (\(> 5\%\)), it is easiest to count fairly long gaps (say \( g_l > 4 \)).

From the formula

\[
\sigma_g = \frac{1}{\sqrt{N_H}} \cdot \frac{1}{\log B/H} = \frac{1}{g_l \sqrt{N_H}}
\]

it is seen that, for a given \( N_H \), the value of \( \sigma_g \) is proportional to \( 1/l \).

However, the track length required for a given number of gaps rises as \( e^{g_l} \). Hence, very large values of \( l \) are never practicable.

**Dipping tracks**

For tracks of dip-angle \( \delta \) (referred to the unprocessed emulsion), the true gap-length is \( l \sec \delta \), if \( l \) is the gap-length projected in the plane of the emulsion.
Thus
\[ g = \frac{1}{\ell \sec \delta} \cdot \log_{eH} \frac{B}{eH}, \]
and the correction is easily applied.

* * *

3348/1P/sm}
DISCUSSION

Hoffmann : The formula for the rate of energy loss $dE/dx$ contains the ionization potential $I$. How is the value quoted for this determination and how constant is it?

Perkins : If you want to calculate the range energy relation of a particle using the formula for $dE/dx$, you need to integrate the expression:

$$ R = \int_{0}^{E} \frac{dE}{(dE/dx)} $$

numerically for various values of the ionization potential $I$ and compare the result with experiment. For G5 emulsion, the best fit to experimental data is obtained for:

$$ \bar{I} = 330 \text{ eV}. $$

This single value of the ionization potential gives a reasonable fit over the whole range of velocities.

Doble : Given the empirical relationship between $dE/dx$ and grain density, how can you find the relation between $dE/dx$ and blob density?

Perkins : The grain density $g$ is the density of developed grains per unit length of track. One should also mention the phenomenon of blobs. What one really sees in a track are blobs. A blob is an unresolved cluster of grains and can consist of one, up to any number of grains.
Perkins: The density of blobs, \( b \), per unit track length is given by:

\[
b = g e^{-gd}
\]

where \( d \) = mean grain diameter. It is obvious that the density of gaps is equal to the density of blobs in a track.

* * *

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I. SINGLE SCATTERING

Using a result we obtained from the theory of ionization loss, the transverse impulse $p_\perp$ received by a particle $m$, $p$, $Z\epsilon$, $v$, from a nucleus $M$, $Z\epsilon$ (where $M \gg m$) at impact parameter $b$ is

$$p_\perp = \frac{2Ze^2}{bv}$$

and the deflection $\Theta$, if $\Theta \ll 1$, is $\Theta \approx p_\perp/p_{\parallel} \approx p_\perp/p$. Hence

$$\Theta = \frac{2Ze^2}{pv} \frac{1}{b}.$$

Consider a foil of thickness $t$, with $N$ nuclei/cc. The number of collisions between $b$ and $b + db$ in traversing the foil is $2\pi Ntbdb$. Setting the total number of collisions

$$n = \int_{b_{\min}}^{b_{\max}} 2\pi Ntbdb,$$

we then have for the probability of collision $b \rightarrow b + db = 2\pi Ntbdb/n$. The probability of deflection $\Theta \rightarrow \Theta + d\Theta$ is

$$f(\Theta) d\Theta = \frac{2\pi Nt}{n} \left[ \frac{2Ze^2}{pv} \right]^2 \frac{d\Theta}{\Theta^3}.$$
Hence, the probability of deflection per unit solid angle at $\Theta$ is (with $d\Omega = \Theta d\Theta/2$):
\[
\frac{d\Phi}{d\Omega} = \frac{4\pi Nt}{n} \cdot \left[ \frac{2Zze^2}{pv} \right]^2 \cdot \frac{1}{\Theta^4} \quad \text{(Rutherford formula)}.
\]

II. MULTIPLE SCATTERING

The result of a large number of deflections $\Theta_i$ is given by:
\[
\bar{\Theta}^2_t = \sum_i \Theta_i^2 = n \int_{\Theta_{\min}}^{\Theta_{\max}} \Theta^2 \cdot F(\Theta) d\Theta = 2\pi Nt \cdot \left( \frac{2Zze^2}{pv} \right)^2 \int_{\Theta_{\min}}^{\Theta_{\max}} \frac{d\Theta}{\Theta}.
\]

(Strictly, $Z = F(\Theta)$ or $F(b)$ because of screening, as properly treated by Molière.)

Thus,
\[
\bar{\Theta}^2_t = 2\pi Nt \left( \frac{2Zze^2}{pv} \right)^2 \cdot \left[ \log_{\Theta_{\min}}^{\Theta_{\max}} \right] = 2\pi Nt \left( \frac{2Zze^2}{pv} \right)^2 \left[ \log_{b_{\min}}^{b_{\max}} \right].
\]

Note: $\Theta_{rms} = \left( \bar{\Theta}^2_t \right)^{1/2} \propto t^{1/2} \propto 1/pv$

(when $v^2 \ll c^2$, $pv = 2 \times$ kinetic energy).

If the number of individual deflections is very large, the distribution in $\Theta$ (multiple) will be Gaussian.

For large $\Theta_i$, the number of deflections is small; hence there will be a "single scattering" tail.

Limits (classical)

$b_{\max}$ due to screening of nuclear charge by electrons (Thomas-Fermi model):

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\[ b_{\text{max}} \approx a = \frac{a_0}{Z^{1/3}} \left( e_0 \text{ Bohr radius} \right) = 137 \frac{\hbar}{m_e} = 137^2 \cdot r_e, \]

\( b_{\min} \) depends on experimental conditions: for fast particles and thick targets, \( b_{\min} \sim \) nuclear radius; for slow particles and thin targets, the breakdown of electrostatic field at the nuclear radius is not important, and \( b_{\min} \) corresponds to \( \Theta_{\text{max}} \sim 1 \) radian, say. For fast particles, we take \( b_{\min} \approx r_e/2 \cdot A^{1/2} \), for example; then

\[ \frac{b_{\text{max}}}{b_{\min}} \approx 2^{4/3} \cdot \frac{137^2 \cdot r_e}{A^{1/2} \cdot r_e^{2/3}}. \]

(For more details see Rossi: High Energy Particles p. 65 et seq.)

N.B. in the above consideration, \( \bar{\Theta} \) is the space angle. The projected angle (in the plane of the emulsion) will be \( \bar{\Theta}/\sqrt{2} \).

III. THEORIES OF SCATTERING

1. Williams

Williams' theory follows approximately along the lines given above. However, the precise value for \( \Theta_{\min} \) depends in fact on the value of \( \Gamma = \frac{Z \varepsilon}{137 \beta} \), depending on whether the Born approximation (\( \Gamma \ll 1 \)) or the classical case (\( \Gamma \gg 1 \)) is valid.

Thus, in the treatment above (classical):

\[ \Theta_{\min} = \frac{2Z\varepsilon^2}{\nu v} \frac{1}{a}, \]

where \( a = \) Thomas-Fermi radius.
In the Born approximation, the relation between $\Theta$ and $p$ is given by:

$$\Theta_{\text{min}} \approx \frac{\lambda}{a} = \frac{h}{pa};$$

(actually $\Theta_{\text{min}} = 1.75 \cdot \frac{\lambda}{a}$ according to Williams).

Thus:

$$\frac{(\Theta_{\text{min}})_{\text{class}}}{(\Theta_{\text{min}})_{\text{Born}}} = \frac{Zze^2}{pva \cdot \hbar} = \frac{Zze^2}{v\hbar};$$

also

$$\frac{e^2}{\hbar c} = \alpha = \frac{1}{137}.$$

Hence

$$\frac{(\Theta_{\text{min}})_{\text{class}}}{(\Theta_{\text{min}})_{\text{Born}}} = \frac{Zzea}{v} = \left[ \frac{Z}{137} \beta = \Gamma \right].$$

When $\Gamma \gg 1$ $(\Theta_{\text{min}})_{\text{class}} >> (\Theta_{\text{min}})_{\text{Born}}$ we must take classical limits; $\Gamma \ll 1$ $(\Theta_{\text{min}})_{\text{Born}} >> (\Theta_{\text{min}})_{\text{class}}$ we must take quantum mechanical limits.

Accordingly, there are two different formulae, depending on $\Gamma$.

2. Molière

The treatment of Molière solved the scattering problem quantum-mechanically. There is no simple expression valid for all $\Gamma$. However, it was found that the following relation is approximately valid for all $\Gamma$ values ($\gg 1$ or $\ll 1$, and intermediate cases):

$$\Theta_{\text{min}} \approx \frac{h}{pa} \left(1.13 + 3.76 \Gamma^2\right)^{1/2} \approx \frac{2^{1/2} \frac{h}{(137 \lambda_0)^2} \cdot (1.13 + 3.76 \Gamma^2)^{1/2}}{pva \cdot \hbar};$$

When $\Gamma \gg 1$, $\Theta_{\text{min}} = (3.76)^{1/2} \frac{Zze^2}{pva} \approx \frac{2Zze^2}{pva}$ (classical case);
when
\[ \Gamma \ll 1, \Theta_{\text{min}} \approx 1.06 \frac{h}{p_a}. \]

The arithmetic mean multiple scattering angle is given in Molière's theory by
\[ \bar{\alpha} = \delta \cdot L, \]
where
\[ \delta = 2 \frac{e^2 (Nz^2)^{1/2} z t^{1/2}}{p v}; \]
and L (the logarithmic term) is
\[ L = B^{1/2} \left( 1 + 0.982/B - 0.117/B^2 \right), \]
where B is the greater solution of the equation
\[ B - \log_e B = \log \frac{\pi \delta^2}{\Theta_{\text{min}}^2} - 0.115, \]
\[ \log \Omega = \log \frac{\pi \delta^2}{\Theta_{\text{min}}^2} \]
is a measure of the number of collisions which the particles make in traversing the thickness t. L is found to increase slowly with increase of t and with decrease of the velocity, v* (tabulated by Goldschmidt-Clermont).

3. Snyder and Scott

This theory is valid for \( \Gamma \ll 1 \) (Born approximation) only. It is restricted to scattering of particles of \( \beta \sim 1 \) in thin foils (t small), and agrees with Molière's theory for \( \beta \sim 1 \).

*) Since \( \delta/\Theta_{\text{min}} \propto t^{1/2} \), and since \( \Theta_{\text{min}} \) depends on \( \Gamma = 2z/137\beta \), one can see that
\[ \frac{\pi \delta^2}{\Theta_{\text{min}}^2} / t = \frac{\Omega}{t} \]
depends only on v.
4. Summary

The mean angle of multiple scattering may be written as

\[ \bar{\Theta} = \frac{K' Z t^{1/2}}{p v}, \]

where \( K' = 2 e^2 (N Z^2)^{1/2} \cdot L. \)

For a mixed medium such as emulsion, we must use

\[ \Sigma N_i Z_i^2 \]

over all elements in the emulsion.

IV. CO-ORDINATE DEVIATIONS

The co-ordinate deviations may be related to the angular deviation as follows:

Let \( \Theta \ll 1 \) always. The contribution \( y_i \) of the \( i \)th deflection, occurring at distance \( (t - x_i) \) from origin is

\[ |y_i| = \Theta_x \cdot x_i, \quad (\Theta_x = i\text{th deflection}). \]

The total deflection is \( E y^2 = y^2 = E \Theta_x^2 \cdot x_i^2. \)

Now \( \Theta_x \) and \( x_i \) are independent, therefore

\[ y^2 = (1/n) E \Theta_x^2 \cdot E x_i^2 = x^2 \cdot \Theta^2. \]
and
\[ x^2 = \int_0^t x^2 \, dx = t^{\frac{3}{2}}; \]
\[ \int_0^t dx \]
therefore
\[ y^2 = \frac{1}{3} t^2 \cdot \bar{\Theta}_t^2. \]

If \( \bar{\Theta}_t^2 \) is the mean square deflection between tangents per cm of path, then
\[ \bar{\Theta}_t^2 = \bar{\Theta}_o^2 \cdot t \]
and
\[ y^2 = (1/3)t^3 \bar{\Theta}_o^2, \]
where
\[ \bar{\Theta}_o^2 = 2\pi \ln \left( \frac{22\pi e^2}{pv} \right) \cdot \log_e \frac{b_{\text{max}}}{b_{\text{min}}}. \]

Also, \( y\text{ rms} \propto t^{3/2} \propto \bar{\Theta}_o \propto \frac{1}{pv}. \)

V. RELATION BETWEEN CHORD AND TANGENTIAL ANGLES

\[ \Phi_1 \]
\[ \Phi_2 \]

Angle between tangents \( \Theta \)

The mean square angle between tangents on cell length \( c \) is:
\[ \bar{\Theta}_{\text{tang}}^2 = c \bar{\Theta}_o^2, \] where \( \bar{\Theta}_o^2 \) = mean square angular deflection/cm;
and \( y^2 = \frac{1}{3} \theta^3 \theta_0^2 \).

**Angle between chords, \( \alpha $$

\[
\alpha_{\text{chord}}^2 = (\varphi_1 + \varphi_2)^2 = 2\varphi^2; \quad \varphi^2 = \frac{\theta^2}{\sigma^2};
\]

therefore

\[
\alpha_{\text{chord}}^2 = \frac{2\varphi^2}{\sigma^2} = \frac{2}{3} \theta_0^2 = \frac{2}{3} \theta^2 \text{tang}.
\]

**VI. Relation between Arithmetic Mean and RMS Deviations**

For the Gaussian distribution, \( f(\Theta) \, d\Theta = A \, e^{-K\Theta^2} \, d\Theta \); it can then be shown that \( \Theta = 1/\sqrt{2K} \), where \( \Theta^2 = \sigma^2 = 1/2K \).

Hence

\[
\left( \frac{\Theta^2}{\Theta_{\text{rms}}} \right)^{\frac{1}{2}} = \Theta_{\text{rms}} / \Theta_{\text{mean}} = \sqrt{\frac{\pi}{2}}.
\]

**VII. Experimental Measurements of Scattering; Scattering Constant; Constant Cell Size**

Both "tangent" and "chord" methods have been used. Generally, the "chord" method is employed, determining the angle between successive chords by measurement of the track coordinates. For convenience, it is customary to measure the arithmetic mean values of the deflection rather than the rms values. It is customary to measure \( \alpha_0^{\circ} \); this is the arithmetic mean angle between successive chords, 100 \( \mu \) long, measured in degrees and projected on to the emulsion plane. Then

\[
\alpha_0^{\circ} = \frac{K}{p^0}.
\]
K is called the scattering constant. If pΩ is given in MeV/c, K varies slowly from 25 to 30, depending on \( \pi \sigma^2/\sigma_{\text{min}}^2 \), and hence on cell size and velocity. To a very good approximation, \( K \) varies with \( \Omega \), the number of collisions, as \( \log \Omega \), using the Molière theory. There is fair agreement of theory with experiment (± 5%).

![Graph of K versus \( \Omega \) with \( x \times 10^3 \) on the y-axis and \( t \) in units of 100\( \mu \) on the x-axis.]

**VIII. CUT-OFF PROCEDURE**

The theories outlined above apply to averages of the scattering angle over large samples. In a restricted sample of readings, the influence of occasional large deflections (single scattering) in one 'cell' may be very great. Several methods have been proposed to deal with such deflections. We shall deal here with the two simplest methods.

1. **Guillotine cut-off**

The method of measuring \( \bar{\alpha} \), from the deviations between successive chords, is the only method used to any extent to-day. Let \( \bar{d} \) denote this deviation.

\[ d = c \cdot \alpha, \quad (c = \text{cell - length}). \]
The average value of $|d|$ is denoted by $\bar{d}$. For a Gaussian distribution,

$$\bar{d} = \frac{2}{\sqrt{\pi}} \cdot \sigma,$$

where $\sigma$ is the s.d. of the Gaussian. The region $d > 4 \bar{d}$ (i.e. $d > 4 \sqrt{2/\pi} \cdot \sigma$) accounts for

$$e^{-(4\sqrt{2/\pi})^2} \approx 0.25\%$$

of the observations. In fact, experimentally about $1.5\%$ of the observations are of $d > 4 \bar{d}$, due to the effects of single scattering.

For a $d\Theta/\Theta^4$ single scattering distribution, the average of readings $> \Theta_0$ is $\frac{1}{2} \Theta_0$; hence the contribution of $d$-values $> 4\bar{d}$ is $1.5 \times \frac{1}{2} \times 4 \approx 9\%$ of the total deflection. For example, on 1,000 cells (independent readings), the statistical error on the mean value of $\bar{d}$ for the gaussian distribution alone will be $100/\sqrt{1,000} = 3.2\%$. The corresponding statistical error on $\bar{d}$ from the single scattering is easily shown to be:

$$\sqrt{\frac{1}{2} \div \frac{1}{4}} \times 9\% = 2\%$$

(since $\Theta_{ss}^2$ about $\Theta_0$ as origin $= 3\Theta_0^2$, and about $\Theta_{ss}^2$ as origin $= 3/4 \Theta_0^2$).

Thus the contribution to the statistical error from single scatters is important. For the Molière distribution, it can be shown that the optimum value at which to apply the cut-off lies between $3.5$ and $4.5 \bar{d}$, $4 \bar{d}$ being the best.

The "guillotine" cut-off consists of removing all readings $> 4 \bar{d}$. 

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Thus, in the accompanying example $\bar{a} = 6.3$; $4\bar{a} = 25$. Thus the penultimate reading is removed. The new value of $\bar{a} = 5.25$.

On looking for further (new) values of $\bar{a} > 4\bar{a}$, we find none. In all extended series of readings, however, it is generally found that further readings must be removed after the first (cut-off), and the procedure must be repeated, converging rather slowly.

2. "Replacement" cut-off

A better method is to replace all readings $\bar{a} > 4\bar{a}$ by the value $4\bar{a}$, i.e. removing only $4(\frac{3}{2} - 1) \times 1\frac{1}{2} = 3\%$ of the total deflection, instead of $\%$. Thus, in the example, the new value of $\bar{a}$ will be $107/17 = 6.3$ again. This method has the advantage of rapid convergence, (only one stage of cut-off being required), and retaining more of the readings.
IX. STATISTICAL ERRORS

(Chord method)

Let \( n \) = number of independent readings of \( \bar{d} \). Then, the rms deviation on \( \bar{d} \) can be calculated.

1. Gaussian distribution with rms deviation from zero = \( \sigma \)

Let rms deviation of
\[ |d| \] about mean \( (\bar{d}) = \Sigma \), then
\[ \Sigma = \left[ \sigma^2 \left( \frac{\bar{d}}{\pi} \cdot \sigma^2 \right) \right]^{1/2} = 0.60 \sigma. \]
Therefore
\[ \frac{\Sigma}{\bar{d}} = 0.60 \cdot \sqrt{\frac{\bar{d}}{2}} = 0.75 \left( = \frac{\pi - 2}{2} \right). \]

2. Actual distribution

An exact calculation of the rms error, also taking into account single scattering, has been made by Scott, who finds \( \Sigma/\bar{d} = 0.80 \), i.e. the standard error on \( \bar{d} \) is \( \approx 80/\sqrt{n} \%), where \( n \) = number of independent cells.

X. NOISE ELIMINATION

When the energy of a particle is to be determined from the observed deflections, both "real" and "spurious" deflections must be taken into account. Using an acoustical analogy (incorrect, however,), the two sources of deflection are called "signal"

\[ \text{Note: if consecutive second differences are used, they are not independent (see below); this fact increases } \Sigma \text{ by } \approx 1.06 \text{ (Holière).} \]
(genuine scattering), and "noise" respectively. Noise arises due to errors:

i) in the measuring apparatus (microscope stage irregularities);
ii) in the emulsion (deviations of the centres of developed grains from the particle trajectory);
iii) of the observer (reading errors).

One can write

\[ d_{\text{observed}}^2 = d_{\text{signal}}^2 + d_{\text{noise}}^2; \]

or \( d^2 = d_0^2 + n^2 \) \( \text{(1)} \)

1. **Method of cross-products**

Nolière has shown that, if \( d_1, d_2, d_3 \ldots \ldots d_i \) are successive values of the deviation observed, \( d \), then

\[ \frac{d_i d_{i+1}}{d_i^2} = \left( \frac{1 - 2\mu}{4 - 3\mu} \right) \]

\( \text{(2)} \)

where \( \mu = \left( \frac{n}{d_0} \right)^2 \)

And \( d^2 = d_0^2(1 + \mu) \), whence one finds \( n \) and \( d_0 \).

2. **Method using different cell-sizes**

A second method, which is used more often, since it involves less numerical computation, is based on the use of different cell-sizes.

a) Assuming \( n \) independent of cell-size, \( c \):

\[ d_1^2 = d_0^2 \left( \frac{c_1}{100} \right)^3 + n^2, \]

\[ d_2^2 = d_0^2 \left( \frac{c_2}{100} \right)^3 + n^2, \]
where \( d_0 \) = second difference for \( c = 100 \mu \).

Then

\[
d_0^2 = \frac{d_2^2 - d_1^2}{\left(\frac{c_2}{100}\right)^3 - \left(\frac{c_1}{100}\right)^3}.
\]

If \( \alpha_{100\mu}^0 \) is the arithmetic mean angle in degrees between chords of 100 \( \mu \) length, then

\[
d_0 = \frac{\left(\alpha_{100\mu}^0\right)}{\left(\frac{180}{\pi}\right)} \cdot 100 = \left(\frac{\pi}{1.8}\right) \alpha_{100\mu}^0 \text{ microns},
\]

Thus,

\[
\frac{\alpha_{100\mu}^0}{\pi^0} = \frac{1.8}{\pi^0} \sqrt{\frac{(d_2)^2 - (d_1)^2}{\left(\frac{c_2}{100}\right)^3 - \left(\frac{c_1}{100}\right)^3}}.
\]

and \( p_0(\text{MeV}/c) = K/\alpha_{100\mu}^0 \), \( (K \approx 25) \), where the deviations \( d \) are in microns, and the cell-lengths \( c \) are in microns. In the above, we have neglected the variation of \( K \) with \( c \). If \( c_2/c_1 \) is not too large \((< 3)\), the error is negligible. The value of \( K \) employed corresponds to the cell-length \( c_2 \) [i.e. the larger cell-length, since this provides most of the signal \((d_2^2 >> d_1^2)\)].

**Ratio of \( c_2/c_1 \); choice of cell-length**

The above procedure of noise elimination will lead to significant results only if \( d_2^2 >> d_1^2 \). Usually, \( c_1, c_2, c_3 \ldots \) are in integral ratios e.g. \( c_1 = 100 \mu \), \( c_2 = 200 \mu \), \( c_3 = 300 \mu \) etc.

If \( n \) is very small, \( d_2^2 = 2^2 \cdot d_1^2 = 8d_1^2 \). 

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If noise and signal are equal for \( c_1 \), i.e. \( n = d_1/\sqrt{2} \), then \((d_2)_{\text{signal}} = 2 \cdot \sqrt{2} \cdot d_1/\sqrt{2}; (d_3)_{\text{noise}} = d_1/\sqrt{2} \), hence
\[ d_2^2 = 4.5 \cdot d_1^2. \]
Usually, the rule \( d_2^2 > 3d_1^2 \) is used. \( d_2^2 \approx 3d_1^2 \) gives approximately the optimum value of \( c_2/c_1 \), when noise and signal on the "prime" cell-size \( c_1 \) are nearly equal. This condition is realized as follows: an approximate value of the energy of the particle can be obtained by a few observations with large cell-size, \( c \). With a knowledge of the noise level likely to occur, (typically \( d_{\text{noise}} \approx 0.1\mu \)), one can calculate the value of \( c_1 \) for which
\[
\frac{(\text{signal})}{(\text{noise})} \approx 1.
\]

Suppose one finds \( c_1 = 72\mu \). Then the use of \( c_1 = 100\mu \) would be suitable, whereas \( c_1 = 50\mu \) would be too small. Large values of \( c_2/c_1 \) should be avoided, since this will result in large statistical errors, the number of independent cells being \( \propto 1/c_2 \).

b) \( n \) dependent on cell-size

In actual measurements, it is found that the noise always increases with cell length, \( c \). For moderate values of the ratio \( c_2/c_1 \), \( n \propto c^m \). Thus:
\[
\begin{align*}
d_1^2 &= d_0^2 \cdot c_1^3 + n_0^2 \cdot c_1^{2m}, \\
d_2^2 &= d_0^2 \cdot c_2^3 + n_0^2 \cdot c_2^{2m},
\end{align*}
\]

\[
c_1^{2m} d_2^2 - c_2^{2m} d_1^2 = d_0^2 \left[ c_2^3 c_1^{2m} - c_1^3 c_2^{2m} \right];
\]
or
\[
d_0^2 = \left[ \frac{d_2^2 - (c_2/c_1)^{2m} d_1^2}{c_2^3 - (c_2/c_1)^{2m} c_1^3} \right].
\]
The value of \( m \) depends on the microscope stage movement. The stage can be calibrated using a Fabry - Perot interferometer, and for most microscopes, with \( 5 \text{ mm} > c > 100 \mu \), \( m \approx 1/4 \).

Spurious scattering (see below) is such that the "noise level" is found to increase more rapidly than \( c^{1/4} \), in some cases even as \( c^{1/2} \) or \( c^{3/4} \). We shall deal with this feature in more detail later on.

Thus, noise = "reading errors" + "stage errors"
\[ \propto c^0 \propto c^{1/2} \]

Actually, \( n \propto c^{1/4} \) to \( 1/2 \) when these two effects are taken into account.

XI. DEVIATIONS BY CO-ORDINATE DIFFERENCE METHOD

The easiest method for determining the deviations between successive chords is by the co-ordinate difference method.

The \( y \) co-ordinates (in the eye-piece scale) are read off at intervals, \( c \). Then

\[ d_1 = (y_3 - y_2) - (y_2 - y_1) = y_3 - 2y_2 + y_1. \]
First difference  Second difference  Third difference
\[ y_1 \quad y_2 - y_1 \rightarrow y_3 - 2y_2 + y_1 \rightarrow y_5 - 2y_4 + 2y_2 - y_1 \]
\[ y_2 \quad y_3 - y_2 \rightarrow y_4 - 2y_3 + y_2 \rightarrow y_6 - 2y_4 + 2y_3 - y_2 \]
\[ y_3 \quad y_4 - y_3 \quad y_5 - 2y_4 + y_3 \]
\[ y_n \quad y_{n+1} - y_n \quad y_{n+2} - 2y_{n+1} + y_n \]

Notice:

i) that consecutive values of second difference are not strictly independent (see below);

ii) that, if \( \epsilon \) represents the contribution of "noise" to a reading \( y \), the contribution to the second difference is

\[
\frac{1}{\sqrt{2}} \sqrt{(1)^2 + (2)^2 + (1)^2} = \epsilon \cdot \sqrt{6};
\]

for higher-order differences, e.g. third difference, the noise contribution is even greater - in the case shown, it is \( \epsilon \sqrt{10} \), i.e. \( \sqrt{\frac{5}{2}} \) times as great as for second difference.

XII. CORRELATION BETWEEN SECOND DIFFERENCES (CO-ORDINATE METHOD)

Adjacent values of the second difference, using the co-ordinate method, are not statistically independent, since they have one reading in common. If \( \alpha_i \) denotes the angle between two chords (= \( \bar{d}_i / c \)), then \( \alpha_i \alpha_k = 0 \) when \( |i-k| > 2 \), and \( \alpha_i \alpha_i = \alpha^2 \).

To find \( \alpha_i \alpha_{i+1} \), consider a track of \( n \) cells of length \( c \) each.

Then \( \alpha^2 = 2/3 \cdot \tan_b^2 \) with the above notation.

The tangential angular deflection, \( \Theta_t \), is given by

\[
\Theta_t^2 = (nc) \tan_b^2 = (\alpha_1 + \alpha_2 + \alpha_3 + \ldots + \alpha_n)^2
\]

\[
= \sum \alpha_i^2 + 2 \sum \alpha_i \alpha_{i+1} + 0
\]

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where \( 0 \) denotes cross products of type \( a_i a_k \), with \( |i - k| > 2 \). Thus, the average of these terms is zero.

\[ \chi_i \chi_i +1 \text{ correlated} \]

\[ \chi_i, \chi_i + 2 \text{ uncorrelated} \]

Hence,

\[
n c \Theta_\delta^2 = \frac{2}{3} n c \Theta_\delta^2 + 2 n \bar{a}_i \bar{a}_{i+1} ;
\]

or,

\[
\bar{a}_i \bar{a}_{i+1} = \frac{c \Theta_\delta^2}{6} = \frac{\bar{a}^2}{4} .
\]

Thus, we have:

\[
\bar{a}_i \bar{a}_k = \bar{a}^2 , \text{ for } i = k;
\]

\[
= \frac{\bar{a}^2}{4} , \quad |i - k| = 1 ;
\]

\[
= 0 , \quad |i - k| > 1 .
\]

The existence of finite cross-products between adjacent terms \( a_i, a_{i+1} \) is typical of the co-ordinate method; it does not apply to the angular method, for example.

One can proceed similarly for computing correlations between adjacent third differences \( \beta \), fourth differences \( \gamma \), etc. We shall omit the algebra and simply quote the results. The essential feature is that the readings may be taken as independent
(superposition of two independent Gaussians) if the cross products are zero for the terms in the column from which the differences are taken. If one uses differences of non-independent readings, however, correlations are introduced. Denoting \( \epsilon/\sqrt{6} \) as the noise level in one co-ordinate reading on the track, one has:

\[
\begin{align*}
\text{Noise} & \\
\text{Independent differences} & \epsilon & \frac{\sqrt{5}}{\sqrt{3}} \epsilon & \frac{\sqrt{11}}{\sqrt{3}} \epsilon & \ldots \\
\text{Consecutive differences} & \epsilon & \frac{\sqrt{10}}{\sqrt{3}} \epsilon & \frac{\sqrt{25}}{\sqrt{3}} \epsilon & \ldots \\
\text{SIGNAL} & \\
\text{Independent differences} & \bar{a} & \sqrt{2} \bar{a} & \sqrt{4} \bar{a} & \\
\text{Consecutive differences} & \bar{a} & \sqrt{\frac{3}{2}} \bar{a} & \sqrt{4} \bar{a} & \\
\end{align*}
\]

Denoting the signal/noise ratio by \( R \), one thus obtains (in units of \( \bar{a}/\epsilon \)):

\[
\begin{align*}
\text{Noise} & \\
\text{Independent differences} & 1 & \frac{\sqrt{6}}{\sqrt{5}} & \frac{\sqrt{12}}{\sqrt{11}} & \ldots \\
\text{Consecutive differences} & 1 & \frac{\sqrt{2}}{\sqrt{20}} & \frac{\sqrt{32}}{\sqrt{35}} & \ldots \\
\end{align*}
\]

Notice, for example, that \( R \) is much lower (by a factor \( \sqrt{6}/\sqrt{5} \approx 1.7 \)) for consecutive 3rd differences as compared with independent ones. Generally, \( n \)th differences are independent \( (\varphi_i \varphi_k = 0) \) when \( |1-k| > 2^{n-1} \) (e.g. for 2nd difference \( 2^{n-1} = 2 \), 4th difference \( 2^{n-1} = 8 \) etc.), where the indices \( i, k \), refer to the original co-ordinate readings.

XIII. OVERLAPPING CELLS

In the manner indicated above for calculating ratios of 2nd, 3rd, 4th differences etc., it is possible to calculate the standard deviations of the distributions of differences - in particular...
calculated with and without overlap. The problem was treated by Molière, who calculated the correlation coefficients for different degrees of overlap.

Thus, scattering measurements are made on the prime cell-size $c_1$, for which there are, say, $n$ readings of $y$, $(n-1)$ 1st differences and $(n-2)$ 2nd differences. There will be, (for $n >> 1$), $n/2$ statistically independent differences for cell $c_2$, $n/3$ for $c_3$, i.e. $n/\lambda$ where $\lambda$ = degree of overlap. Thus, for $c_3$, taking all differences, there are $\approx n$ readings, with three-fold overlap (i.e. every third difference is independent).

Molière has shown that, for all degrees of overlap, the accuracy of $\ddot{\alpha}$ depends on the number of independent cells $(n/\lambda)$. There is hardly any advantage to be gained by taking the overlapping readings, since they are so strongly correlated with the independent readings. Taking the errors on independent cells as $K/\sqrt{N}$ (where $N$ = number of independent cells, and $K \approx 0.8$), one finds, for various values of $\lambda$, that the error is

$$\frac{K}{\sqrt{\frac{\lambda}{n}}}$$

where $n$ = number of "prime" cells, and $n/\lambda = N$ = number of independent cells. $f$ is given as follows:

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f$</td>
<td>1</td>
<td>1.022</td>
<td>1.046</td>
<td>1.078</td>
</tr>
</tbody>
</table>

Thus, there is little to be gained in accuracy from the use of overlapping cells.
XIV. DISTORTION

1. Form of distortion

Suppose the track due to a particle of very high energy (scattering negligible) shows distortion, i.e. the y co-ordinates (and possibly the x co-ordinates) of the developable crystals change during processing. Then, in the unprocessed emulsion we would have:

\[ y = a' + b'x \]

where \( a' \), \( b' \) are constants; and in the processed emulsion, one can represent the track by a polynomial:

\[ y = a + bx + cx^2 + dx^3 + ex^4 + fx^5 + \ldots \]

2. First and higher order distortion (\( n = \text{order} = \text{index of last term in polynomial} \))

First we neglect changes in length of the track (x).

For first-order distortion (change in direction but not in shape):

\[ c = d = e = f = \ldots = 0; \]

\[ \frac{dy}{dx} = b \quad \text{and} \quad \frac{d^2y}{dx^2} = 0. \]

Thus the second difference would represent scattering only. Similarly, when one has second-order distortion, the third difference represents scattering only. In general, the \( n \)th-order distortion is eliminated by taking the \((n+1)\)th order differences.

There are two types of distortion.

i) Gross distortion

The distortion vector \( \vec{\nabla} \) varies regularly in magnitude and direction. Lines of constant \( |\vec{\nabla}| \) are smooth curves as shown; the direction \( \vec{\nabla}/\nabla \) is perpendicular to the contour lines.
ii) Spurious scattering

This is produced by irregular variations in $\vec{v}$.

[Hydrodynamic analogy: i) "laminar flow", ii) "turbulent flow".
(Reynolds number is not known however!)]

3. Uniform shear

In this case either:

i) the distortion vector is constant in direction and changes linearly in magnitude with depth in the emulsion, or

ii) has constant magnitude but changes linearly in direction with depth.

In this case, the originally straight track is bent into a curve; usually such a curve is "C" shaped, or occasionally "S" shaped (reversal of shear vector).

4. Localized distortion

This gives rise to spurious scattering, taking place in a particular plane parallel to the emulsion surfaces - see below.
5. Methods of elimination of distortion from scattering measurements

i) Methods applicable when the distortion is circular

In the case of C-distortion, where the magnitude is not too great, the profile of the track can be approximated by the arc of a circle; i.e. the curvature \( \approx d^2 y/dx^2 \) is constant (i.e. \( y = a + bx + cx^2 \)).

Method of algebraic mean

In calculating the value of \( \bar{d} \) above, we have used the formula

\[
\bar{d} = \frac{1}{n} \sum (d_i - 0)
\]

where \( d_i \) are the deviations from the mean (0) of normal second differences. An alternative quantity is the deviation from the sample (algebraic) mean:

\[
\bar{d}^T = \frac{1}{n} \sum (d_i - \frac{1}{n} \sum d_i)
\]

The deviation, \( D \), due to distortion, if circular, is

\[
D = c \left( 180 - 2(90 - \epsilon/2) \right) = c\epsilon = c^2/\rho.
\]

Also \( b = \rho \left( 1 - \cos \eta \right) \approx \rho \eta^2/2 \)

\( (\eta \ll 1) \), and \( \rho = 2\eta \rho \), therefore

\[
\rho = \ell/2\eta; \quad \text{therefore}
\]

\[
b = \frac{2\eta^2}{2\eta^2} \frac{2\eta}{2} = \frac{2\eta}{2}.
\]

i.e. \( \eta = 4b/\ell \).

Hence,

\[
D = c^2/\rho = 2c^2\eta/\ell = 2c^24b/\ell^2 = 8b(a/\ell)^2 = 8b/n^2
\]

where \( n = \) number of cells.

In an actual case, observations of \( b \) must be made on steeply dipping tracks in the neighbourhood, and one must make sure that use of the algebraic mean does not lead to wrong results, purely because of statistical fluctuations.
Example:
\[ d_i = -4, -1, -4, +1, -6, -5, -1, +3, -2; \]
\[ \frac{1}{n} \cdot \Sigma d_i = -1.9; \]
therefore
\[ \bar{d} = \frac{1}{n} \left[ \Sigma |d_i - 0| \right] = 2.7; \]
\[ \bar{d'} = \frac{1}{n} \Sigma |d_i - \frac{1}{n} \Sigma d_i| = 2.3. \]

A general rule

DISTORTION IS IMPORTANT ON STEEP TRACKS WITH SMALL n (number of cells); THEREFORE IT IS IMPORTANT NOT TO CONFUSE LARGE STATISTICAL FLUCTUATIONS WITH GENUINE DISTORTION; INDEPENDENT EVIDENCE (on other tracks) FOR EXISTENCE OF DISTORTION SHOULD BE OBTAINED BEFORE TRYING TO ELIMINATE IT.

Method of third differences (see above)

Note that, because of increase in noise/signal ratio, using adjacent second differences, it may be necessary to employ a large cell size, and consequently reduced statistical accuracy. Using well-separated differences, to reduce correlations, will also result in a smaller number of cells [the reduction in the number of cells (compared with second difference) is \( 2^{n-1} - 2 \)].

ii) Methods applicable when distortion is not circular

Higher-order differences

For a distortion represented by a polynomial of order \( n \), \((n + 1)^{th}\) differences must be taken. There is usually little justification for proceeding beyond \( 4^{th} \) differences.

Profile method

A better method is the so-called "profile method" particularly applicable in emulsions exposed to accelerator beams. To apply this method, it is necessary to find a steep track,
parallel and close to the track being measured.

\[ S \sim g \]

The profile track must be measured on a very small cell-size; even then some of the readings must be interpolated when "scaling-up" the readings to the actual track being measured.

6. Distortion due to local variations - "spurious scattering"

In addition to "gross" distortion, which depends on differential shearing, which varies with depth, there are local effects which may occur in tracks irrespective of angle of dip. In such a case, the distortion can be represented by a random vector; the magnitude of the displacement is of the order of \(< 5\mu\). Such movements are usually referred to as "spurious scattering", and result, it is thought, from the same process as is responsible for "shifting" of tracks, when one layer of emulsion slides over another. In the case of spurious scattering, the emulsion is strained (e.g. in course of severe treatment during processing and drying) beyond the elastic limit, and plastic flow results.

\[ \begin{array}{c}
\text{600} \\
\text{\mu}
\end{array} \]

The effect is important chiefly for short cell-lengths (of the order of the emulsion thickness, 600 \(\mu\)). Over distances \(> 1 \text{ mm}\), the over-all deflections do not increase appreciably with cell size. The presence of spurious scattering, if severe, prevents any meaningful
scattering measurements, unless nearby comparison tracks of known energy are available (which is now usually the case in machine exposures).

Examples of measurements of the magnitude of spurious scattering:
Bombay group gave \( \approx 5\mu \) at \( 1000\mu \) cell length;
Bristol group obtained \( 0.18\mu \) at \( 500\mu \),
\( 0.45\mu \) at \( 2000\mu \).

For recent results see the Copenhagen report. The measurements were carried out by comparing scattering on
  i) single tracks of \( 4.5 \text{ GeV } \pi^- \);
 ii) relative scattering on neighbouring tracks, < 20\( \mu \) apart.

The spurious scattering can be taken into account by assuming noise varying as a power of the cell length, i.e. \( n \propto \alpha^m \), where the value of \( m \) must be found by calibration; \( m \) is usually \( \frac{1}{2} \) or \( \frac{1}{3} \) for cell lengths 100 - 1000\( \mu \).

XV. CONSTANT SAGITTA MEASUREMENTS

So far, we have considered scattering measurements when the change of velocity of the particle is small along the observed length of track. This is true only for fast particles.

The change in scattering in the interval \( \Delta R \) is given by
\[
\frac{\Delta a}{a} = 0.58 \frac{\Delta R}{R}, \quad (0.58 = \text{index of the range energy relation}).
\]
Thus, if \( \Delta a/a < 10\% \), \( \Delta R < R/6 \).

When \( \Delta R \ll R \), the use of a constant cell size leads to difficulties; for example, the statistical weight of the observations varies along the track, and one must employ a "sliding" cut-off:
\( 4, \tilde{a} = f(R) \). A better method is to use a choice of cell size such that the second differences (sagittas) remain nearly constant.

When the constant-sagitta method is used, the particle is of such velocity that the range-energy relation can be approximated
by a power law.

Then $E \propto R^n$, $n = 0.58$ (valid up to a proton energy of ~ 300 MeV, and $\pi$ energy of 50 MeV).

This may be more generally written

$$f(v) = \left( \frac{R}{m} \right)^n = \left( \frac{E}{m} \right)^n, \quad \left( \frac{E}{m} \right)^2 < 0.3,$$

since the energy loss $dE/dR$ is a function of $v$ only.

When $E \ll m^2$, one can also write $p\beta c = pv = mv^2 = 2E$

(where $E =$ kinetic energy, $m =$ mass of particle).

Then,

$$\frac{-\alpha}{v^2} = \frac{K'}{R} = K'' = \frac{K''}{R} = \frac{K''}{R} \cdot \frac{1}{1 \cdot \frac{A(R/m)^n}{m^{1-n} R^n}} = \text{constant}.$$

The sagitta $\bar{D}$, on a cell length $a_R$ at range $R$ is given by:

$$\bar{D} = \text{const.} \cdot a_R^{3/2} \cdot \frac{1}{m^{0.42}} \cdot \frac{(a_R)^{3/2}}{R^{0.58}} \cdot K(v, \theta),$$

where $K$ = the scattering constant, and is a slowly varying function of cell size, $a_R$, and velocity i.e. $R/m$. If we take $K \approx \text{const.}$, i.e. do not consider large variations of velocity and $a_R$, then

$$\bar{D} \propto \frac{1}{m^{0.42}} \cdot \frac{1}{R^{0.58}}.$$

If $\bar{D}$ is to be independent of cell length, then $a_R \propto (a_R)^{0.58} \propto R^{0.59}$.

We therefore choose a particle of given mass e.g. $m = m_{\text{proton}}$ and a value of $\bar{D} = 1 \mu$, say, and then calculate the value of $a_R$ required for a given large value of $R$. Then working back along the track (towards the end), we calculate new values of $a_{R2}, a_{R3}, \ldots$ by trial and error. Similarly for a particle of any other mass, a scattering scheme can be computed.
For a particle of unknown mass \( m \), the scheme may be used, when

\[
\frac{\bar{d}}{d_{\text{proton}}} = \left( \frac{p}{m} \right)^{0.42}
\]

whence \( m \).

Then a new scheme is calculated for \( m \) (e.g. a K-meson).

**PRECAUTIONS**

In the constant-sagitta method, it is essential to keep the track well aligned (i.e. first differences small) since, because of the changing cell size, a badly aligned track will give a finite second difference even if it is straight.

The necessary condition is that \( \delta \Delta c \ll \text{scattering} \) (where \( \delta \) = angle of misalignment, \( \Delta c \) = change of cell length). When the scheme is applied to tracks very near the end of the range (\( R < 100 \mu \)), it is not possible to keep the track well aligned, but good results can still be obtained by using calibrations with particles of known mass.

**Errors**

Notice that if \( n \) = number of cells, the error on \( \bar{A} = 80/\sqrt{n} \% \) and hence the error on \( m \) is

\[
\frac{80}{0.42 \sqrt{n}} \% \approx \frac{200}{\sqrt{n}} \%.
\]

For particles of 150 \( \mu \) range, \( n \approx 10 \) and the error is \( \approx 60\% \).

* * *

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DISCUSSION

Lock : One has to be careful about equating noise with spurious scattering. In terms of second differences (for measurements on single tracks), spurious scattering is defined as the difference between the measured total signal and the theoretical signal (given by the momentum of the particle which produces the track) together with all known sources of noise. In other words:

\[ \frac{D^2}{\text{meas}} = D_c^2 + D_\epsilon^2 + D_y^2 + D_{ss}^2 \]

where

- \( D_c \) = measured second difference,
- \( D_c \) = theoretical signal from Coulomb scattering,
- \( D_\epsilon \) = irreproducible stage noise + grain and reading noise,
- \( D_y \) = reproducible stage noise,
- \( D_{ss} \) = "spurious scattering" originating in the emulsion.


Bott-Bodenhausen : How does the theoretical value of the scattering constant compare with the measured value?

Perkins : Although the theory involves many approximations the agreement with experiment is good to within \( \sim 5 - 10\% \).

Grote : In the case of a small ratio of \( D_{\text{signal}}/D_{\text{noise}} \) the method of overlapping cells is useful. Although the statistical weight of the measurement...
Grote: does not rise very much, the correction for the noise will be much better in this case because one has two or four times as many measured values, and the "individual" noise for the corresponding track is a better approximation to the mean value of the noise used for the correction.

Perkins: I agree.

Evans: What grain noise does one expect from a track of high energy (i.e. a straight track)?

Perkins: The centres of AgBr grains can lie ~ 0.1 μ on either side of the track. Assuming that on development the position of the centres of the developed grains is unchanged, their spread will be the same. Because some grains will be closer to the track than this, an average value < 0.1 μ is to be expected.

Evans: In fact, on measurements I have made of co-ordinates of grains, and fitting a straight line to them, I get a mean value of 0.04 μ in Ilf emulsion.

Doble: Referring to the expression given by Dr. Lock:

\[
D_{\text{meas}}^2 = D_0^2 + D_\epsilon^2 + D_Y^2 + D_{SS}^2,
\]

it may be mentioned that the use of a particular kind of overlapping cells, i.e. displaced cells (Holière), can be used to eliminate the noise, leaving only the spurious scattering contribution \(D_{SS}^2\) in addition to \(D_0^2\).

* * *
SCANNING METHODS AND STATISTICS OF EMULSION MEASUREMENTS

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I. INTRODUCTION

Let us suppose that an emulsion stack has been exposed and processed. The stack is now a mine of information and the problem is how best to extract this information from the emulsions. This will usually be done for us by trained scanners and we must consider the way in which they will do their work.

II. SCANNING METHODS

In trying to locate events of interest - decays, interactions etc. - in a nuclear emulsion that has been exposed to some kind of radiation, two types of scanning method may be employed. In the "area scan" method a whole volume of emulsion is examined for a specific type of event. In the "line scan" method individual tracks satisfying certain criteria are first selected and these are followed in order to find out whether the particle producing the selected track has decayed or interacted during its passage through the emulsion.

For example, suppose one were wishing to study the production of hyperfragments by a beam of fast π mesons. The probability is very low. Only about one in every 1,000 π meson interactions may produce such an event. When it does occur, however, the appearance of the event is quite striking, producing two interactions usually less than 100 μm apart and joined by a black track. Such 'double stars' can be readily detected in an area scan. It would, however, be a task
involving great effort to find these events by following every $\pi$ meson along its track to see whether it interacted and produced a hyper-fragment. The time required for a line scan could easily be orders of magnitude greater than for an area scan. This is largely because the type of event being sought has such a characteristic appearance.

On the other hand, suppose one was studying the elastic scattering of very high energy protons by free protons in the emulsion. The scattering is predominantly through very small angles, some tens of milliradians at the most, at CERN PS energies. The proton recoil track is short and could easily be missed if its angle of dip is large. Under these circumstances, each field of view would need very careful examination in an area scan and even then many events may be missed, while line scanning can be carried out very rapidly so as to compensate for the great length of track that has to be followed.

Even in some cases where very characteristic events are being sought, it may be advantageous to use a line scanning method. For example, if one is looking in a certain region of the plate for interactions at rest of $K^-$ mesons in a beam of negatively charged particles that contain a much greater number of $\pi^-$ than $K^-$ mesons, many of the stars observed will be due to the unwanted $\pi^-$ mesons and it may take some time to distinguish them. If, however, the individual tracks are examined at a distance of say 1 cm from the position at which the $K^-$ mesons are expected to stop, the $K$ track will have a specific ionization corresponding to $g_s = 3.8 \ g/\text{cm}$, which is readily distinguishable from that of a $\pi^-$ track of 1 cm residual range ($g_s = 2.2 \ g/\text{cm}$). It may then sometimes be quicker to select the tracks on an ionization criterion of this kind and follow them to the end of their range.

Line scanning is generally to be preferred in quantitative work such as the measurement of cross-sections for processes of a specified type, the measurement of branching ratios for different types of processes, etc., since in general it is far freer from biases. We return later to the discussion of the effects of biases.
III. BIASES IN EMULSION MEASUREMENTS

Emulsion measurements rely on the efficiency of the observer. Since there is always the possibility of missing wanted events, it is very necessary to discuss questions of biases and methods of identifying them and estimating the necessary corrections for them.

Biases arise from subjective factors, from lack of concentration, weariness; or from factors associated with the emulsion and its processing - low and uneven sensitivity, uneven development and so on. Some of the most common examples which occur are due to the loss of lightly ionizing tracks near the surface or the bottom of an emulsion pellicle or to the loss of tracks which have a large angle of dip.

Table 1 shows an example of the type of bias that may be met with arising from non-uniform development throughout the emulsion. It shows the depth distribution of \( \pi \) mesons emitted from \( K^- \) stars at different depths in the emulsion. The mesons are divided into two samples according to whether their kinetic energy is less or greater than 60 MeV.

<table>
<thead>
<tr>
<th>Depth in the emulsion (( \mu m ) above glass)</th>
<th>0 - 100</th>
<th>100 - 200</th>
<th>200 - 300</th>
<th>300 - 400</th>
<th>400 - 500</th>
<th>500 - 600</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N_\pi/60 ) MeV ( N_K )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \pi^- (&lt; 60 ) MeV) ( N_K )</td>
<td>10.4 ( \pm ) 1.5</td>
<td>13.4 ( \pm ) 1.7</td>
<td>10.1 ( \pm ) 1.4</td>
<td>13.1 ( \pm ) 1.7</td>
<td>12.7 ( \pm ) 1.7</td>
<td>10.4 ( \pm ) 1.6</td>
</tr>
<tr>
<td>( \pi^- (&gt; 60 ) MeV) ( N_K )</td>
<td>12.2 ( \pm ) 1.6</td>
<td>15.0 ( \pm ) 1.8</td>
<td>17.7 ( \pm ) 2.0</td>
<td>23.0 ( \pm ) 2.3</td>
<td>15.1 ( \pm ) 1.9</td>
<td>15.5 ( \pm ) 2</td>
</tr>
</tbody>
</table>

There is clear evidence here of the loss of \( \pi \) mesons. For low energy \( \pi \) mesons there seems some loss at the bottom and the top of the emulsion. This can probably be satisfactorily corrected for by
omitting the top and bottom 50 μm of the emulsion. For fast π mesons (> 60 MeV), however, the π meson proportion appears to be a maximum near the middle thickness of the plate. One could perhaps try to correct it by including only the central 200 μm of the emulsion. This, however, would greatly reduce the statistical significance of the measurements.

The above measurements are taken from a stack of G5 emulsions studied by the European K− collaboration. These same workers later examined a stack of K5 emulsion containing stopping K− mesons. In this stack the π mesons were easier to see because the grain density for minimum ionization was greater and the background was smaller. The over-all ratio of π/K mesons found was markedly larger than that found in any previous experiment.

The π/K ratio observed in five experimental investigations of stopping K− mesons is shown in Table 2.

<table>
<thead>
<tr>
<th>Authors</th>
<th>π/K ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bachilla et al.</td>
<td>30%</td>
</tr>
<tr>
<td>Chadwick et al.</td>
<td>28.2%</td>
</tr>
<tr>
<td>Eisenberg et al.</td>
<td>32</td>
</tr>
<tr>
<td>European Collaboration (G5)</td>
<td>29.6%</td>
</tr>
<tr>
<td>European Collaboration (K5)</td>
<td>39.2%</td>
</tr>
</tbody>
</table>

The fraction observed in the last work was greater than that observed at any depth of the emulsion in the case of the G5 stack, indicating there had been losses at all depths in the previous work. Further, it was found that the losses were particularly large in the case of K− stars having a large number of baryon prongs. Whilst for all K− stars about one third of the π mesons were missed in the earlier work, for stars containing 3, 4 or 5 stable prongs, one half of the π mesons were missed, indicating a bias against observing lightly ionizing π mesons in large stars.
As already pointed out, area scanning generally introduces larger biases than line scanning. For example, if one were measuring the distribution in size of stars produced by interacting particles at rest, an area scan would be likely to produce a distribution biased towards large stars since the efficiency of detecting small stars will be smaller than for large stars. If the particles are selected on the basis of their grain density at some distance from their end point, however, an unbiased distribution would be expected.

IV. EFFICIENCY OF SCANNING

Under some circumstances it is possible to correct for observational bias by estimating an efficiency factor for individual scanners. The same sample is scanned by two observers, A and B, with efficiency factors $\eta_A$, $\eta_B$ respectively.

Let $N$ be the total number of events present in the sample, and $n_1$ the number of events which both observe, $n_2$ the number of events which A observes but not B and $n_3$ the number B observes but not A.

Then

$$n_1 = \eta_A \eta_B N$$
$$n_2 = \eta_A (1 - \eta_B) N$$
$$n_3 = \eta_B (1 - \eta_A) N$$

so that

$$\eta_A = \frac{n_1}{n_1 + n_3} ; \quad \eta_B = \frac{n_1}{n_1 + n_2}$$

and

$$N = \frac{(n_1 + n_2) (n_1 + n_3)}{n_1} .$$

This method assumes that the events missed are purely random. In practice, however, both scanners will tend to miss the same events because some are more difficult to observe than others. If physical factors can be identified which make one class of event more difficult
to observe than another, separate efficiencies can be estimated for each class of event. For example, the efficiency of observing $\pi$ mesons coming from $K^-$ stars could be estimated in different energy ranges of the $\pi$ meson or for different ranges of dip. The time required to determine the efficiency factor for each observer might then become an important consideration, however, because of the need to collect sufficient statistics.

It is assumed that the efficiency of a given observer is a constant quantity. It is known, however, that extraneous factors such as tiredness, boredom or other pre-occupations may affect the scanning efficiency of an observer. One test of the meaningfulness of an estimated scanning efficiency consists of employing three scanners on the same material. The efficiency of each scanner calculated in the above way against either of the other two should be the same.

V. OPTIMUM SCANNING SPEED

The slower the speed of scanning, the greater in general will be the scanning efficiency. For example, in the experiment referred to above to determine the fraction of $K^-$ captures at rest from which $\pi$ mesons are emitted, great care was taken to minimize the loss of $\pi$ mesons. A given range of azimuthal angle around each $K$ star was carefully examined for a specified time before passing on to the next range of azimuthal angle which was subjected to a similar careful scrutiny. This procedure greatly increased the time of scanning but was rewarding in that it led to a 30% increase in the proportion of $\pi$ mesons observed.

In planning an experiment, a balance has to be struck between the time involved and the extra information obtained. A plot of scanning efficiency against speed of scanning would be expected to look like the following.
If it is decided to observe \( N \) events of a given type, it is reasonable to adjust the scanning speed so that the loss is approximately equal to the fractional statistical error, i.e.

\[
1 - \eta = \frac{1}{\sqrt{N}}.
\]

VI. STATISTICS OF EMULSION MEASUREMENTS

In obtaining quantitative results using visual techniques, one is faced with the problem of deducing the value of certain quantities from a restricted sample of data. It is necessary, therefore, to make an estimate of the limits of error of the determination. Of course, systematic errors may be present as in other physical experiments and there may be certain biases in the selection of events. In this section, however, is discussed the uncertainty due to the limited size of the sample. To obtain the 'best' estimate of a certain quantity and the likelihood that the true value of the quantity falls within a region of specified extent around this 'best' estimate, it is necessary to consider a little elementary statistics.

1. Types of distribution of measurements of physical quantities

The two most important distributions that have to be considered are the Poisson distribution and the normal (or gaussian) distribution.
Suppose a beam of $N$ unstable particles of velocity $v$ and of mean life $\tau$ is passing through an emulsion and the path of each in the emulsion is $L$. Then on the average the number of decays that will be seen in the emulsion will be

$$\bar{n} = \frac{NL}{v\tau} \text{ provided } L \ll v\tau.$$

The probability $P_n$ that in an actual experiment a number $n$ of decays is seen is

$$P_n = \frac{\bar{n}^n}{n!} e^{-\bar{n}}. \quad (1)$$

This distribution is known as the Poisson distribution.

Suppose a particle is passing through an emulsion and is undergoing multiple scattering. We may divide up its path into a large number of cells, each of the same length. Let $\bar{\Theta}$ be the root mean square angle of multiple scattering. Then to a certain approximation, provided we consider the observed scattering in any cell to be due to a large number of very small scatters, the probability $P_\Theta d\Theta$ that the deflection is between $\Theta$ and $\Theta + d\Theta$ is given by

$$P_\Theta = \frac{1}{\sqrt{2\pi} \bar{\Theta}} \exp \left\{ -\frac{\Theta^2}{2\bar{\Theta}^2} \right\} \quad (2)$$

A quantity distributed like $\Theta$ is said to be normally distributed. The distribution (2) is about a mean deflection of zero. This is because in the multiple scattering problem deflections are as probable to the left as to the right. In this case $\bar{\Theta}$ is the rms value of $\Theta$. More generally we have a situation in which the quantity concerned, $x$, has the mean value $x_0$. Then if $\sigma^2$ is the mean square of the difference $(x-x_0)$ between $x$ and its mean value, the distribution in $x$ is

$$P_x = \frac{1}{\sqrt{2\pi} \sigma} \exp \left\{ -\frac{(x-x_0)^2}{2\sigma^2} \right\}. \quad (3)$$
This is the more general form of the normal or gaussian distribution. The quantity \( \sigma \) which determines the 'width' of the gaussian distribution is known as the standard deviation.

The gaussian distribution is of great importance because if a quantity is measured a number of times under the same conditions and the errors of measurement are completely random, the observed results of the measurement will be distributed normally about the mean value \( x_0 \), with rms deviation \( \sigma \).

In Equation (1) above for the Poisson distribution, if \( \bar{n} \) is very large

\[
P_n \to \frac{1}{\sqrt{2\pi} \ n} \exp \left(-\frac{(n - \bar{n})^2}{2\bar{n}}\right). \tag{4}
\]

If \( n - \bar{n}/\bar{n} \ll 1 \), we can replace \( n \) in the part outside the exponential in Equation (4) by \( \bar{n} \). Then Equation (4) reduces to a gaussian distribution with standard deviation \( (\bar{n})^{1/2} \).

For both the Poisson distribution Eq. (1) and the normal distribution Eq. (4), the rms or standard deviation is

\[
\left\{ \text{Av} (n - \bar{n})^2 \right\}^{1/2} = \bar{n}^{1/2}.
\]

This quantity is a measure of the uncertainty in the determination of \( \bar{n} \). For instance, the greater the total number of decays observed, the smaller the fractional uncertainty in its determination

\[
\left(\frac{1}{\bar{n}}\right)^{1/2}
\]

By calculating areas under the gaussian curve Eq. (3), we find the probabilities \( p \), given in Table 3, that a measurement of the quantity will give a value greater than \( \eta \) standard deviations from the true value \( x_0 \).
Table 3

<table>
<thead>
<tr>
<th>n</th>
<th>1.0</th>
<th>2.0</th>
<th>3.0</th>
<th>4.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>p</td>
<td>0.317</td>
<td>0.046</td>
<td>0.03</td>
<td>0.00006</td>
</tr>
</tbody>
</table>

The probable error, $\epsilon = 0.6745 \sigma$, is used when physicists want to make their result look better than it really is. The probability of the result of a measurement leading to a value $x - x_0 < \epsilon$ is just equal to that of obtaining a result $x - x_0 > \epsilon$.

$$\frac{2}{\sqrt{2\pi} \sigma} \int_0^\infty ze^{-z^2/2\sigma^2} \, dz = \left(\frac{2}{\pi}\right)^{1/2} \approx 0.7979.$$

It should be pointed out that in measurements of quantities like the mean lifetime of an unstable particle or the interaction length of a high energy particle, the quantities that are normally distributed are the number of decays in a given time interval or the number of interactions in a given path length (when these are large). These are proportional to $1/\tau$ and $1/\lambda$ respectively. The errors in these reciprocals are appreciatively distributed symmetrically about their mean values. Consequently, the errors in $\tau$ and $\lambda$ themselves are not symmetrical.

2. Maximum likelihood method

It is necessary to find a method of estimating the 'best' value of a physical constant from a set of inexact data. The method employed is that of 'maximum likelihood'. Suppose $x_1, \ldots, x_n$ are a series of measurements of a certain quantity, $x$, whose correct value is $x_0$. Assuming $x$ to be the value of this quantity, let $f(x_0|x)$ be the true normalized distribution function for the measurement of $x$.

Then the likelihood $L(x_0)$ that the set of measurements should have given precisely the set of results $x_1, \ldots, x_n$ is given by

$$L(x_0) = \prod_{i=1}^{N} f(x_0|x_i).$$
For example, the quantities \( x_1, \ldots, x_n \) could be the number per unit-time of \( K^- \) mesons decaying in flight, and \( \alpha \) an estimate of the reciprocal mean life. One would then expect the quantity \( x \) to be distributed about the reciprocal mean-life, \( \alpha \), according to a Poisson distribution. By assuming different values of \( \alpha \) and the same form of distribution, \( L \) can be calculated as a function of \( \alpha \). The maximum likelihood method would then give for the 'best' estimate of \( \alpha \) the value \( \alpha_c \) for which \( L(\alpha) \) is a maximum. It can be shown quite generally that \( \alpha_c \to \alpha_0 \), the correct value, as the number, \( N \), of observations becomes very large.

Suppose the values observed for a certain quantity are normally distributed and that each measurement \( x_i \), has its own error of measurement, \( \sigma_i \). Then

\[
L(\alpha) = \frac{1}{(2\pi)^{N/2}} \prod_{i=1}^{N} \frac{1}{\sigma_i} e^{-\frac{(x_i - \alpha)^2}{2\sigma_i^2}}
\]

\[
w = \ln L(\alpha) = -\frac{1}{2} \sum_{i=1}^{N} \frac{(x_i - \alpha)^2}{2\sigma_i^2} + \text{const.}
\]

The maximum value of \( w \) is given by \( \alpha = \alpha_c \) where

\[
\frac{\partial w}{\partial \alpha} = \sum_{i} \frac{x_i - \alpha_c}{\sigma_i^2} = 0
\]

i.e.

\[
\alpha_c = \frac{\sum_{i} x_i / \sigma_i^2}{\sum_{i} 1 / \sigma_i^2}
\]
where the measurements are weighted according to the inverse squares of the errors. When all the $\sigma_i$ are the same

$$\alpha_0 = \frac{\sum x_i}{N},$$

which is simply the mean.

For large values of $N$, it can be shown that the maximum likelihood function $L(\alpha)$ approaches a gaussian distribution

$$L(\alpha) \propto e^{-\frac{(\alpha - \alpha_0)^2}{2S}}$$

$$w = \ln L(\alpha) = -\frac{(\alpha - \alpha_0)^2}{2S} + \text{const.}$$

$$\frac{\partial w}{\partial \alpha} = \frac{(\alpha - \alpha_0)}{S}; \quad \frac{\partial^2 w}{\partial \alpha^2} = \frac{1}{S}.$$ But the standard error in $\alpha$,

$$\Delta \alpha = S^{1/2} = \left[-\frac{\partial^2 w}{\partial \alpha^2}\right]^{-1/2}.$$

The error in Eq. (6) is obtained by a further differentiation of Eq. (5), giving

$$\Delta \alpha = \left\{ \sum_{i=1}^{N} \frac{1}{\sigma_i^2} \right\}^{-1/2}$$

i.e. in this case the errors are obtained by finding the reciprocal of the rms reciprocal error.

3. Method of least squares

Suppose we wish to fit a function of a given form but containing some undetermined parameters, $y = f(a_j, x)$ to a number of experimental measurements. The problem is to find the best fit to a curve of a given form.
Each measurement is supposed to be normally distributed about its 'true' value.

The likelihood function is now given by

\[ L = \prod_{i=1}^{N} \frac{1}{\sqrt{2\pi} \sigma_i^2} \exp \left[ -\frac{[y_i - f(a_j, x_i)]^2}{2\sigma_i^2} \right] \]

\[ w = \ln L = -\frac{1}{2} M - \sum_{i=1}^{N} \ln \sqrt{2\pi} \sigma_i \]

where

\[ M = \sum_{i=1}^{N} \frac{[y_i - f(a_j, x_i)]^2}{\sigma_i^2} \quad (8) \]

and the condition for the value of \( a_j^0 \) which gives the best fit is given by

\[ \frac{\partial M}{\partial a_j} = \frac{\partial M}{\partial a_j^2} = 0 \quad (9) \]

i.e. we have to minimise the sum of the squares of the departures of the observed measurements from the values given by the curve at a given point divided by the square of the standard error at that point. This is the old method of least squares and it is seen to be justified by maximum likelihood.

The error on the determination of \( a_j^0 \) is given by

\[ \Delta a_j^0 = \left[ -\frac{\partial^2 M}{\partial a_j^2} \right]^{-1/2} \]

4. The \( \chi^2 \) test for goodness of fit

Often some test is needed of how well a given functional form of variation fits a set of experimental points. There may be several different possible forms and one requires some way of distinguishing
between them on grounds of likelihood. The probability distribution
of the likelihood $L$ itself should provide a test of the assumed
form $f(\alpha, x)$. The likelihood function will not approach its maximum
sharply if $f$ is a bad choice.

*  *  *

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emulsion physics.

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A Practical Guide to the Method of
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*  *  *
DISCUSSION

Holthuizen: An improvement of the line scanning method for measuring small angle scattering has been effected in Amsterdam. The work has been divided into two parts. One part is the following of a track, the other is the scanning for angles. This is made possible by using a drawing table coupled to the microscope. Every movement of the microscope is copied on the drawing table in two projections, namely in the xy plane and in the xz plane. The x movement is enlarged by a factor 10, the y movement (for this special case of 12 GeV/c protons) by a factor 200. Thus the tangent of the angle will be enlarged by a factor 20. (This can be changed for another momentum.) The z movement is enlarged by a factor 500 to compensate for the shrinking of the emulsion.

To summarize there are two advantages:

i) the scanner only has to follow tracks; he has not to look for deflections;

ii) the angles are enlarged to a visible extent.

Harmsen: How accurately can you measure angles with this method?

Holthuizen: To about 0.03°.

Gottstein: Is your method not slow because the track has to be kept on a cross-wire, or do you move something in the eyepiece?

Holthuizen: It depends what you call slow. The x movement is controlled by a motor and the y movement is controlled by hand.
Gottstein: Then the accuracy of the method depends on the movement of the y stage, and does this not make the method slow?

Holthuizen: With this method we have scanned 2½ metres per day.

Burhop: How does this compare with shadow scanning?

Zakrzewski: With shadow scanning we achieve something less than ten metres per day.

Burhop: Then this is not true shadow scanning.

Nikolić: The rate of shadow scanning depends on the experience of the scanner, and a very good scanner can achieve ten metres per day.

Burhop: I remember seeing some figures for p-p scattering using shadow scanning, and there was enormous correlation between the rate of shadow scanning and the appearance of white stars. One laboratory had five times as many as another group who were scanning faster. I was looking for this data as an example on biases which I discussed earlier.

Zakrzewski: I do not remember the figures very well, but a few years ago we were interested in this problem. We used specially constructed stages for fast shadow scanning of 9 GeV protons, and we obtained for the interaction length a value of 60 cm, when in fact the interaction length should have been 35 cm. Events of the type containing a track deviating slightly from the primary direction were missed with shadow scanning, and it was also found that large events with a secondary travelling in the same direction as the primary were missed.
Zakrzewski: We therefore concluded that an increase in the rate of scanning causes an increase in the interaction length.

I would like to make a comment in connection with the 'jig-saw puzzle' game proposed by Marquit (Warsaw) to illustrate the effect of speed on scanning efficiency. If you take a number of pieces of a jig-saw puzzle, spread them on a table and ask a 'scanner' to find one piece shown beforehand, you will have a situation which in a way corresponds to that of looking for an event under a microscope in a background of other events. Now, you note down the time needed to find a given piece and repeat this procedure with other pieces. By varying the characteristic features of these pieces you may vary the degree of difficulty in finding them. You may then repeat this procedure with a second 'scanner', compute the efficiency from the formulae given by Professor Burhop and see how this efficiency varies with time of 'scanning'. It is found that this efficiency increases with time and that in general it is higher than the true efficiency. This is due to the fact that it is assumed that the probability of finding each 'event' is the same while in fact it varies according to its features. In other words there are events which are very difficult to detect, even if the time of scanning is very long. Of course one should remember that this is only an analogy. For instance, for a jig-saw puzzle the situation differs from that encountered in the normal scanning in that the scanner knows beforehand that there is a piece which should be found in each case, while in normal scanning many of the fields of view under a microscope do not contain the events one is looking for. I think, however, that this is a rather good illustration of how one can obtain an efficiency which is overestimated as compared with the true efficiency of finding the events in question.
Zakrzewski : Could you make a general comment on the relevance of this type of bias in connection with p-p scattering and indicate how the efficiencies determined are affected by the non-detection of some of the events?

Burhop : This is relevant to the remark I made earlier. If one can distinguish some physical feature of the events one is looking for, then it is clear that one could determine separate efficiencies for different types of events.

In p-p scattering the events do not differ greatly in appearance. In the case of your work on Λ decay in emulsion, the appearance of the events is very different in different cases. I think that this is the type of situation in which the statistical considerations are more relevant. I would not have thought that there would be a large source of error in p-p scattering where all events look alike.

Bull : I would like to add a comparison of line scanning and area scanning for p-p scattering with the normal incidence technique.

a) Area scanning - dry emulsion

When area scanning the dry emulsion at a rate of 10-15 mm² per day, the cross-section for p-p scattering was found to be 5-8 mb.

The distribution of the number of events against scattering angles θ for the events found in the area scanning is as shown:

<table>
<thead>
<tr>
<th>θ (scattering angle)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A'</td>
</tr>
</tbody>
</table>

348/NP/sm
The efficiency of detection of p-p scattering is a function of the ionization and also of the angle of dip of the recoil proton.

It can be seen that events will be missed in region A because the track is very short and therefore difficult to see.

In region B, the ionization is low and this will cause a loss of events due to poor visibility.

Corrections to the efficiency of detection of p-p scattering using inelastic two prong events, cannot be made successfully since the distribution of these events with respect to the scattering angle is very similar to that for p-p elastic scattering. Thus the statistics in regions A and B are poor.

b) Line scanning - water soaked emulsion

Line scanning has been performed at a speed such that 1.5 mm² of emulsion has been scanned each day. The cross-section for p-p scattering found in this way is 10 mb.

The distribution of the number of events against scattering angle θ is as shown:

The efficiency of detection of events in this case is a function of the angle of dip only. Events are lost then in region A only.

In this case a large number of inelastic two-prong events have been found in region A and corrections in this region can be made with confidence.
In a) the optical system contained an x25 objective and an x15 eyepiece, whereas in b) the optical system contained an x45 objective and an x10 eyepiece. It was considered that the combination used in b) was more efficient.

Zakrzewski: This discussion provides a good illustration of a point made by Dr. Gottstein at the beginning of the course. I think one should admit there are cases when it is better to pass a problem over to another technique rather than to use emulsion at all costs. For instance, as mentioned by Professor Burhop, much work has been devoted to the determination of the emission frequency of $\Lambda^0$ hyperons from captures of $K^-$ mesons at rest in emulsion, but only a lower limit for this frequency has been obtained. This is because there are observational losses which are difficult to estimate. In these circumstances one can either repeat the experiment spending a very long time on repeated scanning (but even then one cannot be sure one has not missed some class of events), or one may try to estimate this frequency with another technique. For example, one may use a bubble chamber with heavy liquid exposed to stopping $K^-$ mesons. Here the search for decays of neutral particles is much easier than in emulsion.

Would you agree with these remarks?

Burhop: Yes.

Nikolić: In the application of the statistical formulae given by Professor Burhop, we have, in many practical cases, kinematic and geometrical constraints. The application of the formulae in these cases follows the normal procedure used by bubble chamber groups.
Nikolić: Has any emulsion group applied this statistical treatment, e.g. in the analysis of hyperfragments? We are writing a computer programme *) for the CERN IBM computer for the analysis of hyperfragments. The first programme for this analysis was written by Inman in Berkeley, but the present programme will be more complete than those written previously. We have been wondering if it would be worth while including all these statistical equations into our hyperfragment programme.

Burhop: These equations have not always been applied in emulsion work but are more often used in bubble chamber work, although in early bubble chamber work they were applied wrongly.

Evans: If one obtains a sharp peak in the likelihood distribution but without symmetry, is it possible to determine the spread on either side of the maximum? For instance, can one take the width of the distribution at half-height or at some other point?

Burhop: The equations I have given apply only to the case when the likelihood distribution becomes normal due to a large number of events. I do not know whether what you ask is valid; perhaps Professor O'Calleigh can answer that question.

O'Calleigh: Usually we deal with normal distributions only, and if this is not so we must consider each case on its merits. The non-normal distributions can be treated using methods

*) See Appendix A.
O'Cealleagh: given in most standard text-books on statistics.
(cont.)

Burhop : This method has been used in determining the best value of the lifetime of hyperons. In this case one certainly does not get the type of symmetrical distribution we have considered. With the amount of data available to us, the likelihood function has to be evaluated explicitly.

*   *   *

5348/NF/sng
THE CERN PULSED MAGNET APPARATUS

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I. INTRODUCTION

The aim of every technique used as a detector for high-energy particles is to obtain as much information as possible. Emulsions exposed in high magnetic fields give essentially two additional pieces of information (compared with an exposure without field):

i) the sign of the charge of a particle;

ii) the momentum of a particle determined more accurately than by scattering measurements, if the particle velocity is high.

There are other important applications to experiments to be mentioned in Section IV.

Other visual techniques have for some time already made use of magnetic fields. Cloud chambers, bubble chambers, and also sometimes spark chambers are placed in magnets. However, magnetic fields are rarely used in connection with the emulsion technique, because large fields are needed to provide measurable curvatures of tracks with reference to the properties of the emulsion, i.e. high stopping power and comparatively high Coulomb scattering.

In Section II the main principles and characteristics of pulsed fields will be discussed, since this method seems at present to be the most important in producing high fields over a sufficient volume. In Section III a short review of different magnet types will be given; finally in Section IV we will discuss the application of high fields to emulsions and the expected accuracy of measurements based on the field characteristics.
II. PRINCIPLE OF THE PULSED MAGNET TECHNIQUE

Pulsed magnet systems consist essentially of two parts: a power source and a discharging circuit with an air core coil. Condensers are commonly used for energy storage. The elements of the discharge circuit (capacity C, inductance L, and resistance R) are generally chosen so that the circuit is oscillating. As an example, the condenser bank used for emulsion experiments at CERN\textsuperscript{1}) consists of 250 condensers with a total capacity of 75,000 $\mu$F. At the maximum charging voltage of 2.8 kV, the stored energy is 300,000 joules.

1. Principle of the circuit

We consider the simplified diagram in Fig. 1 and assume a choice of L, R, and C so that the circuit is oscillating. The switch S is closed to charge the condensers. When the condensers, C, have reached the required voltage, switch S is opened and switch A closed to discharge through the coil L. To protect the condensers against the oscillation, switch B is closed just before the voltage changes polarity. Figures 2a and 2b show the voltage across the condenser with and without operating switch B, respectively.

In practice, spark gaps or ignitrons are used as switches A and B. However, peak currents of the order of 100,000 amperes or more are usual. Employing ignitrons as switches, therefore, several ignitrons have to be used in parallel. For example, an ignitron of the type 5555 can switch current pulses of a few $\mu$sec length with a peak current of 20 kA.

2. Operation

The trigger for the switches A and B (Fig. 1) is initiated by a pulse of the magnet cycle of the accelerator or directly by the target trigger. The switches are so commanded that the peak of the magnetic field is reached when the particle burst passes the coil. The synchronization between the field pulse and the particle burst

\textsuperscript{1}
(monitored by a counter) can be checked (and filmed during a run) by feeding both signals into a dual beam oscillograph (Fig. 3).

3. Electrical characteristics

The electrical characteristics of the oscillating circuit can be determined using an oscillograph as shown in Fig. 2b, which gives the ration $U/U_0$ across the condensers. ($U_0 = \text{voltage at the time } t = 0$, and $U = \text{voltage of the first inverse peak of the oscillation after a half-cycle } T/2$.) The capacity is known. Therefore $L$ and $R_{AC}$ can easily be calculated using the two equations:

$$\frac{U}{U_0} = e^{-RT/4L} \quad (1)$$

and

$$\frac{T}{2} = \frac{\pi}{\sqrt{\left(\frac{1}{LC} - \frac{R^2}{4L^2}\right)\gamma_2}} = \frac{\pi}{\omega}. \quad (2)$$

The current of a damped oscillation is given by the integral of

$$L \cdot \frac{d^2q}{dt^2} + R \cdot \frac{dq}{dt} + \frac{1}{C} \cdot q = 0 \quad (3)$$

(wher e$q = \text{charge on the condenser at time } t$), having the solution:

$$i = \frac{dq}{dt} = \omega \frac{q_0}{\sqrt{LC}} \left\{ \omega \cos(\omega t + \delta) - \frac{R}{2L} \sin(\omega t + \delta) \right\} e^{-\frac{Rt}{2L}} \quad (4)$$

where $q_0 = CU_0$.

The peak current is reached for $\omega t = \delta = t_{\text{e}}^{-1}(2\omega L/R)$ and is given by

$$I_{\text{peak}} = U_0 \sqrt{\frac{C}{L}} \cdot e^{-\frac{R}{2L} \cdot t(\delta)} \quad (5)$$

The maximum current passing through the coil is reduced by the factor $U_{\text{coil}}/U_0$ as a consequence of losses in the switching system and the
connections. The voltage ratio $U_{\text{coil}}/U_o$ has to be very near to one in practice. Therefore the connection lines between the condensers and the coil have to be of low inductance and low ohmic losses. 

The ratio $R/L$ in the exponential of Eq. (1) determines the damping of the oscillation and should be kept small for an efficient coil. The efficiency $\eta$, defined as the ratio between the maximum energy stored in the coil and the initial energy stored in the condensers, is given by:

$$\eta = \frac{\frac{1}{2} LI^2_{\text{peak}}}{\frac{1}{2} C U_o^2} = \left(\frac{U_{\text{coil}}}{U_o}\right)^2 \cdot e^{-\frac{R}{L}\cdot t(\delta)}.$$  

(6)

4. **Coils**

Figure 4 shows four basic types of coils:

- a) single-turn coil;
- b) helical coil (Bitter type);
- c) tape-wound coil;
- d) wire-wound coil.

The mechanical strength of the coils listed above decreases in the order a) to d). The single-turn coil is the strongest with respect to the magnetic force. However, the inductance of a single-turn coil is so low that coils of this type are only useful for field pulses of short duration, say less than 100 $\mu$sec. The flexibility from the point of view of inductance increases in the order from a) to d).

Coils of type b) are generally used to obtain field pulses of $\sim$ ms. The application of a Helmholtz pair of coils with an appropriate spacing is the usual method of reaching homogeneity in the centre space. The gap between the two coils can be used for holes which are necessary, for example for beam passage. The appropriate spacing required to obtain homogeneity is calculated so that the second derivative of the field is zero at the origin.
Figures 5 and 6 show coils made from discs of copper-chromium which are soldered together to form a helix. The insulation used is vetronite (glass fibre impregnated with epoxy resin). The two half coils are separated to obtain homogeneity of the field in the centre space. The coils were impregnated in vacuum with epoxy resin. They are cooled by a stream of water passing between an insulating cylinder and the inside surface of the coil. Radial stresses are supported by a number of stainless steel rings split radially to avoid eddy currents. The frame (Fig. 7) consists of two stainless steel blocks to precompress the coil axially. Radial and axial forces for these coils are of the order of a few tons/cm².

5. Field measurements

To measure high magnetic fields two methods are generally used:

a) measurement of the induced voltage in a small solenoid of known dimensions;

b) measurement of the Hall-voltage of a Hall-plate calibrated at fields of 10-20 kgauss.

Figure 8 shows:

a) the induced voltage in a solenoid;

b) the Hall-voltage as functions of time indicated on an oscillograph.

The signal of the induced voltage has to be integrated to give the field pulse. This can easily be done with an appropriate RC-circuit. The Hall-voltage is known 4) to be a linear function of the field in the range between 10 and 180 kgauss for certain types of semiconductors, such as In As. In Fig. 9 some different types of Hall-plate are shown together with a solenoidal search-coil.

Figure 10 gives the measured field distribution along axial and radial directions for the coil shown in Fig. 5.
III. DIFFERENT TYPES OF MAGNETS

1. Pulsed magnets

In Table 1 the characteristics of four different pulsed magnets are compared. These four magnets have been employed for high-energy physics, and types (ii), (iii) and (iv) were used in emulsion experiments. The magnets (i), (ii) and (iii) were constructed for a relatively short radiation burst so that the pulse duration is < 1 ms. However, the inductance in all four cases is of the same order and the short pulse duration for coils (i) to (iii) is given by their lower capacity compared with coil (iv). The stored energy ($\frac{1}{2}Cu^2$) is higher for magnet (iv), but also the length of the magnetic field which is useful for experiments is greater in a transverse field arrangement. Coils (i) and (iii) were used essentially for experiments with the incident beam in the axial direction. In type (iv) a Helmholtz pair of coils is used, allowing holes for beam passage in the gap which is used for the exposures with a transverse field. Coils (i), (ii) and (iii) consist of only one helix.

2. DC-magnets

DC-magnets designed to reach high fields have an extremely high power consumption. The cooling of these magnets is a major problem. In two laboratories (MIT and Oak Ridge) DC-magnets have been constructed for fields up to 100 kgauss. At the MIT Magnet Laboratory a 250 kgauss magnet has been designed with a useful bore of 15 inch diameter. The DC power is rated for 15 megawatt, and 12,000 ft of water per minute are required for cooling. This magnet is divided into three concentric coils, where the contribution to the field from the inner coil is smaller than that from the outer, in order to reduce the stresses in the copper for the smaller coil in the centre. The stresses in all three coils are thereby limited to the same value.
Table 1

<table>
<thead>
<tr>
<th>Type:</th>
<th>(i)</th>
<th>(ii)</th>
<th>(iii)</th>
<th>(iv)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Laboratory:</td>
<td>Argonne\textsuperscript{5)}</td>
<td>UCRL\textsuperscript{6,7)}</td>
<td>Cal Tec\textsuperscript{8)}</td>
<td>CERN\textsuperscript{1)}</td>
</tr>
<tr>
<td>Condenser bank</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Voltage (kV)\textsuperscript{*)}</td>
<td>5.3</td>
<td>13(20)</td>
<td>5</td>
<td>2(2.8)</td>
</tr>
<tr>
<td>Capacity (μF)</td>
<td>900</td>
<td>750</td>
<td>2400</td>
<td>75000</td>
</tr>
<tr>
<td>Coil \textsuperscript{**)}</td>
<td>Cu-Be</td>
<td>Cu-Be(1%)</td>
<td>Cu-3e</td>
<td>Cu-Cr(1%)</td>
</tr>
<tr>
<td>Conductor</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Turns/cm (Total No. of turns)</td>
<td>6.8</td>
<td>5</td>
<td>-</td>
<td>4(2 \times 13)</td>
</tr>
<tr>
<td>Inner diameter (cm)</td>
<td>2.03</td>
<td>5.7</td>
<td>\sim 4.5</td>
<td>7.2</td>
</tr>
<tr>
<td>Outer diameter (cm)</td>
<td>4.06</td>
<td>19.7</td>
<td>-</td>
<td>20</td>
</tr>
<tr>
<td>Cooling</td>
<td>Silicon oil (Dry ice temp.)</td>
<td>Oil</td>
<td>Demineralized Water (12°C)</td>
<td></td>
</tr>
<tr>
<td>Inductance (μH)</td>
<td>18</td>
<td>45</td>
<td>40</td>
<td>31</td>
</tr>
<tr>
<td>ac-resistance (mΩ)</td>
<td>16.5</td>
<td>7.1</td>
<td>-</td>
<td>1.6</td>
</tr>
<tr>
<td>ac-resistance (mΩ)</td>
<td>51.2</td>
<td>-</td>
<td>-</td>
<td>5.9</td>
</tr>
<tr>
<td>Pulse duration \textsuperscript{***)} (μsec)</td>
<td>0.43</td>
<td>0.56</td>
<td>1</td>
<td>4.8</td>
</tr>
<tr>
<td>Peak field (kgauss) \textsuperscript{****)}</td>
<td>200</td>
<td>200</td>
<td>160-200</td>
<td>200</td>
</tr>
<tr>
<td>Peak current (kA)</td>
<td>25</td>
<td>53</td>
<td>\sim 35</td>
<td>81</td>
</tr>
</tbody>
</table>

\textsuperscript{*)} Voltage used in obtaining the coil characteristics following (max. voltage shown in brackets).
\textsuperscript{**)} The insulation used by all laboratories is epoxy/fibre-glass.
\textsuperscript{***)} Half-cycle (T/2).
\textsuperscript{****)} Peak field obtained over a run of a few thousand pulses.
3. **Cryogenic magnets**

Liquids, such as nitrogen and hydrogen, are used for the cooling of dc-magnets as well as for pulsed coils. The advantage of these magnets working at low temperature lies in the decrease of the resistance of the coils, and so the power requirements are reduced. An 80 k gauss-magnet (Los Alamos) with Cu coils for two inch bore needs 25 kW power and 100 l of liquid hydrogen per minute for cooling. Sodium and aluminium are the most appropriate materials since they have the lowest magneto-resistivity.

4. **Super-conducting magnets**

The development of super-conducting coils reduces the general interest in coils cooled with hydrogen. However, engineering and technological problems are not completely solved. These problems include energizing and quenching of the magnets, low resistance joints and constructions for supporting magnetic forces and stresses. Some of the new super-conducting materials are brittle and hard and can only be used if coated with some other material. The decrease of the critical current density in wound solenoids compared with small wire samples is also not yet understood. Figure 11a shows schematically the critical field as a function of temperature, and Fig. 11b) gives the relation between the critical current density and the field for different super-conducting materials.

Maximum fields reached with super-conducting coils are at present of the order of 60 to 80 k gauss over less than 1 cm internal diameter of the coil. As an example, the data for a typical coil are given as follows. The solenoid consists of 15,200 turns of Nb-Zr (25%) - wire of 0.01 inch diameter and a length of 1,500 m. The winding thickness is 2.55 cm and the inner diameter 0.5 cm. At 4.2° K a field of 59 k gauss can be reached, generated by a current of 19 amperes. The current density corresponds to 37,500 Amp/cm².
At the present state of development, the application of super-conducting coils to emulsion experiments with high fields is not yet possible regarding the requirements of Section IV 1.

IV. APPLICATION OF HIGH FIELDS TO EMULSION EXPERIMENTS

1. Momentum measurements

   i) Field characteristics and accuracy

   The main characteristics of the field required for emulsion experiments may be summarized as follows:

   a) a magnetic field strength of 200 kgauss or more over a volume of 100-200 cm$^3$ (which can contain an emulsion stack); in general the field direction should be perpendicular to the plane of the pellicles: the useful area of the emulsions should be 30 cm$^2$ or more;

   b) the variation of the field over the useful volume should be as small as possible;

   c) the peak of the field pulse should be constant within 5% during the time of a radiation burst.

   The accuracy in the absolute value of the field can be obtained to within ± 3%. Moreover the value of the field can be reproduced only with an accuracy of within ± 2 to 3%, due to small changes of the voltage on the condensers from pulse to pulse. The error due to the variation of the field over the volume can be kept very small, but can also be corrected for by the measured field distribution. The last requirement (c) can be fulfilled for field pulses of sufficient length in time compared with the radiation burst, if the radiation burst has a negligible jitter. Taking into account all the sources of errors mentioned, one could expect a standard error of 6 to 7% in the value of the field for all particles passing through the coil space.
ii) Methods of analysis

Measurements to determine the momentum of particles in emulsions exposed in a magnetic field are made essentially by two different methods. Both measurements determine the radius of curvature of the tracks. One method makes use of the change in the angle of successive tangents (to the curved track) at points equally spaced along the track. This method of analysis was employed in connection with the first use of a high field with emulsions, and is described in reference 11. The second method is based on measurements of second differences. The principle and the accuracy obtained using the last method will be discussed briefly.

The radius of curvatures (see Fig. 12a) is given by:

$$\rho = \frac{t^2 + s^2}{2s}$$

(7)

where $t =$ half length of the chord, and $s =$ sagitta (Fig. 12a).
In general we are concerned with cases where $s \ll t$, and therefore the following approximation can be used:

$$\rho = \frac{t^2}{2s}$$

(8)

Further, $t$ and $s$ are connected with the more useful quantities $t' =$ cell length and $s' = D''/2 = [(y_2 - y_1) - (y_3 - y_2)]/2$ by the relations: $t = t' \cos \epsilon$ and $s = s' \cos \epsilon$, where $\cos \epsilon = (y_3 - y_1)/2t'$.
For over-all alignment of the curved track with the microscope stage, $\epsilon$ is small and $\cos \epsilon \approx 1$. Equation (7) can then be written as:

$$\rho \approx \frac{t^2}{D''}$$

(9)

The momentum $p$ of a particle (with a single charge) is given by:

$$p = 0.30 \cdot B \cdot \rho = 0.30 \cdot B \cdot \frac{t^2}{D''}$$

(10)
where p is in MeV/c, ρ, t and D'' in cm and the magnetic field B in kgauss.

Figure 12b shows D'' as a function of p for different values of t as parameter.

iii) Accuracy of the measurements

The cell length t can be determined extremely accurately compared with B and D'', so we may neglect the error Δt/t. We assume further that the errors in the measurement of B and D'' are independent. Then the error in the momentum determination following from Eq. (10) is given by:

$$\frac{\Delta p}{p} = \pm \sqrt{\left(\frac{\Delta B}{B}\right)^2 + \left(\frac{\Delta D''}{D''}\right)^2},$$

(11)

ΔB/B has been estimated to be ± 0.07. If we aim at ΔD''/D'' = ± 0.05, then the error in Δp/p is 8.5%. For example, this requires an accuracy for D'' of ± 1.6 μ, if t = 1 cm, B = 200 kgauss and p = 20 GeV/c. Therefore, the individual values of y have to be measured with an accuracy of ± 0.65 μ.

iv) Correction for distortion

The influence of distortion can be measured, for example, by using vertical tracks obtained in an additional exposure of the same pellisoles to a beam perpendicular to their plane. The displacement Δy due to distortion can be corrected by determining the displacement of vertical tracks near the points used for the curvature measurement. The displacement due to distortion in the other directions can be neglected. For example Δx is in general small compared with the cell length t. The distortion simulates a curvature, if the Δy values do not depend linearly on t. The second difference D'' then has to be corrected by ΔD'' given by ΔD'' = 2Δy₂ - Δy₁ - Δy₃.
v) Deflection by Coulomb scattering

The average second difference due to Coulomb scattering \( \bar{D}_{sc}'' \) is given by:

\[
\bar{D}_{sc}'' = \frac{Kt^{3/2}}{5.7 p\beta},
\]

where \( \bar{D}_{sc}'' \) and the cell length \( t \) are in cm, and \( p\beta \) in MeV/c.

The ratio: deflection due to the magnetic field to the deflection due to the multiple scattering, is then given by Eqs. (10) and (12):

\[
\frac{D''}{\bar{D}_{sc}''} = 6.1 \cdot 10^{-2} \cdot B \cdot \beta \cdot \sqrt{t},
\]

assuming a value for the scattering constant \( K = 28 \), and using \( B \) in kgauss and \( t \) in cm.

For \( t = 1 \) cm and \( B = 200 \) kgauss, \( D_{m}'' / \bar{D}_{sc}'' = 12.2 \), if \( \beta \approx 1 \). Since the track length is confined to a few cm, only the increase of the magnetic field can improve the ratio \( D_{m}'' / \bar{D}_{sc}'' \). Equation 13 also shows that momentum determination based on magnetic curvature is advantageous only for values of \( \beta \) which are not too small.

According to Eq. 13, the cell size, \( t \), has to be made large to improve the ratio \( D_{m}'' / \bar{D}_{sc}'' \). The use of more than two cells for the same track length is no advantage from the point of view of accuracy. However, a set of values of \( D'' \) for the same track helps in recognizing single scatters. For example, a cut-off on values of \( D_{m}'' \) can be applied, which corresponds to the usual cut-off at \( 4D_{sc}'' \) in scattering measurements.

Figure 13a shows the result of curvature measurements on 68 proton tracks of \( 24 \pm 0.4 \) GeV/c. Figure 13b is an integral plot of:

\[
\frac{1}{N_{\text{total}}} \int N \, d\beta.
\]
The asymmetric distribution is expected due to the finite length of the particle burst compared with the field pulse. Particles passing through the coil just before or after the peak field is reached, have a slightly larger radius of curvature.

2. Various experiments using high fields

There are two classes of experiments, in the first of which the emulsions are exposed in the field. As an example we may quote the sign determination and momentum measurements on secondaries from various interactions. In general, transverse fields are used for these experiments.

The second type of experiment uses the high field (at least partly) as an analyser. As an example, the charge and momentum separation of particles may be mentioned. High fields are needed to reach a separation over a short path length; thus this method is especially useful for short-lived particles. Another example is the measurement of the magnetic moment of hyperons. High fields are required to obtain a large precession of the spin between the creation and the decay of the hyperon. In this case, axial as well as transverse fields can be used. The main advantage of a transverse field is the sweeping away of low energy background.

These experiments will be discussed more explicitly in the following lectures.

* * *

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* * *
DISCUSSION

Key : Could you give the approximate accuracy of the small solenoid and the Hall plate methods of measuring the magnetic field?

Hoffmann : The solenoid as well as the Hall plate method give rise to some problems. The first method makes use of a small solenoid whose cross-section must be known very well. On the other hand the Hall plate must be calibrated very carefully to reach an accuracy of \( \pm 3\% \) at high fields. This is usually done with a precision solenoid. The calibration of a Hall plate is rather tricky because the Hall voltage is not a linear function of the magnetic field at low fields. For this reason the calibration is made between 15 kgauss and 20 kgauss. From these values up to 180 kgauss, the In(As-P) Hall plate (type FC 33) has a linear response. Above 180 kgauss the Hall plate is expected to produce saturation effects. By inserting a relatively high resistance (some k-ohms) in series with the Hall plate current, the effects of non-linearity can be reduced. The calibration of the Hall plate can also be done analytically. In this case variation of the field with applied voltage is calculated very accurately and correlated with the Hall plate output, since the magnetic field is a linear function of the voltage applied to the coil.

Key : Will the cross-section of the solenoid be changed during the pulse?

Hoffmann : By fixing the solenoid and its wires to a holder with araldite this effect can be minimized. However, some small errors can still occur.
Doble: What are the comparative lifetimes of multi- and single-turn coils?

Hoffmann: Multi-turn coils have a finite lifetime because we approach the limit of the strength of the materials. Forces as high as 7,000 kg/cm² can be reached. As a result, the copper sheets which constitute the windings are bent around their inner circumference towards the centre of the coil and will eventually cause a short circuit which can destroy the coil. In general it can be said that a multi-turn coil should withstand several thousand pulses, it may be as many as 10,000. However, other defects can occur such as internal breakdown of the insulation or the copper being forced out due to the high radial forces. Single-turn coils usually consist of a single block and have an effectively infinite lifetime. This is one reason for favouring single-turn coils.

Harmsen: What is the repetition rate for the pulsed magnet, and how long does it take to charge the condenser bank?

Hoffmann: The repetition rate of the CERN unit depends on the voltage to which the capacitors have to be charged, i.e. it depends on the magnitude of the field which is desired. This limitation is due to the maximum permissible charging current which is limited at present to 100 A. Some specific values for the charging time are given below.

<table>
<thead>
<tr>
<th>Charging time</th>
<th>Capacitor voltage</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 sec</td>
<td>1300 V</td>
</tr>
<tr>
<td>4 sec</td>
<td>1700 - 1900 V</td>
</tr>
<tr>
<td>9 sec</td>
<td>2400 V</td>
</tr>
</tbody>
</table>
The CERN coil, as an example, needs 1300 volts to reach a field of 200 kgauss, which required 6 seconds of charging time. Thus, only every third pulse of the proton synchrotron can be used at its present repetition rate of ~ 3 sec at 24 GeV.

Declais : What can you say about the curvature caused by distortion?

Hoffmann : One can correct the measurements for distortion. For this purpose one has to expose a plate perpendicularly to the beam to get vertical tracks in the emulsion. The arrangement with the field H is shown in the following sketch.

![Diagram of beam tracks from perpendicular exposure]

These tracks show the distortion and so one can correct the measurements. In our case the distortion at 24 GeV/c was very high, in contrast to our 13 GeV/c exposure.

Göing : Is it possible to get a polarized target using the pulsed magnet?

Hoffmann : Yes, this is possible. But one will get better results using conventional low fields and additional cooling of the target.

Bull : Since λ_{m}/λ_{so} ≈ 10, I would expect the effect of single and slatters to have a negligible effect on the measured second differences D^2. But you indicated that ~ 10% of the second differences would have to be rejected when a cut-off is applied.
Hoffmann: The 10% of second differences rejected do not all come from the effect of single scatters. Distortion and other effects also contribute to the rejection.

Winseler: What is the highest field one has ever reached?

Hoffmann: The highest field ever reached is $10^7$ gauss for one pulse only. This was done in Los Alamos. A coil has the tendency to explode in a radial direction; so one can surround the coil with explosive material which then has the effect of imploding the coil. If the coil operation is synchronized with the explosion, one can reach very high fields.

Key: What are the possibilities of reducing the 5% field variation at the peak of the field pulse?

Hoffmann: This can be done by increasing the inductance L, but this will decrease the peak current. Also one has to make greater efforts in cooling the coil; so it is easier not to change the magnetic pulse, but to ask for the target burst to be as short as possible.
Fig. 1. Simplified circuit diagram for a pulsed magnet.

Fig. 2a. Voltage across the condensers as a function of time. Switches B (Fig. 1) are in operation (crow-bar method).

Fig. 2b. Voltage across the condensers, if switches B are not in operation.
Fig. 3. Field pulse synchronised with a particle burst. The heights are in arbitrary units; the base in msec.

Fig. 4
Basic coil types:

a) single-turn
b) helical
c) tape-wound
d) wire-wound coil

(a)   (b)

(c)   (d)
Fig. 5. Section through a multi-turn coil. The two half coils are separated in order to reach homogeneity in the centre space.

Fig. 6. Multi-turn coil (inside diameter 7.2 cm) with the radial frame of slotted stainless steel rings.
Fig. 7. Axial frame with a multturn coil.

Fig. 8. (a) Induced voltage in a search coil, and (b) Hall-voltage as functions of time (1cm = 2msec).
Fig. 9. Hall-plates and search coil for field measurements.

Fig. 10. Axial and radial field distributions for a Helmholtz pair of coils.
Fig 11
a) Critical field of a superconductor.

\[ H_c = H_{c2} \left( 1 - \frac{T}{T_c} \right)^{1/2} \]

\[ H_{c2} = 0.7 \text{ Tesla} \]

\[ T_c = 18.5 \text{ K} \]

\[ T_{c2} = 17.7 \text{ K} \]

b) Quenching characteristics for superconducting wires at 4.2 K exposed to a transverse magnetic field (Gauster, ReG).

Fig 12a Indication of geometrical quantities involved in curvature measurements.

Fig 12b Plot of the second differences, \( D^2 \), as a function of the particle momentum, with the cell length, \( t \), as parameter.
Fig. 13a Number of tracks as a function of the measured radius of curvature for protons of 24.1 GeV/c momentum.

Fig. 13b Integrated number of tracks (from fig. 12a), normalised and plotted on Gaussian paper, as a function of the radius of curvature.
A very timely warning has been given by Drs. Gottstein and Zakrzewski about not trying to use emulsion technique for experiments that can best be carried out using other techniques. On the other hand, there are some advantageous features peculiar to emulsion, and we must try to devise experiments that exploit these advantages.

One particularly striking feature is the very small distance (of the order of 0.5 µm) that can be resolved using emulsion. In some special circumstances even smaller distances can be resolved. For example, in the measurement of the lifetime of the π⁰ meson it has been claimed that it is possible to detect distances of the order of 0.1 µm between the line of one track and the end point of another. Whether one agrees with such a claim or not, it is certainly the case that actual distances that can be resolved are much smaller than is the case for any other technique.

In relation to experiments with the PS this high resolution property implies that, in principle, emulsion experiments can be carried out with separated beams in which the separation is much smaller than those that can be usefully employed using other techniques. For example, a separated beam is being designed for the East area for particles of momenta up to 10 GeV/c. Quite a high proportion of K⁻ mesons will survive till the end of the beam, but the K⁻ – π⁻ separation will only be of the order of a few millimetres. It should be quite feasible, however, to make use of such a small separation using nuclear emulsion.

The measurement of the π⁰ lifetime shows how the facility to resolve small spatial separations may be used to measure very small time intervals. It is a well-known disadvantage of the emulsion technique, however, in comparison with other visual techniques that the time association of events can be inferred comparatively rarely.
The high resolution property of nuclear emulsion also implies that accurate measurements of the range (and hence of the energy) can be made. It is interesting to compare the accuracy with which the energy of low energy particles can be inferred from their range using nuclear emulsion and bubble chambers. A proton of energy 1 MeV produces a track of length 15μm in emulsion, whilst in liquid helium the length would be 240μm, and in propane 120μm. The length of the track could be measured in emulsion to within 0.5μm, i.e. 3%. In the bubble chamber, however, it could be measured only to within 100μm, giving an accuracy of 40% in liquid helium and 80% in propane. Since the interaction of K mesons with light nuclei or the decay of hyperfragments often leads to the emission of protons of energy 1 MeV or less, studies of the full details of such interactions or decays can be carried out using nuclear emulsion but not with bubble chambers.

With nuclear emulsion it is often possible to work with unseparated but momentum-analysed beams, the tracks of the desired type being identified by their specific ionization. For example, Table 1 shows the expected grain density (in terms of the grain density for 'plateau' ionization for protons, K mesons and π mesons of two different momenta.

<table>
<thead>
<tr>
<th>Momentum</th>
<th>Proton</th>
<th>K Meson</th>
<th>π Meson</th>
</tr>
</thead>
<tbody>
<tr>
<td>400 MeV/c</td>
<td>2 2</td>
<td>1 5</td>
<td>0 9</td>
</tr>
<tr>
<td>3 GeV/c</td>
<td>0 88</td>
<td>0 96</td>
<td>1 0</td>
</tr>
</tbody>
</table>

At 400 MeV/c, therefore, it should be possible fairly readily to distinguish the K meson track from either a proton or a π meson track. It is seen, however, that although it may be possible in principle to separate very high momentum particles using the relativistic rise of ionization, in practice the task involved is a formidable one. An accuracy of the order of 1% would be needed in the specific ionization determination — involving the counting of 10,000 blobs.
The possibility of using nuclear emulsion as a detector of polarization should also not be overlooked. Protons of energy 150 MeV are scattered in the angular range between 7° and 22° with polarization varying between 40 and 70%. One could, for example, make use of this property to study the polarization of recoil protons produced by the elastic scattering of $K^-$ mesons in the momentum range 500-700 MeV/c on a liquid hydrogen target. The polarization of the scattered proton could be measured by observing its subsequent scattering. From an incident flux of $10^6$ $K^-$ mesons on the scatterer about 1,000 protons would enter the stack in the case of a feasible scattering geometry, and about 200 of these would undergo scattering.

It is possible that such polarization studies could best be done using spark chambers. One interesting type of measurement, however, that is very difficult by other techniques, is the elastic scattering of low energy $K^-$ mesons by free protons. For $K^-$ mesons of energy less than 10 MeV, it is difficult to measure the elastic scattering cross-section using the bubble chamber, since the proton recoil is too short to be seen. On the other hand, a 10 MeV $K^-$ meson still has a residual range in the emulsion of 800 $\mu$m, and elastic $K^-p$ scattering can be identified in the emulsion down to $K^-$ energies of 1.5 MeV. It turns out that there is a good deal of physical interest in the measurement of $K^-$ elastic scattering in this low energy region.

In the first place, the threshold for the reaction $K^- + p \rightarrow \bar{K}^0 + n$ occurs in this region. Near this threshold the branching ratios for the different $K^-$ interaction processes as well as the elastic scattering cross-section would be expected to show sharp changes.

Further, Dalitz has suggested that the $Y^*_1$ isobar might be a bound state of a $K^-$-nucleon system. Whether or not a bound state is possible depends a great deal on the behaviour of the $K^-p$ elastic scattering cross-section in this region.

R. D. Hill and his collaborators have reported some results in this field already, but the statistics are very poor. We have requested an exposure at CERN with a large number of stopping $K^-$ mesons.
**K**⁻ mesons would be picked up at a residual range of 5 mm and followed to rest in the search for examples of elastic scattering on free protons. About 1,000 elastic **K**⁻-p scatters in this energy region would be expected for 10⁶ incident **K**⁻ mesons. Since, at the present time, there are only about 10 events in this energy range, this would represent a considerable contribution to our knowledge of this interesting cross-section.

The only other technique with which this experiment is feasible seems to be the high pressure cloud chamber or diffusion chamber with hydrogen gas. It would, however, be a difficult task to design a **K**⁻ beam in which sufficient **K**⁻ mesons stop in the gas of the chamber.

The interaction of **K**⁻ mesons with nuclei provides one of the most fruitful sources of hyperfragments. Studies of hyperfragments have so far only been carried out effectively using nuclear emulsion technique. It is true that a few $^\Lambda_{\text{H}}^3$, $^\Lambda_{\text{H}}^4$, and $^\Lambda_{\text{He}}^4$ hyperfragments have been studied in the helium bubble chamber, but no heavier hyperfragments can be produced in this manner. It is possible that a high pressure cloud chamber with nitrogen or argon might be used for studying hyperfragments, but this has not so far been investigated.

At the present time, about 300 mesic hyperfragments have been identified. About 5% of all **K**⁻ interactions at rest give rise to hyperfragments and a similar high proportion of **K**⁻ interactions in flight - up to about 1 GeV/c at any rate. There is a very interesting difference between **K**⁻ interactions at rest and in flight in that, for **K**⁻ mesons at rest, hyperfragments are predominantly light and come from the light emulsion elements. In this case the $\Lambda^0$ is trapped in a light nuclear fragment of mass number up to 12 or 13 at the most. In **K**⁻ interactions in flight it has been found that the observed hyperfragments are predominantly heavy nuclear recoils containing trapped $\Lambda^0$ hyperons, i.e., with $\Lambda^0$ trapped in the main body of the nucleus. In the case of **K**⁻ at rest, this main body of the nucleus would not have sufficient range to be visible as a track, and so would give rise to a so-called cryptofragment. In the case of **K**⁻ in flight, they will be...
distinguishable as short recoils, enabling a sample of high atomic number hyperfragments to be obtained.

Hyperfragments produced from interactions of K^- mesons in flight come predominantly from the heavy emulsion nuclei. This is evident from Fig. 1, in which the total prong number distribution for non-mesic hyperfragments produced in 800 MeV/c K^- interactions is plotted. This is the sum of the prong numbers of primary and secondary stars, and this number goes right up to 16. A total prong number greater than 7 would not be obtained if K^- interactions occurred in a light nucleus of the emulsion.

One of the problems of considerable physical interest that could be studied is the variation of the binding energy of hyperfragments \( B_A \) with \( Z \). For the light hyperfragments one finds an almost linear relation between \( B_A \) and \( Z \). One knows this is unlikely to continue indefinitely and it would be of great interest to know the value of \( B_A \) in the case of heavy hypernuclei.

Another interesting physical problem concerns the study of hyperfragments of a particular type. For instance, there is great interest in the study of hyperfragments of mass 7, to see if the same sort of isotopic multiplets apply for hyperfragments as for ordinary nuclei. An experiment such as this would involve the detection of excited states of hyperfragments. The frequency of hyperfragments of a given type could be enhanced by loading the emulsion with particular elements (e.g., lithium).

This is a very fruitful field of physics which is only available at the present time to emulsion physicists and which should be exploited. As an illustration an interesting case of hyperfragment production is shown in Fig. 2. In this case the hyperfragment is produced by the interaction of a hyperon in flight, and it is the only event of this type that has been reported up to now.

This is a very inadequate account of the work that could be done on hyperfragments, but there is no time to elaborate further.
The next type of investigation involving $K^-$ mesons in emulsion makes use of the fact that you can actually analyse completely a certain number of stars. Something of the order of one in 200 of the $K^-$ interactions at rest lead to stars that can be analysed completely. The interactions concerned are multinucleon captures; the simplest type one can consider is:

$$K^- + 2N \rightarrow Y + N.$$ 

No $\pi$ meson appears in this interaction and the rest mass of the $\pi$ meson contributes to the kinetic energy of the $Y$ and the $N$. This means that these particles may be very energetic. Because of this, and because of the fact that there exists a negative baryon (the $\Sigma^-$ hyperon), it follows that there are cases where all the products of the interaction are charged. A similar process occurs with $\pi$ meson capture, of course, but one of the products is always a neutron because there is no such thing as a negative nucleon. There are many cases of $K^-$ capture in which all the products are charged and the light nuclei in which the $K^-$ capture occurs can be identified. It appears that at least 50% of the $K^-$ meson multinucleon reactions come from these light nuclei.

Figures 3 and 4 show some examples of such completely analysed events. These events are shown to illustrate what can be done by making use of the fact that, in emulsion, these very short tracks can be detected and their energies measured.

The reason why we are interested in this particular reaction is that it might be used for studying the correlations between nucleons in a complex nucleus. Quite a large momentum is taken by these two particles. In fact, in the centre of mass system of the two product particles, the momentum that has been generated is of the order of 400 MeV/c each. In terms of the uncertainty principle, in order for this to happen, these two particles must be correlated within a distance of about 0.4 fermi apart. We know that this process occurs in less than 1% of all $K^-$ captures in deuterium, in about 17% in helium, and in emulsion it seems to be at least 20% of all $K^-$ captures. If one
could actually measure this quantity for a particular nucleus, C, N, O, then this would give very interesting information about the correlation of nucleons in these nuclei.

I would like to say something about isobars as well. I think there is quite a future for using K^- mesons to study isobars, both \( \Sigma^* \) and \( \Lambda^* \). However, one has to be very careful here not to overlap with bubble chambers. I think in many respects that with a bubble chamber one can study isobars much more effectively than with emulsion. This is certainly the case where \( \Lambda^0 \) or \( K^0 \) are produced. In cases when charged particles are produced, the study of isobars in a bubble chamber has not got much advantage, if any, over that in emulsion. For instance, one of the problems that we want to study with the recent fast K^- exposure at CERN is the search for an isobar of the type \( K_0^* \) of mass about 885 MeV. We know that there is an isobar of the type \( \bar{K}_0^* \) which has been observed in a bubble chamber, and which breaks up into \( K^0 \) and \( \pi^- \): \( \bar{K}_0^* \rightarrow K^0 + \pi^- \). Now there might also be a \( K_0^* \) breaking up into \( K^- \) and \( \pi^+ \): \( K_0^* \rightarrow K^- + \pi^+ \), but whereas it is easy to see \( K^- \) in a bubble chamber, to see the \( K_0^* \) break up is much more difficult. So, in such cases as this, it may be that emulsion has a certain advantage in a study of isobars.

* * *

3348/NP/smg
DISCUSSION

Holthuizen : Concerning the possibility of polarization measurements in nuclear emulsion, we have studied the possibility of measuring the polarization of 12 GeV/c protons. These protons can be polarized if there has been spin-orbit interaction in the production in Be. The polarization could be measured by the left-right asymmetry in the small angle scatterings in emulsion. This asymmetry will be important for the angles which are at a minimum of the diffraction scattering. It turned out that, for the angles with minimum diffraction scattering for Ag and Br, the diffraction scattering of C, N, and O has become a maximum. The asymmetry for scattering on C, N and O will appear at a much larger angle. So the asymmetry in scattering on Ag, Br, will be overlapped by the normal diffraction scattering on C, N, O. We think emulsion is not a good analyser for polarization at high energy.

Nikolić : What do you think of the interest in studying the stimulated decay of hyperfragments?

Burhop : An interesting thing to study with hyperfragments is, of course, the proportion of non-mesic decays as a function of Z. There is some interest in the relative proportion of neutron and proton stimulation of hyperfragment decays. This can be obtained by observing the proportion of fast protons that come from hyperfragment stars. These are interesting points. They give information about $\Lambda^0$-N interaction processes.

Nikolić : There are no new theoretical interpretations during the last three years although there are experimental results.
Burhop: I think this is largely because it is difficult to identify the heavier hyperfragments and to obtain results that refer to a particular type of hyperfragment.

Zakrzewski: Recently we have studied this problem from the experimental point of view. We have examined the methods of determination of the neutron to proton stimulation ratio $R$. From the comparison of results obtained by various authors, it appears that this ratio depends on the selection criteria adopted to collect the sample of non-mesic hyperfragments. Some of these criteria make use of the decay characteristic of hyperfragments, thus introducing the possibility of biases. We have put forward a suggestion that the selection criteria should essentially depend on the analysis of the hyperfragment production in a given reaction, and not on the characteristics of decay stars. By this method, it should be possible to collect a sample of hyperfragments with masses which could be statistically estimated, and study the ratio $R$ separately for light and heavy hypernuclei. If this ratio is well determined, perhaps the theoretical physicists will also show more interest in this problem.

* * *

3/NP/smg
Fig 1  Total prong number distribution for non mesic hyperfragments produced in 800 MeV/c K interactions

Fig 2  The K meson is captured at rest (star A) to produce a $\pi^-$ meson and a $\Xi$ hyperon. The $\Xi$ hyperon interacts in flight (star B) with the production of a $^1\Sigma^-$ hyperfragment which decays at rest (star C) into a $\pi^-$ meson, a proton, and a $^3\Sigma^-$ hyperon (not discernable in the photograph).
Fig. 3 A K meson is captured on a $^{12}$C nucleus to produce a $^{\Sigma^+}$ hyperon (1) proton (2), two $\pi$ particles (3, 4) and a deuteron (5). The $\pi$ particles are probably emitted from the ground state of a $^8$Be nucleus. The $^{\Sigma^+}$ hyperon is seen to decay in flight (at point b) into a very steeply dipping $\pi$ meson. The distance between a and b is 3090 μm.
A K meson interacts with a $^{12}$ nucleus to produce a $^2$ hyperon (1) a proton (1), a triton (3) and a $^2$He nucleus (2). The distance between a and b is 10150 μm.
THE DETERMINATION OF THE LIFETIME OF THE $\pi^0$ MESON

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I would like to discuss our experiment on the $\pi^0$ lifetime in this session on measurements and techniques because it is this feature of the experiment that I particularly want to stress. The persons who are directly involved in the $\pi^0$ lifetime experiment at Berkeley are Dr. Hla Shwe, Francis Smith and Dr. Walter Barkas. As for myself, I was merely a very interested bystander, and what I am able to relate to you is what I have learned from them. There are a lot of details of the $\pi^0$ meson experiment which I confess I do not know, but I think I can describe fairly well the technique that was used, which I think is of particular interest here. Also, the experiment was carried out with a digitized Koristka microscope which is another reason for giving this talk after Dr. Gibson's resume.

There have been a number of groups throughout the continent and the U.S. who have used the emulsion technique to estimate the $\pi^0$ lifetime. Most of these experiments have used those decays of the $K_{\pi^2}$ meson where the $\pi^0$ produces a Dalitz pair. We chose to undertake the $\pi^0$-lifetime measurement from a different point of view; a way that overcomes some of the recognized difficulties of the $K_{\pi^2}$ method. I think the main difficulty in the $K_{\pi^2}$ method is that the gap between the origin of the electron pair and the end of the stopping $K^+$ meson is very difficult to determine. One does, however, have the advantage that the identification of the electron pair is unambiguous. Also, one knows precisely what the momentum of the $\pi^0$ is, and thus the flight time of the $\pi^0$. In our case, we chose to examine statistically the gap distribution of electron pairs that were derived from the decays of $\pi^0$ mesons, via the Dalitz mode, produced in the interactions of 35 GeV/c $\pi$ mesons with emulsion nuclei. The two types of events we
need to consider are schematically shown in Fig. 1. First let us consider a star produced by a 35 GeV/c $\pi^-$ meson where all the near-minimum tracks are charged $\pi$ mesons and/or fast protons (Fig. 1a). This type of event we term a "star-centred" event because the particles are emitted from the point of collision. The star-centred event is by far the most common event. By using a technique which I shall describe shortly, we were able to locate quite precisely the origin of the interaction. By examining all pairs of tracks from star-centred events, including the incoming $\pi$ mesons, we found that we could locate the star origin within an area of about 0 04-$\mu$ radius which we called the "circle of confusion." The object of the experiment was to find pairs of tracks which did not originate within the circle of confusion. To do this, we examined the origins of all pairs of tracks from all stars. Usually, the origins were in the circle of confusion. Once in a while, however, we would pick up a pair whose origin did not lie within the circle of confusion. If the gap were large enough and the errors were small enough, we would identify this as a Dalitz pair (Fig. 1b). The distance between the star centre as determined by at least one star-centred particle and the incoming pion, and the origin of the pair then represented the flight distance of the $\pi^0$. Since we did this experiment in glass-backed emulsions, it was not possible to multiple scatter a suspected Dalitz pair in every case. But in each case where there was proper orientation of a Dalitz pair and scattering measurements could be made, the Dalitz pair indeed turned out to be a true electron pair.

Now, let us discuss the technique we used to identify and measure the flight-paths of the $\pi^0$'s. The measurements were done with a Koristka MS2 microscope. Figure 2 is a photograph of the Koristka microscope which shows some of the modifications we have made on it, remembering that the original MS2 has a very shallow neck and really limits one to small plates. By a very simple method we put an extra block on the back of the microscope base and moved the yoke back about four inches. The objective was replaced on the optic axis.
by inserting an aluminium block behind the optical head. Thus, we increased the depth of our throat by about a factor of two. We now have a nine inch to ten inch deep throat which makes the instrument much more flexible. As a matter of fact, about a year after we made this modification, Koristka came out with a deep-throat version of the MS2. We use a standard Koristka filar micrometer, as shown, but one whose co-ordinate reading is digitized. The digitizer has $10^5$ units. A single division on the filar micrometer corresponds to a few thousands of a micron—about ten times the sensitivity of our measurements. As I shall explain later, we needed a special stage mount that permitted us to rotate the emulsion plate through 90°.

The plates were on the stage with small magnets to hold them in position. The motion of the filar micrometer is initiated by the knob (lower right). Of course, we use this instrument for multiple scattering measurements, but I think for the last year it has been used almost exclusively for measurements of the co-ordinates of silver grains in particle tracks, as was done in this experiment.

So much for the hardware of the experiment; now let us turn to the actual measurements themselves. First, the scanner centred the point of interaction in the field of view. The only selection criteria for stars to be measured was that they have at least three minimum outgoing tracks. The reason for this is that should two of the tracks be a Dalitz pair, we needed at least one star-centred track in addition to the incoming pion to determine the star centre and, hence, the origin of the $\pi^0$. Using the filar micrometer, we determined the relative $x$ and $y$ co-ordinates of the centres of the first six grains nearest the star centre for each fast track, including the primary pion. This was done in two steps, the $x$ co-ordinate first, then, after rotating the plate through 90°, the $y$ co-ordinates. (The $z$ co-ordinates of the grains were not measured.) The stage was not translated at any time throughout these measurements. The readings were punched into IBM cards and the cards were processed by an IBM-650 computer. The computer was programmed to calculate a least-squares straight-line fit to each track. The intersections of all pairs of
tracks and their errors were then calculated. Before the measurements were done, each track was examined, and a rather detailed and accurate drawing of the event was made. Thus, the scanner who was doing the measurements had a well-scaled and detailed drawing in front of him.

Showing some of the results of these measurements does not convey how well a scanner can measure the centres of grains. A good scanner can reproduce these measurements to 0.02µ to 0.03µ. When we fit a least-squares straight line to these data, we find, however, that the centres of grains are distributed about this line by about ± 0.05µ. In other words, the grain noise of the K5 emulsions we used in this experiment is about twice the error of measurement.

Figure 3 shows the actual points, in scale, of the grains of a typical star-centred event. The uncertainty in the origin, i.e., the circle of confusion, of this event is about 0.04µ. The straight lines are fits to the x-y co-ordinates of the grains in each track. Star-centred events such as this serve to calibrate our technique and measurement errors. For instance, how well we can define the origin of a pair of tracks depends on reading error and grain noise. Another difficulty may arise from changes in the temperature during a measurement although we did take some steps to limit this. Other sources of error are distortion and multiple scattering. From the many star-centred events we measured during the course of the experiment, we were able to find out just how well we could measure the origin of a pair of particles. Figure 4 shows the distribution of distance between the origins of pairs for a large sample of star-centred events. The mean distance is 0.06µ. Since all pairs are distributed some 0.06µ from each other, it follows that the origin of the primary interaction star is known to about 0.03µ to 0.04µ. Actually I would say that practically all of the origins are known to be better than 0.1µ, which, as we shall see, is sufficiently accurate to carry out the experiment.

Figure 5 shows the kind of result we get when a pair does not intersect the circle of confusion, and a gap is detected. Here we see the incident π⁻, two star-centred tracks and a pair of tracks.
which do not intersect the origin. The error bar for the origin of the pair is also indicated. In this case, the error is perhaps \( \frac{1}{4} \) of the gap. In order to feel that we definitely had a measurement of a gap, all the events that we used to evaluate the \( \pi^0 \) lifetime were those for which the error in the pair origin was \( \frac{1}{2} \), or less, of the gap length itself. Of course, this selection of events brings in biases that have to be taken into account, as we have done.

Figure 6 then gives the observed distribution of gap lengths between the star centre and the origin of the electron pairs. The typical gap length is about 0.5\( \mu \). This is a very important feature of the experiment and one of the reasons we chose to do it this way. When a \( \pi^0 \) is produced by a 3.5 GeV/c \( \pi^- \), the energy and hence the time dilation of the \( \pi^0 \) is large enough to produce an easily measured gap. The time dilation factor \( \gamma \) is typically ten times larger than can be realized from the \( K_\pi^0 \) decay. The mean gap length of this distribution is 0.5\( \mu \), so with errors typically a few hundredths of a micron, we have a rather good signal and a measurable effect.

In summary then, our technique has been to measure co-ordinate centres of grains, and to use this information for a least-squares fit to the track. From this analysis, we are able to determine the origins of stars and the origins of the decay of \( \pi^0 \) mesons. This technique can be used, in principle, to measure shorter distances and lifetimes by merely increasing the energy of the interaction. The larger \( \gamma \) one has to work with, the smaller lifetime one can measure, in principle at least.

I want to discuss now how we obtain an estimate of the \( \pi^0 \) lifetime from the measurements we have seen. We can write down a simple formula for the probability that a particle decays at distance \( s \) from the origin in the interval \( ds \)

\[
P(s) ds = - \frac{1}{\eta \gamma} \exp \left[ -s/(\eta \gamma) \right] ds,
\]

where \( \eta = \beta \gamma \) is the momentum of the decaying particle divided by its rest mass, and \( \gamma \) is its proper mean life.
Now in our experiment, the momentum $\eta$ is not a constant quantity, but a distribution. Therefore, we have to include in this expression the probability that a particular particle which decays at $s$ has a momentum $\eta$. This we shall denote as $f(\eta)d\eta$. We must now recognize the fact that even though a Dalitz pair is produced, it is not necessarily an observable and measurable event. We can detect a Dalitz pair if its opening angle is large enough and if the gap is greater than a certain minimum distance. This distance depends on the opening angle of the pair.

To account for these limitations on our ability to observe Dalitz pairs we must also include in our expression above, the probability $q(\eta, \alpha)d\alpha$ that a Dalitz pair has an opening angle in the interval $d\alpha$, and the probability $W(s, \alpha)$ that a pair is measurable.

The final expression for the probability for a $\pi^0$ of momentum $\eta = \rho_0/(m_\pi^0)$ to decay in the interval $ds$ at distance $s$ from the star origin and produce

$$P(s)ds = -\frac{1}{\eta \sigma} \exp[-s/(\eta \sigma)] f(\eta) q(\eta, \alpha) W(s, \alpha) d\eta d\alpha ds,$$

The momentum distribution $f(\eta)d\eta$ was obtained from the momentum distribution of the charged $\pi$'s produced in the same interactions. We have assumed the momentum distribution of the neutral pions is the same as that of the charged pions $q(\eta, \alpha)d\alpha$ was calculated and $W(s, \alpha)$ was determined empirically. The only unknown parameter is $\tau$, the mean life of the $\pi^0$.

Figures 7-10 are concerned with the $W$ and $q$ functions. I have just mentioned. Figure 7 reveals the biases we had in detecting Dalitz pairs. Here we have plotted $s$, the gap distance of the pair from the star, versus the product of the distance, $s$, times the opening angle, $\alpha$. This gives a rather interesting result. Practically none of the observed events had a product so less than $4\mu$-deg. This means we could detect and measure a pair with an opening angle of $4^\circ$ if it occurred at a distance of one micron from the origin. Anything less than $4\mu$-deg we missed, or else the error in the gap was so large that we excluded.
it from the sample. Then, too, as we go to larger and larger opening angles, the minimum distance at which a pair can be detected also increases. Pairs with large opening angles at small distances look like star-centred tracks. From these observations we concluded that the function $W$ can be treated simply as a function of $sa$, where $W = 1$ for events lying inside the boundaries shown (i.e., $4.0 < sa < 72.2$), and $W = 0$ for events outside these boundaries. In Fig. 8 the transverse momentum distribution of the charged $\pi$'s is shown. Since we could not determine the momentum distribution $f(\eta)d\eta$ of the $\pi^0$'s, we assumed it to be the same as for the charged pions. The transverse momentum data were obtained by multiple scattering some $1000 \pi^+$ tracks. The distribution of the opening angle of the Dalitz pairs in the laboratory, calculated and observed, is shown in Fig. 9. The circles are the calculated points. Owing to the difficulty in transforming the opening angle distribution of the Dalitz pairs from the centre of mass to the laboratory frame when the $\pi^0$'s have a momentum distribution $f(\eta)d\eta$, some approximations were made. This accounts for the points being a little rough. A smooth curve is drawn through the calculated points. The experimental data are shown in histogram form, and the fit with the expected distribution is quite good. Since we have measured the points of production of the $\pi^0$, we are able to compare the projected angular distribution of the $\pi^0$ mesons with that of the charged pion. These angular distributions are shown in Fig. 10. They are very similar indeed.

A priori it is not possible to state that whenever we detected a gap between a star origin and an electron pair that it was in fact the decay of a $\pi^0$ meson. The data shown, however, gives us confidence that the electron pairs we have detected are actual Dalitz pairs from $\pi^0$ decay. Specifically, the angular distribution of the pairs is consistent with the distribution observed for the charged $\pi$'s, indicative of $\pi^0$'s. Also the distribution of opening angles of the pairs in the laboratory agrees with that expected from $\pi^0$ decay. Another point I should mention is that, if we assume charge independence, that is, the number of neutral pions produced is one-half the number of charged pions,
the total number of Dalitz pairs expected in our sample of events is about 100. We actually observed 109 such events. This evidence justifies the assumption that the moment distributions of the neutral and charged pions are the same.

The evaluation of the $\pi^0$ lifetime from the measured data was done by two methods: curve fitting the integral gap-length distribution (Fig. 6), and the maximum-likelihood method. Figure 11 is the $\chi^2$ fit to the integral gap-length distribution as a function of $\tau$. The lifetime which best fits the data is $2.0^{+0.6}_{-0.4} \times 10^{-16}$ second. When the gap length data are corrected for the $z$ component of direction, the $\pi^0$ lifetime is increased by about 5%. A better estimate of $\tau_{\pi^0}$ is obtained by the maximum-likelihood method. The $\pi^0$ mean life given by this latter method is $2.0^{+0.5}_{-0.3} \times 10^{-16}$ second.

This concludes the outline of our $\pi^0$ lifetime experiment, in which I have highlighted the method by which we measured the small gap distances. Undoubtedly this kind of technique will be important to other measurements of this kind, for example, the identification of events with double star centres in studies of the production of hyper- and cryptofragments.

* * *

\[348/NP/sm\]
DISCUSSION

Holthuisen : I would like to mention that a measurement similar to that described by Dr. Heckman has been performed in Amsterdam to measure the cryptofragments. There is some difference in the apparatus; since the hairlines in our eyepieces can be moved in two independent directions and both of them are digitized, so one can record the x and y co-ordinates simultaneously. We used G5 emulsion exposed to 800 MeV/c K^- mesons. The smallest distances that can be resolved in this way are 0.1 μm, but we hope that it will be better with K5 emulsions.

Combe : I should like to ask a question about the advantage you get in increasing the energy of the π, as you mentioned. Maybe you will have some trouble with the parameter σ or something like that.

Heckman : Actually, this experiment was initially a test experiment. We had to learn how to make these measurements and to learn of the difficulties we might encounter when we got to work on the 16 GeV/c π^- mesons we obtained from CERN. But as the measurements went along, we realized we were getting very good results. We reconsidered the 16 GeV/c experiment, which we thought would really be a better experiment, and the following difficulties became evident. First, the opening angles of the π^0 Dalitz pairs would be smaller, and our estimate of the location of the decay point would have larger errors than those we obtained in the present experiment. Although we do not have any evidence for it, the minimum σ measurable in a 16 GeV/c π experiment may be larger than 4μ-deg, mainly because the star centre would be less well defined. Also, in order to determine the
momentum distribution of the $\pi^+$, I am sure the scattering measurements would be considerably more difficult, and the result would not be as good as we obtained from the lower momentum. We would, none-the-less, have larger values of $\gamma$ and, thus, larger gaps. However, we felt that the advantages and disadvantages of doing the experiment at a higher momentum essentially cancel each other and that 3.5 GeV/c was probably as good a momentum as we would have chosen in the first place.

Zakrzewski: Did you have minimum tracks for determining the centre of the primary star, and if so what degree of difficulty would you find if you used black tracks or grey tracks?

Heckman: In the $\pi^0$ experiment we used only fast tracks and light grey tracks to determine the star origins. In another experiment we have measured slow tracks. We are now involved in an experiment measuring the momentum-energy balance in the capture reaction of $K^-$ mesons at rest by free protons in emulsions. We wanted to find out how collinear the $\Sigma$ was with respect to the $\pi$. The $\Sigma$ hyperon has a momentum $pB \approx 25$ MeV/c, which is quite low. The results of the measurements showed that indeed you can measure the direction of a particle of this momentum over a track section of $\approx 15\mu$. Multiple scattering is the main limitation here, and in fact, you can estimate precisely how much error to expect in the measurement strictly from the multiple scattering formula. The accuracy of centring on the grains in a black track, as the $\Sigma$ is in this case, turns out to be as good as centring on the grains in a minimum track. This is because at a particular point in a dark track there are many more grains on which to centre, hence you have more information on which to estimate the track centre.
It is my experience, then, that this kind of measurement on black and grey tracks is limited primarily by multiple scattering.

Zakrzewski: Did you choose your cell-size according to the energy of the slow particle which you were measuring?

Heckman: We attempted to determine the cell-size, or track segment, empirically. The idea was to take longer and longer cells with the hope that we would see an improvement in our fit to a straight line (because the increase in the number of grains would improve our statistics), but, then, after some distance, multiple scattering would take over and the fit would become increasingly worse with increasing cell lengths. We would then choose the cell length which had the least error in the slope. We found, however, that the error in the direction of the 25 MeV/c Σ always became larger with increasing cell lengths, at least for the cell lengths we felt we could measure with some confidence. Our best estimate then was obtained when we took as small a cell length as possible, 15µ in this case. Ten readings were made on the track. In the π⁰ experiment, multiple scattering was not so large a problem. Here it appeared that measurements on the first six grains of each track - a segment of about 50µ in length - gave the best fit to a straight line.

Bott-Bodenhausen: I see you have a digitizer attached directly on the screw of the microscope. How much noise do you receive from the movement of the screw? An alternative method is to add a digitizer like Feranti directly onto the stage.
Heckman: This problem was looked into. We had several scanners measuring the centre of each grain of a particular track several times, first from one direction, then from the other. I do not believe we saw a significant effect. The digitizer is directly coupled to the filar micrometer screw, so backlash is negligible. Any error due to noise in the filar system was folded into the uncertainty of the origin of the star-centred prongs. In any case, the total measurement noise was always less than the grain-noise of the track itself.

Key: We are trying to do a determination of the \( \pi^0 \) lifetime at Oxford using the \( K_{\pi^0} \) mode of decay. I may mention that the gap length for a time of \( 10^{-16} \) second is only 0.05\( \mu \) which is about \( 1/4 \) of what you get. Our main problem is in determining the exact point of decay of the \( K^+ \) because one gets a large blob at the end of the black K track. It seems difficult to determine, to within 0.1\( \mu \), exactly where the \( K^+ \) decayed. On the other hand, as you say, we do know accurately the dynamics of the event. The method of measurement we use is a double filar eyepiece micrometer so that one can immediately get the two co-ordinates from one setting. At the moment it is not digitized like the one at Amsterdam but we hope to add this. I should like to ask a question. What do you mean when you quote \( +0.5, -0.3 \times 10^{-16} \) sec? What is the definition of your error?

Heckman: The error is approximately \( \pm 1 \) standard deviation. It encompasses two-thirds of the area under the curve of the distribution function (e.g. Fig 11): one-third above the mode, one-third below.
Gibson: I would like to add a small technical point in answer to the question before the last one. If you make a digitizing system properly, you arrange that your filar eyepiece or your stage is controlled by a knob. There may be some backlash between the knob and the stage which you arrange to be the same as the backlash between the knob and the digitizer. In that way you can get the digitizer to reproduce the stage very faithfully. It is a matter of care in manufacture and operation that leads to Dr. Heckman's excellent results.

* * *
Fig 1 Schematic drawings of:
a) typical star centred event
b) star with an associated Dalitz pair

Fig 2 Digitised Korista MS2 microscope Modifications include deep throat rotating stage and encoder assembly mounted on the filar micrometer
Fig 3  Coordinates of grain centres and the least squares straight line fits to the particle tracks for a star centred event

Fig 4  Distribution of gap lengths between the points of intersection of pairs of tracks

Fig 5  An event in which \( \phi \) produced at \( 0 \) decays at \( P \) via the Dalitz mode \( \phi \rightarrow e^+ e^- \mu^+ \mu^- \). Points are centres of grains and the straight lines are least squares fits.
Fig 6  Integral distribution of projected $s^0$ path lengths. Curve is best $\chi^2$ fit to the data points.

Fig 7  Distribution of $s$ vs $m_s$ for observed Dalitz pairs. In the region $40 > s > 72.5$ the events are inaccessible to measurement.

Fig 8  Transverse momentum distribution of secondary charged pions.
Fig 9  Laboratory system opening angle distribution of Dalitz pairs from $\pi^0$ decay. Data are given by histogram; theoretical distribution by smooth curve.

Fig 10  Projected angular distributions for:
(a) secondary neutral pions and
(b) charged pions.

Fig 11  $\chi^2$ distribution for integral gap length data as a function of
SMALL-ANGLE SCATTERING

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This is a rough survey containing no deeper considerations of the problem of small-angle scattering.

I think, the ideal case in doing an experiment is to be confronted first with the physical problem and then to work out the specific way in which to do it best, in our case, either with counters, bubble chambers or emulsions. Though this ideal will only seldom be realizable (since one likes to favour the method one is most familiar with), I include also methods other than emulsion techniques and mention some of their special properties and limitations with respect to the study of small-angle scatterings. This may serve as a source of information, which method might be applied when one wants to investigate specific problems on small-angle scatterings at energies above some GeV.

'Small' is a relative concept. The absolute value depends at least on two factors, namely on the 'profession' and the laboratory of the physicist. Counter people at CERN usually call a small-angle scattering on of the order of degrees; emulsionists at CERN think of the order of some milliradians. So we are mainly considering lab-angles above 1.5 mrad and we are especially interested in elastic scatterings.

I PHYSICS

1 General considerations

It is interesting to study the nature of the elastic scattering amplitude $T(\theta)$ for the following reasons.
In general \( T(\theta) \) can be described by a real and an imaginary part,

\[
T(\theta) = R(\theta) + iI(\theta)
\]

potential diffraction
scattering

with

\[
\frac{d\sigma}{d\Omega} = |T(\theta)|^2
\]

A potential scattering, for instance, happens when an electrical charge moves through an electrical potential field. Diffraction scattering, on the other hand, is a side effect of absorption.

Considering only nuclear forces, the dispersion relations predict that \( R(\theta) \) becomes small in comparison to \( I(\theta) \) at high energies and this has to be checked.

If one now has a measured angular distribution (see Fig. 1) and it is fitted by a 'reasonable'\(^*\) curve, one can immediately calculate a possible real part at small angles with the additional correlation\(^**\):

\[
\text{Im} \left\{ T(\theta = 0) \right\} = I(\theta = 0) = \frac{k\sigma_T}{4\pi}
\]

where \( k = p/\hbar \) and \( \sigma_T = \text{Total cross-section} \). Thus,

\[
R(\theta = 0) = T(\theta = 0) = i \frac{k\sigma_T}{4\pi}
\]

Unfortunately, because of the presence of beam tracks and of the diminishing solid angle one cannot measure \( \theta = 0 \) Therefore.

\(^*\) Meaning that the shape of such a curve should be theoretically suggested

\(^**\) For more details see Massey, Handbuch der Physik, XXXVI, 232
one must go down as far as possible and then extrapolate to zero

So far we have been considering scattering by nuclear forces, and should perhaps mention that the scattering at large angles ($90^\circ$ in c.m.) is very interesting too. The classical picture there is that both collision partners approach very close to each other. From p-p collisions one then obtains indications about the core-core interaction.

If one has a p-p scattering there is, besides this nuclear scattering, the normal Coulomb scattering, which is potential scattering$^*)$ (for which the amplitude goes to $\infty$ with $1/\theta^2$). At first sight this seems to be a bad complication, but we shall see soon that it is a lucky one; the presence of the Coulomb field has an amplifying effect!

2 Interference amplification

With Coulomb field the real part $R(\theta)$ splits up into two components,

$$R(\theta) = C(\theta) + N(\theta)$$

- Coulomb nuclear part

Suppose $I(\theta)$ is known in shape and normalization, \{\text{c.m.} = k\theta^2/4\pi\}, from measurements at other angles and at different energies, or from the normalization point and a theoretical curve$^{**})$

$C(\theta)$ can be calculated with the Möller scattering formula.

$^*)$ Practically completely real amplitude

$^{**}$) At 24 GeV $I(\theta)$ changes very little below 4 mrad

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Figure 2 shows how Coulomb and nuclear scattering add up for 24 GeV protons when one neglects a possible nuclear real part. The unit of the ordinate is proportional to \( \Delta N/\Delta \theta \), i.e., the number of events per space angle interval (integrated over the azimuth angle)

\[
\frac{\Delta N}{\Delta \theta} = \frac{\Delta \sigma}{\Delta \Omega} = 2\pi \sin \theta \frac{\Delta \sigma}{\Delta \Omega}
\]

We now form the expression:

\[
x = \frac{2CN + N^2}{T^2} = \frac{T^2 - I^2 - C^2}{T^2} = \frac{d\sigma/d\Omega_1 - (I^2 + C^2)}{d\sigma/d\Omega_2} \quad ,
\]

where \( d\sigma/d\Omega \) can be measured, and \( (I^2 + C^2) \) is known. \( x \) must be zero if we have no nuclear real part and it is thus the measured relative deviation caused by the nuclear real part. With \( T = C + N + iI = C + \alpha I + iI \) where \( \alpha = N/I \), then \( y = I/C \) follows from

\[
x = \frac{2\alpha I + \alpha^2 I^2}{C^2 + 2\alpha I + \alpha^2 I^2 + I^2} = \frac{2\alpha y + \alpha^2 y^2}{1 + 2\alpha y + \alpha^2 y^2 + y^2}
\]

(1)

Let us discuss this under the assumption that \( \alpha \ll 1 \)

With no Coulomb field (or only a small one), at 24 GeV for 5 mrad (see Fig. 2), one obtains:

\[
x = \alpha^2 \frac{I^2}{\alpha^2 I^2 + I^2} \approx \alpha^2 = \left( \frac{N}{I} \right)^2 = \frac{d\sigma/d\Omega_1 - (I^2 + C^2)}{d\sigma/d\Omega_2} \quad ,
\]

with an error

\[
\Delta x = \frac{I^2 + C^2}{(d\sigma/d\Omega)^2} \quad \Delta \frac{d\sigma/d\Omega_1}{d\sigma/d\Omega} \approx \Delta \frac{d\sigma/d\Omega_2}{d\sigma/d\Omega}
\]

This is simply a consequence of the fact that cross-sections are the squares of amplitudes, and means that, if one wants to measure a 5% effect in \( N/I \), one needs a precision better than 5% in \( d\sigma/d\Omega \). This is very difficult - if not completely impossible - to achieve.
How does the situation change when a Coulomb field is present? We assume that \( \alpha(\hat{\theta}) \) is constant. From (1) it follows that

\[
x = \frac{2ay}{1 + 2ay + y^2}
\]

In this case the effective relative signal still depends on \( y = I/C \). Asking for the point of maximum signal one has to put \( dx/dy = 0 \). That gives

\[
y = 1, \ I = C, \ \text{and} \ x = \alpha = N/I
\]

The error is

\[
\Delta \alpha = \frac{I^2 + C^2}{(\sigma/\partial\Omega)^2} \quad \Delta \sigma/\partial\Omega \approx \Delta \sigma/\partial\Omega
\]

We have thus a gain in precision by a factor \( 1/2\alpha \) at the point where \( I = C \); or, in other words, one measures directly the nuclear real amplitude at this point.

For example, at 24 GeV the point at which \( I = C \) is at \( \hat{\theta} = 2.5 \text{ mrad} \) (see Fig. 2), and it would be very desirable just to explore the 'valley' in Fig. 2. An experiment for this purpose is discussed in Section III 4.

II BEAMS

We shall mainly consider protons since we have here the largest experimental material, and we start with a brief description of the elastically scattered-out proton beam. (For the moment it has the abbreviation 'C3'.)

Given the circulating proton beam hitting an internal
target*) producing all kinds of secondaries within a wide momentum and angular region, the question arises how to separate the protons. For this we remember that, if one wants to separate particles in momentum and mass, one always needs two fields of different nature, for example an electrostatic and a magnet field (cf. Murray separator placed in a magnetically analysed beam, or the method of leading a momentum analysed particle mixture through a layer of Be in which it suffers ionization loss, and then again through a magnetic field)

In our case the idea is to use first the strong nuclear field of the struck target nuclei and second the fringing field of the PS for this separation. In practice this means that the wanted beam does already exist. Protons elastically scattered in the target suffer on average the smallest energy loss. A proton from the inelastic process: \( p + p \rightarrow p + p + \text{pion} \) (24 GeV) has at least 4.00 MeV less energy than from the corresponding elastic process**). Therefore the fringing field treats the elastically preselected protons in a unique manner. In the narrow forward cone (outside the fringing field) there exists for every point in space a distinct and calculable direction of motion which can only belong to elastically scattered protons.

This is illustrated in the series of figures, where Fig. 3 is a general view of parts of the ring, the shielding wall and of the south hall. Target 1 has to be used, and Figs. 4 and 5 show the result of the beam calculations 1) We have divergence in the horizontal plane and convergence in the interesting vertical plane. The angular distribution of this beam (with the simple set-up shown in Fig. 3) at a point in the south hall is given in Fig. 6. One recognizes that it can be very parallel. The standard deviations in this case are:

---

*) This kind of beam extraction has been applied since early 1960. In 1963 it will probably be replaced by the fast ejected beam and later on the slow ejected beam.

**) For this qualitative discussion we neglect any influence of Fermi motion and assume p-p collisions.
\[ \sigma_x = 0.2 \text{ mrad}; \]
\[ \sigma_y = 0.5 \text{ mrad} \]

The intensity is \(0.4 \times 10^4\) protons \(\cdot\) \(\text{cm}^{-2}\) per \(10^{11}\) protons in orbit.

Figure 7 shows a momentum analysis.

In principle it is possible to increase this intensity without loss of parallelism. A cylindrical convex lens close to the target prohibits further divergence in the horizontal plane (see Fig 4) and a concave lens avoids further focusing in the vertical plane (Fig 5). The limits are set by the closest possible quadrupole distance from the target and the opening angle which is limited by the iron of the yoker and by the finite effective target size.

On the other hand one can naturally focus the beam onto a distinct spot in the south hall. This leads to larger angular divergences but to very high intensities. For example Dahl-Jensen, Dayton and Hansen in November 1961 obtained the following beam:

\[ \sigma_x = 0.5 \text{ mrad}; \]
\[ \sigma_y = 1.1 \text{ mrad} \]

with \(1.29 \times 10^9\) protons \(\cdot\) \(\text{cm}^{-2}\) for \(10^{11}\) protons in orbit.

Now for small angle scatterings we need mainly extremely parallel beams. Let us conclude that from the diffracted proton beam angular divergences between 0.1 and 1 mrad are obtainable. The proton beam can be made very clean as far as the contamination from \(\gamma\)'s and neutrons is concerned, when shielding in the ring area is placed in the right position.

To our knowledge, the pion beams have not been studied and developed so systematically either for small angle or for emulsion purposes. A pion beam measured in October 1961 had an angular divergence of \(\sim 2.5\) mrad with an intensity of \(300 \pi^- \cdot\) \(\text{cm}^{-2}\) per pulse.
III TYPICAL EXPERIMENTS

We sketch first some typical events

i) a typical elastic p-p event at high energies has the following properties

One very small angle deviation and one heavy track at about 90°
No particles produced

ii) An elastic p-nucleus event in emulsion *) has the following modes of appearance

One small angle deviation + nothing else visible
One small angle deviation + whole-nucleus recoil or small visible excitation
No particles produced **) 

Illustration of (i) is shown in Figs 8a) and 8b) which belong to p-p scatterings in the 30 cm CERN HBC. The event shown in picture 8a) is on the 'reliability limit' (***) (at small angles) in liquid hydrogen, and is a scattering of 4 mrad. The picture of a p-p scattering in emulsion with an angle of 26' is shown in Fig 9)

In case of p-p scatterings we have a small angle plus most frequently a visible recoil. In case of p-nucleus scattering, however, the recoil can be extremely slow, and short, so that one

---

*) We are here just considering emulsions, because they have the highest resolution for ranges. A proton with an emulsion range of 150μ is on the detection limit in a heavy liquid chamber, for example

**) γ's from excited nuclei are permitted

***) Naturally the 'detection limit' is lower
will a priori expect a larger group of events showing no visible recoil at all. One will therefore apply different methods of investigation, depending on what one is looking for, and following the survey given in Table I we may discuss the effectivenesses of the different methods.

1 Flat exposures

Here one uses preferentially the simple 'pick-up' and 'following-along-the-track' scan

i) 'Cinerama-scan'

When scanning in this way, one can follow several cm per minute*); seeing only the shadow of the track. Irrespective of what happens if the propagation of the original beam track has a change in direction, one will notice this and stop and examine the event.

Naturally there is a lower limit for the detectable angle **). A rough experimental estimate of this is given by the fact that we observed four p-p scatterings where we should have seen eight. At 24 GeV the loss begins below ~ 30'***). At 6 GeV, where multiple Coulomb scattering is larger, this limit is higher, perhaps ~ 1°.

As far as elastic p-p scattering is concerned, the following-along-the-track method is not very effective. At 24 GeV one expects eight of these events among 900 stars. This means that to have the possibility of obtaining eight events, one must follow some 325 m path. So this is a rather tiring kind of

*) If the tracks are flat, parallel to the direction of movement, and if the density is < 10⁵ p's/cm²

**) This is determined by the stability of the stage movement, the following speed, the magnitude of multiple Coulomb scattering (i.e. of the primary energy) and by the observer

***) The detection of limit can be much lower. It is found for example scatterings of 8'
investigation

How does this method work for p-N scatterings? From the point of view of the scan there is no difference since one sees only the shadow of the light track. In spite of this the flat scan is much more efficient for p-N scatterings than for p-p scatterings because elastic p-N scatterings are relatively frequent. Their yield above the cut-off (30′ at 24 GeV) is about 4 to 10%. Figure 10 shows the experimental angular distribution of the stars with one light secondary track for energies of 6.2 and 24 GeV.

An elastic p-N event is on our definition a scattering in which no particles have been produced. It is, however, allowed that the hit nucleus has been destroyed. A priori the fraction of slightly inelastic collisions, of the type: $p + p \rightarrow p + p + n\pi^0$, among the possible elastic ones, is not known and it is extremely difficult to separate them off in the emulsions. This can be done in a heavy liquid chamber in detecting the pairs from the decay $\gamma$'s of the $\pi^0$'s.\(^{2}\) (See Table 1 for 'Heavy Liquid Chamber'.)

Everything with a residual range greater than $\sim 2\mu$ can be distinguished as a track, and within this limitation the charged events can be seen.

Discrimination between p-nucleus and elastic p-p collisions is possible by means of a kinematical analysis. One set of necessary correlations is for example:

\[
\begin{align*}
\mathbf{P}_0 \cdot \mathbf{P}_0 & \quad \mathbf{P}_2 \cdot \mathbf{P}_2 \\
\theta_1, \mathbf{P}_1 & \quad \gamma_2 \mathbf{P}_2 \quad \theta_2
\end{align*}
\]

a) $\tan \theta_1 \times \tan \theta_2 = 1 - \beta_{\text{CM}}^L$;

b) coplanarity: $(\mathbf{P}_0 \times \mathbf{P}_1) \cdot \mathbf{P}_2 = 0$;

c) $p_1 = \frac{2m_p \cos \theta_1}{\sqrt{\gamma^2 + 1} - \frac{\gamma_0^2 - 1}{\gamma_0^2 + 1} \cos^2 \theta_1}$.
Having a larger sample of events which have to be divided into elastic and inelastic ones, one plots the deviation from the calculated values in units of the individually calculated errors. In the case of check (a) this is shown in Fig 11 (The plot was taken from bubble chamber statistics). Check (c) which would be a plot of

\[ \frac{P_1 \text{(rest-range)}}{\Delta(\sim)} - P_1 (\theta_1) \]

is frequently replaced by a plot of

\[ \frac{P_1 \text{(rest-range)}}{\Delta(\sim)} - P_1 (\theta_2) \]

depending on which of the methods summarized in Table I is used.

When the rest-range is not directly measurable, one can take \( P_1 \) (P \( \beta \)) or \( P_1 \) (curvature in magnet field).

ii) Slow flat scan

The techniques of exposure etc. are exactly the same as in III 1 (i) When one follows a track very slowly, however, one has difficulties in detecting very small changes in direction if there is no other track indicating the point of scattering. One expects thus a larger bias for events with no visible recoil. On the other hand, one will certainly find events in which the primary maintains its direction and which have a shorter recoil, for example, one with \( R \approx 5 \mu \), which would belong to a p-p scattering with momentum transfer \( \geq 30 \text{ MeV/c} \). May I mention in connection with this method an instrument called 'The Tractor'? It has been built in Amsterdam and has an amplifier on the Z and Y source. It makes finding of small angles more easy and more reliable.
iii) 'Sagitta'-scan

This survey would not be complete without mentioning a method to discover especially those scatterings which lead to no visible excitation and belong to an extremely small scattering angle. The method consists of going along the track and at the same time performing scattering measurements on a scattering microscope. With this procedure, angles of some $10^{-5}$ radian can be discovered. This can be of considerable interest at energies above $10^{11}$ eV.

At such high energies one has a certain chance of also deciding whether the event was elastic or slightly inelastic by scanning for fast electron pairs in the narrow forward direction.

2 Perpendicular exposures

We come now to another type of exposure which has first been used by Lyubimov et al.\textsuperscript{4} for the study of elastic scatterings. If the particles traverse the emulsion perpendicular to its surface, the primary tracks appear in the ideal case as points in projection. An elastic p-p scatter is in this projection a collinear event, with the recoil relatively flat (dip-angles in most cases < 20°, for energies above some GeV), and grey or black. Thus the possibility of performing a fast preselection is one of the advantages of this method.

Another important feature is the high intensity which is tolerable. We are working preferentially with $(1-2) \times 10^6$ p's/cm$^2$, others with $4 \times 10^5$ p's/cm$^2$, whereas the intensity for a horizontal scan should not exceed $10^5$ p's/cm$^2$. The only point to avoid is the presence of much material in front of the first plate. Figure 12 shows a typical set-up for exposure.

A further advantage of this kind of arrangement for the investigation of small angle elastic scattering is that it is unnecessary to 'stack' the pellicles. Since the elastic p-p scatterings...
always have one large angle, namely that of the heavy track, which is then flat, the proton stops in the same pelliole if one concentrates on the smaller angle scatters. For the special purpose of studying the smallest angles, it is therefore sufficient to use one pelliole or plate. Then, in order to increase the measuring precision for the forward tracks, one puts other plates or emulsions behind the first one, but with air space in between them. The following through is relatively easy, when the plates have been correctly aligned. The beam tracks form groups which can be recognized.

Besides the high exposure intensity, because of the possibility of using single layers, one can increase the original hydrogen content of the emulsion by a factor $\gtrsim 3$, by soaking the plate in water. This has been done by several laboratories. Reference should be made to the session about processing and to the 'round-table-discussion' where methods have been considered to avoid fading in this case.

1) Scanning for recoils

In this kind of scan one concentrates on beginning, or stopping, heavy (grey or black) tracks. The idea in proceeding in this way is that from time to time one finds a star at one end of such a track and one can thus in principle discover all stars with $n_h = 1$. From this sample the two prong stars can easily be picked out and among these, in turn, the possible elastic events. A favourable scanning magnification is $10 \times 1 25 \times 53$ oil.

The detection limit for the recoil scan is at $R \sim 5\mu$, which corresponds to $0.8$ mrad. The reliability limit is usually chosen at about $1.5$ mrad. The measuring precision is usually better than $10\%$. Let us discuss how we analyse this.
Having an event one takes photomicrographs in different planes. From the side the event would appear as shown in the diagram below. This method gives at once the pattern of the beam tracks and its natural divergence (of the order of 0.4 mrad, see Section II). A photomicrograph taken from plane 2 is shown in Fig. 13 (magnification: $10 \times 1, 25 \times 100$).

We go then to the projection apparatus for bubble chamber pictures and reproduce one image after the other using the points of some beam tracks as fiducials. One can easily reach a very precise alignment ($\sim 0.1 \mu$).

The forward track from the scattering deviates in a certain direction and by a certain amount within the fixed pattern of the beam tracks. The appearance of this is illustrated in Figs. 14 and 15. Figures 13 and 14 belong to the same event.

The recoil method naturally also has disadvantages. One is the large scanning loss. In addition, this scanning loss depends on the velocity of the recoil. For ranges between 10 and 10,000 microns, it is approximately constant, above this it drops rapidly. This is nicely shown in Fig. 16, in which G. Kellner and the Vienna Group compare their data with the bubble chamber curve.\(^5\) The scanning loss in bubble chambers for such events is small down to angles of 4 mrad (see Fig. 8a). As already mentioned, evaluation of scanning biases and proper normalization is connected with large uncertainties. One sure way is to normalize both curves (for 24 GeV between 4 and 10 mrad).
against each other. The only uncompensated contribution from this method occurs for the angular interval between 1.5 and 4 mrad (at 24 GeV). Owing to the smallness of the solid angle one will have very few events in this class. The most recent result from the combined emulsion and bubble chamber data (Vienna and Berne) is shown in Fig. 17.

ii) Scanning for deflections

One can, instead of looking for heavy tracks, concentrate on changes in direction of the individual beam tracks. If one turns up and down fast enough in the z-direction, one can see the 'fishes' swimming in the sea (for reasons of distortion the beam tracks never remain fixed as points) and one can have their motion under control. With this 'line-scan' one will also see white stars and deflections on complex nuclei. The detection limit is at 2 mrad. In contrast to the recoil scan, here the main contribution (i.e., the least biased one) comes from larger angles. In this sense the method is complementary with the 'recoil' method of sub-section 2 (i).

3 Comparison of two angular distributions

This method consists of taking the angular distribution of the beam at the entrance into the emulsion and after some distance in the emulsion. It has been used in Berne at 6 and 24 GeV and in Munich at 24 GeV. Both exposures were flat and had all the path in between the measuring regions in emulsion. Figures 18 and 19 show the result for 6 2 GeV (Berkeley, internal beam) and for 24 GeV (PS CERN, Munich). Naturally one has to subtract multiple Coulomb scattering. The angular distribution of the slightly inelastic scatterings has to be determined by another independent experiment and must be subtracted too.

This kind of experiment does not necessarily need a flat exposure with all the path in the emulsions. It could be perpendicular with two plates in front, two plates behind and hydrogen-rich.
material in between (for example polyethylene) The evaluation
could easily be done with the 'photographic' method described in
sub-section 2 (i) The track density can be very high

These are nice aspects, but nevertheless the method is not
very powerful, since one does not have enough information about the
single individual event

4. External target

We have seen in sub-section I 2 that for 24 GeV protons
the optimum measuring region is between 15 and 5 mrad, but consulting
Fig 17 one sees that only very few events have been obtained there
(perhaps some 21 scatterings), even with the strong effort of three
scanners and one physicist for one year One should thus think of a
more powerful method, capable of yielding instead of just 20 events
perhaps some thousands

Such a possibility seems to exist with an experiment using
an external hydrogen target, analysing magnet for the momentum of
the forward track, and a detector with high resolution in position
and direction It has been used at 3 GeV by Preston et al 6)

For 24 GeV the experiment exists only in the form of a
proposal and will perhaps be done at the PS The set-up is shown in
Fig 20

The protons are scattered elastically or inelastically in
the target T Each individual scattering process is characterized by
two parameters, namely scattering angle and momentum after scattering
With an analysing magnet and emulsion as detector, these two parameters
are transformed in a uniquely defined manner into point of entrance
into the detector plate and direction of motion of the particle in
the plate Measurement of points of entrance and of direction of
track therefore give the wanted scattering parameters

The elastic events can then be recognized from their
kinematics This may be illustrated in the following figure with
the Lorentz ellipses for the processes:

a) $p + p \rightarrow p + p$

b) $p + p \rightarrow p + p + \pi^0$

Plotting in a polar diagram $p_2$ as function of $\theta_2$ (see sub-section 1 (i)), one obtains ellipses (for $p_{cm}^2$ = constant). For the elastic process (a), the end point of $p_2$ has to be exactly on the elastic ellipse; for the inelastic process (b), the ellipse limits only the kinematically possible area. (Note for example that a proton from an inelastic process is not only unable to go backwards in the lab system - it must go forwards!)

5 Bubble chambers

The limits in application of bubble chambers for the study of elastic small angle scatterings have been discussed in the preceding text (see Section III). We may mention once again that the heavy liquid chamber is a good tool to study the presence of $\pi^0$.

6 Counter experiments

A typical counter set-up to measure the elastic scattering of 1.5 to 2.5 GeV pions on protons was used by Lai et al. However, as in the case of the set-up of Cocconi at CERN\(^8,9\), the smallest detected angle was relatively large. On the other hand, once the instruments are set up, the counter technique can obtain a large statistics very quickly.

* * *
My thanks, either for indirect contributions to this survey in form of discussions, or for specific information are due to Drs V A Bull, E H S Burhop, B Czapp, B Dayton, Y Eisenberg, D A Garbutt, H Göing, G Kellner, W Koch, E Marquit, J Pahl, H Rollnik, R Schneeberger and the CERN and Berne Emulsion Groups

* * *

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5) G Czapek, G Kellner, G Otter, Institute for Theoretical Physics, Vienna Private communication 1962

6) W M Preston, R Wilson, J C Street, Physical Review 118, 579 (1960)


8/NP/smg
<table>
<thead>
<tr>
<th>Typical experiment</th>
<th>Method of recognition</th>
<th>Scattering nucleus</th>
<th>Can the complete charged event be seen?</th>
<th>Possibility of discrimination between p-Nucleus and p-p scatterings?</th>
<th>In the case of p-p scattering, can inelastic background directly be eliminated and by what means?</th>
<th>In the case of p-Nucleus scattering, can inelastic background be eliminated?</th>
</tr>
</thead>
<tbody>
<tr>
<td>A) Usual flat exposures</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. Canerana scan (shadow scan)</td>
<td>by deflection</td>
<td>complex nuclei + some hydrogen (usually)</td>
<td>yes</td>
<td>yes</td>
<td>yes, by kinematical criteria</td>
<td>no</td>
</tr>
<tr>
<td>2. Slow scan</td>
<td>by recoil</td>
<td>&quot; &quot;</td>
<td>&quot; &quot;</td>
<td>&quot; &quot;</td>
<td>&quot; &quot;</td>
<td>&quot; &quot;</td>
</tr>
<tr>
<td>3. Sagitta scan</td>
<td>by deflection</td>
<td>&quot; &quot;</td>
<td>yes</td>
<td>&quot; &quot;</td>
<td>&quot; &quot;</td>
<td>possibly</td>
</tr>
<tr>
<td>B) Perpendicular exposures</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. Scanning for heavy recoils</td>
<td>by recoil</td>
<td>complex nuclei + hydrogen 51 or 53</td>
<td>&quot; &quot;</td>
<td>&quot; &quot;</td>
<td>yes, by kinematical criteria</td>
<td>no</td>
</tr>
<tr>
<td>2. Scanning for 'deflections'</td>
<td>by deflection</td>
<td>complex nuclei + hydrogen 10:1 or 10:3</td>
<td>&quot; &quot;</td>
<td>&quot; &quot;</td>
<td>&quot; &quot;</td>
<td>&quot; &quot;</td>
</tr>
<tr>
<td>C) Measurement of 2 (flat or perpendicular) angular distributions</td>
<td>&quot; &quot;</td>
<td>complex nuclei + some hydrogen</td>
<td>no</td>
<td>no</td>
<td>no, (only indirectly)</td>
<td>&quot;</td>
</tr>
<tr>
<td>D) External target, pencil beam, emulsion as detector</td>
<td>&quot; &quot;</td>
<td>pure hydrogen or any element</td>
<td>&quot; &quot;</td>
<td>yes</td>
<td>yes, by precise momentum determination of fast p</td>
<td>-</td>
</tr>
<tr>
<td>E) Hydrogen Heavy Liquid Bubble Chamber</td>
<td>by deflection and/or by recoil</td>
<td>pure hydrogen or any other element</td>
<td>yes</td>
<td>&quot;</td>
<td>yes, by kinematical criteria</td>
<td>-</td>
</tr>
<tr>
<td>F) Counter experiment</td>
<td>by deflection and by recoil</td>
<td>pure hydrogen</td>
<td>yes</td>
<td>&quot;</td>
<td>yes, by kinematical criteria</td>
<td>-</td>
</tr>
</tbody>
</table>
DISCUSSION

Bull : I would not agree that the use of the perpendicular incidence technique with normal dry emulsions is rapid. It was first found by Marquit of Warsaw and confirmed by the London group that the scanning should be done very slowly to get reliable results. In fact it would take three scanners about two years to obtain 50 elastic events. If the scanning is done faster than this serious losses occur for very small and very large scattering angles.

Winzeler : The Vienna group has 130 events found by such a technique and I have no reason to doubt the reliability of these results.

Heckman : What are the difficulties one might encounter in a high energy p-p scattering experiment when one chooses to detect the low-energy recoil near 90°? Here the angle of entrance into the emulsion surface is identified with the angle of the recoil proton and the range of the proton can be used to determine its momentum. These two parameters are all one needs to determine the kinematics of the event.

Combe : Suppose one has an emulsion with a 3 mm hole in the centre in which is placed a hydrogen target. The 3 mm of liquid hydrogen is equivalent to about 50-100μ of emulsion. One could not, therefore, observe short recoils originating in the target or know very accurately the energies of longer recoils from their observed ranges in the emulsion.
Private communication received by Heckman after the end of the session

If one uses a low density gaseous hydrogen target with an almost negligible stopping power, the experiment should be feasible

* * *
Fig. 1 Angular distribution for P P elastic scattering at 24 eV.

Fig. 2

Fig. 3

Elastic Single Scattering distribution of events per 1000 eV.
Fig 5 Angular distribution of particles in 24 GeV proton beam

Fig 6

Fig 7) Momentum Analysis of beam 19m behind Collimator 13m behind magnet

6.3 \times 10^{13} \text{ accelerated protons}

\beta = 235 \text{ GeV/c}

Be - target

Be - target

Protons per cm

Aperture of Collimator
Fig 8 a) b) Photographs of $p\;p$ elastic scattering in the 30 cm CERN hydrogen bubble chamber

Fig 9 Photomicrograph of $p\;p$ small angle scattering in emulsion
Fig. 13  That nice graph showing the sample from a scattering as observed in a perpendicular expansion.
**FIG 16**
Angular distribution of scatters found by scanning for recoils in emulsion in comparison with the bubble chamber data.

**FIG 17**
Angular distribution (p-p scattering at 24 GeV) from combined emulsion and bubble chamber data.

**FIG 18**
Projected Angular Distribution at 62 GeV-P to b m.

After 76 cm path length (© 1969)
Fig 19: Angular spread of the proton beam at the entrance and exit of the plates giving 7 cm path length milli.

\[ \frac{\Delta N}{\Delta \theta} \]

0 0.02 0.04 0.06 0.08
0 20 40 60 80

0 0.02 0.04 0.06 0.08
0 20 40 60 80

Set-up for proposed experiment using an external hydrogen target.

- **M₁, M₂**: analysing magnets
- **C**: Collimator (24 mm hole diameter)
- **T**: liquid hydrogen target
- **P**: plate
- **ST**: shielding tunnel for the plate

Length scale 1:200
Width scale a bit a y

Shielding wall of PS

Fig 20
The measurement of the magnetic moment of a particle gives important information about its structure and its interactions. Thus the $g-2$ experiment (magnetic moment of the muon) made it possible to set an upper limit on the field of validity of quantum electrodynamics and on the radius of this particle. The different theoretical models of strong interactions predict a certain number of relations between the magnetic moments of the baryons according to which the magnetic moment of the six hyperons and the transition magnetic moment $\Sigma - \Lambda$ can be expressed as a function of those of the proton and the neutron, $\mu_p$ and $\mu_n$ (Table 1).

<table>
<thead>
<tr>
<th>Model</th>
<th>Global Symmetry</th>
<th>Unitary Symmetry</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>8-fold way</td>
</tr>
<tr>
<td>$\Lambda$</td>
<td>0</td>
<td>$\frac{1}{2} \mu_n$</td>
</tr>
<tr>
<td>$\Sigma^+$</td>
<td>$\mu_p$</td>
<td>$\mu_p$</td>
</tr>
<tr>
<td>$\Sigma^0$</td>
<td>0</td>
<td>$-\left(\mu_p + \mu_n\right)$</td>
</tr>
<tr>
<td>$\Sigma^-$</td>
<td>$-\mu_p$</td>
<td>$-\left(\mu_p + \mu_n\right)$</td>
</tr>
<tr>
<td>$\Sigma - \Lambda$</td>
<td>$\mu_n$</td>
<td>$\sqrt{\frac{3}{2}} \mu_n$</td>
</tr>
<tr>
<td>$\Sigma^0$</td>
<td>$-\mu_n$</td>
<td>$\mu_n$</td>
</tr>
<tr>
<td>$\Sigma^-$</td>
<td>$-\mu_p$</td>
<td>$-\left(\mu_p + \mu_n\right)$</td>
</tr>
</tbody>
</table>
The following relation between the magnetic moments of the $\Sigma$'s is independent of the model chosen and can be deduced from the conservation of the isospin:

$$2\mu(\Sigma^0) = \mu(\Sigma^+) + \mu(\Sigma^-)$$

Thus, in principle, by measuring the magnetic moment of the $\Lambda$, a choice can be made between the three models mentioned, as long as the symmetries postulated are not masked in the low-energy limit. (It should be noted that these relations can be written arbitrarily either between the absolute magnetic moments $\mu_{\Lambda}/2m_\Lambda$, or between the gyromagnetic ratios $g = 2\mu$, the theoretical models being based on the equality of mass of all baryons.)

I PRINCIPLE OF THE MEASUREMENT

Several proposals have been made for experiments to measure the magnetic moment of hyperons\(^2\); some of these are already being carried out at Brookhaven and at CERN. They are all based on the same principle which has already been applied in measuring the magnetic moment of the electron\(^3\) and the muon\(^4\): namely using a beam of particles crossing a magnetic field, the direction of polarization of these particles turns around a given axis at a velocity proportional to their magnetic moment.

Three conditions, therefore, have to be fulfilled:

i) a beam of polarized particles must be available, and the initial direction of the polarization must be known;

ii) these particles must be kept for a sufficiently long time in a magnetic field which is sufficiently intense to obtain an appreciable precession;

iii) It must be possible to determine the final direction of the polarization.
II REVIEW OF DEFINITIONS

Consider a beam of particles of spin $\vec{\sigma}$, and a direction defined by a unit vector $\vec{e}$. If the expectation value of the spin component in the direction $\vec{e}$, $\langle \vec{\sigma} \cdot \vec{e} \rangle$, is not zero for all directions, the beam is said to be polarized, and there is a direction $\vec{e}_0$ for which $\langle \vec{\sigma} \cdot \vec{e}_0 \rangle$ reaches a maximum:

$$\langle \vec{\sigma} \cdot \vec{e}_0 \rangle_{\text{max}} = \langle \vec{\sigma} \cdot \vec{e}_0 \rangle = Pe,$$

where $P$ is the degree of polarization and $\vec{S} = P\vec{e}_0$, the polarization vector. Being defined in terms of expectation values, this vector follows classical equations of motion (Ehrenfest).

III INITIAL POLARIZATION

A polarized beam of particles can be created:

i) by the elastic scattering of a non-polarized beam: Mott scattering on a gold target in experiment $^3$;

ii) by a decay reaction with non-conservation of parity: $\pi \rightarrow \mu$ decay in experiment $^4$;

iii) by inelastic collision of the type $\pi +$ nucleon $\rightarrow Y + K$ in the experiments on hyperons $\Lambda$ and $\Sigma$.

In (ii) the polarization obtained is longitudinal; it is transversal in (i) and (iii), viz. perpendicular to the plane of scattering or production:

$$\vec{S} = P \frac{\vec{p}_{\text{inc}} \times \vec{p}_{\text{sec}}}{|\vec{p}_{\text{inc}} \times \vec{p}_{\text{sec}}|},$$

where $P = |\vec{S}|$. 

3348/NP/smg
In the reactions in which the hyperons are produced:

\[ \pi^- + p \rightarrow \Lambda + K^0 \]  
(1)

\[ \pi^- + p \rightarrow \Sigma^- + K^+ \]  
(2)

\[ \pi^+ + p \rightarrow \Sigma^+ + K^+ \]  
(3)

it is known, or there is good reason to believe, that the polarization obtained is great enough for energies of the incident particle that are not too high above the threshold, (i.e. \( \sim 1 \, 05 \, \text{GeV/c} \) for reaction (1), \( 1 \, 15 \, \text{GeV/c} \) for (2) and (3)) and for certain values of the angle of production (60 to 90 degrees c.m. for 1)\textsuperscript{5})

Not much is known about the production of \( \Sigma \), except that the cross-section is extremely low, which does not give any hope of measuring the magnetic moment in the near future.

IV FINAL POLARIZATION

This can be determined either by further scattering in the case of the electrons or by the observation of decay asymmetry in the case of an unstable particle (muon, hyperon).

It is known that the decay of hyperons is not isotropic, but that the angular distribution of the \( \pi \) mesons emitted takes the form:

\[ dN = \frac{1}{4\pi} \left( 1 + a \frac{\vec{P} \cdot \vec{P}}{|\vec{P}|^2} \right) d\Omega = \frac{1}{4\pi} (1 + aP \cos \chi) d\Omega \]

where \( P \) is the polarization of the initial particles, \( \chi \) the angle between the direction of the polarization and the direction of emission of the \( \pi \) in the centre-of-mass system, and \( a \) a constant which is characteristic of the mode of decay considered and which takes the following values\textsuperscript{6}):

\[ 5348/\text{NP/smg} \]
<table>
<thead>
<tr>
<th>Reaction</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Lambda \rightarrow p + \pi^-$</td>
<td>$0.61 \pm 0.05$</td>
</tr>
<tr>
<td>$n + \pi^0$</td>
<td>$0.7$</td>
</tr>
<tr>
<td>$\Sigma^+ \rightarrow p + \pi^0$</td>
<td>$0.78 \pm 0.12$</td>
</tr>
<tr>
<td>$n + \pi^+$</td>
<td>$0.03 \pm 0.08$</td>
</tr>
<tr>
<td>$\Sigma^- \rightarrow n + \pi^-$</td>
<td>$0.16 \pm 0.24$</td>
</tr>
<tr>
<td>$\Sigma^- \rightarrow \Lambda + \pi^-$</td>
<td>$0.9$</td>
</tr>
<tr>
<td>$\Sigma^0 \rightarrow \Lambda + \pi^0$</td>
<td>?</td>
</tr>
</tbody>
</table>

The decay of the hyperons therefore provides a suitable analyser in the case of the $\Lambda$, the $\Sigma^+$ (only the mode $p + \pi^0$) and the $\Sigma^-$. On the other hand it is not possible to determine the polarization of the $\Sigma^-$ by this method.

This could no doubt be done by studying the angular distribution of the elastic scattering of $\Sigma^- + p \rightarrow \Sigma^- + p$, or the polarization of the $\Lambda$ produced by the reaction $\Sigma^- + p \rightarrow \Lambda + n$.

In order to determine the direction of polarization according to this principle, the decay asymmetry has to be measured with respect to three planes which are perpendicular to each other, in the centre-of-mass system of the particle. Considering an orientated plane, defined by a unit vector $\vec{I}$ (normal to the plane), then the quantity:

$$A(\vec{I}) = \frac{N(+) - N(-)}{N(+) + N(-)}$$

is called the asymmetry with respect to this plane, where $N(\pm)$ is the number of decays in the half-space corresponding to the direction $\pm \vec{I}$. One has:

$$A(\vec{I}) = \gamma_2 \vec{S} \cdot \vec{I}$$

where $\vec{S}$ is the polarization vector.

When the asymmetry $A$ is measured with respect to three planes defined by the three mutually perpendicular unit vectors $\vec{I}$, $\vec{J}$, and $\vec{K}$,
A(\vec{i}), A(\vec{j}), \text{and } A(\vec{k}) \text{ are the components of a vector } \vec{A} \text{ such that:}

\[ \vec{A} = \frac{1}{2} a \vec{a} \]

In practice for instance, the following asymmetries will be measured: forward-backward, left-right and up-down

V PRECESSION OF THE POLARIZATION IN A MAGNETIC FIELD

The general case where the magnetic field \( \vec{H} \), the momentum \( \vec{p} \) and the polarization \( \vec{S} \) of the particle have arbitrary directions will not be studied. We will consider the case where \( \vec{H} \) and \( \vec{p} \) are perpendicular to \( \vec{S} \), and at an angle of \( \vartheta \) to each other. In practice, one generally has \( \vartheta \approx 0 \) (longitudinal field) or \( \vartheta \approx 90^\circ \) (transverse field) (Fig 1)

A system of reference defined by the following unit vectors will be used:

\[ \vec{i} = \frac{\vec{p}}{|\vec{p}|}, \vec{j} = \frac{\vec{p}_{\text{inc}} \times \vec{p}}{|\vec{p}_{\text{inc}} \times \vec{p}|}, \vec{k} = \vec{i} \times \vec{j} \]

For a neutral particle this system is fixed with respect to the laboratory system. For a charged particle it turns around the direction of the magnetic field with an angular velocity:

\[ \omega = \frac{eH}{\gamma m_0} = \frac{1}{\gamma} \omega_0, \]

where \( \omega_0 = \frac{eH}{m_0} \) (cyclotron frequency)

One has:

\[ \vec{H} = H[\cos \vartheta \vec{i} + \sin \vartheta \vec{k}], \]

\[ \vec{S}_0 = P \vec{j} \] (initial polarization)
The polarization vector $\tilde{S}$ satisfies the differential equation:

$$\frac{d\tilde{S}}{dt} = \tilde{S} \times \tilde{\eta},$$

with

$$\tilde{\eta} = \mu_a \frac{e}{mc} \left[ \frac{1}{\gamma} \mathbf{H} - \frac{1}{\gamma} \frac{p}{|p|^2} \mathbf{H} \right]$$

where $\mu_a$ is the anomalous magnetic moment of the particle, expressed in intrinsic magnetons $\hbar e/2mc$:

$$\mu_a(\Lambda) = \mu(\Lambda), \quad \mu_a(\Sigma^+) = \mu(\Sigma^+) - 1$$

In this equation, $\tilde{S}$ is expressed in the centre-of-mass system, all other quantities in the laboratory system.

One easily finds that:

$$\tilde{\eta} = \mu_a \frac{e}{mc} \left[ \frac{1}{\gamma} \cos \phi \mathbf{I} + \sin \phi \mathbf{K} \right],$$

and:

$$\tilde{S}(t) = P \left[ \frac{\mu_a \omega_0}{\Omega} \sin \phi \sin \Omega t \mathbf{I} + \cos \Omega t \mathbf{J} - \frac{\mu_a \omega_0}{\gamma \Omega} \cos \phi \sin \Omega t \mathbf{K} \right]$$

The polarization turns around $\tilde{\eta}$ with a frequency:

$$\Omega = \mu_a \frac{\omega_0}{\sqrt{\sin^2 \phi + \frac{1}{\gamma^2} \cos^2 \phi}}$$

VI MEASUREMENT OF THE ANGLE OF PRECESSION

If the particles cover a distance $l$ in the magnetic field, the polarization turns through an angle:

$$\varphi_0 = \Omega \frac{l}{\beta_0} = \frac{l}{\beta_0} \sqrt{\sin^2 \phi + \frac{1}{\gamma^2} \cos^2 \phi} \frac{Hl}{\beta}$$
If the field is not uniform, \( H \) should obviously be replaced by
\[
\int_0^L H(x) dx
\]
The numerical coefficient \( e/mc^2 \) assumes the following values:

\[
\begin{align*}
\Lambda & : \ 15.40 \text{ degrees/ megagauss cm;} \\
\Sigma^+ & : \ 14.44 \text{ degrees/ megagauss cm}
\end{align*}
\]

The angle of precession \( \varphi_0 \) is found from the asymmetries \( A_1, A_2, \) and \( A_3 \) according to the formula:
\[
\tan \varphi_0 = \frac{\sqrt{A_1^2 + A_3^2}}{A_2}
\]
where:

\( A_1 = \) forward-backward asymmetry,
\( A_2 = \) left-right asymmetry,
\( A_3 = \) up-down asymmetry,
the vectors \( \vec{i}, \vec{j}, \) and \( \vec{k} \) defining the forward, left and up directions.

The statistical error on the angle \( \varphi_0 \) thus determined is:

\[
\delta \varphi_0 \approx \frac{2}{\alpha PV N} ,
\]
which leads to an uncertainty on the magnetic moment, in magnetons:
\[
\delta \mu \approx \frac{2 \rho}{\alpha PV N} \frac{e}{mc^2} \frac{1}{H \ell} \frac{1}{\sqrt{\sin^2 \theta + \frac{1}{\gamma^2} \cos^2 \theta}},
\]
where \( N \) is the number of events observed.

As could easily be foreseen, \( H \) and \( N \) should be as large as possible, and also the polarization \( P \). In addition, it can be seen that there is an optimum distance \( \ell \) such that:

\[
\ell N \sim \ell e^{-\ell/2 \rho \gamma \sigma r}
\]
reaches a maximum:

$$I_{opt} = 2\beta\gamma \varpi$$

For $$l = l_{opt}$$, the error becomes:

$$\delta \mu_{opt} = 1/\alpha \gamma \varpi \sqrt{\gamma^2 \sin^2 \varnothing + \cos^2 \varnothing}$$

In the foregoing treatment, it has always been supposed that a single value of the parameters $$\varnothing$$, $$H$$, $$l$$, $$\beta$$ (and therefore also $$\gamma$$ and $$P$$) corresponded to all the events observed. In fact each of these values fluctuates within limits which depend on the precise conditions of the experiment, for the following reasons:

i) the target and the detector have finite dimensions;

ii) the incident beam is neither monochromatic nor parallel;

iii) the magnetic field is not constant in time, but pulsed, and the height of the pulses is not strictly reproducible;

iv) the target used is generally not hydrogen, but may be polyethylene or any other light hydrogenated substance.

The foregoing analysis can be used, with the average values of the parameters $$\varnothing$$, $$\beta$$ and $$l$$. A more precise procedure would involve the application of the maximum likelihood method of the observed distribution of decay angles.

**VII DESCRIPTION OF ACTUAL EXPERIMENTS OR PROPOSALS**

1 **Brookhaven experiment on the A magnetic moment**

This experiment was done as a collaboration between Argonne and Brookhaven, using the technique of the spark chamber (Fig 2). The geometry used is that of the longitudinal field ($$\vec{H}$$ parallel to $$\vec{P}_{inc}$$); the $$\Lambda$$'s are produced through the reaction $$\pi^+ + n \rightarrow \Lambda + K^+$$ in a Be target; the spark chamber is triggered every time a $$K$$ meson of the right energy is detected in the counter telescope.
The magnetic field decreases from 50 to 20 kgauss between the target and the detector. The path length of the \( \Lambda \) is 17 cm (three mean lives) and their polarization about 50%. The result of this experiment, based on 254 events, is:

\[ \mu_\Lambda = -1.5 \pm 0.5 \] (nuclear magnetons)

**CERN - Lausanne - Bristol collaboration (Fig. 3)**

This experiment, using nuclear emulsions, is scheduled at the CERN PS, although a preliminary run has been done in February 1962 to check the conditions of background, the scanning methods and measurement techniques.

The detector, a stack of 40 pellets 3 cm x 10 cm x 600 microns, is completely contained inside the coil. The magnetic field is perpendicular to the \( \Lambda \) momentum and to the emulsions. The momentum of the incident \( \pi^- \) beam is 1.05 GeV/c, the production angle \( \Theta = 18^\circ \) (\( \sim 80^\circ \) in the c.m. system) corresponding to the maximum possible value of the polarization (near 100% for production on protons). The momentum of the \( \Lambda \) is \( \sim 800 \text{ MeV}/c \).

At this energy, a measurement of the total angle between the decay products is sufficient to determine whether the decay occurred backwards or forwards in the c.m. system. Momentum measurements on the proton and pion tracks might be necessary to discriminate between \( \Lambda \)'s and 2-prong stars, even though the preliminary run indicates that the number of these background events is very small.

Another favourable circumstance is that the angle between the decay-proton track and the \( \Lambda \) line-of-flight is always smaller than 10°. With a field of 150-200 kgauss, all the charged particles originating from the target are deflected to an angle larger than 10°. So, even though these particles cannot be prevented from reaching the stack, they are swept out of the interesting angular region in the emulsions.

This allows us to scan across the plates along lines \( \sim 1 \text{ cm} \) apart, picking up tracks within the right angular region, and of
ionization between 1.5 and three times minimum. These tracks are followed back to their origin or to their entrance in the stack. By this procedure, we expect to find the \( \Lambda \)'s without any bias and much faster than by area scanning.

In the preliminary run, the magnetic field was only 75 kgauss, and the number of background tracks in the direction expected for the \( \Lambda \)-decay protons is very large, making it impossible to scan by this method.

The coil used for this experiment has been built in Lausanne. It gives a field up to 150 kgauss, constant within 10% in a volume \( 4 \times 6 \times 20 \text{ cm}^3 \), the field being parallel to the 6 cm dimension. The coil consists of 20 rectangular plates of elnedur (Cu alloy with 2% Cr) connected in series, insulated with vetronite and held in a stainless steel frame. Both the coil and the frame are water-cooled.

The inductance of the coil is \( \sim 35 \, \mu \text{H} \), its resistance \( \sim 8 \, \text{m\O} \). When connected to the 75 mF CERN condenser bank, its resonance frequency is \( \sim 600 \text{ sec}^{-1} \), corresponding to a current rise-time of 2.5 msec.

The target is a piece of polyethylene, 4 cm long, intercepting the whole incident beam (\( \sim 1 \text{ cm}^2 \) cross section). The number of \( \Lambda \)'s expected to be observed in the plates is \( \sim 1 \) per \( 10^6 \) pions hitting the target. The intensity of the beam used in the test run of February 1962 (\( \kappa_1 \) beam) was \( 1.5 \times 10^5 \) pions per pulse; the total exposure was of 1000 pulses for both signs of the field.

3. **CERN - Lausanne collaboration on the \( \Xi^+ \) magnetic moment**

A first run for this experiment is scheduled at the PS for May 1962 (Fig. 4).

The \( \Xi^+ \)'s are produced in a polyethylene target by 1.15 GeV/c \( \pi^+ \)'s, and detected in two small stacks symmetrically placed with respect to the beam. This geometry has the following advantage: the polarization of the hyperons changes sign from one stack to the other.
so the precession will occur in opposite directions. If there is a bias in the detection of forward decays, for instance, this bias can be eliminated by comparison between the two stacks.

The scanning will be done by picking up tracks at the entrance of the plates and following them down until one finds an angular deflection, $\Sigma^+ \rightarrow p$ decay or scattering. It should be possible, in most cases, to distinguish between the two by ionization, scattering or curvature measurements. The number of proton scatterings is estimated to be about equal to the number of $\Sigma \rightarrow p$ decays.

4 Rome proposal for the $\Sigma^+$ magnetic moment

This experiment will be attempted at the same time as the preceding one, using the same coil (CERN type III, 20C kgauss) but different geometry (longitudinal field).

The $\Sigma$ decays will be found by following grey prongs emitted from stars. It is not known at the moment in which angular region of production one can expect an appreciable polarization.

The parameters referring to the various experiments described are set out for comparison in Table 2.
<table>
<thead>
<tr>
<th>Experiment</th>
<th>Technique, production reaction, target</th>
<th>H (kgauss)</th>
<th>$\varphi$</th>
<th>$P_{inc}$ (GeV/c)</th>
<th>$l$ (cm)</th>
<th>$Hl$ Mgauss x cm</th>
<th>$\beta(\gamma)$</th>
<th>$\phi_0$ for $\mu = 1$ (degrees)</th>
<th>Polarization $\alpha_P$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Lambda$ Brookhaven</td>
<td>Spark chamber $\pi^+ + n \rightarrow \Lambda + K^+$ (Be)</td>
<td>35</td>
<td>$\sim 20^\circ$</td>
<td>1.02</td>
<td>17</td>
<td>0.6</td>
<td>0.70</td>
<td>13$^\circ$</td>
<td>$\sim 0.5$</td>
</tr>
<tr>
<td>CERN, Lausanne, Bristol a)</td>
<td>Emulsions $\pi^- + p \rightarrow \Lambda + K^0$ (CH$_2$)$_n$</td>
<td>75</td>
<td>90$^\circ$</td>
<td>1.05</td>
<td>12</td>
<td>0.9</td>
<td>0.58</td>
<td>24$^\circ$</td>
<td>$\sim 0.6$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>150</td>
<td>80$^\circ$</td>
<td>1.05</td>
<td>12</td>
<td>1.8</td>
<td>0.58</td>
<td>4.8$^\circ$</td>
<td>$\sim 0.6$</td>
</tr>
<tr>
<td>$\Sigma^+$ CERN, Lausanne</td>
<td>Emulsions $\pi^+ + p \rightarrow \Sigma^+ + K^+$ $(\pi^+ + n \rightarrow \Sigma^+ + K^0)$ (CH$_2$)$_n$</td>
<td>200</td>
<td>75$^\circ$</td>
<td>1.15</td>
<td>4</td>
<td>0.8</td>
<td>0.54</td>
<td>28$^\circ$</td>
<td>$\sim 0.57$</td>
</tr>
<tr>
<td>Rome</td>
<td>Emulsions $(\pi^+ + p \rightarrow \Sigma^+ + K^+$ $(\pi^+ + n \rightarrow \Sigma^+ + K^0)$ (Emulsion)</td>
<td>200</td>
<td>10-25$^\circ$</td>
<td>1.15</td>
<td>2-4</td>
<td>0.6</td>
<td>0.7</td>
<td>18$^\circ$</td>
<td>?</td>
</tr>
</tbody>
</table>
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* * *

348/NP/sm
**DISCUSSION**

**Going**: Do you think that the initial polarization of the $\pi^+$'s produced in emulsion is high enough to observe an asymmetry? We measured the polarization of $\pi^+$ in emulsion by means of the up-down asymmetry of the decay. It was small compared to the asymmetry of the $\Lambda$ decay.

**Rosselet**: The polarization of $\pi^+$'s produced in heavy nuclei might be small. In experiment 4 one certainly should try to select hyperons produced on the light nuclei of emulsion.

**Fletcher**: What is the expected error on $\mu_\Lambda$ from experiment 2?

**Rosselet**: If we introduce the numbers given in Table 2 in the formula for $\delta \mu$, and assuming a statistics of 400 events, we find $\delta \mu \approx 0.25$ magneton.

* * *
Fig 1 System of reference

\[ T \frac{\vec{P}_{\text{inc}} \times \vec{P}}{(\vec{P}_{\text{inc}} \times \vec{P})} \]

Fig 2 Brookhaven experiment (Caldwell, Cool, Hill, Jenkins, Kycia, Marshall, Schluter)
Fig 3 CERN-Lausanne experiment Λ magnetic moment
(Rosselet Malamud Rohrbach Combe Hoffmann Vanderhaeghe)

side view

CH$_2$ target

plan

target

Fig 4 CERN Lausanne experiment Σ$^+$ magnetic moment
(Doble Hoffmann Toner Vanderhaeghe Malamud Rosselet)
The Organization of Emulsion Experiments at CERN

W. O. Lock
Nuclear Physics Division, CERN

In this talk we shall discuss the important subject of how one sets about carrying out an experiment at CERN. The first necessity is that one should know a great deal about very many different things: for example how the accelerator works, how all the targetting operates, what possibilities one has with the targets, how to manipulate the different particle beams and so on. It is no good coming for a K-meson exposure if nobody can run the beam transport system and some of you know by hard experience that running a transport system is not always an easy job. For example, one has to learn how to start a separator again when it fails, how to reset the current supplies to all the magnets when all the power supplies go off, etc. One needs to know most of these things before thinking about putting particles into some target, or into some emulsions. During this School the different lecturers have attempted to teach something about these various topics. Let us assume that you have a good knowledge of the PS, how to manipulate beams, and that you have got a group of people in your laboratory, who will come and work for some time at CERN to obtain the necessary particles and experimental conditions.

If you then have a good idea for an experiment, the first thing to do is to make a proposal. In other words you have to write a thousand words (say) about the experiment you want to carry out and give the scientific justification for it: i.e. what new piece of information, and what new data you think you can obtain from the experiment. Further, you must be able to show that you can handle the data when you have got it. It is no good asking for an irradiation of a stack of emulsions of 300 pellicles if you have only one scanner in
your laboratory! Lastly, if your experiment is a complicated one, then you should say what sort of team you are able to send to CERN and for how long. In the past you could write to Combe or myself, asking for some protons to be put into an emulsion stack and in general it could be done relatively easily. These days are gone. For example, to put emulsion stacks in a beam of K mesons has required 15 or 20 people at CERN for a couple of months to run the beam.

Let us assume that you have made your proposal and have sent it to Combe or to myself. If you happen to be out of touch for some reason, and you make a request for an irradiation to $\mu$ mesons, for example, then you might well get a letter back saying that there are no plans for a muon beam in the next year. But supposing this is not the case and you ask for something which should be practicable, then the request has to go to the Emulsion Experiments Committee. There are three committees to receive all the experimental proposals from groups in the member states and in the non-member states. They are the Emulsion Experiments Committee, the Track Chamber Committee, and the Electronics Experiments Committee. The first two are largely composed of representatives from all the member states, nominated by the groups in the member states, while the Electronics Committee is mostly an internal CERN affair.

The Emulsion Committee meets every two months and considers all the proposals that have been received. Now the schedule of experiments on the PS machine is divided up into three periods each year, of approximately three months each. For example, in 1962, period one went from the beginning of the year until the 30th April; period two goes on until the 23rd August, and the last period of this year, period three, starts at the end of October, because of a long shut-down, and goes to February 1963.

For each period the different experimental proposals which have been accepted by the three Experiments Committees are considered by the Nuclear Physics Research Committee (NPRC). The Chairman of each of the Experiments Committees sits on this Committee and it is.
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One important fact, to be stressed here, is the very great demand for time on the PS machine. Of the available time, about 40% goes to electronics experiments, 40% to track chamber experiments, 15% to emulsion work and 5% to machine improvements and development work. Now, not all the time in any one period (usually 13 weeks long) is definitely allocated; the Director-General has about two weeks at his disposal which he can give to experiments of his choice. Further, about three weeks are kept in reserve to allow for breakdowns of the machine or of experimental apparatus. This leaves about eight weeks available for definite scheduling of which between one and one and a half weeks (20-30 shifts) can be devoted to emulsion work. As emulsion experiments become more complex (e.g. involving the use of high magnetic fields), they take more time and thus the total number that can be carried out in any one year is rather limited. Therefore, many of the proposals which come to the Emulsion Committee will never get to the stage of actually being done. Only the best experiments, with a reasonable physics interest, will be carried out. Now and again it is possible to make some simple irradiations in a proton or a pion beam but even this is time consuming, particularly if flux measurements have to be made. On the other hand, one should remark that the time (main user time) devoted to emulsion work at

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The position at the synchro-cyclotron is much simpler. The demand is much less, and there is a definite policy that about half the time goes to outside groups, e.g., Darmstadt, Bologna, etc. Therefore the schedule is rather flexible and is only decided about one or two months in advance. The procedure to be followed to obtain time is the same as for the PS, but the chance of a good proposal being accepted is high.

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Lastly, let me stress how important it is to plan good experiments, which will yield definite results of physical importance. Do not try to do a second rate experiment and do not attempt something which can be done better by a different technique. Think out your proposal in detail, put it forward, and if it is good you have a good chance of obtaining time on the machine.

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3348/NI/SMG
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W. O. Lock
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3348/NP/smam
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* * *
AN IBM PROGRAMME FOR KINEMATIC ANALYSIS OF MESIC AND NON-MESIC DECAYS OF HYPERNUCLEI*)

K Külbüg
Data Handling Division, CERN

M Nikolić
Track Chamber Division, CERN

The working programme of many research groups in Europe, dealing with nuclear emulsions, includes the investigation of hyperfragments. These studies, for which nuclear emulsions appear to be the most adequate technique, are made possible, to a great extent, through the facilities provided by CERN. For example, during this Spring more than 20 laboratories have exposed their emulsions in the K-meson beam (with momentum from 1.3 to 1.5 GeV/c), mainly with the aim of investigating hyperfragments.

The kinematic analysis of the mesic and non-mesic decays of hyperfragments calls for the use of a computer with an appropriate programme.

The first programme for the analysis of hypernuclei was written by C E Violet. Later, this was considerably improved by P W Inman1), who used it in analysing the mesic decays of hyperfragments. A programme for the Bull Gamma 3 computer was employed by the Berne group2;3) for analysing both non-mesic and mesic decays of hyperfragments. It was in principle similar to Inman's programme, but more complete.

CERN has now at its disposal an IBM programme for the kinematic analysis of hypernuclei, which represents an improved

*) To be published as a CERN Yellow Report
version of the one used by the Berne group. The programme is designed for the complete analysis of the non-mesic and mesic decays of hyperfragments at rest. The upper limit for the charge of the hyperfragments to be analysed is eight (oxygen) in the case of the non-mesic and \( \pi^0 \)-mesic decays, and seven in the case of the \( \pi^- \)-mesic decays. The upper limit for the charge of the decay products, as well as the maximum number of the prongs, is eight. If need be, all these upper limits can be increased.

The first part of the programme on the basis of the input information (the range, the azimuth and dip angle with the corresponding experimental errors, the upper and lower limit for the charge, and so on), determines the energy, the total momentum and its components, as well as the errors of all these quantities for each prong and for all its possible identities.

The basic range-energy relations used are those by Barkas, but for slow multiply charged particles, a specific table for each element is applied.

In estimating the uncertainty in the kinetic energy of particles, emitted in the mesic, as well as in the non-mesic decays, account has been taken of all uncertainties considered in reference two. The range straggling is calculated as a function of the normalized range for the appropriate mass value.

The kinetic energy, instead of the range, can be directly imposed for each individual isotope.

The second part of the programme systematically combines all possible prong assignments. The computer calculates for each possible combination the momentum unbalance and the energy balance, by ascribing the former to:

1) a neutron;
2) a \( \pi^0 \), if no charged pion is emitted;
3) no neutral particle, if the momentum unbalance has a value lower than the imposed limit. In that case, in determining the binding...
energy, the energy of the shortest track (recoil), calculated on the basis of momentum conservation is used.

If the binding energy $B_\Lambda$ is within the imposed limits, the corresponding errors are calculated, and the computer prints out the combinations with all output data needed.

A special version of the programme is designed to be of help in analysing hypernuclei in flight.

*   *   *

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3) V Gorgő, W Koch, W Linát, M Nikolić, S Subotić-Nikolić and H Winzeler, Nuclear Physics, 21, 599 (1960)

*   *   *
# APPENDIX B

## LIST OF PARTICIPANTS

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