Compton scattering in scalar electrodynamics

Student Discussion Group

Abstract
The goal of this project is to compute the Compton scattering cross-section for scalar electrodynamics. A comparison with the experimental result is also made.

1 The project
In our group we were concerned about the possibility of predicting the spin nature of electrons by just measuring scattered photons against free electrons. So, we decided to compute the total cross-section for Compton scattering with scalar electrodynamics and compare it with experimental proof.

The starting point is the gauge invariant Lagrangian which describes the interaction of a scalar particle with a massless vector boson.

\[ \mathcal{L} = \frac{1}{4} (\partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu})^2 + \left[ (\partial_{\mu} + ieA_{\mu}) \varphi \right] \left[ (\partial^{\mu} + ieA^{\mu}) \varphi \right]^\dagger, \]

\[ = \mathcal{L}_{\text{free}} + ieA_{\mu} (\varphi^\dagger \partial^{\mu} \varphi - \varphi \partial^{\mu} \varphi^\dagger) + e^2 A_{\mu} A^{\mu} \varphi^\dagger \varphi. \] (1)

We can extract the vertices using the rules given by Ref. [1]:

– For each interaction in the Lagrangian, we introduce a momentum conservation delta, \((2\pi)^4 \delta(\sum p)\).
– Introduce a factor coming from the degeneracy of identical particles in the interaction and the couplings coming from \(i \mathcal{L}_{\text{int}}\).
– For each field derivative \(\partial_{\mu} \phi\), a \(-ip_{\mu}\) factor is associated with the momentum of the incoming particle.

Then, for the photon-scalar interaction, we have

\[ -ie(p_{\mu} + p'_{\mu})(2\pi)^4 \delta^4(p - p' - k), \] (2)

and for the 2-photon-scalar interaction

\[ 2ie^2 g_{\mu\nu} (2\pi)^4 \delta^4(p - p' - k - k'). \] (3)

The next step consists of applying the recipe.

– Draw all topologically distinct diagrams.
– For each internal scalar field of momentum \(k\), we attach the propagator \(D(k) = i/(k^2 - m^2 + i\epsilon)\).
– For each external photon line, we attach a polarization vector \(\epsilon_{\mu}\).

From Fig. 1, we obtain the following amplitude

\[ \text{amplitude} \]

*Work performed as a student project under the supervision of C. Rojas.
\[ iA = -e^2 \epsilon'_\mu (p' + k' + p')^\mu (p + k + p')^\nu \epsilon_\nu \mathcal{D}(p + k) \]
\[ + \quad e^2 \epsilon'_\mu (p - k' + p)\mu (p' - k + p')^\nu \epsilon_\nu \mathcal{D}(p - k') \]
\[ + \quad 2i \epsilon'_\mu g^{\mu\nu} \epsilon_\nu , \]
\[ \equiv -ie^2 \epsilon'_\mu T^{\mu\nu} \epsilon_\nu , \quad \text{(4)} \]

where

\[ T^{\mu\nu} = \frac{(2p' + k')^\mu (2p + k)^\nu}{(p + k)^2 - m^2} + \frac{(2p - k')^\mu (2p' - k)^\nu}{(p - k')^2 - m^2} - 2g^{\mu\nu} , \]
\[ = \frac{(2p' + k')^\mu (2p + k)^\nu}{2p \cdot k'} + \frac{(2p - k')^\mu (2p' - k)^\nu}{2p' \cdot k} - 2g^{\mu\nu} \]
\[ \equiv A^{\mu\nu} + B^{\mu\nu} - 2g^{\mu\nu} . \quad \text{(5)} \]

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In order to compute the scattering cross-section, we take the absolute square value of the amplitude. Since we measure only the number of photons, the final photon polarizations are summed and the initial ones are averaged.

\[ \frac{1}{2} \sum_{\text{pol}} |A|^2 = \frac{e^4}{2} \sum_{\text{pol}} [\epsilon'_\mu T^{\mu\nu} \epsilon_\nu] \left[ \epsilon_\rho T^{\rho\sigma} \epsilon_\sigma \right]^* , \]
\[ = \frac{e^4}{4} T^{\mu\nu} T^{\rho\sigma} g_{\mu\rho} g_{\nu\sigma} = \frac{e^4}{4} T^{\mu\nu} T_{\mu\nu} , \]
\[ = \left( A^{\mu\nu} + B^{\mu\nu} - 2g^{\mu\nu} \right) \left( A_{\mu\nu} + B_{\mu\nu} - 2g_{\mu\nu} \right) \]
\[ = A^{\mu\nu} A_{\mu\nu} + B^{\mu\nu} B_{\mu\nu} + 2A^{\mu\nu} B_{\mu\nu} - 4A^{\mu\nu}B_{\mu\nu} - 8 . \quad \text{(6)} \]

Here we used the orthogonality relation

\[ \sum_{\text{pol}} \epsilon'^*_\mu(k) \epsilon_{\nu}(k) = -g_{\mu\nu} . \quad \text{(7)} \]

We end with

\[ \frac{1}{2} \sum_{\text{pol}} |A|^2 = 2e^4 \left[ m^4 \left( \frac{1}{p \cdot k} - \frac{1}{p' \cdot k'} \right)^2 - 2m^2 \left( \frac{1}{p \cdot k} - \frac{1}{p \cdot k'} \right) + 2 \right] . \quad \text{(8)} \]

For computing the cross-section, we need the phase space factors in a certain reference frame. We choose the laboratory frame, where the incoming photon hits an electron at rest.
\[ p = m(1, 0, 0, 0), \]
\[ k = \omega(1, 0, 0, 1) , \]
\[ k' = \omega'(1, \sin \theta, 0, \cos \theta) , \]
\[ p = (m + \omega - \omega', -\omega' \sin \theta, 0, \omega - \omega' \cos \theta) , \tag{9} \]

then, \( p \cdot k = m\omega \) and \( p \cdot k' = m\omega' \). So, we have the well-known relation
\[
\frac{1}{\omega} - \frac{1}{\omega'} = \frac{1}{m}(1 - \cos \theta) , \tag{10}
\]

ending with the expression
\[
\frac{1}{2} \sum_{\text{pol}} |A|^2 = 2e^4(1 + \cos^2 \theta) . \tag{11}
\]

So, the differential cross-section is
\[
d\sigma = \frac{1}{2} \sum_{\text{pol}} |A|^2 \frac{m}{4(p \cdot k)} \frac{d^3k'}{2\omega'(2\pi)^3} \frac{mdp'}{2p_0'(2\pi)^3} (2\pi)^4 \delta(p' + k' - p - k) \\
= \frac{e^4}{2(4\pi)^2m^2} \left( \frac{\omega'}{\omega} \right)^2 (1 + \cos^2 \theta) d\Omega \\
= \frac{\alpha^2}{2m^2 (1 + \omega/m(1 - \cos \theta))^2} d\Omega . \tag{12}
\]

Integrating out, we obtain the total cross-section (which can also be found in Ref. [2])
\[
\sigma_T = \frac{\pi\alpha^2}{m^2} \left[ 2 \frac{2\omega + 4\omega^2 + 2\omega^3 - \ln(1 + 2\omega) - 3\omega \ln(1 + 2\omega) - 2\omega^2 \ln(1 + 2\omega)}{(1 + 2\omega)\omega^3} \right] . \tag{13}
\]

At the end, in Fig. 2, we compare the scalar cross-section with the ‘experimental’ one (see, for example, Ref. [1]).

\[ \text{Fig. 2: The percentage ratio between the scalar cross-section and the real one as a function of the photon energy in units of the electron mass} \]

We can conclude that for low energies there is no evidence of spin.
References
