RESULTS OF THE LHC PROTOTYPE CHROMATICITY MEASUREMENT SYSTEM STUDIES IN THE CERN-SPS

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Abstract

Tune and chromaticity control is an integral part of safe and reliable LHC operation. Tight tolerances on the maximum beam excursions allow excitation amplitudes of less than 30 $\mu$m. This leaves only a small margin for transverse beam and momentum excitations, required for measuring tune and chromaticity. This contribution discusses the baseline LHC continuous chromaticity measurement with results from tests at the CERN-SPS. The system is based on continuous tracking of the tune using a Phase-Locked-Loop (PLL) while modulating the beam momentum. The high PLL tune resolution achieved, made it possible to detect chromaticity changes well below the nominally required 1 unit for relative momentum modulations of only $2 \cdot 10^{-5}$. The sensitive tune measurement frontend employed allowed the PLL excitation and radial amplitudes to be kept below a few micrometers. These results show that this type of measurement can be considered as practically non-perturbative permitting its use even during nominal LHC operation.

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INTRODUCTION

The nominal stored beam energy of approximately 350 MJ per beam circulating inside a cryogenic environment, tolerating energy depositions in the order of a few mJ/cm³ only, requires an excellent control of particle loss, which for the LHC, is provided by its Machine Protection and Beam Cleaning System [1–3].

The function of these systems depends critically on the stability of orbit, energy, tune (Q) and chromaticity (Q’), and imposes – beyond transverse emittance preservation – significant constraints on the maximum allowed beam excursions that are traditionally required to measure Q and Q’. The transverse oscillation ‘budget’ is shared between several accelerator systems, such as the orbit and energy feedback, the Q PLL and the bunch-by-bunch transverse feedback. As a result, amplitudes of the explicit beam oscillations used by the transverse diagnostic systems for nominal beam operation are limited to a few μm. These constraints led to the development of the LHC Base-Band Tune meter (BBQ, [5]). The same system is further exploited by the LHC PLL system that continuously tracks Q changes by resonantly exciting the tune and a complementary FFT system that provides general-purpose real-time spectral information on beam oscillations [6–8]. The measurement and combined control of orbit, tune, chromaticity and betatron coupling, and the necessary feedback architecture that minimises the inter-loop coupling have been previously discussed in [9].

Due to persistent currents, the related decay and snapback phenomenon (inherent to superconducting magnets) as well as other perturbation sources, values of orbit, energy, Q and Q’ may exceed LHC beam stability requirements by orders of magnitude, as summarised in Table 1 and shown in the case of chromaticity in Figure 1.

### Table 1: Nominal stability requirements and expected (brackets: worst case) ramp-induced perturbations [4].

<table>
<thead>
<tr>
<th></th>
<th>Tune</th>
<th>Chromaticity</th>
<th>Momentum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nom. Value</td>
<td>[frev]</td>
<td>[Q]</td>
<td>[Δp/p]</td>
</tr>
<tr>
<td>Stability</td>
<td>±0.001</td>
<td>±1</td>
<td>&lt; 10⁻⁵</td>
</tr>
<tr>
<td>Perturb.</td>
<td>0.18</td>
<td>70 (300)</td>
<td>±1.5 · 10⁻⁴</td>
</tr>
<tr>
<td>Drift Rates</td>
<td>&lt; 10⁻⁵s⁻¹</td>
<td>&lt; 1.3s⁻¹</td>
<td>&lt; 10⁻⁷s⁻¹</td>
</tr>
</tbody>
</table>

![Figure 1](image-url): Expected un-compensated chromaticity drift (blue) and corresponding drift rate (red) [4].

Assuming that large parts of these perturbations are reproducible from fill-to-fill, these effects may be partially compensated by feed-forward systems. Due to intrinsic uncertainties and the required stability-limited precision of these systems, beam-based measurements and their exploitation in automated feedback systems are mandatory for safe and reliable LHC operation. This contribution focuses on studies on the LHC Q’ tracker. The chromaticity is one of the most critical and more difficult beam parameters to measure, due to the large expected drifts and drift rates, and the fact that it has to be derived from very small tune changes induced by modulating the beam energy via the RF system.
SLOW VS. FAST RF MODULATION

The LHC RF cavities can provide a maximum RF voltage of 16 MV per beam. However, most of this voltage is required to provide sufficiently large bucket area for nominal LHC bunch intensities, and thus only a small fraction, some 0.25-0.5 MV, is actually available for $Q'$ related modulation. In addition to RF system driven constraints, the non-zero dispersion at the collimator location relates the transverse amplitude constraints also to an effective limit on $\Delta p/p$ in the order of $10^{-3}$. The corresponding $Q'$ tracking parameter range is shown in Figure 2.

![Figure 2: $\Delta p/p$ as a function of RF modulation frequency $f_{mod}$. Upper limits imposed by the RF and the LHC Cleaning System are indicated.](Image)

Modulation frequencies equal or close to the synchrotron tune $Q_s$ are excluded by longitudinal emittance preservation, which essentially leaves two choices for RF modulation based $Q'$ tracking methods:

- modulation frequencies well above $Q_s$, as proposed in [10, 11]. As visible in Figure 2, their use in the LHC is impractical due to limited available RF power.

- modulation frequencies well below $Q_s$, commonly referred to as the classical method, which tracks the $Q'$ dependent tune changes $\Delta Q$ as a function of momentum modulation $\Delta p/p$. The underlying relation, also defining the unit of $Q'$, is given by:

$$\Delta Q := Q' \cdot \frac{\Delta p}{p} \quad (1)$$

In most accelerators, the classic $Q'$ measurement relies on $\Delta p/p$ modulations typically in the order of $10^{-4}$...$10^{-3}$. As visible in Equation 1, a $Q'$ resolution of 1 would require a $Q$ measurement resolution in the same order of magnitude ($10^{-4}$). These resolutions can be achieved using traditional methods based on spectral analysis of driven or passive monitoring of residual betatron oscillations. The main limitation of methods using coherent betatron oscillation – besides the available signal-to-noise ratio – is given by LANDAU damping of the collective particle oscillations. The corresponding tune resolution is typically in the order of $10^{-3}$, which can be further improved by at least one order of magnitude by Gaussian fitting of the tune spectral peak [12].

As indicated in Table 1, the base-line LHC $Q'$ tracker diagnostic is required to resolve chromaticity changes in the order of one unit. In combination with the discussed LHC momentum constraints and Equation 1, the tune changes that need to be resolved are thus in the order of $10^{-5}$ or below. Though the classical method is relatively simple, the feasibility and robustness of such a $Q'$ tracker system has not been experimentally demonstrated yet in any accelerator with the given beam parameter constraints.

LHC $Q'$-TRACKER PROTOTYPE

Due to the fast drift rates and long periods over which these drifts need to be monitored and controlled, the LHC requires a continuous measurement of $Q'$ and, consequently, the momentum-driven tune changes. For short time-scales, these tune changes can, for non-zero modulation amplitudes, be approximated by

$$Q(t) = +\beta_0 + \beta_1 \cdot \sin(\omega_m t) + \beta_2 \cdot \cos(\omega_m t) + \beta_3 \cdot t + \beta_4 \cdot t^2 + \beta_5 \cdot t^3 \quad (2)$$

with $\omega_m = 2\pi f_m$ the RF modulation frequency, $\dot{Q} \approx \beta_0$ the unperturbed tune,

$$\Delta Q = \sqrt{\beta_1^2 + \beta_2^2} \quad (3)$$

the modulation amplitude and $\beta_3, ... , \beta_5$ higher order correction terms that compensate for linear, quadratic and cubic tune drifts un-related to the RF modulation, but which are fast with respect to the modulation frequency. There are several suitable techniques to reconstruct the average tune and its modulation. They can be grouped into:

- Classic amplitude de-modulation: The average tune is first removed using a high-pass filter and then – similar to a PLL – multiplied with a sinusoidal reference signal. The higher-order mixing products are rejected through a low-pass filter. The modulation amplitude can be reconstructed according to Equation 3 using the in- and out-of-phase amplitudes. The high- and low-pass cut-off frequencies need to be well below $f_{mod}$, which ultimately limits this scheme’s performance in the presence of fast tune drifts due to the phase lag introduced by the filter.

- Linear regression: Since the modulation frequency $\omega_m$, the time in between tune measurements and, consequently, the harmonic and polynomial terms in Equation 2 are constant. This yields one linear equation for each tune measurement $Q_i$ with $\beta_0, ... , \beta_5$ as free parameters. The collection of $N$ measurements can be re-written in matrix form as

$$(Q_1, \ldots, Q_N)^T = R \cdot (\beta_0, ..., \beta_5)^T \quad (4)$$

$N$ should cover at least half or multiple periods. A universal solution to Equation 4 can be found by computation of the pseudo-inverse matrix $R^{-1}$. Using a
Singular-Eigenvalue-Decomposition, potential singularities can be identified as very small or vanishing eigenvalues and eliminated in the inversion process by setting their inverse to zero. While SVD has a cubic complexity, the final computation to reconstruct the average tune and modulation amplitude consists of a simple matrix-vector multiplication.

- Chi-square fitting: This method also allows the reconstruction of non-linear parameters, such as the frequency. For above discussed case, the solutions using linear regression and chi-square fitting are identical. This method was mainly used as a cross-check in offline analysis.

**CERN-SPS Test Results**

The feasibility of the $Q'$ measurement with the unprecedented small momentum modulation has been demonstrated at the CERN-SPS in 2007. Two typical measurements are shown in Figures 3(a) and 3(b). The reconstruction was based on a sliding window covering two oscillation periods of the tune modulation ($f_{mod} = 0.5$ Hz) and shifted for each measurement by the sampling interval.

The achieved $Q$ resolution was in the order of $10^{-6}$, resulting in a chromaticity resolution of better than 1 unit. As illustrated in Figure 3(b), the $Q'$ tracking loop was able to cope with chromaticity values up to 36 units, which provides some margin for operation in a regime where a classic $Q$ kicker or chirp based measurement using BPMs would fail due to the very fast de-coherence times. The momentum shift has been controlled via changes of the SPS-RF frequency reference. While the achieved tune resolution would have supported smaller momentum modulation amplitudes, the ultimate lower limit of $\Delta p/p = 1.8 \cdot 10^{-5}$ was given by the minimum quantisation of the signal generator that has been used to change the RF frequency. The tune steps are caused by feed-down effects due to the off-centre orbit in the lattice sextupoles, also causing transients in the chromaticity reconstructions because of the trimming-induced tune drifts being faster than those due to RF modulation. For the LHC system, the time-scales of $Q$ and $Q'$ drifts are expected to be small compared to the targeted modulation frequencies of between 1 and 5 Hz. In case of problems and assuming relaxed RF and Cleaning System tolerances, the measurement can be improved by increasing the amplitude or frequency of the modulation.

**CONCLUSIONS**

The LHC will require a continuous, automatic control of orbit, tune, chromaticity, betatron coupling and energy for safe and reliable machine operation. The LHC Cleaning and Machine Protection systems impose tight constraints on the allowed transverse beam oscillations, traditionally required to measure $Q$ and $Q'$. These constraints have led to the development of the high sensitivity direct diode detection technique, further exploited through a $Q$ tracking PLL. Combining this detection technique with small momentum modulation has allowed a tracking of $Q'$ with unprecedented accuracy using minimal excitation in the $\mu$m range. Based on tests at the CERN-SPS, the performance of such a system has been shown to be compatible with nominal LHC requirements during regular operation.

**REFERENCES**