Arnold diffusion lifetime in storage rings

1. Chirikov\(^1\) has given a criterion for the number of revolutions in a storage ring \(N_c\) for which Arnold diffusion becomes important. This can be written in the form:

\[
N_c = \frac{1}{\Delta Q \beta^3} \exp \left[ 3 \left( \beta \gamma \right)^{\frac{1}{3}} \exp \left( \frac{1}{6} \gamma^{\frac{1}{3}} \right) \right]
\]

Here \(\beta\) is a coupling parameter, \(\beta^2\) is the average ratio of the width of horizontal-vertical coupling resonances and the width of uncoupled resonances, \(\Delta Q\) is the linear \(Q\) shift due to beam-beam interactions and \(\gamma\) is given by:

\[
\gamma = \Delta Q n_o^4 \nu^{-1}
\]

Here \(n_o\) is the typical order of resonances which take part in Arnold diffusion and \(\nu\) is the number of regularly spaced identical beam-beam interactions per revolution. An estimate of \(n_o\) may be obtained from the experimental observation of the stochasticity limit \(\Delta Q_s\) in beam-beam collisions:

\[
n_o = \frac{1}{2} \left( \Delta Q_s \beta \right)^{-\frac{1}{3}}
\]

2. There is a certain number of ways for applying this theory to storage rings. We may "guess" values for \(\Delta Q_s\), \(\beta\) and \(\nu\) and calculate \(N_c\) as a function of \(\Delta Q\). The result of such a calculation is shown in Fig. 1.

On the other hand, we may try to obtain an estimate for these parameters from the observations in electron-position storage rings. C. Pellegrini suggested that the beam-beam limit is reached when the Arnold diffusion lifetime and synchrotron radiation damping time are equal. Since the \(Q\) shift \(\Delta Q\) is usually known, one can then find out which value of \(\Delta Q_s\) corresponds to experimental observations in various storage rings.
3. A comparison was made for the beam-beam limit of ADONE\(^2\), and in particular on its dependence on energy. The experimentally observed points for 500 MeV and 1 GeV are marked in Fig. 1. It may be seen that the curves drawn for various values of \(\Delta Q_s\), \(\beta\) and \(\nu\) come quite close to the experimental points.

4. It would be interesting to extend this comparison to other electron-position storage rings.

5. References

1) B.V. Chirikov, Novosibirsk preprint 267 (1969),
2) F. Amman et al, Proc. 8th Int. Conf. High Energy Accelerators,

E. Keil
\[ \Delta Q_1 = 0.025 \]
\[ \gamma = \beta = 1 \]

\[ \Delta Q_2 = 0.025 \]
\[ \gamma = \beta = 1 \]

ADONE 500 MeV

ADONE 1 GeV

Fig. 1 Arnold diffusion lifetime