Measuring nanoampere by Schottky scans

In this note I shall try to estimate the lower limit of unbunched beam current that can be observed by straight longitudinal Schottky scans. A single high-impedance pick-up cavity is assumed to be used.

The r.m.s. "signal" current is given by\(^1\)

\[
I_{\text{rms}} = \sqrt{2ef\delta I}
\]

(1)

where \(e\) is the elementary charge, \(f\) the average revolution frequency and \(\delta I\) the fraction of total circulating current carried by particles whose revolution frequency is within the resolution, \(\delta f\), of the spectrum analyzer. Assuming a rectangular distribution of full width \(\Delta f/f\) in frequency and \(\Delta p/p\) in momentum, one obtains:

\[
I_{\text{rms}} = \sqrt{\frac{2eIp\delta f}{n\Delta ph}}
\]

(2)

where \(I\) is the total circulating current, \(h\) the harmonic number and

\[
n = \left| \frac{1}{Y_t^2} - \frac{1}{Y_e^2} \right|
\]

(3)

The available "signal" power from a pick-up cavity of unloaded shunt impedance \(R\) (peak voltage gain squared over two times power-dissipation) is given by

\[
P_s = \frac{I_{\text{rms}}^2R}{4}
\]

(4)

This has to be compared with the available thermal noise power from the same source

\[
P_n = kT\delta f
\]

(5)

with \(kT = 4 \times 10^{-21}\) Ws at room temperature. Hence, the "signal"-to-noise power ratio is given by
\[ \varepsilon = \frac{P_s}{FP_n} = I \frac{eR}{2kTFh} \frac{p}{\eta \Delta p} \quad (6) \]

where \( F \) is the noise factor of the amplifier, and

\[ I = \varepsilon \frac{2kTFh}{eR} \eta \frac{\Delta p}{p} \quad (6a) \]

is the minimum detectable current for given \( \varepsilon \).

The "signal" is statistical noise just like the thermal and amplifier noise, but it is concentrated in bands \( h \eta \Delta p/p \) so that the spectrum analyzer will show a hump of fractional amplitude \( \sqrt{\varepsilon} \) on top of a pedestal of equipment noise. Averaging over a few minutes - the Fast Fourier Transform device should be much superior to the sweeper - one may hope to recognize quite small values of \( \varepsilon \), say \( \varepsilon = 0.01 \) or a 10% hump in amplitude.

For antiprotons filling the radial aperture of the ISR at 3.5 GeV/c

\[ \frac{\Delta p}{p} \sim 0.05 \]

\[ \eta = 0.055 \]

i.e. the particles cover the unusually large frequency range \( \Delta f/f = 2.75 \times 10^{-3} \), making real high-Q cavities problematic.

The harmonic number \( h \) in the numerator of (6a) suggests that a ferrite "cavity" (or resonant transformer) at \( f_r = 318 \text{ kHz} \) could be advantageous. A core built of blocks of low-frequency ferrite \( (\mu \sim 3000) \) with 1 m length, 40 mm core thickness and about 1 mm air gap, might reach 40 \( \mu \)H inductance and a Q-factor of 100, hence a shunt impedance \( R = 8 \text{ k}\Omega \). With \( \varepsilon = 0.01 \) and \( F = 10 \) one finds a minimum detectable current of

\[ I = 1.72 \text{ nA} \]

As the device measures density, not current, the beam can be scraped down to a smaller momentum spread, giving a narrower hump on the spectrum analyzer, without loss of sensitivity.
Using an ISR stacking cavity is another possibility. This would mean \( h = 30 \) and, approximately, \( Q = 1200, R = 20 \, \text{k}\Omega \) (information from H. Frischholz and S. Hansen), perhaps \( F = 5 \), and hence

\[ I = 10.3 \, \text{nA} \]

if scaled to the full 5% momentum spread. In reality the beam can and should be scraped down to bring \( \Delta f/f \) inside the cavity bandwidth. Alternatively the cavity could be tuned across the beam during the measurement.

A higher energy would be desirable because of the rapidly decreasing value of \( n \).

As an extreme example of both bunched and unbunched beams one may consider a single particle. The peak RF current at each revolution harmonic,

\[ I_{RF} = 2eF \]

equals \( 10^{-13} \, \text{A} \). Now - before resorting to superconductivity - one could at least use a real high-Q accelerating structure made of copper, of the kind that is used in electron accelerators. For instance a five-cell, \( \pi \) mode, iris loaded structure at 350 MHz will yield 10 MQ/m (as defined above), or \( R = 20 \, \text{M}\Omega \). The Q-factor will be about 30,000, so that the structure will have to be mechanically tuned across the aperture to search for the particle. Assuming \( F = 2 \) and \( \Delta f = 10 \, \text{Hz} \) (with the Fast Fourier Transform device, so that all particles circulating within the bandwidth of the cavity can be seen simultaneously) one obtains a signal-to-noise power ratio

\[ \epsilon = \frac{(2eF)^2 R}{8FkT\Delta f} = 0.32 \]

i.e. a 57% amplitude hump that should be clearly visible. It is true that no such accelerating structure is available at the ISR at present.

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Reference