2.2.65 NSC A + NRC A

11.1.65

MEASUREMENTS AND ANALYSIS OF MUONIC X-RAY SPECTRA IN THE
SPHERICAL NUCLEI Au, Pb and Bi AND THE DEFORMED
NUCLEI W AND U

CERN LIBRARIES, GENEVA

CM-P00073704

This is to be attached to the Memorandum to the NRC and
NSC concerning :

Muonic X-ray experiment, request for
machine time 1965

Erratum in this Memorandum:

1st page, 7th line from bottom read: In August 1964
we have contacted E. Baldinger

MEASUREMENTS AND ANALYSIS OF MUONIC X-RAY SPECTRA IN THE
SPHERICAL NUCLEI Au, Pb and Bi AND THE DEFORMED
NUCLEI W AND U

H.L. ACKER

Physikalisches Institut der Universität, Freiburg, Germany

G. BACKENSTOSS, C. DAUM*, J.C. SENS and S.A. de WIT*
CERN, Geneva

Geneva - 11 January, 1965

(Submitted to Physics Letters)

* On leave of absence from the Instituut voor Kernfysisch Onderzoek,
Amsterdam, Netherlands.

MEASUREMENTS AND ANALYSIS OF MUONIC X-RAY SPECTRA IN THE
SPHERICAL NUCLEI Au, Pb AND Bi AND THE DEFORMED
NUCLEI W AND U

H.L. ACKER

Physikalisches Institut der Universität, Freiburg, Germany

G. BACKENSTOSS, C. DAUM^{*}, J.C. SENS and S.A. de WIT^{*}

CERN, European Organization for Nuclear Research, Geneva

Muonic X-ray spectra, in contrast to electronic X-ray spectra, have a pronounced sensitivity to the distribution of nuclear charge. In spherical nuclei the transition energies are at present adequately described by a nuclear charge distribution characterized by two parameters: radius and skin thickness. In non-spherical nuclei the quadrupole interaction between the nucleus and the muon gives rise to an additional shift and splitting of the fine structure spectrum. It has been pointed out by Jacobsohn¹⁾ and Wilets²⁾ that such effects are expected to be quite pronounced in muonic X-ray spectra of permanently deformed nuclei, since here the fine structure splitting of the muonic energy levels is in near-resonance with the excitation energy of the first nuclear rotational level.

In the following, we report preliminary data on the energies and relative intensities of muonic X-ray spectra in natural Au, Pb, Bi, and in natural W and U. The data were taken with a Li-drifted Ge-detector.^{**} The results are compared with calculations which are specified below.

Negative muons of 68 MeV from the muon channel of the (stochastically operated) CERN Synchro-cyclotron were slowed down through a Cu moderator and brought to rest in the target. The targets are typically 5 g/cm² thick and 50-100 cm² in cross section. A stopping muon is identified in the customary way by a yes-signal in scintillation

* On leave of absence from the Instituut voor Kernfysisch Onderzoek, Amsterdam, Netherlands.

** Preliminary data on muonic X-rays using a similar detector have been reported by H.L. Anderson (Int. Conf. on High Energy Physics, Dubna, 1964).

counters before and after the moderator in coincidence with a no-signal in an anticounter behind the target. The X-rays are detected by a $2.8 \text{ cm}^2 \times 2 \text{ mm}$ Li-drifted Ge-detector (RCA SJGG-2), cooled by liquid nitrogen and centred behind the anticounter. A fast and a slow signal is obtained from the detector; the slow Ge-signal is accepted by a 1024-channel pulse-height analyser in case of a coincidence between a stopping muon and the fast Ge-signal. Typical rates e.g., for Pb, are 6800 stopping muons per sec, about 0.12 X-rays per sec in the double-escape peaks of the two 2p-1s lines, and 0.3 X-rays per sec in the photo peaks of the 5g-4f lines. Hence the experimentally measured efficiency (solid-angle times absorption due to pair production or photo effect) is 2×10^{-5} in the 5-6 MeV region and 4.5×10^{-5} at about 500 keV. Frequent calibrations with sources gave the necessary checks on stability and linearity of the electronics and provided reference points and line shapes in various parts of the energy scale. Figures 1 and 2 show typical parts of the spectra of Pb, Bi, W and U.

The X-ray energies are determined by interpolating linearly between the two nearest calibration lines and using one of these lines as reference. Table 1 lists the experimental data and the calibration lines. Our values are in substantial agreement with earlier measurements^{3,4,5} of the spherical nuclei. The nuclei Au, Pb and Bi have been analysed by solving the Dirac equation for the 1s, 2p and 3d levels numerically using a Fermi distribution for the nuclear charge. In this way, the energy values are obtained as a function of the radial constant c and the skin thickness t (90% to 10% value). The calculation includes the correction due to vacuum polarization, following Ford and Wills⁶; no correction is made for nuclear polarization. The relationship between c and t can be represented by iso-energetic lines in the $c-t$ diagram for any given energy difference. The intersection of two iso-energetic lines then determines a point (or, if errors are taken into account, an area) in the $c-t$ plane corresponding to values c and

and t compatible with the two chosen transitions. If, as appears to be the case, all transitions determine a common area, the chosen charge distribution is consistent with the data.

Figure 3 shows the iso-energetics in the $c-t$ diagram for Pb corresponding to the measured energy values ($\pm$ error) of table 1. The diagram contains four iso-energetics: $E(2p_{1/2} \rightarrow 1s_{1/2})$, $E[(3d_{5/2} + 3d_{3/2}) \rightarrow 2p_{3/2}]$, $E(2p_{3/2} \rightarrow 1s_{1/2}) - E(2p_{1/2} \rightarrow 1s_{1/2}) \equiv \Delta(2p)$ and $\Delta(2p) - \Delta(3d)$. The $E(2p_{1/2} \rightarrow 1s_{1/2})$ band is narrow and most sensitive to the parameters. Of all the other iso-energetics, the $E(3d_{5/2} \rightarrow 2p_{3/2})$ differs most strongly in slope from the $E(2p_{1/2} \rightarrow 1s_{1/2})$. Therefore the combined values $E(2p_{1/2} \rightarrow 1s_{1/2})$ and $E(3d_{5/2} \rightarrow 2p_{3/2})$ are more restrictive in determining c and t than for instance, $E(2p_{1/2} \rightarrow 1s_{1/2})$ and $\Delta(2p)$. The $\Delta(2p)$ and $\Delta(2p) - \Delta(3d)$ bands do not reduce the common area (dashed area in fig. 2) of the $E(2p_{1/2} \rightarrow 1s_{1/2})$ and the $E(3d_{5/2} \rightarrow 2p_{3/2})$ bands any further. In fig. 3 and table 1 are indicated the central values c_0 , t_0 with errors and the extreme values c_1 , t_1 and c_2 , t_2 .

The higher transitions ($4f \rightarrow 3d$ and $5g \rightarrow 4f$) were obtained from the computed $3d$ levels including vacuum polarization and from point nuclear energy values for the $4f$ and $5g$ levels. Comparison with the experimental transition energies shows that the theoretical $4f \rightarrow 3d$ values are slightly too high due to the omission of the vacuum polarization correction and finite size effects on the $4f$ levels.

The X-ray spectra and $c-t$ diagrams of Au and Bi are in all respects similar to those of Pb (fig. 3, table 1). The effect of the small quadrupole moments has been neglected; no clear evidence for such effects was found in the spectra. For comparison the values of c and t obtained from electron scattering data⁷⁾ are given in brackets in table 1 (see also fig. 3). For a more restrictive determination of the charge distribution, a detailed comparison of muonic X-rays and electron scattering must be performed.

In addition to the energies some crude information was obtained concerning the intensity ratios of the two components of the $2p \rightarrow 1s$ and the $3d \rightarrow 2p$ doublets (see table 1). In general there is agreement with previous data^{3,4}). An interesting case is the $3d \rightarrow 2p$ spectrum of Bi where in addition to the X-ray doublet, a delayed nuclear capture γ -ray is observed; this line results from the reaction³)
 $\mu^- + {}^{209}\text{Bi} \rightarrow {}^{209}\text{Pb}^* + \nu$; ${}^{209}\text{Pb}^* \rightarrow {}^{208}\text{Pb}^*(3^-) + n$; ${}^{208}\text{Pb}^*(3^-) \rightarrow {}^{208}\text{Pb}(0^+) + \gamma(2615 \text{ keV})$.
 In a "delayed spectrum" (see fig. 1b) the X-rays are suppressed, while the capture γ -ray is retained. Contrary to an earlier suggestion³), the intensity ratio in the Bi $3d \rightarrow 2p$ spectrum is low even when accounting for the delayed γ -ray.

Figure 2 shows the spectra of the strongly deformed nuclei U and W; one sees a pronounced difference with respect to the spectra of the spherical nuclei. The elements U and W (which consist mainly of 0^+ spin nuclei) were analysed as follows. The energy eigenvalues of the muon in the spherical part of the nuclear coulomb field were calculated for a Fermi-type charge distribution, i.e., in the same way as for the spherical nuclei described above. The quadrupole interaction, as formulated by Wilets²) is treated by diagonalizing a matrix which contains elements of the structure

$$H_{\phi} \sim e^2 Q_0 \times \langle \phi' | f(r) | \phi \rangle \times (\text{ang. mom. factors})$$

The radial part of the muon wave function $\phi(r)$ are approximated by functions given by Pustovalov⁸) for the $2p$ levels and by wave functions for a point nucleus in the $3d$ levels. In the other levels the quadrupole interaction is negligible. For the radial function $f(r)$ of the quadrupole interaction, the nucleus is assumed to be a uniformly charged spheroid, with intrinsic quadrupole moment Q_0 . This treatment follows closely that of Acker et al.⁹) The theoretical spectra are shown in fig. 2. They were obtained by varying c , t and Q_0 in a search for the best fit to the data. Although no exhaustive analysis for the best set of parameters has been made so far, the values given in table 1 seem to be in good agreement with the experiment.

In W the spectra were calculated for $Q_0 = +6, +7, +8$ and -7 barn, varying c and t to obtain a fit for each Q_0 . The best fit was obtained for $Q_0 = +7$ barn with values of c, t as listed in table 1. The fits for $Q_0 = +6$ and $Q_0 = +8$ deviate significantly from the data. A negative Q_0 is excluded by the data (fig. 2).

In U the spectra were calculated for $Q_0 = +9.5, +10.5, +11.5, +12.5, +13.5, +15$ and -11.5 barn in the same way. A negative Q_0 is excluded by the data, in agreement with previous results^{4,5}). The best fit was obtained for $Q_0 = 12.5$ barn with values for c, t as listed in table 1; $Q_0 = 10.5; 11.5$ and 13.5 barn are in nearly equally good agreement with the data, while $Q_0 = 9.5$ and 15 barn deviate significantly.

The values of the quadrupole moments derived above are subject to reservations, since only the product $e^2 Q_0 \times \langle \phi' | f(r) | \phi \rangle$ is determined. Improvements in the calculation of the radial matrix elements (e.g., relativistic effects) may alter the values of Q_0 .

We wish to thank Professor A. Wapstra for active collaboration in making this experiment possible; Dr. F. Scheck for making available his programme for electron scattering; Professor H. Marschall and Dr. T. Ericson for stimulating discussions; Mr. M. Arbet for his assistance during the preparation and execution of the experiment; and the Institut für angewandte Mathematik, Freiburg, for use of the Siemens 2002 computer. This work was supported in part by the Stichting voor Fundamenteel Onderzoek der Materie (F.O.M.), which is supported by the Nederlandse Organisatie voor Zuiver Wetenschappelyk Onderzoek (Z.W.O.).

REFERENCES

1. B.A. Jacobsohn, Phys.Rev. 96 (1954) 1637.
2. L. Wilts, Mat.Fys.Medd.Dan.Vid.Selsk. 29, No. 1 (1954).
3. W. Prati and J. Rainwater, Phys.Rev. 128 (1962) 2360.
4. G. Backenstoss, K. Goebel, B. Stadler, U. Hegel and
D. Quitmann, Nucl.Phys. (to be published).
5. R.D. Ehrlich, R.J. Powers, V.L. Telegdi, J.A. Bjorkland,
S. Raboy and C.C. Trail, Phys.Rev.Letters 13 (1964) 550.
6. K.W. Ford and J.G. Wills, Report IAMS 2387 (1960).
7. U. Meyer-Berghout, K.W. Ford and L.D.S. Green, Annals of
Physics 8 (1959) 119.
8. G.M. Pustovalov, Soviet Physics (JETP) 36 (1959) 1288.
9. H.L. Acker, H. Marschall, G. Backenstoss and D. Quitmann,
Nucl.Phys. (to be published).

TABLE 1. EXPERIMENTAL MUONIC X-RAY ENERGIES AND INTENSITY RATIOS AND NUCLEAR PARAMETERS

	$2p_{3/2} \rightarrow 1s_{1/2}$	$2p_{1/2} \rightarrow 1s_{1/2}$	$3d_{5/2} \rightarrow 2p_{1/2}$	$(3d_{5/2} + 3d_{3/2}) \rightarrow 2p_{3/2}$	$4f \rightarrow 3d$	$5g \rightarrow 4f$	t_0 ; t_0 [fm] (this exp.)	t_0 ; t_0 [fm] (e^- scatt.)
^{79}Au	E ΔE R	5772 ± 9^a 170 ± 2 $1.4^5 \pm 0.3$	5602 ± 9^a	2478 ± 4^b 136 ± 6 1.1 ± 0.5	878 ± 3^e (888) - -	4.02 ± 5^e (400) - -	6.58 ; 2.18	6.46 ; 2.4
^{82}Pb	E ΔE R	5972 ± 7^a 185 ± 2 1.8 ± 0.4	5787 ± 7^a	2643 ± 4^b 141 ± 5 1.6 ± 0.5	953 ± 3^d (957) - -	4.37 ± 4^e (432) - -	6.68 ; 2.13	6.58 ; 2.4
^{83}Bi	E ΔE R	6041 ± 7^a 194 ± 5 1.2 ± 0.3	5847 ± 7^a	2699 ± 7^c 146 ± 6 $1.3^3 \pm 0.5$	974 ± 8^b (983) - -	4.47 ± 3^e (443) - -	6.61 ; 2.44	6.59 ; 2.4
		I $2p \rightarrow 1s$ II III	I $3d \rightarrow 2p$ II	I $3d \rightarrow 2p$ II	$4f \rightarrow 3d$	$5g \rightarrow 4f$	t_0 ; t_0 [fm] $2p$ states $3d$ states	Q_0 [b]
^{74}W	E	5418 ± 7^a 5308 ± 7^a 5214 ± 7^a	2211 ± 3^b	2048 ± 3^b	768 ± 3^f (776)	352 ± 3^f (351)	6.60 ; 1.8	435.59 58.50 $+7$
^{92}U	E	6442 ± 9^a 6137 ± 7^a	3254 ± 3^b	3006 ± 3^b	1214 ± 4^b (1208)	552 ± 4^e (546)	6.98 ; 2.7	959.02 192.45 $+12.5$

Energies in keV; values of E in brackets are point nuclear values (vacuum polarization and finite size effects included for $3d$ states only); R = intensity ratio $I(2p_{3/2} \rightarrow 1s_{1/2})/I(2p_{1/2} \rightarrow 1s_{1/2})$, respectively $II(3d_{5/2} + 3d_{3/2}) \rightarrow 2p_{3/2}/I(3d_{5/2} \rightarrow 2p_{3/2})$; I , II , III refer to the main groups in the spectrum of U and W ; t_0 , t_0 are the central values of the dashed area in fig. 1 for the radial constant and skin thickness in the Fermi-type charge distribution; Q_0 is the intrinsic quadrupole moment; the following reference lines have been used: a) double escape line of 6134 keV of ^{16}N , b) 1367.6 keV of ^{24}Na , c) 2615 keV of ^{208}Pb , d) 4273.6 keV of ^{22}Na , e) 511 keV annihilation line present in the X-ray spectrum, f) 511 keV annihilation line taken from ^{22}Na .

Figure captions

Fig. 1 : Experimental spectra of Pb and Bi.

Pb : The double escape lines of the $2p \rightarrow 1s$ and $3d \rightarrow 2p$ transitions and the photo lines of the $4f \rightarrow 3d$ and the $5g \rightarrow 4f$ transitions are shown. The energies corresponding to the centre of gravity of the lines are indicated. The dashed lines indicate the background. For the $2p \rightarrow 1s$ transitions the line shape obtained from the ^{16}N calibration line is drawn.

Bi : $3d \rightarrow 2p$ transitions. In the curves a capture γ -ray (Pb^{208*}) is present at 2615 keV between the ($3d_{5/2} \rightarrow 2p_{3/2}$) and the ($3d_{3/2} \rightarrow 2p_{1/2}$) transitions. In the lower curve the X-ray transitions are strongly suppressed by delaying the signal of the counter telescope by 50 nsec.

Fig. 2 : $2p \rightarrow 1s$ and $3d \rightarrow 2p$ transitions in $_{92}\text{U}$ and $_{74}\text{W}$. The double escape lines are shown, the energy corresponding to the centre of gravity of the groups is indicated. The solid line shows the spectrum calculated with the values c , t , and Q_0 of table 1. The line shape of the calibration line has been folded into the theoretical spectrum.

Fig. 3a : $c-t$ diagram for Pb. The dashed area is the overlap between the $E(2p_{1/2} \rightarrow 1s_{1/2})$ band and the $E(3d_{5/2} \rightarrow 2p_{3/2})$ band. All energies in keV. Electron scattering point from ref. 7.

Fig. 3b : $c-t$ diagram for Au.

Fig. 3c : $c-t$ diagram for Bi.

FIG. 3a

Fig. 3b