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AUXILIARY FIELDS AND ULTRAVIOLET DIVERGENCES

IN SUPERGRAVITY

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ABSTRACT

The relation between the supergravity counterterms and the coupling of auxiliary fields to matter is investigated. As a result we determine the infinities due to one and two internal-matter loops for the supersymmetric Yang-Mills-Einstein system. Furthermore, the couplings of the auxiliary fields to the massive scalar multiplet are obtained through order κ^2 , and the contribution to the counterterm Lagrangian trilinear in the auxiliary fields is given.

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1. INTRODUCTION

Recently various auxiliary-field structures for supergravity have been proposed ^{1,2,3)} which lead to a closed algebra of gauge transformations without use of the equations of motion. The structure proposed in Ref. 2 is particularly simple, and it allows one to couple supergravity to certain matter configurations in a "minimal" fashion ^{3,4)}. The one-loop counterterms for supergravity can also be given in this formulation ³⁾. It is known that there exist two different off-mass-shell counterterms, one of which is only known to lowest order in the fields ^{5,6)}.

In this paper we will discuss some of the consequences of the auxiliary-field structure of these counterterms. If the coupling of matter to supergravity is known in this formulation, like for the Maxwell-Einstein theory ^{3,4)} then one can determine the coefficients of the two counterterms due to the matter corrections in an easy way, even to higher orders in the matter self-couplings. We shall, for instance, present the coefficient of the counterterm in the Yang-Mills-Einstein supergravity theory up to two matter loops.

On the other hand, if the coupling of matter to supergravity is not known, then the structure of the counterterms determines to some extent the coupling of the auxiliary fields to the matter fields. In this way we obtain trilinear couplings for the scalar multiplet without any assumptions about the matter-field transformation rules. The thus obtained information can then be shown to have implications on the structure of the counterterms in higher orders.

Our method is based on the fact that in the counterterms the auxiliary fields enter in a simple way, at least for the terms quadratic in the fields. We can determine the coefficients of the counterterms by calculating selfenergy diagrams of the auxiliary fields. It is, however, crucial in this approach that we have an auxiliary-field formulation at our disposal in which the supergravity-field transformations are completely

insensitive to the presence of matter.

This paper is organized as follows. In Sect. 2 we present the known results relevant to our work, in particular the form of counterterms and some standard quantum-gravity results. In Sect. 3 we discuss the Maxwell- and the Yang-Mills-Einstein supergravity theory. The massless and the massive scalar multiplet, coupled to supergravity are considered in Sect. 4 and 5, respectively. Sect. 6 contains our conclusions.

2. PRELIMINARIES

In this section we summarize several known results on supersymmetric counterterms and ultraviolet divergences in quantum-gravity which we shall use later.

There exist two off-mass-shell counterterms for pure supergravity, which lead to the following contribution quadratic in the fields ³⁾

$$\Delta \mathcal{L}_1 = a_1 \epsilon^{-1} e \left[R_{\mu\nu}^2 - \frac{1}{3} R^2 - k^2 \bar{\gamma}_\mu (\square \delta_{\mu\nu} - \partial_\mu \partial_\nu) (R^\nu - \frac{1}{3} \gamma^\nu \gamma \cdot R) - \frac{1}{3} k^2 F_{\mu\nu}(A)^2 \right], \quad (1)$$

$$\Delta \mathcal{L}_2 = a_2 \epsilon^{-1} e \left[R^2 + k^2 \bar{R} \cdot \gamma \not{\partial} \gamma \cdot R - 4k^2 (\partial_\mu S)^2 - 4k^2 (\partial_\mu P)^2 + 4k^2 (\partial_\mu A^\mu)^2 \right]. \quad (2)$$

These are invariant under the local supersymmetry transformations of Ref. 2. The first counterterm is conformally invariant and is known to all orders in the fields ⁵⁾. The second one is only known up to terms quadratic in the fields, and was constructed in Ref. 6. In the formulae (1) and (2) we have used

the following notation. The fields S , P and A_μ are the scalar, pseudoscalar and axial auxiliary fields, respectively, which have been introduced in Ref. 2. $R_{\mu\nu}$ and R are the contracted Riemann tensors, and R^μ is the Rarita-Schwinger field equation

$$R^\mu = e^{-1} \epsilon^{\mu\nu\rho\sigma} \gamma_5 \gamma_\nu D_\rho \psi_\sigma. \quad (3)$$

$F_{\mu\nu}(A)$ is the field strength of the axial-vector field

$$F_{\mu\nu}(A) = \partial_\mu A_\nu - \partial_\nu A_\mu. \quad (4)$$

We use the conventions of Ref. 3.

The ultraviolet divergences in the counterterms (1) and (2) have been expressed in terms of the dimensional regularization parameter ϵ , defined by

$$\epsilon = 8 \pi^2 (n-4), \quad (5)$$

where n is the number of space-time dimensions. Of course, the dimensional regularization method is strictly speaking not applicable to supergravity, but since we are only interested in the divergent part of Feynman diagrams, we do not expect this to cause complications at the one-loop level. However, since we shall also present two-loop results, we should caution the reader that those may have only formal significance.

For future purposes we now list some of the matter contributions to the counterterms of pure gravity ⁷⁾.

$$\Delta \mathcal{L}_g^{(0)} = \frac{1}{120} \frac{1}{\epsilon} e (R_{\mu\nu}^2 - \frac{1}{3} R^2) + \frac{1}{144} (1-\alpha)^2 \frac{1}{\epsilon} e R^2, \quad (6)$$

$$\Delta \mathcal{L}_g^{(1/2)} = \frac{1}{40} \frac{1}{\epsilon} e (R_{\mu\nu}^2 - \frac{1}{3} R^2), \quad (7)$$

$$\Delta \mathcal{L}_g^{(1)} = \frac{1}{10} \frac{1}{\epsilon} e (R_{\mu\nu}^2 - \frac{1}{3} R^2) . \quad (8)$$

Eqs. (6), (7) and (8) correspond to the one-loop contribution from a massless scalar, Majorana fermion, and vector (gauge) field, respectively. For the scalar field we have allowed an improvement term in the Lagrangian:

$$\mathcal{L}_{\text{imp}} = \frac{1}{12} \alpha e R \phi^2 \quad (9)$$

If the scalar or Majorana spinor are massive we have the following additional contributions

$$\Delta \mathcal{L}_g^{(m)}(0) = \frac{1}{12} (1 - \alpha) \frac{1}{\epsilon} m^2 e R \quad (10)$$

$$\Delta \mathcal{L}_g^{(m)}(\frac{1}{2}) = \frac{1}{12} \frac{1}{\epsilon} m^2 e R \quad (11)$$

where m is the mass parameter of the spinless or spinor field. This counterterm Lagrangian can be extended to a supersymmetric expression, which is proportional to the pure-supergravity Lagrangian. In addition the presence of the dimensionful parameter m allows in principle the occurrence of supersymmetric extension of the cosmological term as a counterterm Lagrangian³⁾. Hence we find

$$\begin{aligned} \Delta \mathcal{L}_3 = a_3 e^{-1} [& e R + \kappa^2 \epsilon^{\mu\nu\rho\sigma} \bar{\psi}_\mu \gamma_5 \gamma_\nu D_\rho \psi_\sigma \\ & + \frac{2}{3} \kappa^2 e (S^2 + P^2 - A_\mu^2)] \end{aligned} \quad (12)$$

$$\Delta \mathcal{L}_4 = a_4 e^{-1} e [2\kappa S + \kappa^2 \bar{\psi}_\mu \sigma^{\mu\nu} \psi_\nu] \quad (13)$$

The coefficients a_3 and a_4 are proportional to m^2 and m^3 , respectively.

3. MAXWELL-EINSTEIN SUPERGRAVITY

We consider the supersymmetric Maxwell-Einstein system in the formulation with auxiliary fields of Refs. 3 and 4. The supersymmetric Maxwell Lagrangian with its coupling to the supergravity fields $e_{\mu a}$, ψ_μ , S , P and A_μ , is given by

$$\begin{aligned} \mathcal{L}_M = & -\frac{1}{4} e F_{\mu\nu}(V)^2 - \frac{1}{2} e \bar{\lambda} \tilde{D} \lambda \\ & + \frac{1}{4} e \kappa \bar{\psi}_\mu \sigma^{\alpha\beta} \gamma^\mu \lambda F_{\alpha\beta}(V) + \frac{1}{2} e D^2, \end{aligned} \quad (14)$$

where $\tilde{D}_\mu$ denotes

$$\begin{aligned} \tilde{D}_\mu \lambda = & (\partial_\mu + \frac{1}{2} \omega_\mu^{ab} \sigma_{ab} + \frac{1}{2} i \kappa A_\mu \gamma_5) \lambda \\ & - \frac{1}{2} \kappa F_{\alpha\beta}(V) \sigma^{\alpha\beta} \gamma_\mu \lambda - \frac{1}{2} \kappa^2 (\bar{\psi}_\alpha \gamma_\beta \lambda) \sigma^{\alpha\beta} \gamma_\mu \lambda, \end{aligned} \quad (15)$$

and ω_μ^{ab} denotes the spinor connection field given in Refs. 2 and 3.

This Lagrangian (14) is invariant under local supersymmetry transformations, and can be thought of as representing a quantum matter field in the presence of the classical background supergravity fields $e_{\mu a}$, ψ_μ , A_μ , S , P . This will give rise to loop corrections to the background field supergravity action, which are manifestly invariant under the local supersymmetry transformations on the supergravity fields. To prove this result is straightforward, but it is crucial that one has an auxiliary-field formulation, in which the supergravity-field transformations do not involve the matter fields. In principle there is a

potential source of trouble because the quantization of the Maxwell field requires the introduction of a gauge-fixing term, which will break local supersymmetry as well. However, one can easily show by using a theorem given in Ref. 8 that Green's functions of operators that are invariant under the electromagnetic gauge transformations are not affected by gauge-quantization complications. This follows from the fact that the electromagnetic and the supersymmetry transformations commute.

We conclude that, since the matter-induced corrections are supersymmetric invariant, their ultraviolet divergences are invariant as well, and are therefore precisely given by a linear combination of the two counterterms (1) and (2). In particular if we wish to calculate the coefficients a_1 and a_2 it suffices to calculate explicitly the divergence of the two-point function corresponding to one of the terms in $\Delta\mathcal{L}_1$ or $\Delta\mathcal{L}_2$. In the present example the simplest way to find these coefficients is to calculate the two-point functions of the auxiliary fields. Since the auxiliary fields S and P do not couple to the vector multiplet we can immediately conclude that

$$a_2^{(V)} = 0 \quad (16)$$

This conclusion is consistent with the conformal invariance of the Lagrangian (14). The auxiliary field A_μ couples to the spinor field λ in a minimal way, $-\frac{1}{2}i\kappa\bar{\lambda}\gamma^\mu\gamma_5\lambda A_\mu$. Since the field λ is massless this interaction can only give rise to a counterterm of the form $F_{\mu\nu}(A)^2$, which is only present in $\Delta\mathcal{L}_1$. To find the precise coefficient is straightforward, and requires only to calculate the axial-vector selfenergy diagram of Fig. 1. The result is

$$a_1^{(V)} = \frac{1}{8} \quad (17)$$

This coefficient can be compared to the one obtained by adding the conventional quantum-gravity counterterms $\Delta\mathcal{L}_g(\frac{1}{2})$ to $\Delta\mathcal{L}_g(1)$, as given by Eqs. (7) and (8). Their sum represents the

contribution from a supersymmetric vector multiplet and gives a result in agreement with our value for $a_1^{(V)}$. We have also computed the selfenergy diagrams for the Rarita-Schwinger fields, and found complete agreement with Eqs. (16) and (17).

We would like to emphasize that using the auxiliary-field formulation allows a much simpler determination of the coefficients $a_1^{(V)}$ and $a_2^{(V)}$ in this case. To generalize these results for the Yang-Mills-Einstein system is now trivial. This case is interesting because it has matter selfinteractions which contrary to the Maxwell-Einstein system give higher-order corrections to Eq. (17). For example, the two-loop contribution to $a_1^{(V)}$ can be calculated from the diagrams given in Fig. 2. In fact the result to this order can be read from the Jost-Luttinger calculation for the corresponding vacuum polarization diagrams in Q.E.D., modified by group-theoretic factors⁹⁾. The axial coupling is no complication in this case because the fermions are massless. The result is

$$a_1^{(V)} = \frac{1}{8} \left(C_1 - \frac{3}{4} \frac{g^2}{4\pi^2} C_2 \right) , \quad (18)$$

where the coupling constant g of the internal gauge group is defined by the following interaction between the gauge fields V_μ^a and the fermions λ^a .

$$\mathcal{L}_{int} = \frac{1}{2} e g f^a_{bc} \bar{\lambda}_a \gamma^\mu \lambda^b V_\mu^c . \quad (19)$$

The coefficients C_1 and C_2 are the values of the Casimir operators in the adjoint representation.

$$C_1 = \delta_a^a \quad (20)$$

$$C_2 = f^a_{bc} f^{cb}_a$$

where the f_{bc}^a are the structure constants of the internal symmetry group. For instance, for the group $SO(3)$ we find $C_1 = 3$, $C_2 = -6$.

As a prelude to the next section we observe that if the coupling to the auxiliary fields was unknown we could have determined the various coupling constants in the Lagrangian (14) by requiring that the infinities of the two-point functions of ψ_μ , A_μ , S and P precisely match the result of the standard quantum gravity calculations (7) and (8) in a way that is consistent with the structure of the counterterms (1) and (2). This will be our starting point in the next sections where we will determine several couplings of the scalar multiplet to the auxiliary fields.

4. THE MASSLESS SCALAR MULTIPLY

We consider the massless scalar multiplet. As we have mentioned already we will now turn our strategy around to constrain the coupling of the supergravity fields to matter, using our knowledge of the counterterm structure. Since at the present time the auxiliary-field coupling of the scalar multiplet is only known in one particular case ¹⁰⁾ these constraints may be useful for the construction of the general case. Our approach has three important ingredients: the form of the counterterms, as given in Eqs. (1) and (2), the assumption that the supergravity-field transformations are not affected by the presence of matter, and chiral symmetry and dimensional arguments to restrict the form of the couplings.

As a starting point we take the Lagrangian of the free massless scalar multiplet in flat space

$$\mathcal{L} = -\frac{1}{2}(\partial_\mu A)^2 - \frac{1}{2}(\partial_\mu B)^2 - \frac{1}{2}\bar{\chi}\not{\partial}\chi. \quad (21)$$

We can covariantize this Lagrangian with respect to general-coordinate transformations, and we may add an improvement term

of the form (9). We first consider the couplings quadratic in the matter fields and linear in the auxiliary fields. Their general form is

$$e^{-1} \mathcal{L}_1 = \kappa \bar{\chi} (g_S S + i g_P P \gamma_5 + i f_1 \not{A} \gamma_5) \chi + 2 \kappa (f_2 A^\mu A^\nu \partial_\mu \partial_\nu B + g_A \partial_\mu A^\mu \partial_\nu B) \quad (22)$$

Since the auxiliary fields have dimension 2 this is the most general coupling linear in κ . We can exclude couplings such as SA^2 , $\kappa^2 S(\partial_\mu A)^2$, since they would lead to divergences which are not present in the counterterms (1) and (2). The Lagrangian $\mathcal{L}_1$ leads immediately to divergences in the two-point functions of the auxiliary fields, which require the following counterterm

$$\Delta \mathcal{L} = -\frac{1}{3} \kappa^2 \frac{1}{\epsilon} (2 f_1^2 + f_2^2) F_{\mu\nu}(A)^2 - 2 \kappa^2 \frac{1}{\epsilon} (g_S^2 (\partial_\mu S)^2 + g_P^2 (\partial_\mu P)^2 - g_A^2 (\partial_\mu A^\mu)^2) \quad (23)$$

In the scalar multiplet the coefficients $a_1^{(S)}$ and $a_2^{(S)}$ of the counterterms (1) and (2) are already known from the quantum-gravity results (6) and (7). Adding $\Delta \mathcal{L}_g^{(0)}$ and $\Delta \mathcal{L}_g^{(1/2)}$ with coefficients 2 and 1, respectively, leads to

$$\begin{aligned} a_1^{(S)} &= \frac{1}{24} \\ a_2^{(S)} &= \frac{1}{72} (1 - \alpha)^2 \end{aligned} \quad (24)$$

Comparing Eqs. (1), (2), (23) and (24) we find directly

$$\begin{aligned} 2 f_1^2 + f_2^2 &= \frac{1}{24} \\ g_S^2 &= g_P^2 = g_A^2 = \frac{1}{36} (1 - \alpha)^2 \end{aligned} \quad (25)$$

If we choose $\alpha = 1$ the coefficient of the second counterterm vanishes, so that the S and P fields decouple. Furthermore the auxiliary field A_μ must couple minimally to the fields A, B and χ , since the first counterterm (1) is known to be locally chiral invariant ⁵⁾. This situation corresponds to the model of Ref. 10, which describes a conformally invariant version of supergravity with a compensating scalar multiplet. This model can be coupled to Einstein supergravity ^{3,9)} and precisely as for the Maxwell-Einstein supergravity theory it requires only a minimal coupling of A_μ to the matter fields.

By means of the same procedure we can now calculate the infinities in the selfenergy diagrams of the Rarita-Schwinger fields. Assuming that this field couples to a conserved matter current we find

$$e^{-1} \mathcal{L}_2 = \frac{1}{2} \kappa \bar{\psi}_\mu \gamma^\nu \gamma^\mu \partial_\nu (A + i\gamma_5 B) \chi + \kappa g_I \bar{\psi}_\mu \tilde{\partial}_\nu \sigma^{\mu\nu} (A + i\gamma_5 B) \chi, \quad (26)$$

with the coefficient g_I subject to the condition

$$(g_I + \frac{2}{3})^2 = \frac{4}{9} (1 - \alpha)^2 \quad (27)$$

The second term in (26) represents the improvement term of the supersymmetry current ¹¹⁾. Again for $\alpha = 1$ this agrees with Ref. 10.

The Lagrangians (21), (22) and (26) are invariant with respect to the following global chiral transformations of the fields

$$\begin{aligned} \delta \psi_\mu &= i\gamma_5 \psi_\mu \\ \delta \chi &= -i\gamma_5 \chi \\ \delta S &= -2P \end{aligned} \quad (28)$$

$$\delta P = 2S$$

$$\delta e_{\mu a} = \delta A = \delta B = \delta A_{\mu} = 0$$

Only this set of chiral transformations is consistent with the supersymmetry transformations of the supergravity fields ¹²⁾. We remark that when $g_S = g_P = g_A = 0$ we recover the second chiral invariance of the scalar multiplet, which only acts on the matter fields ¹³⁾.

If this chiral invariance (28) holds to all orders in κ then we can further constrain some of the couplings. For instance, we find

$$g_S = g_P, \quad (29)$$

and that the four-point couplings of order κ^2 quadratic in the matter fields are of the form

$$\kappa^2 (S^2 + P^2) A^2, \kappa^2 (S^2 + P^2) B^2, \kappa^2 A_{\mu}^2 A^2, \kappa^2 A_{\mu}^2 B^2 \quad (30)$$

As we have mentioned in Sect. 2 the counterterm $\Delta \mathcal{L}_1$ is completely known and can be obtained from Ref. 5, whereas for $\Delta \mathcal{L}_2$ only the terms quadratic in the fields are known at present. However, we can now use the information obtained above to determine all the terms trilinear in the auxiliary fields. In principle $\Delta \mathcal{L}_2$ could contain the following terms

$$\kappa^2 S^3, \kappa^2 S^2 P^2, \kappa^2 A_{\mu}^2 S, \kappa^3 \partial_{\mu} A^{\mu} S P, \kappa^3 A^{\mu} S \overleftrightarrow{\partial}_{\mu} P \quad (31)$$

(The structure of the matter couplings obtained so far is sufficient to exclude higher-derivative trilinear terms in the counterterm). The first three terms can not be present because they are not invariant under the chiral transformations (28), and because matter loops can only lead to contributions of order κ^3 . The fourth term is not allowed by chiral invariance, whereas the last term is indeed present since it must cancel the

divergence of the fermion triangle diagram given in Fig. 3. From an explicit calculation we find that the counterterm $\Delta \mathcal{L}_2$ should contain a term $-\kappa^3 \epsilon^{-1} 16 g_P g_S f_1 A^\mu S \overleftrightarrow{\partial}_\mu P$, which (after proper normalization and using Eq. (29)) can be obtained from Eq. 2 by the "minimal" substitution

$$\begin{aligned} \partial_\mu S &\rightarrow \partial_\mu S - 4 f_1 \kappa A_\mu P \\ \partial_\mu P &\rightarrow \partial_\mu P + 4 f_1 \kappa A_\mu S \end{aligned} \quad (32)$$

However, the structure of the counterterms is completely independent from the kind of matter to which supergravity has been coupled. In other words f_1 must be a universal constant, which we can determine by comparing to the model of Ref. 10. Using Eq. (25) we then find

$$\begin{aligned} |f_1| &= \frac{1}{12} \\ |f_2| &= \frac{1}{6} \end{aligned} \quad (33)$$

5. THE MASSIVE SCALAR MULTIPLY

We will now use the same method as in the previous section to determine some of the couplings of the auxiliary fields to the massive scalar multiplet. Because of the presence of a new dimensionful parameter m we must consider two more counterterms, given in Eqs. (12) and (13), with coefficients proportional to m^2 and m^3 , respectively. To restrict the possible interactions we require all couplings that do not explicitly depend on m to be invariant under the chiral transformations (28). This leads to the following couplings of order κ^2 and $m\kappa$, quadratic in the matter fields

$$\begin{aligned}
 e^i \mathcal{L}_3 = & \kappa g_S \bar{\chi} (S + i \gamma_5 P) \chi + i \kappa f_1 \bar{\chi} A \gamma_5 \chi \\
 & + \kappa (f_2 A^\mu A^\nu \partial_\mu B + g_A \partial_\mu A^\mu A B) \\
 & + \kappa^2 (S^2 + P^2) (h_1 A^2 + h_2 B^2) + \kappa^2 A_\mu^2 (h_3 A^2 + h_4 B^2) \\
 & + \kappa m (d_1 S A^2 + d_2 S B^2 + d_3 P A B)
 \end{aligned} \tag{34}$$

This expression includes the interactions that we have found in the preceding section.

Using the quantum-gravity results (10) and (11) we find that the coefficient of the counterterm Lagrangian $\Delta \mathcal{L}_3$, given in Eq. (12), is equal to

$$a_3^{(S)} = \left(\frac{1}{4} - \frac{1}{6} \alpha \right) m^2 \tag{35}$$

We now discuss the mass-dependent divergences in the two-point functions of A_μ , S and P . The diagrams that contribute are generically indicated in Fig. 4. Since a term of the form $\partial_\mu A^\mu P$ is not present in $\Delta \mathcal{L}_3$, we find the condition

$$g_A d_3 = 4 g_S f_1 \tag{36}$$

The coefficients of the remaining divergences should be compatible with Eq. (35). Therefore we have the relations

$$\begin{aligned}
 d_1^2 + d_2^2 - \frac{1}{2} d_3^2 &= 8 g_S^2 \\
 h_1 + h_2 &= \frac{1}{18} (2\alpha - 3) - 4 g_S^2 + \frac{1}{2} d_3^2 \\
 h_3 + h_4 &= -\frac{1}{9} \alpha
 \end{aligned} \tag{37}$$

where we have used Eq. (25). Notice that when $\alpha \neq 1$ we can combine Eqs. (25) and (36) to find

$$|d_3| = \frac{1}{3} \quad (38)$$

Furthermore from the absence of counterterms S^3 , SP^2 and SA_μ^2 we conclude that the corresponding three-point functions should be finite. An explicit calculation which involves the diagrams shown in Fig. 5 then leads directly to

$$d_1 h_1 + d_2 h_2 = -8 g_S^3 \quad (39)$$

$$d_1 h_3 + d_2 h_4 = -16 g_S f_1^2 - 2 (d_1 + d_2) f_2^2$$

At this point it is worth mentioning that the Eqs. (25), (33), and (36) - (39) allow the solution $\alpha = 1$, $h_1 = h_2 = 0$, $h_3 = h_4 = -\frac{1}{18}$, for the mass-independent couplings. This solution implies the absence of mass-independent couplings of S and P to the scalar multiplet, and a minimally coupled axial-vector field A_μ , which is in agreement with Ref. 10. The mass-dependent couplings which are not considered in Ref. 10 are then constrained by $d_1^2 + d_2^2 = \frac{1}{2} d_3^2 = \frac{1}{18}$.

Before commenting on these results any further we discuss the divergences in the selfenergy diagrams of the Rarita-Schwinger fields. We again assume that ψ_μ couples to a conserved matter current, with a possible improvement term.

$$\begin{aligned} e^{-1} \mathcal{L}_4 = & \frac{1}{2} k \bar{\psi}_\mu \gamma^\nu \gamma^\mu \partial_\nu (A + i \gamma_5 B) \chi \\ & + \frac{1}{2} k m \bar{\psi}_\mu \gamma^\mu (A - i \gamma_5 B) \chi \\ & + k g_I \bar{\psi}_\mu \tilde{\partial}_\nu \sigma^{\mu\nu} (A + i \gamma_5 B) \chi \end{aligned} \quad (40)$$

To make the divergences in the selfenergy diagrams compatible with the structure of the counterterms (12) and (13) requires the following interactions quadratic in the fields ψ_μ :

$$\begin{aligned} \mathcal{L}_5 = & \kappa^2 (h_5 A^2 + h_6 B^2) \epsilon^{\mu\nu\rho\sigma} \bar{\psi}_\mu \gamma_5 \gamma_\nu D_\rho \psi_\sigma \\ & + e \kappa^2 m (d_4 A^2 + d_5 B^2) \bar{\psi}_\mu \sigma^{\mu\nu} \psi_\nu \end{aligned} \quad (41)$$

An explicit calculation leads to the following equations for the coefficients

$$\begin{aligned} h_5 + h_6 &= \frac{1}{6} \\ d_4 + d_5 - \frac{1}{2} (d_1 + d_2) &= -2g_S \end{aligned} \quad (42)$$

To obtain the second equation we have used the fact that coefficient of the counterterm $\Delta \mathcal{L}_4$ follows from the divergences of the tadpole diagrams with a single external S-field (see Fig. 6). This gives the following value for $a_4^{(S)}$:

$$a_4^{(S)} = m^3 (2g_S - \frac{1}{2} d_1 - \frac{1}{2} d_2) \quad (43)$$

For $\alpha = 1$ the result for the mass-independent couplings, $h_5 = h_6 = 1$, agrees again with Ref. 10.

We have thus shown how our method restricts both the form and the coefficients of the auxiliary-field couplings to the scalar multiplet. The restrictions do not yet allow a complete determination of the coefficients, mainly because we have left the coupling strengths of the scalar and pseudoscalar fields A and B unrelated. It is known from the existing realizations of the scalar multiplet ^{13, 14, 15} that these couplings are usually related, unless a self-interaction is present. In that case the scalar potential is a function of $\kappa^2 (A^2 + B^2)$. To recover this result after elimination of the auxiliary fields requires

$$\begin{aligned}
 |d_1| &= |d_2| \\
 d_3^2 &= (d_1 - d_2)^2 \\
 h_1 &= h_2 \\
 h_3 &= h_4 \\
 h_5 &= h_6
 \end{aligned} \tag{44}$$

However, the equations (37), (39) and (42) then imply that

$$\begin{aligned}
 d_1 &= -d_2 = d \\
 d_3^2 &= 4d^2 \\
 \alpha &= 1 \\
 h_1 &= h_2 = -\frac{1}{36} + d^2 \\
 h_3 &= h_4 = -\frac{1}{18} \\
 d_4 &= -d_5 \\
 h_5 &= h_6 = \frac{1}{12}
 \end{aligned} \tag{45}$$

We remind the reader that in this case the axial-vector auxiliary field is coupled minimally to the matter fields. This result suggests that the scalar-multiplet coupling to supergravity in the auxiliary-field formulation requires in general an improvement term

$$\mathcal{L}_{imp} = \frac{1}{12} (A^2 + B^2) \left\{ eR + \kappa^2 \epsilon^{\mu\nu\rho\sigma} \bar{\psi}_\mu \gamma_5 \gamma_\nu D_\rho \psi_\sigma \right\} \tag{46}$$

To compare this to the standard formulations of the scalar multiplet one has to make field redefinitions of the type mentioned in Ref. 15.

6. CONCLUSIONS

We have shown that in an auxiliary-field formulation of supergravity one can obtain information about the coupling to matter from a partial knowledge of the one-loop counterterms. This information can then be used to further determine the structure of these counterterms. In this procedure the local-supersymmetry transformation laws of the matter fields are irrelevant. This is somewhat different from the standard procedure, which simultaneously constructs order by order in the Lagrangian and the transformation rules. Given the auxiliary-field couplings to matter we were also able to easily determine the coefficients of the counterterms due to matter-loop corrections, including higher-order effects in the matter self-interactions.

We emphasize again that the following aspects are crucial in obtaining our results. We need the supersymmetric counterterms, or at least that part which is quadratic in the fields, combined with some standard quantum-gravity results. Furthermore to obtain supersymmetric results the (background) supergravity-field transformations should not depend on the matter fields.

One may wonder whether calculations of the type presented here can also be done for the case of pure supergravity. In other words can one include the one-loop effects of internal supergravity fields as well? The difficulty is that if such contributions are included conventional field-theory calculations like the ones we have been doing will lead to off-mass-shell infinities which are gauge dependent and need not have a locally supersymmetric structure¹⁶⁾. To illustrate this one can calculate the infinities of the two-point functions of the auxiliary fields A_μ , S and P . While in the classical Lagrangian (12) these fields are essentially noninteracting interactions can arise in the quantized Lagrangian from the gauge-fixing procedure. If the gauge-fixing terms are independent of the auxiliary fields then these fields will couple only to the Faddeev-Popov ghost fields. Therefore the divergences in the

auxiliary-field Green's functions will arise from ghost loops.

For example, if we consider the standard supersymmetry gauge-fixing term, $F = \gamma \cdot \psi$, then the auxiliary fields couple to the ghost fields according to

$$\mathcal{L}_{int} \sim \kappa \bar{c}^* (S - i\gamma_5 P - \frac{i}{4} A \gamma_5) c \quad (47)$$

Since the axial-vector field couples minimally to the ghost fields the resulting infinities in the auxiliary-field two-point functions require a counterterm which is a linear combination of

$$(\partial_\mu S)^2 + (\partial_\mu P)^2 \quad \text{and} \quad F_{\mu\nu}(A)^2$$

Because of the absence of a term $(\partial_\mu A^\mu)^2$ this result can not be part of a linear combination of the supersymmetric counterterms (1) and (2). A different choice of gauge, for instance $F = \partial \cdot \psi$ does, however, generate a term $(\partial_\mu A^\mu)^2$, but nevertheless the resulting counterterm is not supersymmetric. This illustrates our remarks concerning the lack of supersymmetry invariance off-shell.

In principle supersymmetric counterterms could be obtained using the background-field method, provided that it is possible to find a background-field invariant gauge-fixing procedure. It would be interesting to further explore these possibilities, although there are indications that this approach may not work *).

Let us now briefly summarize our results. We have considered the Maxwell, Yang-Mills, and the scalar multiplet coupled to supergravity. For the first two theories we have demonstrated that the counterterm coefficients due to matter corrections can be obtained very easily in a formulation with auxiliary fields. In particular we have given the coefficients up to two closed matter loops for the Yang-Mills-Einstein system. Subsequently we

*) P.K. Townsend and P. van Nieuwenhuizen, private communication.

have studied the constraints on the coupling of the auxiliary fields to the scalar multiplet. This enabled us to determine the contributions to the counterterms trilinear in the auxiliary fields. Our method does not lead to a complete determination of the couplings, although it severely restricts the various coefficients. We find indications that the conventional form of the scalar-multiplet coupling to supergravity is not the most natural one in view of the auxiliary-field formulation, but that instead a version with improvement terms (46) is preferable. The auxiliary fields may be particularly useful in the case of the scalar multiplet, since they will probably remove the non-polynomial terms which are usually present in the conventional parametrizations.

Note added:

When this paper was being prepared for publication we received a preprint by S. Ferrara and P. van Nieuwenhuizen who construct a tensor calculus for supergravity which enables a direct construction of locally supersymmetric actions with matter fields.

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FIGURE CAPTIONS

- Fig. 1 The selfenergy diagram with one matter loop for the axial-vector auxiliary field in the Maxwell-Einstein and Yang-Mills-Einstein theory.
- Fig. 2 The selfenergy diagrams with two matter loops for the axial-vector auxiliary field in the Yang-Mills-Einstein theory. The solid and wavy lines denote the fermion and vector gauge-field propagators, respectively.
- Fig. 3 Fermion triangle diagram which leads to the determination of the counterterm trilinear in the auxiliary fields. The solid lines denote the fermion propagators of the scalar multiplet.
- Fig. 4 Selfenergy diagrams of the auxiliary fields with one matter loop for the scalar multiplet. The dashed and solid lines denote the scalar and spinor propagators, respectively.
- Fig. 5 Vertex diagrams of the auxiliary fields with one matter loop for the scalar multiplet. The notation is as in Fig. 4.
- Fig. 6 Tadpole diagrams of the scalar auxiliary field with one matter loop for the scalar multiplet. The notation is as in Fig. 4.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 6