Unitarity implies that the evaporation of microscopic quasiclassical black holes cannot be universal in different particle species. This creates a puzzle, since it conflicts with the thermal nature of quasiclassical black holes, according to which all of the species should see the same horizon and be produced with the same Hawking temperatures. We resolve this puzzle by showing that for the microscopic black holes, on top of the usual quantum evaporation time, there is a new time scale which characterizes a purely classical process during which the black hole loses the ability to differentiate among the species and becomes democratic. We demonstrate this phenomenon in a well-understood framework of large extra dimensions, with a number of parallel branes. An initially nondemocratic black hole is the one localized on one of the branes, with its high-dimensional Schwarzschild radius being much shorter than the interbrane distance. Such a black hole seemingly cannot evaporate into the species localized on the other branes that are beyond its reach. We demonstrate that in reality the system evolves classically in time, in such a way that the black hole accretes the neighboring branes. The end result is a completely democratic static configuration, in which all of the branes share the same black hole and all of the species are produced with the same Hawking temperature. Thus, just like their macroscopic counterparts, the microscopic black holes are universal bridges to the hidden sector physics.

We shall now briefly reproduce the BH evaporation argument. Consider a theory with $N$ light species $\Phi_j$, where $j = 1, 2, \ldots, N$ is the species label. Let us show that the gravity cutoff in such a theory cannot exceed (1). We can prove this by showing that the opposite assumption will inevitably lead us to a contradiction. Thus, let us assume the opposite, that the cutoff $M_*$ is much above the scale $M_p/\sqrt{N}$. Then, at distances $\gg l_*$, gravity, by default, must be in the classical Einsteinian regime. In particular, a BH of an intermediate size

$$l_* \ll r_g \leq \sqrt{N}/M_p$$

is also in a quasiclassical regime. The quasiclassicality implies that such a BH should evaporate on the time scale much longer than its size $r_g$. If this condition is not satisfied, the rate of the temperature change exceeds the temperature square, which would imply that such a BH cannot be considered as a thermal state with a well-defined Hawking temperature $T$ and, thus, cannot be quasiclassical. For quasiclassicality of any given BH, the necessary condition is

$$\frac{dT}{dt} \ll T^2.$$  

But this condition is impossible to satisfy for any Schwarzschild BH of size $\leq \sqrt{N}/M_p$. Indeed, if the black hole were quasiclassical and Einsteinian, its half-evaporation time would be

$$\tau_{\text{BH}} \sim r_g^3 M_p^2/N \leq r_g,$$  

I. NONDEMOCRACY PUZZLE

By now, it is understood [1,2] that from the consistency of the large-distance black hole (BH) physics it follows that, in an effective field theory coupled to Einsein gravity, there is the following ultimate connection between the gravity cutoff $M_*$ and the number $N$ of the particle species below it:

$$M_* = M_p/\sqrt{N}. \quad (1)$$

The physical meaning of the parameter $M_*$ is similar to that of $M_p$ in ordinary gravity with $N \sim 1$. It marks the fundamental scale at which gravity becomes strong. In particular, it sets the lower bound on the size and the mass of a black hole that can be treated quasiclassically. For example, any sufficiently compact source of mass $M \gg M_*$ represents a quasiclassical BH with a generalized Schwarzschild radius $r_g \geq l_* \equiv M_*^{-1}$.

Perhaps the simplest proof of relation (1) comes from the consistency of BH evaporation [1,2], but exactly the same bound follows from a number of other BH-related fully nonperturbative arguments, such as the BH entanglement entropy [3] and the BH quantum information considerations in the presence of species [4]. Another indication comes from the perturbative renormalization of the gravitational coupling [2,5,6]. In our analysis, however, we shall focus only on the former exact nonperturbative treatments, which are immune against the artifacts of the perturbation theory, such as, e.g., cancellations among the different contributions.
where the last inequality is obtained by taking into account (2). Thus, the black hole has a half-lifetime of the order of or smaller than its inverse temperature. The quasiclassicality condition (3) is inevitably violated, which is a clear indication that such a black hole cannot be regarded as a quasiclassical state with a well-defined temperature. Thus, we are lead into the contradiction with our initial assumption that the black holes of size \( r_g \ll \sqrt{N}/M_P \) are normal Schwarzschild black holes. The only resolution of this inconsistency is that the gravity cutoff is (1). Notice that this conclusion is absolutely insensitive to what happens to the black hole at the later stage. It is unimportant whether the black hole evaporates completely or whether some new physics sets in and stabilizes the black hole at some size \( \ll l_s \).

A well known example in which relation (1) is manifest from the fundamental physics is the framework of large extra dimensions. If we consider a \( 4 + n \)-dimensional theory with a fundamental Planck mass \( M_\ast \) and \( n \) extra dimensions compactified on a certain manifold, relation (1) simply translates as the familiar relation between the \( n + 4 \)-dimensional and four-dimensional Planck masses

\[
M_p^2 = M_\ast^2 V_n, \tag{5}
\]

where \( V_n = N_{KK} \) is the volume of the extra space measured in \( l_s \) units. Obviously, this quantity counts the effective number of four-dimensional Kaluza-Klein (KK) species with masses \( \ll M_\ast \).

For example, consider a \( 4 + n \)-dimensional theory with \( n \) space dimensions compactified on an \( n \) torus and \( 4 \) non-compact dimensions forming the Minkowskian geometry. Without affecting any of our results, for simplicity, we shall set all of the compactification radii equal to \( R \gg l_s \). The fundamental high-dimensional theory has one dimensionful parameter, the \( 4 + n \)-dimensional Planck mass \( M_\ast \), which is the cutoff of the theory. Assume that the only species in the theory is a \( 4 + n \)-dimensional graviton. From the four-dimensional point of view, the same theory is a theory of the tower of spin-2 KK species. For any \( R \), the relation between the four-dimensional Planck mass and the cutoff of the theory is [7]

\[
M_p^2 = M_\ast^2 (RM_\ast)^n. \tag{6}
\]

The key fact establishing the connection between (1) and (6) is that the factor \( (RM_\ast)^n \) measures the number of KK species:

\[
N_{KK} = (RM_\ast)^n. \tag{7}
\]

Thus, relation (6) is a particular example of relation (1) in which \( N \) has to be understood as the number of KK species.

As pointed out in Ref. [8], the fact that in \( N \)-species theory the gravity cutoff is much below \( M_P \) inevitably implies the existence of a second length scale \( \geq l_s \), which marks the distance at which gravity starts departing from the Einsteinian regime. We shall denote this scale by \( \mathcal{R} \).

In the interval \( l_s \ll r \ll \mathcal{R} \), gravity is still in a weakly coupled classical regime, but it is non-Einsteinian (see Fig. 1). We define non-Einsteinianity in the obvious sense that at such distances gravitational interaction is no longer mediated by a single massless spin-2 state but also by some new degrees of freedom. As a result, in general, neither the gravitational potentials nor the BH parameters are obliged to obey the usual Einsteinian or Newtonian laws.

Correspondingly, the black holes with \( r_g \) in this interval are still classical but non-Einsteinian. In the large extra-dimensional example, the length scale \( \mathcal{R} \) has an obvious meaning; it is the same as the radius of extra dimensions \( \mathcal{R} = \mathcal{R} \), beyond which the classical gravity becomes high-dimensional. Obviously, the BHs of the intermediate size \( l_s \ll r \ll \mathcal{R} \) are still classical but high-dimensional. In particular, the gravitational radius of such a BH is related to its mass \( (M) \) in the following way:

\[
r_g \approx l_s (M_\ast)_{1/n+1}. \tag{8}
\]

Notice that in certain \( N \)-species theories the two scales \( \mathcal{R} \) and \( l_s \) can be close to each other, but this proximity is an illusion. To understand this, it is useful to go into the energy-momentum space instead. Even if the length scales are close to each other, the mass interval in which the BHs are in non-Einsteinian regime is nevertheless huge:

\[
M_\ast \ll M_{\text{non-Einstein}} \ll \mathcal{R} M_p^2. \tag{9}
\]

The lower bound of this interval is obvious, and the upper bound comes from the fact that, for the Einsteinian BHs of gravitational radius \( r_g = \mathcal{R} \), the mass and the size are related through the usual Schwarzschild formula. As an illustrative example, consider a Kaluza-Klein theory with \( n \gg 1 \) extra dimensions. Obviously, in the large \( n \) limit, the compactification radius approaches \( l_s \) from above. Nevertheless, the volume of extra space stays constant in units of \( l_s \). A high-dimensional BH becomes Einsteinian after its \( 4 + n \)-dimensional gravitational radius (8) becomes equal to the four-dimensional gravitational radius of a usual Schwarzschild BH with the same mass. This happens when \( r_g = \mathcal{R} \), that is, only after the BH fills up the whole high-dimensional volume, and thus it has to be very massive.

- **Quantum regime**
- **Quasiclassical Non-Einsteinian regime**
- **Einsteinian regime**

\[
l_s = \frac{\sqrt{N}}{M_P}, \quad \mathcal{R}
\]

FIG. 1. Regimes of gravity in the presence of \( N \) species. For the particular case of large extra dimensions \( N = N_{KK} \), the gravitational cutoff is the higher-dimensional Planck length \( l_s = M_c^{-1} \), and the scale at which the first deviations from Einsteinian gravity show up is the compactification radius \( \mathcal{R} \).
The existence of the two critical length scales \( l_* \) and \( R \) in the light of unitarity and large-distance BH properties leads us to the two powerful conclusions:

1. The evaporation of the quasiclassical microscopic BHs with size \( \ll R \) is nondemocratic in the species. That is, the micro BHs carry “hair” that distinguishes among the different species.

2. The democracy is gradually regained for the larger BHs and becomes complete for \( r_g > R \).

In order to understand the above properties, it is enough to consider a thought experiment with BH production and subsequent evaporation. A neutral BH of mass \( M \geq M_* \) and gravitational radius \( r_g \) can be produced in a particle-antiparticle collision within the given (say, \( j \)th) species, at the center of mass energy \( E \sim M \) and an impact parameter \( R_{\text{imp}} \sim r_g \). At the threshold of the smallest BH creation, the dynamics is governed by a single mass scale \( E \sim R_{\text{imp}} \sim M_* \), and the production rate obviously is \( \Gamma_{j-BH} \sim M_* \). By unitarity, the rate of BH decay into a pair of the same \( j \)th species should be similar:

\[
\Gamma_{\text{BH} \rightarrow j} \sim M_* . \tag{10}
\]

But then, by the same unitarity, it is impossible for the BH to have the same partial decay rates into the other species individually. Since the total rate of decay into all of the other species cannot exceed \( \sim M_* \),

\[
\sum_{i \neq j} \Gamma_{\text{BH} \rightarrow i} \leq M_* , \tag{11}
\]

the decay rate into most of the other individual species must be suppressed by a \( \sim 1/N \) factor. Thus, the decay of the microscopic BH cannot be democratic, and the original BH has to carry some information that allows one to distinguish among the different species. Since the BH by construction was neutral, this information cannot correspond to any conserved charge measurable at infinity. In other words, we are led to the conclusion that the micro BHs must carry species hair.

At the same time, the level of nondemocracy is size-dependent. By increasing the gravitational radius gradually, we must recover the complete democracy at \( r_g = R \), since above this size the BHs become Einsteinian.

Both properties stated above acquire a clear geometric meaning in the case of the extra-dimensional example. Indeed, imagine an extra-dimensional theory with \( n \) dimensions compactified on a manifold of radius \( R \gg l_* \) and with two parallel 3-branes placed at some intermediate distance \( d (l_* \ll d \ll R) \) from each other. Both branes are pointlike on the compact \( n \)-dimensional manifold. Assume that there are two different four-dimensional particle species \( \Phi_1 \) and \( \Phi_2 \) localized on the first and the second branes, respectively.

Now we can form a quasiclassical black hole in a collision (or a collapse) of species \( \Phi_1 \) on the first brane. If the size of the BH is \( r_g \ll d \), there is no way for this BH to evaporate into the species \( \Phi_2 \) that are localized on the second brane. Seemingly, this fact is pretty natural from the point of view of the five-dimensional observer and is entirely due to locality in the extra-dimensional space. Yet, from the point of view of a four-dimensional observer, the same effect is rather puzzling. For such an observer, measuring the four-dimensional Hawking flux at large distances, the flux will include only \( \Phi_1 \) species, and this fact would be hard to reconcile with the thermal nature of the quasiclassical BHs.

This puzzle is not limited to the extra-dimensional example only and is generic for BHs in theories with \( N \) species. The nondemocracy of the BH evaporation implies that effectively such BHs have different temperature with respect to the different species, but this contradicts the notion of thermality.

The ability of a BH to distinguish among the different species is also puzzling from the point of view of the BH no-hair theorems [9]. Indeed, a nondemocratic BH can be labeled according to its interaction with species, and, thus, it carries a species hair. However, since in the above thought experiment the BH produced in the collision of particles was neutral by design, its species hair cannot correspond to any conserved quantum number that can be detected from outside. This fact goes in contrast with the no-hair properties of the Einsteinner BHs [9], which can be labeled only by charges that can be detected at infinity, either classically or quantum-mechanically [10].

A quick and naive resolution (or rather a dismissal) of the above puzzle would be to argue that, because the BHs of the size \( r_g \ll R \) are non-Einsteinian, they have no obligation to obey the usual universal thermal rules. Nor they should be subjected to the classical no-hair theorems. However, such reasoning is incorrect, since the thermal nature of BHs is a result of quasiclassicality rather than of Einsteinianity. The former requirement is obviously satisfied in the region of the interest. For example, the high-dimensional BHs that are smaller than the compactification radius, but larger than the fundamental Planck length, are obviously non-Einsteinian, but they are still quasiclassical. Such BHs must be the thermal states. Then, how can such BHs be nondemocratic in species?

It is the purpose of the present work to answer this question. We shall argue that the resolution of the puzzle is in the time dependence of the nondemocratic BHs. In other words, the microscopic BHs can exhibit nondemocracy only for a limited time interval, after which they lose the ability to distinguish among the species and become fully democratic. This democratization is a classical process. In other words, the species hair is time-dependent and falls off within a finite time. At the end of this process, all of the interactions between the BH and the species are universal, and there is a single temperature with respect to all of them. Thus, we conclude that nondemocracy is only a property of classically time-dependent micro BHs, and the
Furthermore, as we will see in Sec. IV B, the time scale for hair is not measured by the disappearance of any charge. The loss of this kind of charge, as they are small enough, is simply the degree of asymmetry in the local charge—it exists even for completely neutral BHs so long as the two. The species hair is not associated with any global or intrinsic characteristic classical time scale on which they lose the species hair and become democratic. Although nondemocracy turns out to be a temporary property, for the microscopic BHs it plays the crucial role in maintaining unitarity. This is because, in the presence of many species, the consistent gravitational dynamics is always such that the BH finds the way to get in contact with all of the existing species within the finite time. The end result is always a fully democratic BH.

For example, in the codimension-1 case, we localize a classical BH away from the brane by attaching it to the latter by a string (flux tube), using the setup of Ref. [12]. By choosing the appropriate string tension, one can balance the gravitational repulsion created by the brane with the attractive force due to the stretched string and stabilize the BH in a neutral equilibrium at an arbitrarily large distance. In this way, one can seemingly isolate the BH from the modes localized on the brane. However, this is not what happens. We show that, in reality, it is energetically favorable for the string to shorten and be replaced by a chunk of the brane itself (see Fig. 2). As a result, the BH is attached to the brane by the deformed brane portion, and all of the brane modes are produced democratically.

The second example is the setup with parallel codimension-2 3-branes in six-dimensional space-time. The transverse metric produced by these objects is very similar to the one produced by the cosmic strings in four dimensions and amounts to a locally flat space with a conical deficit. As long as the deficit angle is less than $2\pi$, the four-dimensional metric produced by these defects is Minkowskian. We consider a BH that initially is localized on one such brane, so that only the modes from the piercing brane see the BH horizon and can be emitted thermally. We show that the end result of the classical evolution of the system is that the BH is pierced through by all of the existing branes symmetrically, and, thus, all of the localized modes see a common BH horizon and temperature. This process is described in Fig. 4.

The results of this paper have phenomenological implications for the microscopic BHs that can be created at LHC, since they indicate that such BHs will have an intrinsic characteristic classical time scale on which they lose the species hair and become democratic. Although nondemocracy turns out to be a temporary property, for the microscopic BHs it plays the crucial role in maintaining unitarity. This is because, in the presence of many species, the consistent gravitational dynamics is always such that the BH finds the way to get in contact with all of the existing species within the finite time. The end result is always a fully democratic BH.

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mass of the BH, the evaporation rate decreases, whereas the democratization process becomes more efficient. The interplay between the two processes is the characteristic property of any micro BH, and this fact has important phenomenological implications for the microscopic BHs that can be produced at LHC.

The democratization phenomenon, discussed in this paper, demonstrates that the universality is not exclusively a property of the Einsteinian classical BHs but is shared also by the microscopic non-Einsteinian ones. Thus, just like their macroscopic counterparts, the micro BHs represent universal bridges to the hidden sector physics.

II. THE PRINCIPLE OF MICRO BLACK HOLE DEMOCRACY

We shall now formulate the principle of micro BH democracy, explicit manifestations of which will be discussed in the following sections.

We shall restrict our considerations to backgrounds that at large distances represent asymptotically flat four-dimensional Minkowski spaces. On such a background, let us consider an arbitrary classically static and stable BH configuration, for which the backreaction from the Hawking radiation is small (i.e., such a BH can be consistently treated as a quasiclassical state). Then, all of the species that can be treated as the pointlike elementary particles, at least up to the distances of the order of the BH size, are produced in the evaporation of the BH at the same temperature. What makes the above formulation more general with respect to the usual universality of the Hawking radiation for macroscopic BHs is the absence of the assumption that gravity is Einsteinian. Instead, what is important is that the BHs are the quasiclassical states, implying that they correspond to stable static solutions of a classical gravity theory operating at the intermediate distances $l_\gamma \ll r_g \ll R$, for which the quantum backreaction is small. This theory by default is not Einsteinian gravity but rather its classical short-distance completion, such as, for example, a large extra-dimensional theory of gravity.

Applying to such completions, the above principle will suggest that any four-dimensional zero mode, with the localization width shorter than the high-dimensional BH gravitational radius, must be produced with the same Hawking temperature. In other words, any static quasiclassical BH must be seen by all of the zero modes in the same way.

This is a pretty strong statement, which naively seems to be easily circumvented due to locality in the extra space. Indeed, it would imply that no static configuration, in which modes are localized away from the position of a high-dimensional BH, is possible. Nevertheless, things are more subtle, as we shall see.

First, let us briefly discuss what happens for micro BHs in a model with large extra dimensions, consisting of a tower of KK gravitons. For small enough BHs, these modes cannot be treated as pointlike elementary particles. The reason is that they are completely delocalized in the bulk, so the evaporation rate into each KK mode is suppressed by the overlap between the BH and the KK wave function, as observed in Ref. [13] (note that the same “form factor” for each KK is also crucial in order to reproduce the correct BH entropy [3]). Hence, the BH evaporation in this case is clearly undemocratic. This is why our principle includes the assumption of elementarity of the species. Nevertheless, it is interesting to point out that the departure from democracy in this case arises as a probabilistic factor that normalizes the overall evaporation rate for each KK, but still all of the modes see the same horizon and so are produced with the same temperature.

We shall now consider some seeming counterexamples to the above principle, in which, at least naively, the locality in high dimensions should prevent the localized modes from being produced in the BH evaporation, and show why what happens is exactly the opposite.

III. CODIMENSION-1 CASE

It has been known for a long time [14,15] that gravity of positive-tension codimension-1 branes (domain walls) is repulsive. So, naively, it seems straightforward to stabilize a classical BH away from the brane at an arbitrarily large distance by balancing the repulsive gravity by some attractive force. One would expect then that in such a case the BH would not be able to evaporate in the modes that are localized on the brane, in contradiction to our democracy principle. We shall now discuss why this naive intuition is false.

The relevant mechanism for stabilizing a classical BH away from the brane was discussed in Ref. [12]. So let us follow this construction. We shall discuss the case of a positive-tension 3-brane, with the energy-momentum tensor $T^I_A = T \text{diag}(1, 1, 1, 1, 0) T > 0$, embedded in five-dimensional space-time. The solution with a locally Minkowskian bulk metric can be written as [15]

$$ds^2 = (1 - |y|\kappa)^2 [dt^2 - e^{2\pi\kappa} dx^2] - dy^2,$$

(12)

where $\kappa = T/(3M_5^2)$ is the gravitational curvature radius of the brane. Because of the decreasing warp factor, the force exerted by the brane on test particles is repulsive in the $y$ direction. However, the four-dimensional induced metric is not Minkowski but rather de Sitter with the Hubble radius set by $\kappa$. In order to create a static metric, one needs a source that would absorb the gravity flux, and such a source is the negative bulk cosmological constant $\Lambda$. By carefully tuning $\Lambda$ versus the brane tension, in such a way that $\kappa$ coincides with the bulk anti-de Sitter (AdS) curvature scale

$$\kappa = \sqrt{|\Lambda|/6M_5^2},$$

(13)
one can create a configuration, with a static 4D Minkowski metric \[ ds^2 = e^{-2|y|\kappa}(dt^2 - dx^2) - dy^2. \] (14)

In such a metric, a BH is repelled from the brane, and it looks like we can create a counterexample, provided we stabilize the BH at some finite distance. We shall now discuss the method of stabilization considered in Ref. [12].

**A. Step one: Compactification**

In order to obey the conditions of our micro BH democracy principle, we first need to obtain the four-dimensional gravity at large distances. We thus need to compactify the extra space, that is, that the y coordinate in (14) has a finite range. However, as it is known [17] that such compactification requires the introduction of a negative-tension brane, say, at \( y = y_0 \). The latter requirement on the tension directly follows from the matching of the warp factor, at the brane location. In Ref. [12], it was noticed that, by increasing \( y_0 \), such a setup allows one to create a quasi-classical BH even for a small BH mass, due to a blueshift of the local Planck length \( l_{\text{local}} = l_0 e^{y_0|\kappa|} \). In order to be in a five-dimensional quasiclassical regime, the BH size must be smaller than the 5D AdS curvature radius \((\kappa^{-1})\) but larger than the local (blueshifted) Planck length:

\[
\kappa^{-1} \gg r_g \gg l_{\text{local}}. \quad (15)
\]

In such a case, it seems that the BH can be easily localized far enough from \( y = 0 \) and cannot emit radiation in the modes localized at the positive-tension brane, in seeming contradiction with our claims. Before declaring victory, however, one has to address a crucial subtlety, which as we shall see will reverse the above naive conclusion.

The negative-tension brane can be considered as an effective description only at large distances. But, in order to fully understand its possible effect on the nearby localized BHs, this large-distance description does not suffice. The negative-tension brane has to be resolved by some short-distance physics, and the BH behavior in its vicinity will crucially depend on the nature of this regulating physics. Currently, the dynamics of such resolving physics is not well understood. Therefore, to be on the safe side in our analysis, we will stabilize the BH far away from the negative-tension brane, in the region that is not influenced by the ultraviolet physics that resolves the latter.

**B. Stabilization by the stretched string**

Reference [12] also suggested an alternative way of stabilizing the BH away from the positive-tension brane, by attaching the BH to the latter by a string. The string in question can either represent a QCD-type electric flux tube or be solitonic. It could even be a fundamental string. The precise short-distance nature is unimportant for our purposes. The explicit realizations of the above setup were discussed in Ref. [12]. For example, the appearance of such a string is automatic whenever the BH carries a charge under a massless gauge field localized on the positive-tension brane [18], and it is an inevitable consequence of charge conservation in 4D. In the latter case, the string in question is an electric flux tube. For our present purpose, this method of the BH stabilization has a clear advantage that we can avoid any encounter with the negative-tension brane physics. Choosing a small \( \kappa \), one can place a negative-tension brane arbitrarily far, while keeping the BH attached to the positive-tension brane at a fixed distance. To obtain the static configuration, we have to balance the repulsive gravitational force acting on the BH by the attractive one induced by the tension of the stretched string. This amounts to the following condition:

\[
\mu = \kappa M_{\text{BH}}. \quad (16)
\]

where \( \mu \) is the string tension and \( M_{\text{BH}} \) is the BH mass. Under the above condition, the BH is in a neutral equilibrium and can be placed at an arbitrary distance away from the brane. Such a configuration, however, is exactly in agreement with the BH democracy, since the latter will evaporate through the modes localized on the brane, because the Hawking radiation from the BH is conducted to the brane through the string. As a result, a 4D brane observer will see the Hawking radiation in the form of the brane modes. Notice that the condition (16) can be rewritten in the following way:

\[
\mu^2 = (r_g^2 \mu) T. \quad (17)
\]

Taking into account that the string and brane widths are \( \ell_{\text{brane}} \sim T^{-1/2} \) and \( \ell_{\text{string}} \sim \mu^{-1/2} \), respectively, we see that when the BH is wider than the string the string tension exceeds the brane tension. So it is energetically favorable for the string to shorten and pull out the stringy chunk of brane (see Fig. 2). As a result, the original string will be replaced by the stretched part of the brane, and the BH becomes attached to the brane by the tube of the stretched brane itself. Hence, all of the brane modes are produced democratically.

In the opposite case, when \( r_g^2 > \mu \), both brane and string are wider than the BH, and the conditions of our principle are automatically violated, since the mode localization width becomes wider than the BH size.

**IV. CODIMENSION-2 CASE**

The second piece of evidence in favor of the principle of micro BH democracy arises from a model with two compact extra dimensions and a number of codimension-2 branes placed at different locations in the extra space. From the 4D perspective, the modes localized in each of the branes are species that interact only through gravity. Naively, it seems that it should be possible to have a stable situation in which a mini BH is pierced by only one of the
branes. Then, by the locality in the extra space, only the modes localized on that brane would be thermally produced by the BH, which from the 4D point of view would look like a BH evaporating into only some of the species. Hence, such a BH would carry a classical species hair.

We shall now see that the classical dynamics drives the branes + BH system to a configuration in which the BH becomes pierced by all of the branes. This transition happens within a finite time. As a result, the modes localized on all of the branes see the same horizon and are emitted with the same temperature. Thus, the species hair is lost classically, in a characteristic time that depends on the model.

We demonstrate this by finding the attractive force between the BH and the codimension-2 3-branes. Because the gravitational interaction of codimension-2 branes is very similar to that of cosmic strings in 4D, from now on we shall simply refer to them as “cosmic strings” (CSs). It is well known [15] that a CS produces no (static) force on test particles. However, this is not the end of the story, since there are nontrivial higher order effects that become important both for a BH and for a point particle, whenever one goes beyond the “test particle” limit.

A. Brane-BH interactions

Let us now find the force between a BH and a codimension-2 brane. It is straightforward to see that the force exerted by the CS on a test particle at linear level in $G_N$ vanishes. This follows from the single graviton exchange amplitude, which in $D$ dimensions is

$$\mathcal{A} \sim G_N \int d^D x T^{iMN} \frac{1}{\Box_D} \left( T_{MN} - \frac{1}{D-2} T^R R_{MN} \right),$$

where $T_{MN}$ and $T^i_{MN}$ are the stress tensors of the codimension-2 brane and of the point particle (or vice versa), respectively. The above amplitude is automatically zero for a codimension-2 brane, because $T^i_{MN}$ has $D-2$ nonzero and equal entries. This fact can also be seen from the exact solution representing a straight CS, which is flat space with a wedge removed [14]. This solution is locally flat and has a zero Newtonian potential.

However, there is a nonzero attractive force at order $G^2_N$. In fact, it is possible to identify two independent contributions to the force at this order. The first was discussed long ago in the context of cosmic strings in 4D [19–22], and, even though it is a $G^2_N$ effect, it does not account for the graviton self-interactions. Rather, this effect arises because the CS imposes nontrivial boundary conditions. We describe this phenomenon in more detail in Sec. IVA 1.

The other $O(G^2_N)$ contribution to the CS-BH force arises by taking into account the space-time curvature induced by the BH (or by the localized particle, the exterior solution of which is a BH metric). This can be most easily extracted by considering a probe CS on a fixed BH background, as was done in Ref. [23] (see also [24]). For completeness, we review this computation in Sec. IVA 2 (see also Appendix B). Let us add now only that one can view this contribution to the force as arising from a two-graviton exchange diagram with one graviton vertex. Hence, this contribution arises from the nonlinear structure of higher-dimensional general relativity (GR). In contrast, the previous contribution has nothing to do with the nonlinearities of GR, as it would be present even in a linear gravity theory.

Finally, in Sec. IVA 3 we further argue that the end result of the BH-CS system must be the configuration in which the BH “eats up” a segment of the CS. This result is supported by the thermodynamical properties of the known exact solutions representing a BH pierced by a CS [25].

1. Interacting with the images

As mentioned above, CSs do not exert any force on point particles in the test limit, in which any gravitational effect of the particle is neglected. A simple way to go beyond this limit is to compute the gravitational field of the point particle on the exact CS background in a quasi-Newtonian approximation, that is, to compute the Newtonian potential due to a point source on a conical space. This simple logic already leads to an $O(G^2_N)$ attractive force between a pointlike source and a CS [19–22] (even though as we argue in Sec. IVA 2, this does not capture all of the contributions to the force in the given order).

The key point is that the Newtonian potential $\phi_N$ due to a point source of mass $m$ in a conical space (i.e., with a conical singularity along the $x$ axis) is equivalent to that produced by the same source plus a few image sources of the same mass, with the number of images depending on the CS tension $\mathcal{T}$. This is a consequence of Gauss’ law: In the conical space with deficit angle $\delta$, the field lines from a particle spread through a volume that is “smaller” by a factor of $(1 - \delta/2\pi)$ as compared to without the CS. Hence, the gravitational field is stronger than for the particle in isolation. The additional contribution is, of course, proportional to $\delta$ for small tensions and can be thought of as due to a number of images distributed around the CS.

This is especially clear for the values of the CS tension for which the deficit angle is of the form

$$\delta = 2\pi \frac{k}{k+1}, \quad k = 1, 2, \ldots.$$  

The resulting conical spaces for $k = 1$ and 3 are represented in Fig. 3. It is clear that $k$ is the number of images. For general values of the tension, the image sources are not so neatly distributed, but still a qualitatively similar picture should hold, with an effective number of images given by

$$\frac{T}{2\pi M^2_e - T},$$

where $M_e$ is the 6D Planck mass.
Now, if \( \phi_N \) takes the same form of the usual potential plus the one due to the images, then for all practical purposes the picture should be as if the images are there, and, in particular, the point source itself should feel the gravitational potential from them. For small tensions, this immediately leads to an attractive force towards the location of the CS

\[
F \sim \frac{1}{M_5^2} \frac{T m^2}{z^2},
\]

where \( z \) is the perpendicular distance between the CS and the particle.

This force is analogous to the one that an electric charge \( Q \) feels in the presence of a conducting plate. The boundary conditions imposed by the plate are such as if there were an image charge on the opposite side of the conductor. Hence, the particle feels a force exerted from the plate, even if the latter were electrically neutral in the absence of the particle. This can also be understood as the fact that the charge polarizes the medium that makes up the plate so that the induced charge interacts with the induced charge distribution on the plate. Since the induced charge itself is proportional to the original charge \( Q \), the resulting force is quadratic in \( Q \). In our CS-massive particle example, one could say that the mass induces a gravitational dipole distribution along the CS, and this is what attracts the particle towards it.

2. **Probe cosmic strings on BH backgrounds**

We can obtain another contribution to the CS-BH force by taking the limit that is opposite to the one considered in the previous section. We shall take a probe brane in an exact BH background. This is a well-defined problem, and it allows us to extract the different contribution to the force.

Let us briefly sketch how this computation is done and quote the result while leaving the details for Appendix B. This method closely follows Ref. [23]. The idea is to compute the energy of the brane configurations where the brane asymptotes to a fixed perpendicular distance \( z \) to the BH. From this, one can extract the potential energy \( V(z) \) that the brane stores for every such configuration. Of course, the total energy stored in each configuration is divergent because the brane has infinite extent, but the energy difference between any two configurations is finite.

For a generic \( p \)-brane in \( 4 + n \) dimensions, one finds that the potential is already nonzero at linear level and behaves as

\[
V(z) \sim -\frac{T m^2}{M_5^{n+2}} \frac{1}{z^{n+2}},
\]

where

\[
q \equiv n + 3 - p
\]

is the codimension of the brane. This leads to a brane-BH force that is attractive for \( q > 2 \), zero for \( q = 2 \), and repulsive for \( q = 1 \), as expected.

For codimension 2, the leading nonzero contribution arises at the next order, and the result is

\[
V(z) \sim -\frac{T m^2}{M_5^{2(n+2)}} \frac{1}{z^{n+1}},
\]

This also leads to an attractive force, which is of the same order as the one due to the images (18).

3. **Full nonlinear problem and thermodynamics**

In general, obtaining the form of the localized BH metric including the brane gravity is a difficult task. However, the codimension-2 case is much more tractable and in fact is the only known example that allows the explicit construction of a localized brane BH [26]. Indeed, this case is equivalent to a BH pierced by a cosmic string, which can be simply obtained by reducing the range of an appropriate angular coordinate. Focusing on our 6D example (and the nonrotating case), this leads to a higher-dimensional version of the Aryal-Ford-Vilenkin metric [25]

\[
ds^2 = -f dr^2 + \frac{dr^2}{f} + r^2[d\theta_1^2 + \sin^2 \theta_1 d\theta_2^2 + \cdots + b^2 \sin^2 \theta_1 \sin^2 \theta_2 \sin^2 \theta_3 d\phi^2],
\]

\[
f(r) = 1 - \frac{r_g^3}{r^3}, \quad b = 1 - \frac{\delta}{2\pi},
\]

where \( \delta = 8\pi G_N^{(6)} T \) is the deficit angle and \( r_g \) depends on the BH mass in the usual 6D fashion without the CS:

\[
r_g^3 = \frac{3}{2\pi^2} G_N^{(6)} m.
\]

Notice that the Newtonian potential of the solution \( f(r) - 1 \) includes the appropriate enhancement from the CS tension, which can be understood in terms of the images produced by the string, as discussed in Sec. IVA 1. Thus, even if the BH mass itself does not increase when it “swallows” a segment of the CS, nevertheless the BH
produces a stronger gravitational field. For $\delta \ll 1$, this is equivalent to an increase in mass $\sim (6/2\pi)m = (8\pi/3)Tr_g^2$, which is twice the mass of a three-dimensional ball of radius $r_g$ and energy density $T$, that is in the segment of CS that the BH naively swallows.

Hence, upon accreting a string of tension $T$, the BH radius increases, by a factor of $b^{-1/3}$. Accordingly, the area of the pierced BH is

$$S_{cs} = bb^{-4/3}S_{\text{isolated}}.$$  

where the first factor comes from the smaller range of one of the angles and the other “four” to the larger radius.

As a result, in the accretion process the area and the entropy increases (at least) by a factor of $b^{-1/3}$. This is another indication that the interaction between the CS and the BH is attractive. In an energy-conserving process, the entropy will grow if the BH accretes the CS but not otherwise.

**B. Democratization as brane accretion**

The CS-BH accretion effect clarifies the puzzle of the micro BH nondemocracy. The accretion process (see Fig. 4), as seen from the point of view of the four-dimensional large-distance BH physics, is the process of democratization. During this process, the BH gradually loses the ability to distinguish among the different species, i.e., loses its hair. As demonstrated above, this is a fully classical process, the end result of which is the static BH which is pierced by all of the strings. Such a BH interacts with all of the four-dimensional modes democratically. All of these modes share the same horizon and correspondingly are emitted with the common Hawking temperature.

Obviously, democratization is fully consistent with the existence of the *temporary* hair that is suggested by the unitarity arguments [2,8]. The democratization time scale is automatically such that unitarity is always maintained in the BH production and evaporation processes.

In the above-studied codimension-2 system, the democratization time is the same as the accretion time, which for a given BH-CS system is

$$t_d \sim z_0 \left( \frac{z_0}{r_g^2 \delta} \right)^{1/2},$$

where $z_0$ is the initial BH-CS distance.\(^2\)

On the other hand, the evaporation time of a six-dimensional BH localized on a given brane and evaporating into a single species is

\[^2\text{The extension of this estimate to the branes of codimensions } q \text{ higher than 2 is straightforward. The interaction potential in that case is nontrivial already at the linear level and is given by Eq. (19). It follows from this potential that the democratization time scale is } t_d \sim (z_0^2/(TG_{k+\delta}))^{1/2}. \	ext{Note that, to this order, it is independent of the BH mass.}\]
time-dependent and BHs are free to enjoy the nondemocracy, without conflicting with the notion of universality of the Hawking radiation.

Our resolution of the micro BH nondemocracy puzzle is not limited to the extra-dimensional theories but should apply to a general class of theories with multiple species. The net result then is that, in any theory with \( N \) light species, the micro BHs have the two characteristic time scales. The first one, \( t_{\text{BH}} \), is the usual quantum evaporation time, due to Hawking radiation. The second time scale \( t_j \) is a characteristic time of a purely classical process of the BH democratization, during which the BH loses its ability to differentiate between the different species and all of the species start seeing the same BH horizon.

V. IMPLICATION FOR THE ACCELERATOR
EXPERIMENTS

In any theory which includes Einsteinian gravity as its low energy limit, the ultimate consequence of the elementary particle collisions, at sufficiently high center of mass energies and a sufficiently small impact parameter, is the production of BHs. The threshold where the BH formation starts happening depends on the parameter \( M_* \), the scale where gravity is getting strong. The interesting physical property of the large extra-dimensional framework is that this scale can be reachable in the near-future collider experiments. In such a case, the lightest accessible BHs will be essentially quantum [27]. However, production of the larger quasiclassical BHs is also possible [12,28].

However, as we know by now, the primary suppressor of the strong gravity scale is not a geometry but the number of species \( N \). So similar properties should be shared by the BHs in all of the theories with sufficiently large \( N \), irrespectively of their underlying geometric origin.

Thus, the general results of our paper are applicable not just to extra-dimensional theories but to a more general class with particle species. Properties of the microscopic BHs that we have uncovered have direct observational consequences and have to be taken into account in their experimental search. In particular, our results indicate the following picture.

First, in agreement with the previous work, the quasiclassical BHs produced in the collision of elementary particles will start out with the full memory of their “parent” species. However, in time they start losing this memory classically and become democratic over a certain time scale. This classical democratization process will compete with the quantum Hawking evaporation, and the outcome depends on the details of the underlying theory. In any case, a typical signature [8] is the correlation between the softening of the amplitudes characteristic to BH production (see [29] and references therein) and the democracy of the evaporation products.

For example, in an extra-dimensional scenario, in which other species are localized on the nearby branes, the democratization rate can exceed the evaporation rate, and the latter process can quickly become universal in all of the species. Of course, the whole transition will proceed in an unitary way. Monitoring the level of BH democracy and the evaporation rate can be an interesting observational bridge between the standard model particles and the hidden sector species.

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APPENDIX A: MORE ON LARGE EXTRA
DIMENSIONS WITH \( N \) BRANES

Let us describe in more detail the models with large extra dimensions and a number of branes in the bulk. For the moment, we will keep full generality and assume that the bulk is \( 4 + n \)-dimensional. As mentioned before, we shall add \( N \) 3-branes, and we populate them with the localized modes. Among these modes there are the fluctuations of the brane positions. These already give \( nN \) light species in the 4D effective theory. In each brane we have \( n \) localized modes corresponding to Goldstone bosons of \( n \) independent translations that are spontaneously broken by the brane. In the approximation of noninteracting branes, these would be exactly massless. Brane interactions are expected to generate masses to \( Nn - n \) of them. These masses will be below the cutoff, so all of the modes will count in our argument. Thus, having only these modes is enough to illustrate our point. The Hawking radiation into these modes translates into a thermal excitation of the transverse displacement of the brane location, which occurs when it crosses the BH horizon [30].

The \( N \) branes can be modeled as topological defects, made out of some higher-dimensional fields. For example, in the 6D case, we could model them as Abelian-Higgs vortices, in which case one needs a complex scalar and a vector, but these details are not going to be important in our discussion.

In order to have these light modes interacting through gravity only, we shall assume that branes are made out of independent fields (one per brane) that couple to each other through gravity. In this case, we need of the order of \( N \) independent \( 4 + n \)-dimensional fields. For simplicity, we can assume that the bulk theory consists of \( N \) copies of the field theory that supports a brane as a smooth solution. This immediately tells us that the actual gravitational cutoff \( \Lambda_* \) of the \( 4 + n \)-dimensional theory is [2]

\[
\Lambda_*^{2+n} \leq \frac{M_*^{2+n}}{N}.
\]  

(A1)
Note that the relation between the 4D Planck mass and the higher-dimensional one,
\[ M_4^p = R^n M_4^{2+n}, \]
is not changed by the presence of the branes, since it relies only on the Gauss law. However, in our setup, the actual number of KK modes (that is, the number of KK modes below the cutoff) is
\[ N_{KK} = (\Lambda_s R)^n, \]
which is a factor of \( N^{-(n/(n+2))} \) smaller than without the branes.

Note that Eq. (A2) can be written as
\[ M_4^p \geq N N_{KK} \Lambda_s^2, \]
which indicates that the total number of degrees of freedom is \( N_{total} = N N_{KK} \), as is indeed the case since each of the \( 4+n \)-dimensional fields gives rise to \( N_{KK} \) KK modes. Note that this counting of modes is quite independent of the actual value of the masses of the 6D fields, as long as they are not too close to \( \Lambda_s \).

The main point concerning the micro BH democratization transition discussed above is unrelated to the hierarchy problem. However, for completeness in this appendix, we shall also consider the possibility that the higher-dimensional model solves the hierarchy problem. This is accomplished by setting the cutoff \( \Lambda_s \approx \text{TeV} \). It is apparent from (A4) that, in this case, then the number of modes in each KK tower is
\[ N_{KK} \leq \frac{10^{32}}{N}. \]

1. Crowded extra dimensions

In this section, we describe the “crowded” limit of the models with large extra dimensions [7] and \( N \) branes, with the maximal possible values of \( N \). Let us start by obtaining the constraints arising from including the branes as (solitonic) dynamical objects. Generically, one can characterize the 3-branes by their tension
\[ T = \mu^4 \]
and their thickness \( \varepsilon \). Typically, in weakly coupled field theories, the tension scale \( \mu \) is larger than \( 1/\varepsilon \), but to simplify matters we will assume that they are about the same, \( \varepsilon \approx 1/\mu \). Hence, one obtains the thinnest possible branes for \( \mu \approx \Lambda_s \). So the maximum number of branes that one can fit in the bulk is
\[ N_{max} \approx (R \Lambda_s)^n = N_{KK}. \]

Hence, for a maximally populated bulk, there are as many branes as KK modes in the tower.\(^3\)

As we already emphasized, requiring that the model solves the hierarchy problem is orthogonal to the main point of this paper. However, let us briefly consider this possibility. First of all, the above arguments mean that it suffices to have of the order of \( 10^{16} \) high-dimensional species in order to solve the hierarchy problem. Of course, this is completely equivalent to having \( 10^{32} \) 4D species, because each high-dimensional field brings in a tower of \( 10^{48} \) modes.

Let us also point out that the phenomenological constraints on models with \( N \) bulk species are automatically milder as compared to large extra-dimensional models without them. The reason is that the compactification radius must be smaller as compared to the \( N = 1 \) case. Combining (A2) and (A3), one finds
\[ R'' = \frac{M_4^p}{M_4^p \mu^2} \leq \frac{1}{N} \frac{M_4^p}{\Lambda_s^{n+2}}. \]

By setting \( \Lambda_s \approx \text{TeV} \) in order to solve the hierarchy problem, this gives a compactification radius smaller than in the scenario without the \( N \) branes by a factor of \( N^{-n} \).

This is especially interesting for the \( n = 1 \) case, since with the maximal brane population this kind of (unwarped) model can be rescued. Indeed, plugging in the numbers, one finds that \( R \) must be millimetric or less in this case. Of course, a more detailed study of the phenomenology is required to see the viability of this case.

2. Micro black holes

In general, by “micro” BHs we mean non-Schwarzschildian BHs that are quasiclassical solutions of the theory of gravity that operates at short distances \( r < \mathcal{R} \). Their masses are from around the cutoff \( \Lambda_s \) to some scale at least of the order of \( \sqrt{N_{total}} M_P \) [8]. In the present theory, there are two distinct kinds of micro BHs. The BHs with masses close enough to \( \Lambda_s \) cannot behave as semiclassical \( 4+n \) BHs, because in the high-dimensional theory there is a large number of species as well, and the cutoff correspondingly gets lowered as compared to the high-dimensional Planck mass. However, for large enough masses of the BHs, eventually a semiclassical high-dimensional regime sets in.

Arguing as in Ref. [8], the threshold where the smallest BHs will start behaving as quasiclassical high-dimensional BHs is found by imposing that the \( (4+n) \)-dimensional relation between the mass and radius \( \mathcal{M} = M_4^{n+2} \mathcal{R}_n^{n+1} \)

\(^3\)In principle, there are other constraints on the total number of branes (or their tension) coming from the gravitational effect of the branes. In the 6D case, for example, this amounts to restricting the total tension to be less than critical: \( N \mu^4 < M_4^p \) (so that the total deficit angle is less than \( 2\pi \)). It is clear that this only demands \( \mu \approx \Lambda_s \), so at least in this case it does not lead to any further constraint.
holds. Here $\mathcal{R}_*$ denotes the BH radius for which the high-dimensional quasiclassical regime starts. The lowest possible bound on $\mathcal{R}_*$ is $1/\Lambda_*$, so the smallest possible window where the BHs are not even semiclassical $(4+n)$ BHs is

$$\Lambda_* \leq M_{\text{non-(4+n)Schwarzschild}} \leq N \Lambda_*.$$  

On the other hand, the threshold of having the quasiclassical 4D BHs is found by setting that the 4D mass-radius relation holds for the BH horizon being equal to the classical 4D BHs is

$$M^2 \mathcal{R} \leq N^{-1/n} \left( \frac{M_p}{\Lambda_*} \right)^{(n+2)/n} M_p \approx N^{-1/n} 10^{16(1+(2/n))} M_p.$$  

where we used (A1) and (A7), and in the last equation we assumed that $\Lambda_* \approx \text{TeV}$. Note that (as it happens in the large extra-dimensional model without branes) this is a factor of $N^{1/\Lambda_{KK}}$ above the absolute lower bound for this scale: $N^{1/\Lambda_{total}} M_p$ [8].

Hence, the window where we have an approximately $(4+n)$-Schwarzschild BHs is

$$M_* \sim N^{1/(n+2)} \Lambda_*$$

$\Lambda_* \sim \text{TeV}$

$\mathcal{R}$

FIG. 5. Schematic distribution of the relevant scales in the large volume compactifications with $N$ branes. For small enough masses, there are two types of non-Schwarzschildian BHs: For moderate masses, they are well approximated by $(4+n)$-dimensional BHs. For smaller masses, the BHs are not even $(4+n)$-dimensional Schwarzschildian.

The resulting set of scales is arranged as shown in Fig. 5. Notice that, in the limit $N = 1$, we recover the usual large extra-dimensional picture, where the high-dimensional BH window is $M_* \leq M_{(4+n)\text{Schw}} \leq N^{(1/n)+(1/2)} M_p$.

Notice as well that, for the maximal number of branes allowed $N \sim 10^{16}$, the mass of the heaviest non-$(4+n)$-Schwarzschildian BH reaches out to precisely $M_p$. However, there is always a higher-dimensional window.

APPENDIX B: BRANE-BH EFFECTIVE POTENTIAL

In this appendix, we derive the force between a BH and a test brane and show that it takes the form advanced in Sec. IVA.2. For the sake of generality, we shall consider Nambu-Goto branes of arbitrary dimension in a generic $(4+n)$-dimensional bulk space-time. This discussion follows Refs. [23,24].

Let us start with a test 3-brane in the background given by a $(4+n)$-dimensional Schwarzschild BH. To better visualize the brane embedding in the BH background, it is convenient to use the so-called isotropic coordinates

$$ds_4^2 = -f^2(R) dt^2 + h^2(R) [d\mathbf{x}^2 + dz^2],$$

where $\mathbf{x}$ and $z$ are “Cartesian” three-dimensional and $n$-dimensional coordinates, respectively,

$$R^2 = r^2 + z^2, \quad r^2 = x^2, \quad z^2 = \bar{z}^2$$

and

$$f(R) = 1 - \frac{1}{3} \left( \frac{r_g^2}{R} \right)^{n+1},$$

$$h(R) = \left( 1 + \frac{1}{3} \frac{r_g}{R} \right)^{n+1},$$

with $r_g$ the usual Schwarzschild radius

$$r_g^{n+1} = \frac{16 \pi G_{4+n}}{(n+2) \Omega_{n+2}} M_{\text{BH}}$$

and $\Omega_{n+2}$ the volume of an unit $(n+2)$-dimensional sphere.

The brane embedding in this space can be parameterized as $\bar{z} = \tilde{z}(x^\mu)$, with $x^\mu = \{t, \mathbf{x}\}$ the coordinates along the brane. This leads to the following induced 4D geometry:

$$ds_4^2 = ds_{(0)}^2 + h^2(\partial_\mu \bar{z} dx^\mu)^2, \quad \text{with}$$

$$ds_{(0)}^2 = -f^2 dt^2 + h^2 d\mathbf{x}^2,$$

and here $f$ and $h$ evaluated at $R = \sqrt{r^2 + z^2(x^\mu)}$.

In the probe approximation, the brane dynamics is described by the Nambu-Goto action

$$Na_* \leq M_{(4+n)\text{Schwarzschild}} \leq 10^{16} \left( \frac{10^{16}}{N} \right)^{1/n} M_p.$$
\[ S^{NG} = -T \int d^4x \sqrt{-g_4} \]
\[ = -T \int d^4x f \sqrt{1 - h^2 g^{\mu \nu}_{(0)} \partial_\mu \tilde{z} \cdot \partial_\nu \tilde{z}}, \]
where as usual \( T \) is the tension. The generalization to a \( p + 1 \)-dimensional brane is straightforward:
\[ S^{NG}_p = -T \int d^{p+1}x f h^p \sqrt{1 - h^2 g^{\mu \nu}_{(0)} \partial_\mu \tilde{z} \cdot \partial_\nu \tilde{z}}, \]  
(B1)
where the number of \( \tilde{z} \) coordinates equals the codimension \( q \equiv n + 3 - p \).

The energy of static configurations is
\[ E[\tilde{z}(x)] = T \int d^p x (1 - \psi)(1 + \psi)^{(p+2-q)/(p-2+q)} \times \sqrt{1 + \Delta^2 \tilde{z} \cdot \partial \tilde{z}}, \]  
(B2)
where we introduced the notation
\[ \psi \equiv \frac{1}{4} \frac{r_0^{p+1}}{(r^2 + z^2)^{(n+1)/2}}. \]

Far enough from the BH, the gravitational potential is small, \( \psi \ll 1 \), everywhere on the brane, and the gradients of \( \tilde{z} \) are also small. Up to a (divergent but irrelevant) constant, the energy of the brane configurations is
\[ E[\tilde{z}(x)] \approx T \int d^p x \left( -2 \frac{q-2}{p+q-2} \psi + \frac{1}{2} \tilde{z} \cdot \Delta \tilde{z} \right) + \text{const}, \]  
(B3)
where higher order terms in the (high-dimensional) Newton constant \( G_{4+n} \) are neglected. For codimension 2, the potential energy term in (B3) vanishes at linear order \( G_{4+n} \). This is, of course, a manifestation of the fact that codimension-2 branes do not either attract or repel the test particles. In this case, we have to include in the expansion of the energy the next order terms. Thus, for codimension 2 we have
\[ E_{\text{cod}2}[\tilde{z}(x)] \approx T \int d^p x \left( -\psi^2 - \frac{1}{2} \tilde{z} \cdot \Delta \tilde{z} \right) + \text{const}. \]  
(B4)

Recall that we are interested in the potential energy of the brane configurations that are asymptotically separated from the “equatorial” plane of the BH by a certain distance \( Z \). As done in Ref. [23], one way to obtain this is the following: We first solve for the profile of the brane with the boundary condition that \( \tilde{z} \) approaches a constant \( (Z) \) at infinity \( \left( r \to \infty \right) \) and then evaluate the energy of these configurations.

The first step requires one to extremize the functional (B4). Denoting by \( z \) the nonzero component of \( \tilde{z} \), the extremization leads to \( \Delta z = v_z \), with \( v(r, z) \) the potential appearing in (B3) and (B4). Hence, the energy of the minimum energy configurations is of the form
\[ T \int d^p x [v - (1/2) \tilde{z} v_z], \]
with \( v \) and \( v_z \) evaluated on the profile \( z(r) \). However, if we are interested in the potential to leading order in \( G_{4+n} \), it suffices to replace the actual profile by the constant \( Z \) to which \( z(r) \) asymptotes. Thus, for \( q \neq 2 \) the leading order potential is
\[ E(Z) \approx -\frac{(q \Gamma(p/2)\Gamma(q/2)}{8 \Gamma((p+q)/2)} \Omega_{p-1} T r_0^{p+1} Z^{p+2}. \]  
(B5)
Equation (B5) encodes the well known gravitational effect of branes of different codimension: For codimension 3 or more, there is an attractive force and for codimension 1 it is repulsive, whereas for codimension 2 there is no force.

Hence, what sets the sign of the force for \( q = 2 \) is the next term, of order \( G_{4+n} \). One obtains
\[ E_{\text{cod}2}(Z) \approx -\frac{(\Gamma(p/2)\Gamma(2+p/2))}{16 \Gamma(p+1)} \Omega_{p-1} T r_0^{2(p+1)} Z^p, \]  
(B6)
which gives an attraction. These results reproduce the behavior quoted in Sec. IV A 2.


[27] See the second reference in [7].

