SCATTERING AND PRODUCTION AT HIGH ENERGIES

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A summary is given of some of the developments in the elastic scattering and production processes at high energies. First, Professor Yang's geometrical model of hadronic and nuclear collisions is reviewed. When there was some preliminary experimental evidence that the total cross section for proton-proton collision might not approach a constant at very high energies, theoretical studies were initiated to determine the high-energy asymptotic behavior in relativistic quantum gauge theory. With this starting point, the successive stages of development are:

- I. Theoretical prediction of increasing total cross sections;
- II. Development of phenomenology and quantitative predictions that were verified experimentally afterwards; and
- III. Theory and phenomenology of production processes, especially that of the Higgs particle, at the Large Hadron Collider.

Since the Large Hadron Collider (LHC) is still being built, the last topic is at an early stage of development, and unexpected results may be forthcoming.

Keywords: Scattering, production, high energy, cross section, Yang-Mills non-Abelian gauge theory, phenomenology, Large Hadron Collider, Higgs particle

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I. INTRODUCTION

Professor Yang has opened up numerous important fields in physics. The best known one is [1, 2]:

Yang–Mills non-Abelian gauge theory.

This is the basis of our present understanding of just about everything in particle physics, including both the strong and electroweak interactions.

In [2], the authors stated explicitly that they had “not been able to conclude anything about the mass of the b quanta.” This turned out to be prophetic: now we know that the Yang–Mills particle (the gluon) for the strong interactions is massless, while those (the W and Z) for the electroweak interactions are massive.

During the Symposium in 1999 on the occasion of Professor Yang’s retirement from the State University of New York at Stony Brook, I gave a talk on the Yang–Mills theory [3]. On this happy occasion of the 85th birthday celebration of Professor Yang, I would like to discuss another field opened up by Professor Yang:

High-energy hadron–hadron collisions.

II. GEOMETRICAL MODEL OF HADRONIC COLLISIONS

Starting in 1965, Professor Yang has published over twenty papers on this topic. Some of his collaborators are: J. Benecke, N. Byers, A. W. Chao, T. T. Chou, T. T. Wu, and E. Yen.

The first paper [4], published in 1965, is actually on high-energy large momentum transfer processes. The starting points for this paper are the following experimental facts:

- (i) The total proton–proton cross section remains essentially constant at high energies;
- (ii) above 300 MeV of excitation energy the nucleon has many excited states; and
- (iii) the large-angle elastic proton–proton cross section drops down spectacularly with energy.

These facts suggest picturing the nucleon as an extended object, and that the difficulty in making large momentum transfers is due to the *difficulty in accelerating the various parts of the nucleon without breaking it up*.

Shortly thereafter, Professor Yang realized that this picture of the nucleon should have a small-momentum aspect in addition to the large-momentum aspect emphasized in this paper. For this reason, Professor Yang has considered Ref. [4] to be a precursor which launched a geometrical model of hadronic and nuclear collisions, p. 60 of Ref. [5].

This geometrical picture by Professor Yang has led to numerous predictions. In particular, it has been found [6] that, on the basis of this geometrical picture, for very-high-energy collisions the lab system (L) and the projectile system (P, where the incoming projectile is at rest) are to be preferred over the center-of-mass system, because in these systems, some of the outgoing particles approach limiting distributions.

As an example, the following limit exists:

$$\lim_{E \rightarrow \infty} \left(\begin{array}{l} \text{partial cross section that a particle of mass } m_1 \text{ and momentum } \\ \vec{p}_1, \text{ and a particle of mass } m_2 \text{ and momentum } \vec{p}_2 \text{ are emitted,} \\ \text{together with any number of other particles.} \end{array} \right) \\ = \rho_2(\vec{p}_1, \vec{p}_2) d^3 p_1 d^3 p_2, \quad (1)$$

where E is the center-of-mass energy, and $\rho_2 > 0$.

In particular, for the total cross section and elastic differential cross section,

$$\sigma_0 = \lim_{E \rightarrow \infty} \sigma_{\text{tot}} \quad (2)$$

and

$$\Sigma(t) = \lim_{E \rightarrow \infty} \left. \frac{d\sigma}{dt} \right|_{\text{elastic}}, \quad (3)$$

both exist in the limit of very high energies.

III. DO THE LIMITING VALUES EXIST?

Let us consider Eqs. (2) and (3) in more detail. Of these two, Eq. (2) is more important. Because the total cross section is the sum of the contributions from all the sub-processes, this total cross section is one of the best candidates for having a limiting value. Furthermore, Eq. (2) is closely related to the special case $t = 0$ of Eq. (3).

Shortly after Professor Yang developed the geometrical picture of hadron-hadron collisions, the question was asked: What are the experimental evidences for or against the existence of a limiting value for the total cross section? At that time, above the center-of-mass energy of about 6 GeV, the measured proton-proton total cross section was barely changing, decreasing slightly. The proton-antiproton total cross section was decreasing more rapidly, but approaching that of proton-proton. The π^+p total cross section is also nearly constant, but that for K^+p is increasing significantly.

In particle physics, it is often useful to question long-held beliefs. Such inquiry is especially likely to lead to progress when there are some, even preliminary, experimental data giving counter-indications to the belief.

Is there any other quantity that is sensitive to the issue whether the total cross sections approach limiting values or not? One of the most promising quantities is the ratio of the

real and the imaginary parts of the forward scattering amplitude. The reasoning is as follows. The forward scattering amplitude is expected to satisfy a dispersion relation. This means that the real part of the forward scattering amplitude can be written as an integral over the imaginary part, which is essentially the total cross section. To avoid questions of normalization, it is usual to consider the ratio

$$\rho = \frac{\text{Real part of } pp \rightarrow pp \text{ in the forward direction}}{\text{Imaginary part of } pp \rightarrow pp \text{ in the forward direction}} \quad (4)$$

in the case of proton-proton collisions.

If the proton-proton total cross section does approach a finite limit, then this ratio ρ must approach zero at high energies. At that time, the experimentally measured values of ρ were negative and increasing toward zero.

However, the rate of increase seemed to be a little too fast, i.e., this ρ seemed to have a tendency to overshoot to become positive. This is a preliminary indication; if this indication is taken seriously, then it is desirable to study whether a high-energy limit does exist for the proton-proton total cross section.

IV. RELATIVISTIC QUANTUM GAUGE THEORY

How could we study such a problem theoretically? Clearly we need a model. How do we choose a model for this purpose?

Interactions between elementary particles share the following basic features:

- relativistic kinematics,
- unitarity, and
- particle production,

in addition to the space-time dimension being $3 + 1$.

Even though each of these three basic features may be considered to be “trivial,” it is nevertheless not easy to have a model with all these features. The simplest way to have these features is to have a relativistic quantum field theory. It remains to make a choice which relativistic quantum theory to study.

As already mentioned in Sec. I, Professor Yang has taught us that all interactions in particle physics are described by gauge theories, Abelian or Yang-Mills (non-Abelian) [1, 2]. Therefore, in order to study the high-energy behavior of various quantities, including especially the total cross sections, we chose the model of four-dimensional relativistic quantum field theory with a gauge invariance of the second kind.

There is no known example of such a theory that is exactly solvable. For this reason, it is necessary to study the perturbation series of such a theory at high energies, renormalized when needed.

On the basis of this model, the problem is to calculate asymptotically, order by order, each term of the perturbation series. Here “asymptotically” means the leading term when the ratio

$$\frac{E}{m} \text{ is large,} \quad (5)$$

where m is the mass of the incident particles, typically that of a proton. After these asymptotic terms are obtained for each order, these leading terms are summed over the order. This procedure will be illustrated in the next section.

The summation of these leading terms, carried out by Cheng and me, [7, 8] turned out to require a rather complicated and lengthy calculation. In the context of the high-energy behavior of the relativistic quantum gauge theory, the argument given at the end of Sec. III above is supplemented by the following observation and question. In any relativistic quantum field theory, there is not only direct channel (s -channel) unitarity, but also cross channel (t -channel) unitarity. How does the cross channel unitarity affect the high-energy behavior of, for example, the total cross section?

V. THEORETICAL PREDICTION OF INCREASING TOTAL CROSS SECTIONS

For definiteness, consider fermion-fermion elastic scattering in a relativistic quantum gauge theory. As a first step in trying to understand the high-energy behavior of such a theory, consider the exchange of Yang-Mills gauge particles directly between the two fermions. The Feynman diagrams for second order, fourth order, and sixth order are shown in Figs. 1(a), 1(b) and 1(c), respectively, while the general diagram is given schematically in Fig. 1(d). Actually, there is another set of diagrams where the two outgoing fermions are exchanged. In each of this second set of diagrams, the Yang-Mills gauge particles have to carry a large momentum, and hence the contribution at high energies is negligible.

For high energies and fixed momentum transfers, the diagrams of Fig. 1 have been evaluated. When the gauge particle is massless, then there is the usual divergence in the total cross section due to the one-“photon” exchange of Fig. 1(a). If the gauge particle is not massless, for example through the Higgs mechanism [9], then this divergence disappears. Such trivial divergences are ignored in the following discussions.

By the way, the name Higgs mechanism should more accurately be called the Englert-Brout-Higgs (or EBH) mechanism, because the paper of Englert and Brout [10] was actually submitted for publication four weeks earlier than that of Higgs [9]. In the rest of this paper, to be fair to Englert and Brout, the term EBH or EBH particle shall be used.

All results from the diagrams of Fig. 1 are in perfect agreement with the geometrical picture of Professor Yang. In particular, the existence of the high-energy limit is verified for σ_{tot} and $(d\sigma/dt)|_{\text{elastic}}$, as given by Eqs. (2) and (3).

In this verification of the geometrical picture of Professor Yang, two of the three basic

FIG. 1: Feynman diagrams for the direct exchange of Yang-Mills gauge particles between two fermions. (a) second order; (b) fourth order; (c) sixth order; (d) general.

FIG. 2: A Feynman diagram for the production of a single Yang-Mills gauge particle.

features listed in Sec. IV on the interaction between elementary particles have played central roles: they are relativistic kinematics and unitarity. The question to be raised at this stage is thus: What is the role of particle production in the total cross section at very high energies?

The high-energy behaviors of a large number of production processes have been investigated. It turned out that the simple diagram of Fig. 2 plays a crucial role; it involves the production of a single Yang-Mills gauge particle. Because of the importance of this process, it may be of interest to discuss it in some detail.

In a high-energy collision of elementary particles, additional particles may be produced with a wide range of energies and momenta. For example, a produced particle may have a relatively low momentum in the projectile system; see Sec. II. At high energies, such a particle necessarily has a high momentum in the so-called lab system, i.e., where the other incoming particle is at rest. Besides the produced particles that have relatively low momenta in the projectile or lab system, there may be others that have relatively high momenta in both these systems; for example, they may have relatively not high momenta in the center-of-mass system. In cosmic ray events, particles satisfying this description have been seen; they are mostly pions and are therefore referred to as pionization products. In this language, the Yang-Mills gauge particle produced as shown in Fig. 2 is a pionization product. It is

FIG. 3: Feynman diagrams for the production of (a) two, (b) three, and more generally (c) n Yang-Mills gauge particles.

known [8] that the integrated cross section for the production process shown in Fig. 2 is proportional to the available rapidity range, which is of the order of $\ln(E/m)$.

This is the first clear evidence of an increasing partial cross section, and hence an increasing total cross section. This does not mean that the total cross section should increase as $\ln(E/m)$; the actual situation is much more interesting.

Since the appearance of one pionization product leads to an extra factor of $\ln(E/m)$ in the integrated cross section, the appearance of two independent pionization products is expected to lead to $[\ln(E/m)]^2$. This is indeed the case, and the relevant diagram is shown in Fig. 3(a). Similarly, the diagram of Fig. 3(b) gives an integrated cross section that increases as $[\ln(E/m)]^3$.

But such an increase is impossible because it violates the bound of Froissart [11] and Martin [12]. Therefore, factors such as $[\ln(E/m)]^3$, or more precisely a positive power of E/m when these various powers of $\ln(E/m)$ from the diagrams of Figs. 2 and 3 are summed, must be regarded as representing a strongly absorptive potential. This shows the importance of particle production, the third basic feature listed at the beginning of Sec. IV.

The incident plane wave is a superposition of waves with the impact distance b ranging from zero to infinity. If b is small, the wave is prone to create pionization products and is then lost to the beam. The resulting physical picture is shown schematically in Fig. 4, where the black core of nearly complete absorption and increasing radius is surrounded by a gray fringe of partial absorption.

FIG. 4: Schematic representation of the appearance of a high-energy particle [$s = E^2$].

VI. PHENOMENOLOGY AND PREDICTIONS

Physics is basically an experimental science. This theoretical result of increasing cross sections *must be followed by quantitative predictions that can be verified or refuted by future experiments.*

In order to obtain such quantitative predictions, it is necessary to develop a phenomenological model which satisfies the following: (a) it must possess the properties obtained from relativistic quantum gauge theory for very, very high energies; and (b) it must be sufficiently realistic that direct comparison with experimental data is possible.

At very high energies, the elastic scattering amplitude is given approximately by the impact-distance representation

$$\mathcal{M}(s, \vec{\Delta}) \sim \frac{is}{2\pi} \int d\vec{x}_\perp e^{-i\vec{\Delta} \cdot \vec{x}_\perp} [1 - e^{-\Omega(s, x_\perp^2)}], \quad (6)$$

where $\vec{\Delta}$ is the momentum transfer and all spin variables have been omitted. In view of the increasing cross sections as discussed in Sec. V, the absorption $\Omega(s, x_\perp^2)$, or more precisely the real part of $\Omega(s, x_\perp^2)$, should increase with increasing energy. A natural and simple choice for this $\Omega(s, x_\perp^2)$ is

$$\Omega(s, x_\perp^2) = \mathcal{S}(s) F(x_\perp^2). \quad (7)$$

As discussed in Sec. IV, relativistic quantum gauge theory is used to arrive at the conclusion that cross sections increase at high energies. It is therefore essential to choose this $\mathcal{S}(s)$ to be crossing symmetric. The particular choice used is

$$\mathcal{S}(s) = \frac{s^c}{(\ln s)^{c'}} + \frac{u^c}{(\ln u)^{c'}}, \quad (8)$$

where u is the third Mandelstam variable and c and c' are two constants to be determined. Note that this $\mathcal{S}(s)$ is complex.

This choice (8) is far from being straightforward and has a rather deep basis. A competing possibility is for the $\mathcal{S}(s)$ of Eq. (7) to be replaced by the Fourier transform of

$$\bar{\mathcal{S}}(s, t) = s^{\alpha(t)}, \quad (9)$$

or its crossing symmetric version. This form (9) is similar to the usual Regge pole, except that the singularity is above 1. In the Regge terminology, (9) is a moving Regge pole while

(8) is a fixed cut. In order to choose between these two possibilities, among others, we appeal once more to relativistic quantum gauge field theory. On the basis of the high-energy behavior of such theory, it has been found that Eq. (8) holds but not Eq. (9), with or without any dependence on t . The underlying reason for Eq. (8) is that such gauge field theory in four dimensions is renormalizable but not super-renormalizable.

For Abelian gauge theory, the value of the c' of Eq. (8) has been found to be

$$c' = \frac{3}{2} \quad (10)$$

to leading order; see Appendix B of Ref. [8]. The corresponding value of c' has not been successfully calculated for Yang-Mills theory yet. It has been conjectured that, to leading order,

$$c' = \frac{3}{4} \quad (11)$$

for SU(3) Yang-Mills non-Abelian gauge field theory—quantum chromodynamics. See below.

We now turn to the choice of the $F(x_\perp^2)$ for phenomenology. In the first realistic phenomenology for pp and $p\bar{p}$ elastic scattering [13], developed in 1978 and published in 1979 in collaboration with Bourrely and Soffer, the choice is that $F(x_\perp^2)$ is the Fourier transform of

$$\tilde{F}(t) = f[G(t)]^2 \frac{a^2 + t}{a^2 - t} \quad (12)$$

with

$$G(t) = \frac{1}{\left(1 - \frac{t}{m_1^2}\right) \left(1 - \frac{t}{m_2^2}\right)}. \quad (13)$$

Thus there are six parameters: c , c' , m_1 , m_2 , a and f ; these are determined by an overall fit using existing data at that time.

Six years later, when there were significantly more experimental data at high energies, Bourrely, Soffer and I repeated the overall fit [14]. The values of these parameters are listed in Table 1. That these values do not change much implies that this phenomenological model is quite robust.

Of these six parameters, perhaps the most interesting one is the second one, c' . That this value, determined by an overall fit of the experimental data, is close to 0.75 is the basis for the conjecture (11).

Since the phenomenological formulas are very simple, numerous computations have been carried out using the 1984 values. For the lower part of the energy range, Regge backgrounds are also included. A few examples are given in the next three figures.

Figure 5 shows a comparison of the experimental data with the result of the phenomenological model for the pp and $p\bar{p}$ total cross sections. This figure is taken from Ref. [14], the

Table 1. Parameters for the Phenomenological Model

Parameter	1978 value [13]	1984 value [14]
c	0.151	0.167
c'	0.756	0.748
m_1 (GeV)	0.619	0.586
m_2 (GeV)	1.587	1.704
a (GeV)	2.257	1.953
f (GeV) ⁻²	8.125	7.115

same paper that gives the 1984 values of the parameter values in Table 1. The difference between the pp and $p\bar{p}$ total cross sections is due entirely to the Regge background. The predicted total pp cross section for the Large Hadron Collider, being built at CERN and to be described in the next section, is

$$\sigma_{\text{tot}} = 103.6 \text{ mb.} \quad (14)$$

This is more than twice the total pp cross section measured at the highest energy in the mid-sixties.

Figure 6 shows a similar comparison for the ratio of the real to the imaginary part of the proton-proton forward scattering amplitude; see the definition (4). In the mid-sixties, only the data below $\sqrt{s} \sim 10$ GeV were available. As seen from Fig. 6, these data points gave a negative value for ρ , but were increasing quite rapidly for increasing $\sqrt{s}$, with a tendency to become positive. This is the preliminary indication discussed in the last two paragraphs of Sec. III.

For the Large Hadron Collider, the predicted value is

$$\rho = 0.122. \quad (15)$$

The phenomenology is in good agreement with proton-proton experimental data, not only for the total cross section but also for the elastic differential cross section. This pp elastic differential cross section has a dip somewhat above a momentum transfer of 1 GeV/c. Since the dip is due to the cancellation of various contributions, it gives a severe test to the phenomenology. Figure 7 shows such a comparison. Note the good agreement in the dip region. The depth of the dip in the phenomenology is intimately related to the fact that the crossing symmetric $\mathcal{S}(s)$, as defined by Eq. (8), is not real.

FIG. 5: Comparison of the result from the phenomenological model with the measured total cross sections for pp (black points) and $p\bar{p}$ (open points).

VII. PRODUCTION PROCESSES AT THE LHC

In the last two sections, we have presented results that are reasonably well understood. A more detailed summary for these results may be found in Ref. [15]. In this section, some ongoing work is to be discussed.

So far we have been concerned almost exclusively with elastic scattering and directly related quantities such as the total cross section. That the total cross section is increasing with energy must have far-reaching consequences beyond elastic scattering, production processes being an obvious candidate. It is, however, believed that, for the effect of Sec. V to be important, even higher energies are required for production processes than for elastic scattering. Such higher energies have not been available so far from particle accelerators; this situation is expected to change in 2008 when the construction of the Large Hadron Collider (LHC) will be completed at CERN, Geneva, Switzerland.

The LHC is a proton-proton collider with two 7 TeV beams, i.e.,

$$\sqrt{s} = 14 \text{ TeV} = 14,000 \text{ GeV}. \quad (16)$$

The values of the total cross section and ρ in Eqs. (14) and (15) are both specifically for this energy. At LHC, there will be four experiments: ATLAS, CMS, ALICE, and LHC-B.

FIG. 6: The ratio ρ for pp (black points) and $p\bar{p}$ (open points) elastic scattering.

The ATLAS detector is the largest of the four, and CMS the second largest.

There are many exciting experiments to be carried out at LHC. One of them is on the production and decay of the EBH particle already mentioned in Sec. V. Experiments at LEP, which is an electron-positron colliding accelerator at CERN, have given first possible experimental evidence [16, 17] for the EBH particle, whose mass M is about

$$M \sim 115 \text{ GeV}/c^2. \quad (17)$$

Gastmans and I have been studying the production of the EBH particle at LHC on the basis of the high-energy behavior of relativistic quantum gauge field theory. Compared with the work presented in Sec. V, there are the following two major differences:

(1) While the problem of Sec. V is to determine the asymptotic behavior of a four-point function, here the problem is to treat that for a five-point function; and

(2) While there is one large parameter, as specified by (5), for the problem of Sec. V, there are here two large parameters, namely,

$$\frac{E}{M} \quad \text{and} \quad \frac{M}{m} \quad \text{are both large.} \quad (18)$$

As is well known, asymptotic calculations with two independent large parameters are very much more difficult than those with one large parameter. We are now only beginning

FIG. 7: Proton-proton differential cross section at $p_{\text{lab}} = 2060 \text{ GeV}/c$. Note the good agreement in the dip region.

to know how to deal with the two independent large parameters of (18) for the present problem. Using the LEP preliminary result of (17) and taking m to be the mass of the proton, then it follows from the LHC energy (16) that

$$\frac{E}{M} \sim \frac{M}{m} \sim 122. \quad (19)$$

That these two independent large parameters happen to be nearly equal is, of course, accidental.

It has been seen from Sec. V and especially Fig. 3(c) that the increasing total cross section at high energies is intimately related to the production through pionization of Yang-Mills gauge particles. In Fig. 8(a) is shown the simplest elastic scattering through the exchange of one Yang-Mills gauge particle, a process that leads to a constant total cross section at high energies. From Fig. 3, this simplest scattering, when modified by the production of additional Yang-Mills gauge particles as shown in Fig. 8(b), leads to the increasing total cross section discussed in Sec. V.

What is the corresponding situation for the production of the EBH particle at LHC? The simplest production process is shown in Fig. 8(c). In this process, each of the incident fermions (quarks) emits a Yang-Mills gauge particle (gluon) and the two gluons combine to produce the EBH particle. This process is commonly referred to as gluon fusion and is believed to be the most important EBH production process. In this case, each of the

FIG. 8: Schematic comparison of increasing cross sections for elastic scattering and EBH production at high energies.

two originally emitted Yang–Mills gauge particles can produce additional Yang–Mills gauge particles. Very similar to Fig. 8(a) leading to Fig. 8(b), this gluon fusion process Fig. 8(c) leads to the diagram of Fig. 8(d). [In both Fig. 8(b) and Fig. 8(d), each group of three particles stands symbolically for an arbitrary number of particles.]

As seen from Fig. 8, especially from 8(b) and 8(d), there is a great deal of similarity between the diagram for elastic scattering and *half* of the diagram for EBH production. There are also major differences: for elastic scattering there are of course two incident particles, but, for half of EBH production, there is only one incident particle unless somehow the triangle due to a heavy fermion can serve as the second incident particle. This is one of the major problems being studied. It should also be re-emphasized that the diagrams of Fig. 8(b) are evaluated for one large parameter as given by (5), while those of Fig. 8(d) are for two large parameters (18).

Next let us turn our attention briefly to phenomenology. That for elastic scattering, as discussed in Sec. VI, depends critically on experimental data for pp and $p\bar{p}$ scattering. In particular, these data are used to determine the values of the parameters given in Table 1. But for the production of the EBH particle, there are no experimental data at all. Nevertheless, Gastmans and I are optimistic that this major obstacle to phenomenology can be overcome. For this and other reasons, this work on LHC physics is an ongoing project.

VIII. CONCLUSION AND DISCUSSIONS

Already for some time, the increasing total cross section, together with its immediate consequences on elastic cross sections, is well established, both theoretically and experimentally. Indeed, Professor Yang has incorporated this new phenomenon into his geometrical picture of scattering [18].

It is also clear that the increasing cross section has direct consequences on inelastic processes. One way to see this point is as follows. As already discussed in some detail in Sec. V, an integrated cross section that increases as $[\ln(E/m)]^3$, given by the diagram of Fig. 3(b), violates the bound of Froissart[11] and Martin [12] and is hence impossible. Therefore, such results from the diagrams of Fig. 3 must be regarded as representing a strongly absorptive potential. Nevertheless, the problem still needs to be understood: What are the high-energy behaviors of the integrated cross sections for pionization?

One way to put this issue succinctly is as follows. Since pionization leads to increasing cross sections, the increasing cross section must in turn affect pionization cross sections. Precisely how does this come about and what are the resulting pionization cross sections? This and many related questions about production processes have not yet been answered satisfactorily.

Of the numerous processes that can be and will be studied at the Large Hadron Collider—which is scheduled to begin operation at CERN in the summer of 2008—the production and decay processes of the EBH particle are surely among the most exciting. [This EBH particle is commonly called the Higgs particle; see Sec. V.] The remainder of this section is to be devoted to some comments on these production and decay processes.

Consider the diagram of Fig. 8(d): the upper part is similar to the pionization diagram 8(b), and so is the lower part. If both parts are removed, what remains is the diagram of Fig. 9, EBH production by gluon fusion. In the standard model of Glashow, Weinberg, and Salam [19], by far the most important contribution to this gluon fusion is from the top triangle.

If this diagram of Fig. 9 is turned around and the gluons are reinterpreted as photons, then the result is that of Fig. 10, the decay of the EBH particle into two photons. Once again, in the standard model, the most important contribution to this two-photon decay is

FIG. 9: EBH production by gluon fusion.

FIG. 10: The decay $EBH \rightarrow \gamma\gamma$.

from the top triangle.

The situation is much more interesting if the standard model is not the whole story. In this case, there may well be additional heavy particles that contribute significantly to these triangles. In general, the list of new particles that give such significant contributions may not be the same for the diagrams of Figs. 9 and 10: for the triangle of Fig. 9, the particle must have color so that it couples to the gluons; and for the triangle of Fig. 10, the particle must have charge so that it couples to the photon. If the contributing particle is of spin-0, then the triangle is, of course, not the only diagram.

This situation bears some resemblance to that of the LEP Collider in 1989. The first major experimental result from LEP was the determination of the number of generations with light neutrinos from the width of the Z. Thus the number of light neutrinos, and hence the number of such generations, was found to be 3. This was accomplished without observing experimentally any neutrino at all.

In the present case, from the production cross section of the EBH particle, or more precisely from the decay of the EBH into two photons, information can be obtained about the possible existence of new heavy particles. In the case of the decay into two photons, the rate may be normalized to any decay process that does not depend on the triangular loop, one example being the leptonic decay

$$EBH \rightarrow l^+ l^-. \quad (20)$$

Similar to the LEP case, information about these heavy particles can be obtained without observing them experimentally. Because of this possibility, the triangular loop in Figs. 9 and 10 has been sometimes referred to as the magic triangle.

Finally, the following even more exciting possible scenario may be mentioned briefly: there is more than one physical EBH particle, leading to various magic triangles. This possibility follows from noting that, in the standard model, the presence of only one EBH doublet is an assumption based on simplicity.

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