Supersymmetry without the little hierarchy

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We propose a simple gauge extension of the minimal supersymmetric standard model, where the fine-tuning in the Higgs mass parameters is highly reduced. The Higgs boson is insensitive to high energies because of supersymmetry and also because it is a pseudo-Goldstone boson of a global symmetry (referred to as “double protection” or “super-little Higgs” mechanism). A large shift in the Higgs quartic self-coupling is obtained via a nondecoupling $D$ term coming from an extra gauge group, resulting in a Higgs mass that can be as heavy as 135 GeV with a tuning milder than 10%. With an appropriate choice of quantum numbers one can achieve that the additional quartic is generated without a corresponding shift in the Higgs mass, thus preserving the double protection of the Higgs. The model predicts the existence of several top-partner fermions one of which can be as light as 700 GeV, while the ordinary minimal supersymmetric standard model states could be as light as 400 GeV. In addition to many new particles in the multi-TeV range there would also be an axionlike state very weakly coupled to standard model matter, which could be in the sub-GeV regime, and sterile neutrinos, which could be light.

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I. INTRODUCTION

Supersymmetry (SUSY) is a very attractive solution to the hierarchy problem. The electroweak scale is only logarithmically sensitive to the cut-off scale $\Lambda$, while it is quadratically sensitive to the soft SUSY breaking parameters $m_s$. Thus, if a SUSY model is to be natural, then the generic prediction would be that the soft breaking scale has to be around the electroweak symmetry breaking (EWSB) scale $v \approx 175$ GeV. However, the lack of discovery of superpartners at LEP2 and the Tevatron, and the bounds from electroweak precision tests (EWPT) and flavor physics, frustrate this natural expectation, reintroducing a tension between these two scales $m_s$ and $v$ [1]. This is usually referred to as the little hierarchy problem.

The way the little hierarchy problem manifests itself in the context of the minimal supersymmetric standard model (MSSM) is via a tension between the lightest physical Higgs mass and the mass of the Z boson: in order for the Higgs mass to be above the LEP2 bound of 115 GeV, one needs to radiatively enhance the Higgs quartic self-coupling, which requires a large stop mass $m_{\tilde{t}} \gtrsim$ TeV. However, the same heavy stop will then yield the dominant radiative corrections to the quadratic Higgs mass parameter through the stop/top loops. The natural size of the electroweak scale would then be at

$$ (3m_s^2/4\pi^2) \ln \Lambda/m_s, \quad (1.1) $$

which generically results in a percent level (or worse) fine-tuning.

This supersymmetric little hierarchy problem stimulated several authors to depart from SUSY, and to look for alternatives. In little Higgs (LH) models [2] (for reviews see [3]), the quadratic sensitivity to the cutoff scale is removed by same spin particles whose couplings are fixed by a global approximate symmetry broken at some scale $f$: these models recast the old idea that the Higgs is naturally light because it emerges as a pseudo-Goldstone boson of a broken global symmetry [4]. This global symmetry breaking is usually linked to a new strongly interacting sector [5]. However, LH models generically generate tree-level contributions to electroweak precision observables so that the scale $f$ has to be in the multi-TeV range to evade the electroweak precision bounds [6]. Thus, the natural EWSB scale of order

$$ (3f^2/4\pi^2) \ln f/v \quad (1.2) $$

in generic LH models is still too large. LH models with $T$ parity can accommodate EWPT with a lower $f$ [7,8], but they do not provide a solution to the (big) hierarchy problem between $f$ and $\Lambda$, unless complicated additional layers of structures are added [9].

One prominent idea is to combine the broken global symmetry and supersymmetry, to enforce a “double protection” (or “super-little Higgs”) mechanism [10–15] on the EWSB scale whose natural value is then expected to be of the order
where the cutoff \( \Lambda \) of the MSSM is replaced by the global symmetry breaking scale \( f \sim \text{few} \times \text{TeV} \). All the good features of SUSY are retained: it solves the hierarchy problem between \( f \) and \( \Lambda \) and satisfies EWPT’s. However, for a completely natural theory all superpartners (including the stop) should be in the few hundred GeV range, which would generically result in a Higgs mass that is too light [15]. Therefore, a new contribution to the Higgs quartic self-interaction is needed in order to raise the Higgs mass above the LEP bound. However, the source for the new quartic should not spoil the double protection with an unwanted large correction to the quadratic Higgs coupling. Existing models have to include a complicated new sector with several light gauge singlets to achieve this [13,14].

In this paper we realize this program via nondecoupling \( D \) terms of extended gauge symmetries [16,17], where the effective gauge couplings, hence the Higgs quartic, are enhanced.\(^1\) With an appropriate choice of quantum numbers under the extended gauge group we can ensure that the only nondecoupling effects are in the form of an additional contribution of the hypercharge \( D \) term. This will guarantee that the quadratic Higgs terms remain unaffected, and the double protection mechanism continues to work. The nondecoupling is driven by a sizable soft breaking mass term for a field that does not carry standard model (SM) quantum numbers (but carries only charges under the extended gauge group). The presence of this field will modify (via interactions through the \( D \) terms) the effective Lagrangian for the light fields: the supersymmetric low-energy limit for the usual \( D \) terms of the unbroken generators is replaced by \( D \) terms with enhanced gauge couplings [16,17]. This framework provides both the residual approximate global symmetry and the enhancement of the tree-level Higgs quartic coupling. The explicit model we discuss here is a supersymmetric version of the “simplest” LH model [20] based on an SU(3)\(_W\) gauge symmetry, which extends the weak SU(2)\(_W\). The charge assignment of the matter content is anomaly free and generation universal. The model turns out to be natural over a wide range of the input parameters, requiring a fine-tuning of better than 10\%. The two main constraints on how little fine-tuning one can get away with are from the requirements that the \( Z' \) is sufficiently heavy to avoid electroweak precision bounds, and that the Landau pole of the new U(1) gauge group is separated by several orders of magnitude from the mass scale of the new particles introduced here. Within the same region without tuning (and satisfying these bounds) the Higgs mass can be raised up to about 135 GeV. The MSSM masses can be as light as around 400 GeV (as low as the direct detection bounds allow them to be). There will be a large variety of new states beyond the MSSM in the spectrum. The top partners can be as light as 700 GeV, while most other “little-partner” states will be in the multi-TeV regime, including a \( Z' \). The model also predicts a light axionlike state, which is very weakly coupled to SM fields, and sterile neutrinos.

The paper is organized as follows: in Sec. II, we review the simplest super-little Higgs model, and explain that without additional gauge extension the theory is still fine tuned. In Sec. III, we further extend the gauge group with an additional U(1) factor, and show that the mechanism of [16,17] can be easily implemented. In Sec. IV, we include the matter fields into the theory, and calculate the leading loop corrections of the full model. Section V will be devoted to quantify the fine-tuning in the theory and to give some interesting signals for the LHC. We conclude in Sec. VI.

II. THE SIMPLEST SUPER-LITTLE HIGGS

Let us start considering the supersymmetric version of the simplest LH model. The electroweak gauge group SU(2)\(_W\) \( \times U(1)\) of the SM is promoted to a larger group SU(3)\(_W\) \( \times U(1)\), under which the Higgs fields are chiral superfields.\(^2\) We use a couple of Higgs fields, \( \Phi_{u,d} = 3_{+1/3}, \bar{3}_{-1/3} \), to break the gauge group down to the SM at the some high scale \( F \approx 10 \) TeV to evade all the experimental constraints on new heavy gauge bosons. Another copy of Higgs fields, \( \Phi_{u,d} = 3_{+1/3}, \bar{3}_{-1/3} \), takes vacuum expectation values (VEVs) at a lower scale, \( f = 1 \) TeV; if the \( \hat{H} \) VEVs point in the same direction than \( \Phi \) VEVs there is no EWSB, while a misalignment of the \( \langle \Phi \rangle \) and \( \langle \hat{H} \rangle \) VEVs will lead to EWSB.

As long as the fields \( \Phi \) and \( \hat{H} \) do not communicate with each other, there is an enlarged SU(3)\(_1\) \( \times \) SU(3)\(_2\) global symmetry with the diagonal SU(3)\(_W\) gauged. At the scale \( F \) both SU(3)\(_1\) and SU(3)\(_W\) are broken, while the global SU(3)\(_2\) acting on \( \hat{H} \) is still preserved down to the scale \( f \). Thus, at low energies we are left with 5 physical Goldstones living (mostly) in \( \hat{H} \). The other 5 longitudinal Goldstones are eaten by the gauge bosons corresponding to the broken SU(3)\(_W\) generators. One of physical Goldstone bosons is an electroweak singlet \( \eta \), which does not play any role in the following. We will discuss it later in Sec. V. The other 4 Goldstones form an SU(2)\(_W\) doublet with the quantum numbers of the Higgs boson.

To be more concrete, we consider the following superpotential for the Higgs sector:

\[
\mathcal{W} = \kappa N (\Phi_u \Phi_d - \mu^2) + \mathcal{W}_3f (\mathcal{H}_{u,d}),
\]

\(^1\)Other known mechanisms for enhancing the Higgs quartic include the NMSSM [18] and fat Higgs [19] type models.

\(^2\)We work with the normalization \( Y = T^3/\sqrt{3} + X \), where \( X, Y, \) and \( T^3 \) are the U(1)\(_z\), U(1)\(_y\), and SU(3)\(_W\) generators, respectively, with \( T^{3,8} \) diagonal, and \( T^3 = \lambda^2/2 = 1/\sqrt{3} \times \text{diag}(1/2, 1/2, -1) \). Color SU(3)\(_{\text{QCD}}\) is left untouched and is assumed to be understood everywhere. The SU(3)\(_W\) \( \times U(1)\) gauge couplings are \( g \) and \( g_s \).
with no mixing terms like $\Phi_u \mathcal{H}_d$ that would spoil the $SU(3)_1 \times SU(3)_2$ symmetry. Here, the singlet field $N$ and the superpotential $\mathcal{W}$ provide just one of the possible realizations of the symmetry breaking pattern we described above. Taking equal soft terms $m_{\Phi_u}^2 = m_{\Phi_d}^2 = m^2 > 0$ the scalar potential will be

$$V = \kappa^2 |\Phi_u |^2 - \kappa^2 \mu^2 (\Phi_u \Phi_d + H.c.)$$

$$+ m^2 (|\Phi_u|^2 + |\Phi_d|^2) + \kappa^2 |N|^2 (|\Phi_u|^2 + |\Phi_d|^2),$$

(2.2)

$$\langle \Phi_{u,d} \rangle = (0, 0, F), \quad \langle |N| \rangle = 0,$$

(2.3)

$$F^2 = \mu^2 - m^2 / \kappa^2 > 0.$$

We will comment later on the effect of having nonequal VEVs $F_u^2 - F_d^2 = m_{\Phi_u}^2 - m_{\Phi_d}^2$.

Integrating out at tree level the heavy modes around the VEVs of $N$ and $\Phi_{u,d}$, we get the effective potential for the light fields $\mathcal{H}_{u,d}$

$$\mathcal{H}_u^T = (H_0^T, S_u), \quad \mathcal{H}_d = (H_d, S_d).$$

(2.4)

It is given by the $SU(3)_2$-symmetric $F$ term from $\mathcal{W}_3 f$, and the effective $D$ term potential

$$V_D = \frac{g_f^2}{8} \sum_{a=1}^{3} [H_u^a \sigma^a H_u - H_d \sigma^a H_d^\dagger]^2 + \frac{g_f^2}{8} [|H_u|^2$$

$$- |H_d|^2]^2 + \left(\frac{2m^2}{2m^2 + m_W^2}\right) \frac{g_f^2}{8} \sum_{a=1}^{3} [\mathcal{H}_u^a \lambda^a \mathcal{H}_u - \mathcal{H}_d \lambda^a \mathcal{H}_d^\dagger]^2$$

$$- \mathcal{H}_u \lambda^a \mathcal{H}_d^\dagger]^2 + \left(\frac{2m^2}{2m^2 + m_Z^2}\right) \frac{g_f^2}{8} [\mathcal{H}_u^a \lambda^a \mathcal{H}_u - \mathcal{H}_d \lambda^a \mathcal{H}_d^\dagger]^2$$

$$- \mathcal{H}_u \lambda^a \mathcal{H}_d^\dagger]^2 + \frac{g_f^2}{18} [\mathcal{H}_u^2 - |\mathcal{H}_d|^2]^2],$$

(2.5)

where

$$\frac{1}{g_f^2} = \frac{1}{3g_1^2} + \frac{1}{g_2^2}, \quad g_f^2 = \frac{g_1^2}{1 + (2m^2/m_Z^2)},$$

$$m_W^2 = g_f^2 F^2, \quad m_Z^2 = \frac{4}{9} (3g_1^2 + g_2^2) F^2.$$

(2.6)

In the supersymmetric limit $m^2 / F^2 \to 0$ one gets the MSSM $D$ terms. For the hard breaking, $m^2 / F^2 \to \infty$, the $D$ terms corresponding to broken generators do not decouple, and the full $SU(3)_W \times U(1)_Y$ expression is recovered. In this case, the additional (non-MSSM) $D$ terms induce both quadratic and quartic terms for the physical Higgs at tree level, and the additional explicit mass terms would ruin the double protection mechanism.

Thus, we are forced to take the limit $m^2 / F^2 \ll 1$, in which case the global $SU(3)_2$ is softly broken by MSSM-like $D$ terms, so that the Higgs is actually a pseudo-Goldstone boson. This source of global symmetry breaking contributes to the tree-level quartic self-interaction of the Higgs $\lambda_0 = (g_1^2 + g_2^2)/8$, while the leading sources for the Higgs mass terms (as well as for additional quartics) come from loops of the top(stop) sector. This way the large log contribution to the Higgs mass parameter is replaced by a smaller effective one, $\delta m^2 \sim (3m_Z^2/4\pi^2) \ln f / m_t$, as result of the double protection of the global symmetry and of supersymmetry, which forbid a log $\Lambda$ dependence of the cutoff [10–12]. However, the contribution to the Higgs quartic is even smaller than in the MSSM [15], and a large stop mass $m_t = \text{TeV}$ is needed. This is especially true when one includes two loop effects, which usually lower the effective Higgs quartic further [21]. From the expression (2.5) for the $D$ terms, we see that increasing the soft breaking parameter $m^2$ is even worse because it decreases the effective $U(1)_Y$ coupling $g_Y^2 < g_Y$, and also introduces large tree-level corrections to the Higgs mass parameter, since this is breaking both supersymmetry and the $SU(3)$ global symmetry in a hard way.

III. DOING THE SUPER-LITTLE HIGGS MODEL

We propose a simple way to improve the simplest superlittle Higgs model. We present a mechanism that enhances the Higgs quartic coupling without introducing additional Higgs mass terms. We have seen above that in the supersymmetric limit the global symmetry breaking via $D$ terms induces only a Higgs quartic coupling, but no quadratic term. This gives us the hint: if we find a way to increase the effective low-energy gauge coupling $g$ or $g_Y^2$, then we increase the tree-level quartic $\lambda_0$ too, keeping the quadratic tree-level Higgs coupling untouched. This can be achieved by extending the gauge group once more. For example, assume that we have a gauge extension $G_1 \times G_2$ with gauge couplings $g_1$ and $g_2$, respectively. Breaking spontaneously to the diagonal subgroup $G_1 \times G_2$ at some scale $\Omega$, the low-energy gauge coupling is $1/\sqrt{g_{1,2}^2} = 1/g_1^2 + 1/g_2^2$. However, as we saw above, the effective $D$ terms in the low-energy theory are determined by the same low-energy gauge coupling only in the supersymmetric limit. If we keep the ratio $m_{\text{soft}}^2 / \Omega^2$ finite, where $m_{\text{soft}}$ is a soft breaking term of a field charged only under one of two $G_i$, the effective gauge coupling $g_{\text{eff}}$ in front of the $D$ terms turns out to be slightly bigger, $g_{\text{eff}} \approx g_{1,2} [16,17]$, achieving our goal of raising the Higgs quartic couplings.

The simplest way to take advantage of this effect is by adding an extra $U(1)_Y$. In order for the $U(1)_Y$ to have an effect on the Higgs couplings, the triplets $\Phi, \mathcal{H}$ should be charged under the $U(1)_Y$. We also need another field $\Psi$ to break the additional $U(1)_Y$ via its VEV $\langle \Psi \rangle = \Omega [\text{the triplet VEVs would leave an extra unbroken U(1) left over at low energies}]. In order for this $\Psi$ not to introduce new global symmetry breaking terms we will take it to be a singlet.

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3Here, $H_{u,d}$ and $S_{u,d}$ are $SU(2)_W \times U(1)_Y$ doublets and singlets, respectively.
under SU(3)\(_w\) \(\times\) U(1)\(_x\). The resulting field content is shown in Table I.

### A. Quartic without quadratic Higgs terms

In order for the new \(D\) terms to be nondecoupling, the soft breaking mass \(m_\Psi\) for the new field \(\Psi\) should be sizeable, \(m_\Psi/\Omega = \mathcal{O}(1)\). The main worry is whether these nondecoupling effects will only enhance the quartic or also introduce a large Higgs mass correction reintroducing the fine-tuning. Next, we argue that for the case when the U(1)\(_x\) charges of the two triplets are chosen to be equal \((q' = q)\) there will be no quadratic terms introduced. This can be seen as follows: The low-energy Lagrangian for the light fields should be expressible in terms of the various \(D\) terms \((D_8, D_y, D_u, D_z)\) formed from the light field \(\mathcal{H}\), where

\[
\begin{align*}
D_8 &= \mathcal{H}_u^a \bar{\lambda}^8 \mathcal{H}_u - \mathcal{H}_d^a \bar{\lambda}^8 \mathcal{H}_d^a, \\
D_y &= |\mathcal{H}_u^a|^2 - |\mathcal{H}_d^a|^2, \\
D_u &= |H_u|^2 - |H_d|^2, \\
D_z &= D_x.
\end{align*}
\]

(3.1)

Using the embedding of hypercharge we also find that

\[
D_y = \frac{1}{\sqrt{3}} D_8 + \frac{2}{3} D_x.
\]

(3.2)

So the actual expression of the low-energy Lagrangian for the scalars should be of the form

\[
AD_y^2 + BD_y D_x + CD_x^2.
\]

(3.3)

\(D_x\) does not depend on the Goldstones; however, it will have a nonzero expectation value, so a cross term \(D_xD_y\) would indeed generate a large mass correction to the Higgs. We will show that this cross term is avoided with the choice \(q = q'\).

Since the nondecoupling \(D\) terms from the triplet \(\Phi\) give a large mass to the Higgs, we will take the limit \(m/F \to 0\). In this limit the triplet sector is supersymmetric, and all their effects should decouple. Similarly, we know that in the supersymmetric limit \(m_\Psi/\Omega \to 0\) for the new field \(\Psi\) one should simply obtain the hypercharge \(D\) term at low energies \([22]\), that is \(A \to 1, B, C \to 0\). So one can express these coefficients as

\[
\frac{m_\Psi^2}{\Omega^2} a(m_\Psi^2/\Omega^2), \quad \frac{m_\Psi^2}{\Omega^2} b(m_\Psi^2/\Omega^2),
\]

(3.4)

To fix \(b, c\) let us now consider another limit, where \(m_\Psi/\Omega \to \infty\). In this case the extra scalar is infinitely heavy, so it cannot have any effect on the low-energy Lagrangian, thus it should decouple. So the low-energy \(D\) terms should just be the high-energy \(D\) terms with the heavy \(\Psi\) eliminated. However, the \(m_\Psi/\Omega \to \infty\) limit has another important effect as well: since effectively the breaking VEV is set to zero, one ends up with two unbroken U(1)’s in this limit: the usual hypercharge \(Y = T^8/\sqrt{3} + X\) and \(Q = T^8/\sqrt{3} + Z/3q\). So the \(D\) terms in this limit should be \(D_y + D_x\). For a generic choice of \(q'\) for the Higgs \(z\) charge there will indeed be a contribution different from \(D_y\), which will in turn imply the existence of a large mass term. However, for \(q' = q\) the low-energy Lagrangian in the \(m_\Psi/\Omega \to \infty\) limit is \(\propto D_y\), which implies that \(b = c = 0\), and so there will be no mass term introduced for the Higgs (but only a quartic).

### B. The enhanced quartic and tree-level Higgs mass bound

Let us now explicitly calculate the enhancement of the Higgs quartic for this doped model. We can take for \(\Psi\) a simple superpotential like \(\mathcal{W}_\Psi = \kappa_\Psi S(\Psi_u \Psi_d - w^2)\), with equal soft masses, \(m_{\Psi_u}^2 = m_{\Psi_d}^2 = m_\Psi^2 > 0\), so that \(\langle |\Psi_u| \rangle^2 = \Omega^2 = w^2 - m_\Psi^2/\kappa_\Psi > 0\). Integrating out the heavy fields, we are again left at low energy with the SU(3)\(_2\) symmetric potential coming from \(\mathcal{W}_\Psi\), and the effective \(D\) terms

\[
V_D = \frac{g_\Psi^2}{8} \sum_{a=1}^3 [H_u^a \sigma^a H_u - H_d^a \sigma^a H_d^a]^2 + \frac{g_\Psi^2}{8} [\langle H_u^a \rangle^2 - |H_d^a|^2]^2.
\]

(3.5)

Here, we neglect contributions of order \(m_\Psi^2/F^2 \ll 1\) but we keep the ones from \(m_\Psi^2/\Omega^2 \ll 1\) entering in the effective U(1)\(_y\) gauge coupling\(^4\)

\[
g_\gamma = g_\gamma \left[ 1 + \frac{1}{t_W} \left[ 1 + 18 \frac{r_\gamma}{g_\Psi^2} \left( \frac{m_\Psi^2}{\Omega^2} / m_\Psi^2 + r_\gamma^2 (3 - r_\gamma^2) \right) \right] \right]^{-1/2},
\]

(3.6)

where \(t_W = g_\gamma/g_\gamma\), \(r_\gamma = q g_\gamma/g_\gamma\), \(r_\gamma = q g_\gamma/3q\). The main point here is that the Higgs quartic coupling provided by

\(^4\)The gauge coupling \(g_\gamma\) is still given by Eq. (2.6).
such doped $D$ terms is bigger than the MSSM one. In order to see this explicitly, we parameterize the Higgs triplet fields $H$ (which determine where the VEVs point is) using a nonlinear sigma model\(^5\)

\[
\begin{align*}
\mathcal{H}_u &= \sin \beta (H_u \sqrt{f^2 - |H|^2}) \\
\mathcal{H}_d &= \cos \beta (H_d \sqrt{f^2 - |H|^2}).
\end{align*}
\]

(3.7)

Once we plug this parameterization into the full potential, we get the new tree-level upper bound on the Higgs mass $m_h$

\[
m_h^2 \leq (1 - v^2/f^2) \left[ m^2_\text{Higgs} 2 \beta + m^2_\text{Higgs} \cos^2 2\beta \frac{r_1^2 (3 - r_1^2)}{[1 + 18r_1^2 (g^2/\sqrt{\lambda} + r_1^2 g^2/\sqrt{\lambda}) + r_1^2 (3 - r_1^2)]} \right].
\]

(3.8)

The overall suppression factor $(1 - v^2/f^2)$ comes from the wave-function normalization. We will see in the next section, that with these enhanced tree-level Higgs mass moderately low stop mass of few hundred GeV will be allowed, so that the quadratic Higgs coupling generated at one-loop has the natural order of magnitude, $v^2$.

Figure 1 shows the tree-level Higgs mass on the plane $(r_q, r_{\tilde{q}})$ for fixed $\tan \beta = 10$, $F = 10$ TeV and $\Omega = 12$ TeV. The two main constraints on the parameter space comes from the requirement that the $Z'$ is sufficiently heavy (to avoid electroweak precision constraints), and also from ensuring that the Landau pole of the new $U(1)_c$ is at a sufficiently high scale. With the fermion matter content presented in the next section the one-loop expression for the Landau pole is given by

\[
\Lambda_{\text{Landau}} \sim M_{Z'}^{(8\pi^2/[g^2 f^2 (138 + 18r_q^2)])}.
\]

(3.9)

In Fig. 1 we show the parameters corresponding to $U(1)_c$ Landau poles of $10^3, 10^6,$ and $10^9$ TeV. After imposing the $Z'$ and Landau-pole constraints, the tree-level Higgs mass is still quite large over a large fraction of the plane (and can even surpass the LEP bound).

### IV. THE TOP SECTOR

In this section we discuss the embedding of the quark and lepton fields into the theory, focusing ourselves on the radiative contributions to the Higgs potential from the top/stop loops.

#### A. Matter fields and top Yukawa coupling

Our assignment of quantum numbers, summarized in Table II for the third generation, is the same as in [14]. This choice of charges is anomaly free and generation universal.

The main difference compared to [14] are the Yukawa couplings.\(^6\) The most general superpotential for the top sector is

\[
W_{\text{top}} = m_Q \tilde{Q}' Q' + y_t \tilde{T}' H_u U + y_2 Q H_u Q' + \tilde{y}_1 \tilde{T}' \Phi_u U + \tilde{y}_2 Q \Phi_u Q',
\]

(4.1)

We see the “collective” nature of the symmetry breaking pattern: if only one of the four Yukawa couplings $\tilde{y}_i$ or $y_i$ is turned on at a time, the global $SU(3)_c \times SU(3)_L$ symmetry is preserved, while simultaneous nonvanishing couplings $\tilde{y}_i, y_i \neq 0$ leave only the diagonal symmetry. To maintain

\(^6\)In [14] a different representation for the Higgs sector was used.
the full global symmetry we will later be working in the $y_1 = y_2 = 0$ limit.

As long as the Higgs triplets $H_u$, $\Phi_u$ have their VEVs aligned $(0, 0, f_u)$ and $(0, 0, F)$, respectively (i.e. no EWSB occurs), the top stays massless, while the heavy top partners $T_{\pm 1, 2}$ and the heavy bottom partner $B_1$ get large masses

$$m_{T_1}^2 = m_{B_1}^2 = m_{T_2}^2 = m_{Q_2}^2 + (y_2 f_u + \tilde{y}_2 F)^2,$$

$$m_{T_1}^2 = m_{T_2}^2 + (y_1 f_u + \tilde{y}_1 F)^2.$$  

When EWSB occurs via the misalignment of the VEVs, $(H_u) = (0, v_u, f_u - v_u^2)$, the top quark gets a mass proportional to the physical Higgs VEV

$$m_t = y_t v_u = \frac{v_u (F m_Q |y_1 \tilde{y}_2 - y_2 \tilde{y}_1|)}{m_{T_1} m_{T_2}}.$$  

Note, that the combination $|y_1 \tilde{y}_2 - y_2 \tilde{y}_1|$ is simply the determinant of the coupling matrix $(y, \tilde{y})$, i.e. the two sets of couplings have to be misaligned as expected by the collective symmetry breaking. Note also, that the Lagrangian in (4.1) would still have a full SU(3) × SU(3) global symmetry even for the generic choice of the couplings $y_i$, $\tilde{y}_i$, except those would be acting on the linear combinations of the Higgs triplets $X = y_1 H_u + \tilde{y}_1 \Phi_u$ and $Y = y_2 H_u + \tilde{y}_2 \Phi_u$. In order for the global symmetries not to be misaligned with the original global symmetries we have to choose $y_1 = \tilde{y}_2 = 0$ otherwise the top/stop contribution would induce terms like $\sim |x|^2 \log A$ spoiling the double protection when $y_1 \neq 0$. Considering the simplest mass spectrum $m_{T_1} \approx m_{T_2}$ with $y_2 f_u \approx \tilde{y}_1 F \approx m_{Q_2}$ we get the typical sizes of Yukawa couplings: $\tilde{y}_1 = 1/5$ and $y_2 = 2$, if we assume that the supersymmetric mass parameter $m_{Q_2}$ is of the order of a few TeV.

For the other light SM fermions, we can add the following superpotential

$$W = \alpha_i Q H_d D_i + \tilde{\alpha}_i Q \Phi_d D_i + \beta_i E H_d L_i + \tilde{\beta}_i E \Phi_d L_i,$$  

where flavor indices are understood. In the down quark sector there is another heavy bottom partner $B_2$

$$m_{B_2}^2 = (\alpha_1 f_u + \tilde{\alpha}_1 F)^2 + (\alpha_2 f_u + \tilde{\alpha}_2 F)^2,$$

$$m_{B_1} = \frac{u (F m_Q |\alpha_1 \tilde{\alpha}_2 - \alpha_2 \tilde{\alpha}_1|)}{m_{B_1} m_{B_2}}.$$  

We see that even removing $Q'$ and $\tilde{Q}'$ from the spectrum all down quark flavors remain massive. Thus, only the up and charm quarks need nonrenormalizable operators to get masses. A similar situation occurs for the leptons, where all the charged states get mass

$$m_{L_L}^2 = (\beta_1 f_u + \tilde{\beta}_1 F)^2 + (\beta_2 f_u + \tilde{\beta}_2 F)^2,$$

while the SM neutrinos and the sterile neutrinos $\nu_s$ remain massless. We will discuss more on $\nu_s$ in Sec. V.

Gauge coupling unification is not easy to maintain in models based on SU(3) × U(1) extensions of the electroweak group. Interestingly, it was found in [14] that the $\beta$ functions of the matter content presented here are actually suitable for one-loop unification into an SU(6) group, except that with the addition of the vectorlike $Q'$, $\tilde{Q}'$ one loses asymptotic freedom, and QCD hits a Landau pole before reaching the unification scale. Here, we have added one more U(1) group, whose Landau pole is potentially even lower than that of QCD. It remains to be seen whether a simple extension of these ideas can be made consistent with perturbative unification.

B. Top/stop loop contribution to the Higgs potential

Next we calculate the radiative potential $\Delta V = \delta m_H^2 |H|^2 + \delta \lambda |H|^4 + \ldots$ generated for the Higgs by the top/stop loops. Using the Coleman-Weinberg formula [23], one can explicitly see that $\Delta V$ has no logarithmic dependence on $A$, due to the double protection. For instance, setting for simplicity all soft masses equal, $m^2_2 (|Q|^2 + |Q'|^2 + |U|^2 + |Q|^2)$, we get

$$\delta m_H^2 = -\frac{3 y_i^2 \sin^2 \beta}{8 \pi^2} (\frac{m_{T_1}^2 m_{T_2}^2}{m_{T_1}^2 - m_{T_2}^2}) \ln (\frac{m_{T_1}^2 + m_{T_2}^2}{m_{T_1}^2})$$

$$- \frac{m_{T_1}^2}{m_{T_2}^2} \ln (\frac{m_{T_2}^2 + m_{T_2}^2}{m_{T_1}^2}) + \ln (\frac{m_{T_1}^2 + m_{T_2}^2}{m_{T_1}^2}) + \delta,$$  

where $\delta$ is the contribution of the other sectors (like the gauge/gaugino sector). The top/stop contribution in (4.7) vanishes for $m_t \rightarrow 0$ or $m_{T_1} \rightarrow 0$. The expression of the correction $\delta \lambda$ to the Higgs quartic coupling is quite long and we report it in Appendix A.

The degenerate limit $m_{T_1} = m_{T_2} \equiv m_t$ is relatively simple:
\[ \delta \lambda = \frac{3y^4 \sin^4 \beta}{16 \pi^2} \left[ \ln \left( \frac{m_1^2}{m_1^2} \right) + \frac{m_2^2 (7m_3^2 + 8m_2^2)}{6(m_2^2 + m_4^2)^2} \right. \\
- \ln \left( \frac{m_1^2 + m_2^2}{m_1^2} \right) - \frac{4m_2^2}{m_2^2} \ln \left( \frac{m_1^2 + m_2^2}{m_3^2} \right) \right] \] (4.8)

The nonlog dependent contribution in (4.8) is generated from the interplay of a divergent coefficient \((m_1^2 - m_2^2)^{-3}\) (in the degenerate limit), which is in front of a vanishing log in the same limit (see Appendix B).

Another simple limit corresponds to taking hierarchical masses for the little partners of the top, \(m_{T_1} \ll m_{T_2}\)

\[ \delta \lambda = \frac{3y^4 \sin^4 \beta}{16 \pi^2} \left[ \ln \left( \frac{m_{T_1}^2}{m_{T_1}^2} \right) + \ln \left( \frac{m_{T_2}^2}{m_{T_1}^2} \right) + \frac{2m_3^2}{m_{T_1}^2} \right. \\
\times \ln \left( \frac{m_2^2 + m_3^2}{m_2^2 + m_3^2} \right) \right] \] (4.9)

A plot of the shift in the quartic due to the top/stop loops for both limiting cases is shown in Fig. 2.

FIG. 2 (color online). Contour plots of 100 \(\times \delta \lambda\), where \(\delta \lambda\) is the radiative correction to the Higgs quartic coupling in the degenerate limit \(m_{T_1} = m_{T_2} = m_T\) (left) and in the hierarchical one \(m_{T_1} \ll m_{T_2}\) (right).

Figs. 3 and 4, we see that Higgs masses as heavy as 135 GeV are allowed, with less than 10% fine-tuning. The stop can be rather light, for example \(m_s = 400\) GeV is allowed. In addition, the little partners of the top are relatively light; they can be as light as \(m_{T_1} \approx 700\) GeV.

V. TUNING AND LHC SIGNALS

A. Possible sources of tuning

Let us now discuss how natural this model is, and how the tuning is reduced compared to the MSSM and other extensions. Figure 2 shows the enhanced quartic coupling due to the \(U(1)_z\) D term. Fine-tuning in the MSSM is usually measured by the quantity

\[ \Delta_1 = |\delta m_H^2|_{\delta=0}/(m_H^2/2), \] (5.1)

which shows how much cancellation is needed to compensate the top/stop loops with the other contributions \(\delta\). We are presenting \(\Delta_1\) for three different choices of the charges...
and couplings in the U(1)g gauge sector in Figs. 3–5 (we actually plot 100/Δ1%). The three choices correspond to cutoff scales (determined by the U(1)g, Landau pole) of 10^3, 10^6, and 10^9 TeV. Clearly, the bigger one takes the U(1)g charges, the bigger the shift in the Higgs mass; however, the Landau pole will obviously hit earlier. As explained in the Introduction, in the MSSM there are two origins for the tuning in Δ1: the large logarithm from running from a heavy scale, and the large loop from a heavy stop needed to raise the Higgs mass. In our case the stop can be significantly lighter due to the Higgs mass. In our case the stop can be significantly lighter due to the Higgs mass. In our case the stop can be significantly lighter due to the Higgs mass. In our case the stop can be significantly lighter due to the Higgs mass. In our case the stop can be significantly lighter due to the Higgs mass. In our case the stop can be significantly lighten due to the Higgs mass. In our case the stop can be significantly lighter due to the Higgs mass. However, due to the extended structure there could be other sources for fine-tuning in this model. For example, the soft terms m_{\Psi,q}^2 have to be nearly universal at the scale F, since their difference contributes to the Higgs quadratic coupling via D terms. The same argument applies for m_{\Psi,q}^2. Assuming universality at the cut-off, we need to tune by Δ_2 = (m_{\Psi,q}^2 - m_{\Psi,q}^2)/\nu^2 = 3\gamma^2/(8\pi^2)m_2^2/\nu^2\ln\Lambda/F \quad \text{and} \quad \Delta_3 = (m_{\Psi,q}^2 - m_{\Psi,q}^2)/\nu^2 = g_s^2/(8\pi^2)S\ln\Lambda/F, \quad \text{where} \quad S = \text{Tr}(Z_i^2 m_i^2), \quad \text{and} \quad Z_i \quad \text{is the U(1)g generator. For typical values of couplings and soft masses, the tuning Δ_2 is totally under control resulting in Δ_2 = O(1), even for Λ = 10^15 GeV. The source Δ_1 identically vanishes if we assume the renormalization group invariant condition S = 0, which can be imposed by a Z_2 symmetry in the Ψ sector. Even allowing a nonvanishing S as large as S = (5 TeV)^2, Λ_1 corresponds to 10% of fine-tuning if supersymmetry is broken at a low scale, Λ = 100 TeV, like in gauge mediation. Note that the hierarchy among m_\Psi and any other soft mass m_{soft} is not a problem because the induced loop corrections are suppressed by the charges, the gauge couplings, and the loop factor δm_{soft}^2/m_{soft}^2 \sim (m_\Psi^2/m_{soft}^2) \times (q^2 q_\Psi g_s^2)/(16\pi^2)\log\Lambda/f = O(1).

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Finally, we comment here about the superpotential \( W_{3f} \) for the light triplets. As long as it is SU(3) symmetric, its specific form does not really matter for the low-energy physics of the lightest Higgs boson \( H \). The only important question is whether there is sizable tuning hiding in this sector in order to get the VEV \( f \) much smaller than \( F \) at large \( \tan \beta \). Indeed, the effective \( D \) terms (3.5) cannot provide for a quartic coupling when \( H_{u,d} \) point along the third direction of the triplets. However, a sizable quartic coupling (together with a negative quadratic driving the breaking) for \( H_u \) is actually radiatively generated by the Yukawa coupling \( y_z \). The tuning associated to this breaking is milder than 10% if \( y_z \gtrsim 1.2 \) (or even less depending on the scale \( \Lambda \) where the soft terms are generated). Such values of \( y_z \) are small enough to maintain perturbativity up to scales bigger than \( \Lambda = 10^9 \) TeV.

An alternative way to get a sizable quartic for \( H_u \) is to extend the Higgs sector as in [15], introducing symmetric representations \( Z_{u,d} \) of SU(3) getting small VEVs \( f_z \ll f \). While those fields do not modify the \( D \) terms and the tree-level Higgs mass formula, they slightly lower the U(1)\_Landau pole. Keeping the same benchmark values \( \Lambda = 10^3, 10^6, 10^9 \) TeV as above, means that we have to choose a slightly smaller \( g_z \), decreasing in fact the tree level Higgs mass by only of \( 2 \div 3 \) GeV.

**B. Particles within the LHC reach**

The model predicts various new particles potentially within the reach of the LHC. Here, we summarize the main features, which are generically present for a large part of the parameter space. In Fig. 6 we sketch a typical mass spectrum focusing only on the most important properties. The familiar MSSM states can all be light, with soft breaking masses of the sfermions as low as 400 GeV. We are not including most of these states in Fig. 6, but rather focus on the modes present due to the gauge extension. A detailed study of the full mass spectrum varying the input parameters is beyond the scope of this paper.

The enlarged gauge structure leads to the presence of new gauge bosons. Besides the \( W' \) and \( Z' \) with masses of order \( \sim gF \) in the multi-TeV range, there is a \( Z' \) with mass of few TeV, controlled by \( \sim g\phi\bar{\phi} \), \( \Omega \). Its lower bound (from electroweak precision constraints) puts the only serious constraint on \( \Lambda \gtrsim 5 \) TeV.

The best candidate for observing the SU(3) structure is the fermionic “little partner” of the top, \( T_1 \). Its mass is directly linked to the global symmetry breaking scale \( f \), and can be as light as 700 GeV (at which point one may have to start worrying about loop-level electroweak precision constraints). Since \( T_1 \) is even under \( R \) parity (which is assumed throughout the paper), it can be singly produced at the LHC via \( Wb \) fusion or Drell-Yan, likely decaying into \( ht, Zt \) or \( Wb \) [24].

Among the extra fermions there are two SU(2)_W sterile neutrinos \( \nu_s \), per generation from the third component of \( L \).

They arise from the embedding of SU(2)_W singlets inside SU(3)_W triplets rather than SU(3)_W singlets [14], which is the only known generation independent charge assignment for the SM matter content that ensures anomalies cancellation. As a result, these \( \nu_s \)'s are not completely sterile and could in principle be produced at accelerators. The model can be modified by adding more singlets to give Dirac masses to the \( \nu_s \)'s of order \( \sim \mathcal{O}(f) \). Such scalars are already naturally present in the SU(6) embedding discussed in [14].

The Higgs sector is very similar to the MSSM in the decoupling limit and large \( \tan \beta \), with \( m_{\tilde{g},\tilde{b}} \gtrsim 500 \) GeV, where \( \tilde{H} \) are the radial modes of \( \tilde{H} \). It provides a natural framework for incorporating the mechanism of [25] for eliminating the \( \mu - B_\mu \) problem in the context of gauge mediation. The Higgs can be as heavy as 135 GeV due to the tree-level enhancement of the quartic governed by the ratio \( m_{\phi}/\Omega \sim 1 \) in the effective \( D \) terms. Thus, a smaller contribution from the top/stop loop is needed and a quite light stop around \( m_s \sim 400 \) GeV is allowed. Hence, the tuning of generic SUSY theories is drastically reduced here, to levels better than 10%.

The main difference with respect to the MSSM Higgs sector is the presence of an axionlike state \( \eta \) coming from a residual global U(1) symmetry acting on the third components of the triplets \( \tilde{H}_{u,d} \). This symmetry is explicitly broken by the Yukawas, which will generate a mass at two loops. Alternatively, if both \( y_1 \) and \( \tilde{y}_1 \) are switched on (but still with one much smaller than the other to preserve the collective symmetry breaking), a one-loop mass \( \mathcal{O}(1) \) GeV for \( \eta \) is generated, avoiding all the astrophysical constraints.
VI. CONCLUSIONS

We presented an extension of the MSSM, which is free of the usual percent level (or worse) of fine-tuning of the Higgs mass parameters. This insensitivity of the Higgs mass to high scales is achieved via a double protection (or “super-little Higgs”) mechanism, whereby the supersymmetric Higgs is also a pseudo-Goldstone boson of a global symmetry broken around the TeV scale. While such models are in principle very appealing, they have usually suffered from the reduction of the Higgs quartic coupling from the extended sector. The novelty of our model is to use nondecoupling $D$ terms in order to enhance the quartic coupling. For this we enlarge the gauge structure to incorporate an additional U(1) gauge group, where the VEV of the field breaking this symmetry is comparable to the SUSY breaking mass for this field. This will automatically generate an extra contribution to the Higgs quartic coupling. With an appropriate choice of the gauge charges one can achieve that the extra contribution to the Higgs potential does not contain a quadratic term, thus preserving the double protection mechanism.

The particular model we constructed is an extension of the previous super-little Higgs models (or supersymmetrized versions of the simplest little Higgs models) based on the gauge group $SU(3)_W \times U(1)_X \times U(1)_Y$ with an extended global symmetry $SU(3) \times SU(3)$. This model can be extended to include SM fermions in anomaly free generation universal representations.

We have carefully analyzed the Higgs potential of this theory, taking into account the effect of the nondecoupling $D$ terms and the one-loop corrections from the top/stop sector. We have found that it is fairly easy to find regions of the parameter space where the Higgs mass is well above the LEP bound (and could be as heavy as 135 GeV) with a fine-tuning of less than 10%. The MSSM superpartners could be as light as 400 GeV, while $Z^\prime$s dangerous for electroweak precision corrections can be pushed into the multi-TeV regime. The Landau pole of the new $U(1)$ symmetry can be separated from the new physics scales relevant here by several orders of magnitude. The lightest non-MSSM states that carry SM quantum numbers are the baryon or lepton number violating states, which are potentially light sterile neutrinos (which are not completely sterile, since they transform under the extended gauge group). It remains to be seen whether the ideas presented here can be extended to a model incorporating perturbative unification.

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APPENDIX A: THE RADIATIVE HIGGS QUARTIC COUPLING

We report here the full expression for the radiative correction to the Higgs quartic coupling from the top/stop sector, still assuming universal soft masses $m_s$,

$$
\delta \lambda = \frac{3y_t^4}{16\pi^2} \left\{ \ln \left( \frac{m_t^2}{m_s^2} \right) + \frac{2m_t^2 m_s^2 (m_t^2 - 2m_s^2)}{m_t^2 m_s^2 (m_t^2 - m_s^2)^2} \right\} 
\times \ln \left( \frac{m_t^2}{m_s^2 + m_t^2} + \frac{(m_s^2 + m_t^2)^2}{m_t^2 m_s^2 (m_t^2 - m_s^2)^2} \right) 
\times \ln \left( \frac{m_t^2}{m_s^2 + m_t^2} + \frac{(m_s^2 + m_t^2)^2}{m_t^2 m_s^2 (m_t^2 - m_s^2)^2} \right). \quad (A1)
$$

The degenerate limit $m_{T_1} = m_{T_2} = m_T$ is finite and is given in formula (4.8). This expression has been derived using the Coleman-Weinberg potential [23]

$$
\Delta V(H) = \frac{1}{32\pi^2} S \text{Tr} \left\{ M^4|H|^4 \left( \ln \frac{M^2|H|^2}{\Lambda^2} - \frac{3}{2} \right) \right\} = \text{const} + \delta m_{T_1}^2 |H|^2 + \delta \lambda |H|^4 + \ldots, \quad (A2)
$$

where the Higgs field $|H| = |H_u|/\sin\beta$ measures the misalignment between the $SU(3)$ breaking VEVs, $H_u = (0, |H_u|, \sqrt{f^2_u - |H_u|^2})$. To determine $\delta \lambda$ one needs the eigenvalues of the matrix $M$ up to the fourth order in $|H|$. This calculation is reported in Appendix B.

APPENDIX B: PERTURBATION THEORY: THE FOURTH ORDER

Given a diagonal real matrix $H$, organized in blocks of degenerate eigenvalues $E_n$, we determine the approximate eigenvalues of $H + \lambda V$ up to the fourth order in $\lambda$

$$
E_n = E_n^0 + \lambda \Delta_n^{(1)} + \lambda^2 \Delta_n^{(2)} + \lambda^3 \Delta_n^{(3)} + \lambda^4 \Delta_n^{(4)} + \ldots \quad (B1)
$$

Here, $V$ is a Hermitian permutation matrix, which is diagonal inside each block of $H$ (completely removing the degeneracy). Following any standard quantum mechanics textbook, we get

$$
\Delta_n^{(1)} = V_{nn}, \quad (B2)
$$

$$
\Delta_n^{(2)} = \sum_{k \neq n} \frac{V_{nk} V_{kn}}{(E_n^0 - E_k^0)^2}, \quad (B3)
$$
\[ \Delta_n^{(3)} = \sum_{k,j\neq n} \frac{V_{nk}V_{ij}V_{jn}}{(E_0^n - E_k^n)(E_0^n - E_j^n)} - V_{nn} \sum_{k\neq n} \frac{V_{nk}V_{kn}}{(E_0^n - E_k^n)^2}, \]
\[ \Delta_n^{(4)} = \sum_{k,j,l\neq n} \frac{V_{nk}V_{kl}V_{lj}V_{jn}}{(E_0^n - E_k^n)(E_0^n - E_k^n)(E_0^n - E_l^n)} - V_{nn} \sum_{k,l\neq n} \frac{V_{nk}V_{kl}V_{lj}V_{jn}}{(E_0^n - E_k^n)(E_0^n - E_k^n)^2} - V_{nn} \sum_{l\neq n} \frac{V_{nk}V_{kn}}{(E_0^n - E_k^n)^2} \frac{V_{nl}V_{ln}}{(E_0^n - E_l^n)^2}. \]

In order to use this result in the calculation of the Coleman-Weinberg potential, we need to rotate into a basis in which \( V = M^2(|H|) - M^2(0) \) is diagonal in each degenerate block.