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STUDY OF MU-MESONIC X-RAYS : ELEMENTS FROM RHODIUM TO URANIUM

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ABSTRACT

The energies of the μ -mesonic $2p-1s$ transitions and differences of these energies for 19 elements between rhodium and uranium were measured. The apparatus, the experimental procedure and the data analysis are described in detail. The measured $2p-1s$ energies agree with those calculated under the assumptions of a constant maximum density of the nuclear matter and a constant surface thickness for the elements studied. For europium and gadolinium a broadening of the $2p-1s$ transition, probably due to quadrupole interaction, is observed. The K_{α} - and L_{α} -spectra of lead and bismuth were measured especially carefully, and the spin doublet components were analysed for their spacing and intensity ratio.

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1. INTRODUCTION

We report on measurements of the X-ray transition energies of μ -mesonic atoms which have been performed in 1961.

The aim of these measurements was primarily to obtain more and better data for the charge radius of the nuclei throughout the periodic system which can be derived from determinations of the μ -mesonic $2p-1s$ transition energies. Part of the results have been published previously^{1,2,3)}

In the present paper we describe the experimental arrangement and discuss the measuring methods. The final results obtained from the elements between ^{16}S and ^{42}Mo have been published already in a preceding paper⁴⁾ quoted in the following as (I). The results for the elements from ^{45}Rh to ^{83}Bi and partly for ^{92}U are given here, whereas more information about the elements ^{90}Th and ^{92}U will follow in a subsequent paper⁵⁾.

The results of the elements between ^{45}Rh and ^{64}Gd are presented after a description of the evaluation of these spectra. The elements ^{82}Pb and ^{83}Bi which necessitated a special analysis are treated separately.

2. EXPERIMENTAL ARRANGEMENT AND METHOD

Figure 1 shows the experimental layout and a block diagram which are conventional except for the points to be stressed below.

A muon beam of $140\text{ MeV}/c$ was extracted from the μ -channel⁶⁾ of the CERN synchro-cyclotron (SC) and was stopped in a target of the element under investigation. The stopping of a muon in the target was signalled by a coincidence (1, 2, 3, $\bar{4}$). The energy of the X-rays emitted from the target was measured in a $3" \times 3"$ NaI(Tl) crystal viewed by an EMI 9530A photomultiplier. Two power supplies were used, one between the photocathode and the 7th dynode, which was at ground potential, the other between the anode and the 7th dynode. The pulse height analyser (PHA) was opened when a fast pulse from the anode of the

multiplier connected to the NaI crystal was in coincidence with the signal originating from a stopped muon. The linear signal was taken from the 7th dynode of the photomultiplier, amplified in a bipolar pulse shaping amplifier (Model Laben G40), and analysed by a PHA Model Laben C31. The rate of coincidences (1, 2) was used as a monitor. The coincidences (1, 2, 3, 4, NaI), as well as the number of events accepted by the PHA, were registered.

The X-rays and the calibration lines have to be measured under conditions as identical as possible, in order to avoid or minimize corrections due to the following points which have been considered carefully: Effects due to the overloading of the linear line, the linearity of the system, stability of the gain in time and under changes of the beam intensity, and the proper timing of the counters. We will then discuss possible distortions of the spectra measured, due to a structure of the background as well as the fact that for the calibrations spectra of radioactive γ -sources have to be compared with X-ray spectra.

2.1 Overloading Problems - Muon Beam

The pure and intense muon beam available from the CERN μ -channel contributed essentially to the solution of the problems mentioned, particularly to the overloading problems. All pulses occurring in the NaI crystal cause a wrong measurement of the pulse height if they arrive too soon before the X-ray event to be measured. Since the longest time span needed in the linear amplifier was $\sim 2.5 \mu\text{sec}$, the single counting rate in the NaI crystal had to be kept sufficiently below $4 \cdot 10^5/\text{s}$ in any instant.

The vibrating target⁷⁾ produced a favourable duty cycle of about $\frac{1}{3}$, the repetition rate being 54/s. Figure 2 shows a measured time distribution of particles producing a coincidence (1, 2). The proper adjustment of the vibrating target was checked before and during every run on a scope where integrated counter pulses were displayed.

The background originating from the general radiation near the SC which can be reduced by proper shielding proved to be negligible. The background radiation directly associated with the beam mainly comes from neutrons and γ -rays following meson capture. It was reduced by using a pure muon beam

of a narrow momentum band. Pions having the same momentum and being captured in the absorber may contribute to the background as well as the Bremsstrahlung produced by the electron contamination.

A typical range curve is shown in fig.3 . The number of incoming particles, of which at least 98% are muons⁸⁾, is about $2-3 \cdot 10^4/s$. Of these, about 2000/s could be stopped in the target. The targets consisted of discs (8 cm diameter, 5 g/cm² thick). In the cases where the elements were available as powder or grains and could not be melted, they were pressed into thin plexiglas boxes to have the same diameter and weight. Only the rhodium powder filled a box of 5 cm diameter, which was centred in an Al disc of the standard size in order to simulate the normal geometry. The number of single counts in the NaI crystal above about 200 keV was measured. For the least favourable running conditions the contribution from the beam was less than 12.000 events/s, leading to an instantaneous counting rate of about 36.000/s and an additional steady counting rate from the calibration γ -source of about 6000/s. The resulting total counting rate amounted therefore to about 10% of the $4 \cdot 10^5/s$ mentioned above. Since most spectra have been taken, however, with a considerably smaller load on the NaI crystal, e.g. a half or third of this, pile up was negligible. As a check, some measurements have been made with the NaI crystal in the beam axis and below the beam (see fig.1). No significant change of the energy has been seen, e.g. for Fe less than 2 keV.⁴⁾

2.2 Linearity

The energy scale was set to cover a convenient energy range which contained at least two well known γ -lines. The γ -sources which have been used are listed in table 1, sources marked with C could be measured in coincidence. Between the calibration lines, interpolation was made with a mercury pulser of 1‰ linearity. The remaining non-linearity has been measured by H. Artus⁹⁾ to be less than 2‰ over the energy range from 700 keV to 1400 keV.

2.3 Gain Stability

The stability of the gain has to be considered in its two aspects. Firstly, drifts should be small during the time necessary to perform a complete measurement cycle. Secondly, the effect caused by changes in the counting rate should be known and should give rise only to a small correction.

The gain stability in time was checked by recording the calibration lines immediately before and after each X-ray measurement. We have concentrated on those types of measurements which mainly require the comparison of a nearby γ - or X-ray line. In order to reduce further an effect of drifts during these measurements, the two spectra to be compared (X-ray spectrum and γ -line of radio-isotope or two X-ray spectra) have been measured in an interchange, the period of which was small compared with the total measuring time. E.g., the two X-ray targets were fixed on flags normally positioned left and right of the beam. By remote control they could be flipped alternately into the same position in the beam. The total measuring time required is given by the statistical error aimed at. A number of 500 events, e.g., registered in a Gaussian shaped line with 5 channels half width, allows to determine the centre of gravity to 0.1 channel. Always more than 2000 events were accumulated in the photopeak which took about 10-15 min, while the time between interchanges was about 1 min. Figure 5 shows an example. The two X-ray spectra were recorded in the two parts of the PHA memory, whereas the linear line remained absolutely unchanged. This method of comparing two almost identical spectra was used for three different purposes:

- a) to compare a mesonic X-ray spectrum with a γ -ray spectrum,
- b) to compare two different mesonic X-ray spectra,
- c) for controls, e.g. measuring the energy of a γ -ray when the SC burst is present or absent (see below).

The dependence of the electronic gain on changes in the counting rate was measured⁹⁾ by the following method. The signal of a mercury pulse generator (50 pulses/s) was mixed to the inputs of the spectrometer pre-amplifier and the fast coincidence. For additional counting rates from a

γ - source increasing from 0/s to 5000/s, the overall change was mostly due to a change in the PHA bias level leading to a decrease of about 0.5 channel, or about 4‰. With the muon beam switched on, the recorded pulse height was 0.1 - 0.2 channel lower than without beam, the counting rates being 7000/s and 3000/s respectively. Changes in the amplifying parts were normally less than 2‰. However, the actual measurements took place under conditions such that these shifts were considerably smaller.

The most sensitive and direct test to check whether a shift in line position due to changing pulse rates occurred was made as follows. A suitable radioisotope emitting two γ -rays in coincidence was placed near the NaI crystal next to an additional scintillator (P in fig.1). The spectrum produced on the PHA by a coincident event (P, NaI) was observed without the SC in operation. Then the SC was switched on and the γ -ray spectrum was recorded again, but only during the time the SC bursts arrived. A comparison of the two spectra showed that shifts between 0 and 1 channel occurred varying with the experimental conditions (i.e., range curve, counting rate, etc.). The time constant involved in this shift is supposed to be smaller than the time between SC bursts. This shift was measured in each run and corrections were applied where necessary. For many X-ray measurements, however, this shift could be eliminated practically completely by measuring the reference γ -line in the presence of the muon beam. The only difference was that the NaI crystal, in one case, was put in coincidence with the additional counter (P) and in the other case with the counter telescope. Therefore, we preferred as energy references γ -rays emitted in coincidence with another γ -ray whenever such γ -rays were available. The X-ray lines neighbouring these γ -lines are measured with reduced systematic errors (see I).

2.4 Timing

The proper adjustment of the coincidence requirements plays an important rôle. A coincidence circuit with a high resolving power would best suppress accidental coincidences. However, the arrival time of the fast pulses from the NaI crystal depends somewhat on their pulse height,

and prevents therefore too high a resolution. This can be seen in fig.4 where the coincidence rate (1, 2, 3, $\bar{4}$, NaI) is shown as a function of the delay between the telescope and the NaI counter for the mesonic K_{α} -X-rays of ^{40}K , ^{42}Mo and ^{82}Pb ; these have been selected by the PHA. The 6 MeV X-rays from the 2p-1s transition in ^{82}Pb triggered the coincidence about 25 ns earlier than the pulses of 0.7 MeV from ^{40}K , whereas the resolution time of the coincidence circuit was about 35 ns. Therefore, for each energy range the proper timing curve was determined and the delay of the NaI counter was set to the middle of the plateau. For comparison, the timing curve between the NaI crystal and the counter P, using a radioactive source emitting two γ -rays in coincidence is also shown.

2.5 Background

The two most important contributions to the background (see 2.1) can be seen from fig. 4. The time delay for the region indicated by "a" is such that prompt events, i.e. γ -quanta simultaneously produced with the stopping of a muon, are registered. The delay curve appears to be asymmetric. There are considerably more counts in region "b" than in region "c". The fact that in region "c" the counting rate is small shows that almost no accidental events due to general background occur. This proves that the efficiency of the anticoincidence counter measured to be $\sim 10^{-3}$ and the general shielding are quite adequate. The purely accidental background had no structure. It may cause trouble mainly by loading the NaI crystal which effect was discussed in 2.1.

The tail of the curve "b" is due to delayed γ -rays following muon capture. The gamma rays originating from the de-excitation of the nuclei which have captured a muon are emitted after capture took place. Therefore they are distributed exponentially with a capture time¹⁰⁾ of 540 ns for ^{32}S and 88 ns for ^{92}U . Since the time resolution of the coincidence amounted to ~ 35 ns for the NaI pulse and could not be reduced (see 2.4), a noticeable contribution of capture γ -rays in the X-ray spectra was unavoidable. By changing the delay of the coincidence signal from the NaI counter appropriately one can, however, suppress coincidences

with the promptly produced X-rays and we have thus taken systematically the background spectra for all elements studied. For example, the background spectrum to the μ -Bi 3d-2p transition (fig. 12) may be considered, which was measured by decreasing the delay in the NaI line by 50 ns. The capture γ -ray spectra are shown in fig. 6 for the elements $_{45}\text{Rh}$ to $_{64}\text{Gd}$. The arrows indicate the position of the 2p-1s X-ray transition. No disturbing structure of the background occurs in the vicinity of these transitions. The background spectra not displayed did not show any structure. For the light elements the spectra are given in (I).

2.6 Comparison between X-ray and γ -ray Sources

Special care was taken to make sure that differences in geometry, especially the behaviour of a γ -ray point source and an extended X-ray source did not introduce errors. This was checked⁹⁾ by a comparison of a point source of Na^{22} ($E_{\gamma} = 1275 \text{ keV}$) the energy of which is close to the 2p-1s energy in μ -Fe, with a Na^{22} source distributed uniformly between

two iron plates which simulated a μ -Fe X-ray target. Similarly a ThC'' point source ($E_{\gamma} = 2615 \text{ keV}$) was compared with a source distributed between two plates of molybdenum. For 1275 keV an energy decreased by $(0.45 \pm 0.25)\%$ was measured whereas for the higher energy (2615 keV) the measurements revealed with $(-0.25 \pm 2.2)\%$ no significant shift. No corrections had to be applied but these shifts have been included in the systematic errors of the transition energies measured.

An inherent difficulty originates from the fact that the 2p-1s X-ray lines have a fine structure. Only for the heaviest elements (Pb, Bi) could this be resolved. For the other elements a systematic error in determining the centre of gravity of a line must be included which was of the order of 0.2 channel (see 3.1 below, and (I)).

3. RESULTS

In this chapter, the evaluation of the data is described and the results are presented. Firstly, the results are discussed which have been obtained for the elements between $_{45}\text{Rh}$ and $_{64}\text{Gd}$ where a fine structure cannot be resolved and the centre of gravity of the $2p-1s$ transitions is the only accessible quantity. Later the heavier elements $_{82}\text{Pb} - _{92}\text{U}$ are dealt with in more detail.

3.1 Elements from $_{45}\text{Rh}$ to $_{64}\text{Gd}$

Contrary to the measurements on lighter elements (see I), only two γ -rays suitable for calibration are available between the K_{α} -energy of $_{45}\text{Rh}$ of 2.98 MeV and the K_{α} -energy of $_{64}\text{Gd}$ of 4.53 MeV. Essentially the entire energy region had to be calibrated by the γ -energy of Na^{24} at 2.753 MeV and a PoBe source at 4.433 MeV, of which the latter could not be measured in coincidence. Therefore, the more accurate measurements of the transition energies against nearby coincident γ -rays as described in section 2.3, were not possible. Furthermore, at these energies the spectrum produced by a single γ -line in the NaI crystal is not a simple line as can be seen from fig. 7, where the K-line of Cd is shown as a typical example. Since the absorption by pair production gets more important, escape peaks appear of which the first one can be seen 511 keV below the photo line (Fig. 7). The spectrum is more difficult to evaluate. Thus the accuracy reached is necessarily less than for the light elements.

3.1.1 Evaluation

Essentially two methods have been used to determine the position of the X-ray lines from the spectra obtained.

- i) The simplest method was to determine the centre of gravity of the line graphically. A large scale drawing of the spectrum has been made in order to separate out the photo peak. This was done by extrapolating the background above the photo peak and the Compton edge linearly into the region underneath the photo peak. The centre of gravity of the remaining photo peak has been determined analytically. By plotting some of these photo lines on probability paper, it was checked that the lines

had a Gaussian shape. This graphical method was useful to investigate how sensitive the centre of gravity is to slight changes in the subtraction of the background. It was the only method suitable also in those cases where the assumptions of the following method were not given, e.g., when comparing γ -lines of rather different energies as in establishing the energy calibration.

- ii) For the evaluation of the energy differences of elements with $\Delta Z = 1$ or 2, which are of the order of 80-90 keV per $\Delta Z = 1$, it was assumed that the shape of the lines is identical. The distance v between two of these spectra was calculated on a computer, by demanding

$$\sum_k [N_1(k) - wN_2(k-v)]^2 = \text{Min} \quad (1)$$

$N_1(k)$ is the number of events in channel k of spectrum 1 and $N_2(k-v)$ is the number of events in channel $k-v$ of spectrum 2. $w = \sum_k N_1(k) / \sum_k N_2(k)$ is the normalization factor.

a) On an IBM 704 computer, v was calculated from the solution of (1) using only the regions of the two photo lines. For this we put $dN/dk = N_i(k+1) - N_i(k)$ and utilized both spectra ($i = 1$ and $i = 2$). Constant numbers of counts per channel have been subtracted in both spectra. These constant numbers were treated as a parameter in the programme.

b) Between the elements Rh and Te and for Pb and Bi, the following method has been applied exclusively and for the other elements partly to check method a): A programme for the CERN Mercury computer was written which calculated expression (1) by using not only the photo peak, but the whole spectrum down to about 1.4 MeV below the photo peak. The background was not subtracted, since its variations from element to element are small. In the vicinity of the minimum of eq. (1), spectrum 2 was displaced in steps of 1/10 channel with respect to spectrum 1.

The rates at fractional channels were interpolated linearly between the full channels. In order to avoid errors due to linear interpolation, the same procedure was repeated three times, each time spectrum 1 having been displaced by 1/4 channel ($N_1(k)$ of eq.(1) is replaced by $N_1(k - 0.25)$). An example is displayed in fig.8, where the left side of expression (1) is plotted as a function of v for the case of the K_α -lines of Pb and Bi.

The error in finding the best value of v depends on the statistics as well as on the separation of the two lines. It may also be derived from a comparison between distances obtained which are not independent, e.g., Ag-Cd, Ag-In, Cd-In and from the deviations between the distances obtained by method 1) and 2). A conservatively stated error of ± 0.1 channel corresponding to about 2.3 keV is used in the following.

3.1.2 Results

Table 2 gives the measured differences of 2p-1s transition energies. The error due to the calibration sources Na^{24} and PoBe amounted to 1‰ (see table 1). The non-linearity has been corrected for by using a pulser, and the remaining error is less than 1‰. The error in determining the centre of gravity of the line was estimated above to about 0.1 channel. So the error ϵ_i of a single measurement was calculated by adding these contributions quadratically. In a typical case (e.g., Ag-Cd) it is about 5 keV. In column 3, the arithmetic mean and the error σ of the mean is given. The latter was calculated somewhat arbitrarily by adding to the deviations from the mean energy $|x_i - \bar{x}|$ the above estimated error ϵ_i for a single measurement by forming

$$\sigma = \sqrt{\frac{\sum_{i=1}^n (|x_i - \bar{x}| + \epsilon_i)^2}{n(n-1)}} \quad (2)$$

Quite often differences of elements with $\Delta Z > 1$ have been measured. Such measurements yield also indirect information about the $\Delta Z = 1$ differences. A weighted average is given in column 4 of the table, where the weight was inversely proportional to $\Sigma |\Delta Z|$ for the differences used to obtain the $\Delta Z = 1$ difference in question.

The energies of the 2p-1s transitions are subject to additional errors. Particularly the higher transitions 3p-1s (K_β) and np-1s will produce escape peaks in the vicinity of the 2p-1s photopeak which may introduce a systematic error in the comparison of the X-ray line and the calibration γ -line. No corrections were applied but a contribution of 2‰ to the systematic error was taken into account. The evaluation was based on method 1) (see, however, ref.3)).

In table 3, the 2p-1s energies are listed. The errors were mainly determined by possible systematic errors which amount to $\pm 1\%$ due to the error of the non-linearity correction, $\pm 5\%$ due to the uncertainty of the correction for the dependence of the gain on rate changes, $\pm 2\%$ due to the evaluation of the centre of gravity of the line, $\pm 1\%$ due to the error of the calibration lines, $\pm 2\%$ due to the error in correcting amplification changes. These contributions, quadratically added, yielded a systematic error of $\pm 0.6\%$. To be on the safe side, this error for a single measurement was again added quadratically to the observed variance (cf. eq.(2)) to obtain the values given in column 3 of table 3.

Furthermore, the complete sequence of differences of adjacent elements (table 2) measured with a considerably smaller absolute error allows to add up these differences to obtain other values for the transition energies, which are listed in column 4. The errors noted there are the quadratically added errors of all the differences involved. Since these two methods of determining the energies are not independent - the same spectra and the same calibrations have been used - the error is not reduced and the data have not be combined. Therefore the values of column 3 are to be considered as our final results. The reasonable agreement of the two methods is very gratifying and suggests the conclusion that the systematic errors predominant in column 3, and the evaluation errors mainly responsible for the errors in column 4, are not under-estimated.

In the region $Z > 59$ where the nuclei have big static quadrupole moments, the question arises whether quadrupole splitting can be observed. A rough inspection of these spectra shows that no fine structure can be resolved. As an example the spectrum of ^{64}Gd is shown in fig. 9. However, we believe to have seen a broadening of the lines of ^{63}Eu and ^{64}Gd . The Eu spectrum has been compared (see 2.3) with the Ba spectrum as a standard in order to make sure that no broadening of the lines was introduced by the apparatus.

In fig. 10 the widths of the photo lines produced by γ -rays in the energy region between 0.5 MeV and 6 MeV are plotted. They lie on a straight line in the log-log scale, except for the γ -rays of PoBe (C^{12*}). Here, however, a Doppler broadening²²⁾ may be the reason. Above about 3 MeV the line widths of the mesonic transition begin to increase systematically beyond the line width of the γ -lines. The effect amounts to about 10-15% at 4.2 MeV, the K_{α} energy for ^{59}Pr , i.e. about 30 keV. The fine structure splitting in this region amounts to about 90 keV explaining thus the observed increase in line width. From these observations one would expect a line width for ^{63}Eu and ^{64}Gd increased with respect to a γ -line of the same energy by about 30 keV. However, an increase of about 100 keV was observed. It is tempting to attribute this to the big quadrupole moments of these two elements.

3.2 The Elements ^{82}Pb and ^{83}Bi

For heavy elements, the mesonic 1s - and 2p-levels are more influenced by the extended nuclear charge, and in addition the relative line width of the NaI(Tl) spectrometer becomes better with higher energies (see fig. 10). With a resolution of about 5% at 2.7 MeV and 3% at 6 MeV our spectra show the fine structure of the μ -mesonic L_{α} - and K_{α} - lines for Pb and Bi. Unfortunately the analysis of the γ -spectra in the range of 6 MeV is even more difficult than in the 3 or 4 MeV region. This may be seen from fig. 14 which shows a spectrum of the γ -radiation of N^{16} which has no fine structure and was used as a reference line.

a) The μ - Pb Spectra

The 3d - 2p transition in Pb was measured five times in three different runs. The target thickness used was 5 g/cm². A typical spectrum is shown in fig. 11. The 2753.5 keV γ -line of Na²⁴ was used as direct reference as described in section 2.3. The energy scale of the spectrometer was calibrated by the γ -lines of Na²⁴, ThC'' and Y⁸⁸.

To determine the energy of the 2p - 1s transition in Pb, two measurements with a target of 5 g/cm² and one measurement with a target of 10 g/cm² were performed in one run. The 6135 keV γ -line of N¹⁶ was used as reference, the energy scale was fixed by this line and the 4433 keV γ -line of a PoBe source. These measurements took place in frequent interchanges of about 1 minute for the recording of the Pb- and the N¹⁶-spectra, the muon channel being blocked for the N¹⁶-recording. We have also measured the 2p - 1s transition in Pb in an interchange with the 2p - 1s transition in Bi. This will be discussed below.

Each spectrum contained about 20 000 events in the range between 4 and 6.3 MeV which were collected in about 25 min. Since from the N¹⁶ spectra no drift in the spectrometer could be detected (i.e. less than .2 channel), these four spectra have been added before evaluation. The spectrum is shown in fig. 13.

The N¹⁶ - source was produced by irradiating water by neutrons ($O^{16}(n, p) N^{16}$) inside the cyclotron. A container with a piece of uranium in front, 1 cm thick, was placed near the vibrating target. Water was pressed through and was flowing in about 15 s to a source box in front of the NaI spectrometer. The source box had the same diameter as the Pb target. The N¹⁶ decays by β - emission with a half life of 7.3 s to O¹⁶; 68% of the decays lead to the 6135 keV level and 5% to the 7.12 MeV excited state of O¹⁶ producing the corresponding γ - rays. The 7.12 MeV γ -line was subtracted from the measured spectrum in order to obtain the pure 6135 keV γ -spectrum. Figure 14 shows the two spectra.

Since the N^{16} γ -ray could not be measured in coincidence, the energy shift due to changing counting rates was checked separately. Therefore, two N^{16} - spectra were recorded by interchanging in cycles of 1 min in the following two conditions: either the muon beam was blocked, the rate of pulses in the spectrometer corresponding to more than 200 keV amounted to about 5000/s; or the muon beam was in the usual way hitting a 5 g/cm² Pb target. The contribution from N^{16} remained the same as above, but the counting rate due to the beam increased during its time of arrival by 9000/s. With the high counting rate of 14000/s, the N^{16} - spectrum was 0.38 ± 0.20 channels higher than measured at the low rate of 5000/s. Since in the actual measurements the counting rate changed approximately from 5000/s to 9000/s, a correction of (-0.25 ± 0.20) channel corresponding to (11 ± 9) keV has been applied.

The evaluation of the K_{α} - as well as the L_{α} - spectrum was done by comparing them with an analogue spectrum obtained from the reference spectrum, the N^{16} γ -line or the Na^{24} γ -line, respectively. The analogue spectrum was constructed by superposing two reference γ -spectra using their energy separation and intensity ratio as variables to simulate the fine structure doublet of the mesonic spectrum. Besides these two parameters, the energy difference of the reference γ -line to the $2p_{3/2}$ - 1s component of the mesonic doublet was to be determined. It was assumed that the reference spectrum and each component of the mesonic spectrum has the same shape, whereas the background could be different. From the reference line, a background was subtracted which was assumed to be a smooth function of the energy and, therefore, was extrapolated from the high-energy side of the spectrum to the Compton continuum. To the analogue spectrum a background was added to give a good fit to the mesonic spectrum at the high-energy side as well as at the region of the Compton continuum. An exponential and a parabolic shape were tried for this background, since it was checked experimentally that no structure due to capture γ -lines appeared (see section 2.5, figs. 11 and 13).

The mesonic L_{α} - spectrum consists of 3 components, the transitions $3d_{5/2} - 2p_{3/2}$, $3d_{3/2} - 2p_{3/2}$ and $3d_{3/2} - 2p_{1/2}$ with an intensity ratio of 9 : 1 : 5 which is based on a population of the states proportional to the statistical weight $2j + 1$. The two transitions $3d_{5/2} - 2p_{3/2}$ and $3d_{3/2} - 2p_{3/2}$ could not be separated. We therefore treated those two transitions as one single component, the centre of gravity of which is 4.3 keV lower than the pure $3d_{5/2} - 2p_{3/2}$ transition¹¹).

The comparison of the measured with the constructed spectrum was done with the help of the CERN Mercury computer by computing the expression

$$\sum_i [X(i) - W[Y(i + \epsilon) + AY(i + \epsilon + \delta)] + \gamma(i)]^2 = \text{Min} \quad (3)$$

where $X(i)$ and $Y(k)$ denote the number of counts in the i^{th} and k^{th} channel of the mesonic and the reference spectrum, respectively. A is the intensity ratio of the two components of the constructed spectrum, and ϵ and $\epsilon + \delta$ the displacement of the reference spectrum to fit the two components of the mesonic spectrum. $\gamma(i)$ represent the background counts mentioned above, and W is the normalization factor (as in eq. 1). The parameters ϵ , δ , A and the two parameters fixing $\gamma(i)$ were varied systematically to find the minimum of the expression (3). The latter should be of the order of

$$\sum_i [X(i) + W^2 Y(i + \epsilon) + W^2 A^2 Y(i + \epsilon + \delta) + \gamma(i)] \quad (4)$$

since the statistical uncertainties of each number $N(i)$ is $\sqrt{N(i)}$. This was used as a control whether the procedure finds the correct minimum. The quantities δ and ϵ were varied by fractions of a channel. The $X(i)$, $Y(i)$ were calculated in steps of 0.1 channel by linear interpolation between adjacent channels. For the quantity A steps of 5% around the statistically expected $(2j + 1)$ value have been used.

The errors of the parameters ϵ , δ and A can be estimated from the width of the minimum of (3) as a function of the various parameters. For the summed up Pb K-spectrum we assume for the quantities ϵ and $\epsilon + \delta$ an error of ± 0.1 channel since the sum of the squares increases at these limits by about 10%. The quantity A was estimated to be accurate within $\pm 5\%$. For the L_{α} -spectrum the statistics of a single spectrum, each containing 10 000 to 20 000 counts in the energy range between 1.8 and 3 MeV, was worse than for the evaluated K_{α} -spectrum, but there are 5 independent measurements taken in 3 different runs as shown in table 4. Therefore we took in this case the mean values of the parameters found. A 5% correction has been applied to A , caused by the different photo efficiency of the two components. To the errors mentioned, the errors due to the energy scale, the reference γ -line and in the case of the K_{α} -spectrum, those due to the burst shift have been added quadratically. The results are listed in table 4.

b) The μ -Bi spectra

The 2p-1s spectra of μ -Bi and μ -Pb were measured in interchange in three runs. One pair of these spectra is shown in fig. 15. The same target thickness was used for both elements (5 g/cm^2). The investigation of the background spectra revealed a flat behaviour in both cases. One of the Bi-spectra was subject to the analysis described already for Pb. Results are given in table 4.

Within the errors stated, one obtains the same parameters for the Pb- and the Bi-spectra. Therefore, it is justified to apply the difference programme described under 2b in section 3.1.1, which gives smaller limits for the energy difference of the two spectra than the comparison with the constructed analogue spectrum. We obtained $(62 \pm 5) \text{ keV}$ for the energy difference (table 4), which represents the distance between the centres of gravity of the 2p-1s transitions.

The corresponding comparison was made between the 3d-2p transitions of Bi and Pb. Here the background spectrum of Bi showed clearly a capture γ -line in the energy region of interest at 2615 keV which can be attributed to the process

the γ having an energy of 2615 keV.

Figure 12 shows the L_{α} -spectrum of Bi beside the corresponding Pb-spectrum. The capture γ -line lies between the two main components of the L_{α} -line. To establish the energy difference between the L_{α} -spectra of Bi and Pb, it was assumed that the distances between, and the intensities of, the components of the L_{α} -lines are equal. Also the contributions to the L-lines caused by the Compton continuum of the K-lines can be assumed to be equal. A slightly modified difference programme as described in 3.1.1,2b was applied, which included a further parameter to vary the Bi background. The solid line in Fig. 12 shows the best fit found. An energy difference for the centre of gravity of the two 3d-2p transitions of 66.5 keV was obtained.

The value found for expression 4 was $53 \cdot 10^3$ compared with an expected value of $(52 \pm 5) 10^3$. This seems to justify the assumption of equal fine structure splitting and equal intensity ratios. The error for the best set of values is influenced by the linear interpolation between two adjacent channels used to obtain fractional channels, which is a bad approximation near the peak of the line. One can check this effect by using various linearly interpolated values also for the reference spectrum (Pb). In this way it could be shown that the energy difference could be determined to ± 0.1 channel. The result is given in table 4.

3.3 The Element $_{92}\text{U}$

From a rough inspection of the spectrum shown in fig. 16, one sees that its structure is different from the spectra obtained for $_{82}\text{Pb}$ or $_{83}\text{Bi}$. It cannot be understood by assuming two components only. Falling short of a more detailed discussion to be published separately⁵⁾, only the centre of gravity of the spectrum will be discussed here.

It was obtained by comparison with the Pb - spectrum. Treating the two spectra identically in the way the background is subtracted and neglecting changes in line shape due to the different energy, the centre of gravity of the U - spectrum turned out to be 348 keV above the one for the Pb - spectrum. Two different prescriptions for the background subtraction changed this result by less than 1 keV. The method was checked further by applying it to the Pb- and the N^{16} - spectrum. These two spectra are at least as different as the Pb- and U-spectra. This rough graphical method yields for the centre of gravity a value 10 keV higher than the one calculated from the results obtained in section 3.2 and table 4. Therefore, we conclude for the centre of gravity of the 2p-1s transition of ${}_{92}\text{U}$

$$E(2p - 1s) = (6288 \pm 40) \text{ keV} .$$

4. CONCLUSION

The data presented in section 3 can be discussed in various respects. The energies of the 2p-1s transitions of all elements should be interpreted in terms of one parameter of the size of the nuclear charge distribution. For the strongly deformed nuclei Eu, Gd and U, such a straight-forward interpretation will be difficult since quadrupole interactions may enter the picture. For the heaviest undeformed nuclei (Pb, Bi) the spin doublet is partly resolved and therefore also the intensity ratio of the two components can be discussed.

The 2p-1s transition energies have been used to fix the equivalent nuclear charge radius $R_{eq} = \sqrt{\frac{5}{3} \langle r^2 \rangle}$ as described in (I). In fig. 13 of (I) the R_{eq} of the elements presented in this part of the work are already shown. In table 5 the values of $R_{eq}/A^{1/3}$, based on the data of table 3,

column 3, are given for elements ^{*)} between $_{45}\text{Rh}$ and $_{83}\text{Bi}$ as well as the values $\delta R_{\text{eq}}/A^{1/3}$ which denote the amount by which the value chosen by Ford and Wills¹²⁾ must be changed to get agreement with our measured energies. In table 5, the deformed nuclei $_{63}\text{Eu}$, $_{64}\text{Gd}$ and $_{92}\text{U}$ have been omitted.

Comparison with existing X-ray measurements can be made only for $_{50}\text{Sn}$ where a measurement by Anderson et al¹³⁾ is 3.6 keV lower, and for $_{82}\text{Pb}$ and $_{83}\text{Bi}$ where measurements by Frati and Rainwater¹⁴⁾ are lower by 12 keV and 11 keV, respectively, than our $2p_{3/2} - 1s$ energies. The Sn values agree well within the errors quoted, also the $3d_{5/2} - 2p_{3/2}$ transitions of Pb and Bi. The error of the $2p_{3/2} - 1s$ energy is stated in ref. 14) to be ± 11 and ± 9 keV for Pb and Bi, respectively, leading only to a partial overlapping of the errors. As can be seen from fig. 13 in (I), the agreement in the general trend between the μ -mesonic X-ray data, the electron scattering data¹⁵⁾ and the smooth curve deduced from the "Elton" parameters¹⁶⁾ is remarkable. Only between $45 < Z < 50$ a slightly increased R_{eq} might be seen. Also the agreement with the values calculated by Ford and Wills can be seen from the δR_{eq} values of table 5.

*) New calculations have been made using the computer programme of Ford and Wills for the following elements (symbols as in reference 12):

Z	45	46	49	52	53	55	58	59
$r_1/A^{1/3}$ [10^{-13} cm]	1.092	1.080	1.080	1.086	1.087	1.089	1.090	1.091
n	7.1	7.1	7.35	7.36	7.58	7.80	7.70	7.70
$R_{\text{eq}}/A^{1/3}$ [10^{-13} cm]	1.240	1.227	1.217	1.224	1.217	1.212	1.216	1.218
$E(2p_{3/2} - 1s)$ [keV]	2995	3099	3384	3620	3736	3916	4172	4266
2p splitting [keV]	32.1	34.6	42.9	51.6	55.2	62.2	73.0	77.1
1s state [keV]	4427	4596	5082	5531	5720	6051	6543	6719

The energies quoted are for the finite nucleus and include the vacuum polarization correction. For A we used the atomic weight. - For $_{82}\text{Pb}$ and $_{83}\text{Bi}$ the set of parameters $[r_1/A^{1/3}, n; R_{\text{eq}}/A^{1/3}]$ chosen range from $[1.11, 10.0; 1.1869]$ to $[1.090, 7.5; 1.2231]$, and from $[1.095, 9.5; 1.1790]$ to $[1.090, 7.0; 1.2423]$, respectively.

The energy differences between neighbouring elements have been measured directly (table 2) and are therefore more accurate than the differences calculated from the energies themselves. This was done in order to look for unusual changes between neighbouring elements. Greiner and Scheck¹⁷⁾ have calculated these energy differences based on the single particle shell model state of the added proton and have compared them with our measurements. To facilitate a further comparison of the measured energy differences, two columns have been added in table 2. In column 5 the energy differences are given as calculated for a smooth variation of nuclear radius with the mass number A as given in reference 16) and also shown in (I), fig. 13 and eq. 15. Similar values can be taken from the work of Pustovalov¹⁸⁾. In column 6 the energy difference was calculated assuming that the charge radius of the heavier element is the same as for the lighter one. The fact that these values are greater than the measured energy differences means that the increase of the charge radius due to the few, in some cases only one, nucleons added can be clearly observed. The values of column 5 indicate that the actual increase of the nuclear radii is within our accuracy described satisfactorily by the assumption of a smoothly varying nuclear size.

That a structure occurs at $Z \geq 63$ can be seen from fig. 10. The increase of the measured line width compared with the expected line width taking into account only the spin doublet splitting is equal to zero within an error of ± 20 keV for the elements ^{56}Ba , ^{57}La and ^{59}Pr . The increase amounts to (70^{+30}_{-20}) keV for the elements ^{63}Eu and ^{64}Gd . The intrinsic quadrupole moments, deduced from the spectroscopic ones, of ^{56}Ba , ^{57}La , ^{59}Pr , ^{63}Eu and ^{64}Gd , averaged over the isotopes, are about 1.0^* , 0.6 , 0.2 , 4.8 and $7.0^*) \times 10^{-24} \text{ cm}^2$ respectively. Therefore, from the correlation between line width and quadrupole moment, one can conclude that quadrupole effects are just noticeable in this region of the periodic system. In this connection we want to point out that also the $2p-1s$ energy difference between Eu and Gd is anomalously small (cf. table 2). Further measurements would be useful. The presence of quadrupole effects in heavier elements (^{90}Th , ^{92}U) will be discussed elsewhere⁵⁾.

*) For Ba and Gd only two isotopes with abundances of 18% and 31% respectively have $I \geq 1$ and have been measured.

The transition energies of the heavy undeformed nuclei can also be compared with the calculations by Ford and Wills¹²⁾ and Hill and Ford¹¹⁾. For $_{82}\text{Pb}$ agreement is found for the $2p_{3/2} - 1s$ transitions ($E_{\text{exp}} = 6002 \pm 16$ keV, $E_{\text{calc}} = 6023 \pm 50$ keV). Furthermore, the data for the $3d_{5/2} - 2p_{3/2}$ transitions ($E_{\text{exp}} = 2500 \pm 4$ keV, $E_{\text{calc}} = 2502$ keV) and the 2p splitting show that the shape of the charge distribution now commonly used¹¹⁾ does not contradict our data. It may be noted, however, that the experimental errors now have reached the limit of the theoretical uncertainties^{11, 22)}.

For $_{83}\text{Bi}$ our measurements do not agree with the calculations of Ford and Wills¹²⁾ based on the charge distribution resulting from electron scattering. This can be seen most clearly from the energy difference between the $2p - 1s$ transitions in Pb and Bi. The calculations yield $E(\text{Bi}) - E(\text{Pb}) = -30$ keV compared with our measurements of $+(62 \pm 5)$ keV. Ford and Wills use a "family II" charge distribution. A Fermi distribution as used by Crannell et al¹⁵⁾, was not fully consistent with the electron scattering data. The calculations of Greiner and Scheck¹⁷⁾ and Marshall and Scheck¹⁹⁾ however, yield $E(\text{Bi}) - E(\text{Pb}) = 60$ keV, in excellent agreement with our data. Recent calculations by Clark, Herman and Ravenhall²⁰⁾ who used various charge distributions compatible with the electron scattering experiments¹⁵⁾ give values for the $2p - 1s$ transition between 5984 keV and 6125 keV. Our value of (6062 ± 20) keV provides therefore additional information suitable to reduce the choice of charge distributions. The same calculations yield energies for the $3d - 2p$ transition between 2537 keV and 2552 keV. Our value of (2566 ± 7) keV may indicate that the charge distributions used for Bi so far have to be slightly modified.

For the deformed $_{92}\text{U}^{238}$ nucleus our value of the centre of gravity of the K_{α} - spectrum is (6288 ± 40) keV which disagrees with the 6457 keV found by Ford and Wills, since there are no quadrupole effects taken into account (see however ref. 5)).

Our intensity ratio $I(2p_{3/2} - 1s) / I(2p_{1/2} - 1s)$ of the spin doublet agrees poorly for Pb with the data of Frati and Rainwater¹⁴) of 2.04 whereas in the case of Bi their value of 1.33 disagrees with our data. The theoretical value based on the statistical weight $(2j + 1)$ of the states would be 2 in both cases. However, only for the non-relativistic treatment of line intensities the population of the states is given by the statistical weight. In the case where the relativistic doublet splitting cannot be neglected, the term E^3 (E = transition energy) as well as the radial matrix elements of the transitions have to be considered. In our case of a Dirac particle and an extended nucleus the value of 2 for the intensity ratio would be purely accidental. Within the accuracy of our measurement no difference in the intensity ratio of the K_α - doublet can be seen between Pb and Bi. The intensity ratios for the K_α - and L_α - doublets of Pb (table 4) are identical within the errors quoted. If there would be a considerable feeding to the 2p levels from higher s levels, such transitions, particularly the 2s - 2p transitions, would favour the lower $p_{1/2}$ level. As a consequence of this, $I(2p_{3/2} - 1s) / I(2p_{1/2} - 1s)$ would be smaller than $I[(3d_{5/2} + 3d_{3/2}) - 2p_{3/2}] / I(3d_{3/2} - 2p_{1/2})$. This is only very slightly suggested by our data. For Pb, we could not find a 2s - 2p transition in our spectra.

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Table 1
Calibration γ - Lines

Isotope	Energy [keV]	Isotope	Energy [keV]
Positron annih.	511.0 $\pm$ 0.0 c)	Co ⁶⁰	1332.8 $\pm$ 0.4 c)
Cs ¹³⁷	661.6 $\pm$ 0.1	Na ²⁴	1367.9 $\pm$ 1.0 c)
Nb ⁹⁵	767 $\pm$ 3	Y ⁸⁸	1840.0 $\pm$ 2.0 c)
Sc ⁴⁶	887 $\pm$ 3 c)	ThC''	2615.0 $\pm$ 1
Y ⁸⁸	898.8 $\pm$ 1.2 c)	Na ²⁴	2753.5 $\pm$ 1.0 c)
Zn ⁶⁵	1114.0 $\pm$ 1	PoBe	4433 $\pm$ 5
Sc ⁴⁶	1118 $\pm$ 2 c)	N ¹⁶	6135 $\pm$ 10
Na ²²	1273.6 $\pm$ 1.8 c)		

Lines marked c) could be measured in the presence of the muon beam (see text).

Table 2

Energy Differences of K_{α} X-Ray Lines from μ -Mesonic Atoms

Measurement			ΔE : Direct Measurement [keV]	Mean Value [keV]	Weighted Energy Differences [keV]	ΔE : "Elton Assumption" [keV]	ΔE : "No Radius Change" [keV]
Elements	ΔZ	Date					
$42\text{Mo} - 45\text{Rh}$	3	IX	271.0	271.0 ± 10	271.0 ± 10		
$45\text{Rh} - 46\text{Pd}$	1	IX	86.0	86.0 ± 7	88.2 ± 5	91.7	
$45\text{Rh} - 48\text{Cd}$	3	V	276.5	276.5 ± 10			
$46\text{Pd} - 47\text{Ag}$	1	V IX	94.3 93.5	93.9 ± 6	93.5 ± 4	97.6	104.2
$46\text{Pd} - 48\text{Cd}$	2	V V	179.7 182.0	180.8 ± 7			
$47\text{Ag} - 48\text{Cd}$	1	V IX XI XI XII	93.8 85.0 88.6 86.6 89.4	88.6 ± 3	88.6 ± 3	83.6	105.6
$48\text{Cd} - 49\text{In}$	1	V V VIII XI XII	97.7 98.8 101.8 98.9 99.8	99.5 ± 3	99.5 ± 3	94.6	103.4
$48\text{Cd} - 50\text{Sn}$	2	V	191.5	191.5 ± 8			
$48\text{Cd} - 51\text{Sb}$	3	V	288.5	288.5 ± 10			
$48\text{Cd} - 56\text{Ba}$	8	V XI XI	731.0 720.0 750.0	733.0 ± 20			
$49\text{In} - 50\text{Sn}$	1	V VIII XI	92.0 93.0 95.2	93.4 ± 4	93.0 ± 4	86.7	103.8
$50\text{Sn} - 51\text{Sb}$	1	V VIII XI	91.0 90.2 88.8	90.0 ± 4	90.0 ± 4	93.3	103.4
$51\text{Sb} - 52\text{Te}$	1	V V	86.1 83.0	84.5 ± 6	84.5 ± 5	74.6	102.6
$51\text{Sb} - 53\text{J}$	2	V V	170.5 171.8	171.2 ± 7

Table 3

Energies of K_{α} X-Ray Lines from μ -Mesonic Atoms

Measurement		Energy	Mean Value	Calc. from Table 2
Element	Date	[keV]	[keV]	[keV]
$_{42}\text{Mo}$		a)	2711.6 ± 8	2712 ± 8
$_{45}\text{Rh}$	V	2980	2982 ± 13	2983 ± 13
	V	2978		
	IX	2989		
	IX	2983		
$_{46}\text{Pd}$	IV	3073	3077 ± 10	3071 ± 14
	V	3079		
	V	3074		
	V	3072		
	IX	3077		
	IX	3088		
$_{47}\text{Ag}$	V	3171	3172 ± 13	3165 ± 14.5
	V	3161		
	IX	3178		
	IX	3172		
	XII	3176		
$_{48}\text{Cd}$	VIII	3262	3258 ± 13	3254 ± 15
	IX	3259		
	XII	3250		
	XII	3249		
	XII	3258		
	XII	3270		
$_{49}\text{In}$	IV	3366	3358 ± 12	3353 ± 15
	V	3352		
	V	3353		
	VIII	3358		
	VIII	3355		
	XII	3361		
$_{50}\text{Sn}$	V	3447	3449 ± 15	3446 ± 16
	V	3445		
	VIII	3449		
	VIII	3454		
$_{51}\text{Sb}$	IV	3553	3544 ± 15	3536 ± 16
	V	3537		
	V	3543		
	V	3539		
	VIII	3545

Table 3 (cont'd)

Energies of K_{α} X-Ray Lines from μ -Mesonic Atoms

Measurement		Energy	Mean Value	Calc. from Table 2
Element	date	[keV]	[keV]	[keV]
$_{52}\text{Te}$	IV	3621	3622 ± 17	3621 ± 17
	V	3632		
	V	3619		
	V	3615		
$_{53}\text{J}$	V	3712	3712 ± 16	3710 ± 18
	V	3713		
	X	3702		
	X	3720		
$_{55}\text{Cs}$	X	3882	3888 ± 15	3896 ± 19
	X	3888		
	X	3889		
	X	3894		
$_{56}\text{Ba}$		b)	3981 ± 30	3982 ± 20
$_{57}\text{La}$	X	4066	4065 ± 15	4074 ± 21
	X	4058		
	X	4061		
	X	4072		
	X	4070		
$_{58}\text{Ce}$	X	4146	4146 ± 25	4162 ± 22
	X	4146		
$_{59}\text{Pr}$	VIII	4264	4244 ± 35	4251 ± 23
	X	4227		
	X	4245		
$_{63}\text{Eu}$	VIII	4220	4523 ± 25	4524 ± 26
	VIII	4528		
	VIII	4514		
	VIII	4523		
	VIII	4528		
$_{64}\text{Gd}$	VIII	4531	4531 ± 35	4554 ± 26

Table 3 (cont'd)

- Col. 1 : Element, Roman figures denote the month of measurement in 1961.
- Col. 2 : Measured energy of the centre of gravity of the 2p-1s transition.
- Col. 3 : Mean energy for the 2p-1s energy (centre of gravity), error stated as explained in subsect. 3.1.2. These are our final results.
- Col. 4 : Energy values of the X-ray lines, obtained by adding up the energy differences from table 2, are given for comparison.
- a) The energy value for Mo has been taken from reference 4).
- b) For Ba the result is taken from reference 3) (mean value of 11 measurements). In this reference the error is increased to ± 30 keV for unknown systematic effects.

Table 5

Equivalent Radii of Elements between Rh and Bi

Element	$\delta R_{eq}/A^{1/3}$ [10^{-16} cm]	$R_{eq}/A^{1/3}$ [10^{-13} cm]	Element	$\delta R_{eq}/A^{1/3}$ [10^{-16} cm]	$R_{eq}/A^{1/3}$ [10^{-13} cm]
$_{45}\text{Rh}$	+ 1.9 ± 12.4	1.242 ± 12	$_{53}\text{I}$	+ 3.5 ± 9.9	1.220 ± 10
$_{46}\text{Pd}$	+ 9.2 ± 8.8	1.236 ± 9	$_{55}\text{Cs}$	+ 3.9 ± 8.4	1.216 ± 8
$_{47}\text{Ag}$	+13.9 ± 10.8	1.237 ± 11	$_{56}\text{Ba}$	- 3.7 ± 16	1.206 ± 16
$_{48}\text{Cd}$	+17.0 ± 10.2	1.232 ± 10	$_{57}\text{La}$	- 2.2 ± 7.8	1.211 ± 8
$_{49}\text{In}$	+ 8.6 ± 9.0	1.226 ± 9	$_{58}\text{Ce}$	+ 0.6 ± 12.5	1.217 ± 12
$_{50}\text{Sn}$	+ 1.3 ± 10.7	1.220 ± 11	$_{59}\text{Pr}$	- 1.9 ± 17	1.216 ± 17
$_{51}\text{Sb}$	-11.8 ± 10.2	1.215 ± 10	$_{82}\text{Pb}$	+ 8.6 ± 7	1.195 ± 7
$_{52}\text{Te}$	-12.2 ± 11.1	1.211 ± 11	$_{83}\text{Bi}$	-20.6 ± 9	1.200 ± 9

The R_{eq} are based on the transition energies calculated by Ford and Wills for a "family II" charge distribution¹²). In column 2 the corrections are shown necessary to apply to the values of Ford and Wills in order to obtain the values of column 3 which agree with our data (table 3).

For Pb and Bi, some additional calculations of the $2p_{3/2} - 1s_{1/2}$ energies were made (see footnote on page). They fall along a straight line within about ± 6 keV when plotted vs R_{eq} . The errors quoted in the table contain the experimental errors and this contribution. No correction for nuclear polarization is included.

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FIGURE CAPTIONS

Figure 1: Experimental arrangement and block diagram

- Part I NaI crystal positioned in the axis of the μ -beam
Part II NaI crystal positioned below the axis of the μ -beam.
- 1-3 Coincidence scintillation counters; 4 anticoincidence scintillation counter, NaI NaI crystal.
- C Graphite stopper
T Target
Fe Iron hood to shield magnetic field
Pb Pb shielding to reduce low-energy background, 2 mm thick
* Radioactive γ -source for calibration
P Scintillation counter to record the γ -rays in coincidence with NaI
L Scintillation counter to observe burst shape on the scope after integration (I) took place
 D_i Delay units
 C_m, C_s Coincidence circuits
DS Dynode signal of NaI spectrometer
AS Anode signal of NaI spectrometer
A Linear amplifier (Laben Model G40)
PHA Pulse height analyser (Laben Model CISEC31)
PG Pulse generator to gate PHA to the arriving SC bursts
 $\rightarrow$ Muon beam axis
 S_m, S_e, S_a Scaler monitor rate, event rate, accepted event rate, resp.

Figure 2: Structure of muon burst as measured with a 1 ms wide gate shifted in steps of 1 ms. Repetition rate of the SC: 54/s.

Figure 3: Range curve in Cu. Number of particles stopped in 5 g/cm² Fe per $5 \cdot 10^4$ monitor counts (C_m). The peak is due to μ -mesons of 145 MeV/c. The arrow indicates the range of pions of the same momentum.

Figure 4: Timing curves. Abscissa: Relative delay in ns between the pulses from the NaI crystal and the pulses from either the beam telescope (curves at the right), or the pulses from the P counter (curve left hand side). Ordinate: number of accepted event curves are shown for K, Mo and Pb. In region a) prompt coincidences are registered, in b) γ -rays delayed with respect to the stopped muon are registered, and in c) only accidental coincidences appear (a,b,c refer to the Pb curve). All curves are taken from the same run.

- Figure 5 : Spectra of the 2p-1s transitions of $_{50}\text{Sn}$ and $_{51}\text{Sb}$ as measured in frequent interchanges of the 2 targets (10 interchanges Sn-Sb, each interval about 2 minutes).
- Figure 6 : The background spectra of elements between Rh and Gd. Contents of two channels have been added to obtain the ordinate values. The arrows indicate where the 2p-1s transitions of the respective elements would appear.
- Figure 7 : Spectrum of the 2p-1s transition of $_{48}\text{Cd}$.
- Figure 8 : The values of $\sum_k [N_1(k) - wN_2(k-v)]^2$ (see eq.(1)) are plotted vs. displacement v . The curves are shown for four displacements of spectrum 1 in steps of $1/4$ channel. In the given example, the spectra of the 2p-1s transition of $_{83}\text{Bi}$ and $_{82}\text{Pb}$ have been compared. At a displacement of 1.58 channels the two normalized spectra coincide best.
- Figure 9 : Spectrum of the 2p-1s transitions of $_{64}\text{Gd}$. The contents of 2 channels have been added.
- Figure 10: Full width of photolines at half height in [keV] as a function of the energy of the lines.
- γ -lines of radioisotopes, including PoBe (C^{12*}), $\times$ 2p-1s transitions,
 - [x] expected line width for the Eu 2p-1s transition taking into account its $p_{3/2} - p_{1/2}$ splitting.
- Figure 11 : The 3d-2p transition of $_{82}\text{Pb}$ as measured in 17 min. The curve drawn is the spectrum synthesized from the γ -line of Na^{24} . Underneath the background spectrum measured in 6 min. is shown.
- Figure 12 : The 3d-2p transition of $_{83}\text{Bi}$ as measured in 18 min. The solid line gives the best fit. Underneath the background spectrum measured in 13 min. is shown. The arrow is put at 2615 keV the energy of the Pb^{208} capture γ -line.
- Figure 13 : The 2p-1s transition of $_{82}\text{Pb}$. The arrows indicate the position of the fine structure components. The curve drawn is the spectrum synthesized from the γ -line of (O^{16*}) N^{16} .

Figure 14 : The spectrum of the 2 simple γ -transitions of O^{16*} of 6135 keV and 7120 keV (intensity ratio 68:5). Full circles connected by a solid line show the spectrum of the 6135 keV γ -line after subtraction of the 7120 keV γ -line from the original spectrum (open circles).

Figure 15 : Spectra of 2p-1s transitions of $_{83}\text{Bi}$ and $_{82}\text{Pb}$. The lines connect the measured points.

Figure 16 : Spectrum of the 2p-1s transition of $_{92}\text{U}$. The line connects the measured points.

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