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THE MIXING OF QUARK'S FLAVOURS IN NEUTRAL MESONS

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ABSTRACT

If quarks are coloured and interact by exchanging gluons, this have simple implications on the mechanism which mix $q\bar{q}$ pairs inside neutral mesons. These implications seem to be verified in first approximation for the masses and quark compositions of the pseudoscalar, vector and tensor mesons. The experimental intensity of the mixing is particularly weak for vector mesons, which have negative C-parity, as expected. In the general case, the exception being the pseudoscalar mesons, the mixing process is of small amplitude. It is then the mass hierarchy of quarks which governs essentially the composition of neutral mesons.

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1. INTRODUCTION

The discovery of charmed particles have given a strong support to the theory of hadrons composed of quarks.

However, a long way remains to understand strong interactions. Even the success of SU_3 classification is far from complete: for example three meson multiplets only are established; the distinction between octet and singlet states seems to be the exception and not the general rule. In the other multiplets 0^+ , 1^+ ... - some states are missing or so broad (ϵ , A_1 , Q) that it is difficult to identify them with resonances.

The merit of the coloured quarks approach is to start from a fundamental interaction between quarks. We want to show that this theory allow to link many aspects of strong interactions, among them properties of mesons.

In this article we want to concentrate on the mesons which appear as the superposition of several $q\bar{q}$ flavours, and the mechanism responsible of this mixing.

A parallel article [1] will be devoted to the decay of mesons belonging to 0^{++} , 1^{++} , 2^{++} multiplets, that is $q\bar{q}$ systems in p-wave.

2. WHY COLOUR AND GLUONS? [2]

If quarks exist, it is essential to know about the force between them: this force which makes turn, bend, create, annihilate these lines we draw in quarks' diagrams.

It was proposed first by Fuji [3], Okun [4] and Gell-Man [5] that strong interactions were carried by neutrally charged vector mesons. Later it was suggested by Greenberg [6] that quarks may have an additional degree of freedom to make them obey to the Fermi statistic. This new degree of freedom, which is to a large extent transparent to observation, is now called colour.

There exist several descriptions of colour. We shall adopt the most classical one, in which the different coloured varieties of a quark are equivalent. This section is devoted to a brief recall of colour hypothesis and to the definition of our notations.

2.1 Coloured quarks and gluons

2.1.1 Quarks and Anti-quarks (fig. 1)

Quarks can be distinguished first by their usual nature or flavour (flavour is a global name of the properties which distinguish the quarks: charge, strangeness, charm etc.).

Our notation will be for the four established flavours: u (up), d (down), s (strange), c (charmed) and $\bar{u}$, $\bar{d}$, $\bar{s}$, $\bar{c}$ for the anti-quarks.

Each q (or $\bar{q}$) flavour may exist in three equivalent sub-varieties. Our notations will be

B (Blue), W (White) and R (Red),

or $\bar{B}$, $\bar{W}$ and $\bar{R}$ for anti-quarks.

2.1.2 Hadrons

They are $q\bar{q}$, qqq and $\bar{q}\bar{q}\bar{q}$ systems of any flavour (it is flavour which brings variety in the hadron world) building SU_3 multiplets of given spin-parity.

They are colour singlets (the three colours are equally represented):

$$\begin{aligned} \text{mesons} & \quad \frac{1}{\sqrt{3}} \left[|B\bar{B}\rangle + |W\bar{W}\rangle + |R\bar{R}\rangle \right] , \\ \text{baryons} & \quad \frac{1}{\sqrt{6}} \left[|BWR\rangle - |BRW\rangle + |WRB\rangle - |WBR\rangle + |RBW\rangle - |RWB\rangle \right] . \end{aligned} \quad (1)$$

2.1.3 Gluons

Coupled to $q\bar{q}$ systems, they are the carrier of the strong force. Their characteristics are:

- Spin parity $J^{PC} = 1^{--}$, that corresponds to a $q\bar{q}$ pair in s-wave^(*), and spin-state, $s = 1$.

(*) If we neglect a possible admixture of d-wave.

- Flavour scalar: gluons emission do not modify the nature of a quark

$$\frac{1}{\sqrt{3}} \left[|u\bar{u}\rangle + |d\bar{d}\rangle + |s\bar{s}\rangle + |c\bar{c}\rangle + \dots \right] \quad (2)$$

- Gluons build an octet of colour states. If we represent them by the $q\bar{q}$ colour states to which they are coupled, we obtain the traceless tensor

$$\Gamma = \begin{vmatrix} \frac{2B\bar{B} - W\bar{W} - R\bar{R}}{3} & B\bar{W} & B\bar{R} \\ W\bar{B} & \frac{2W\bar{W} - R\bar{R} - B\bar{B}}{3} & W\bar{R} \\ R\bar{B} & R\bar{W} & \frac{2R\bar{R} - B\bar{B} - W\bar{W}}{3} \end{vmatrix} \quad (3)$$

The off-diagonal gluons of tensor Γ carry a colour change and are coupled to a unique $q\bar{q}$ colour state. The three diagonal terms carry no colour change and are coupled to the three states $B\bar{B}$, $W\bar{W}$, $R\bar{R}$. They are not orthogonal^(*) and represent only two independent gluons.

We shall rely on two fundamental hypotheses:

- Strong interactions are strictly invariant under rotations in colour space (exact SU_3 colour).
- The $q\bar{q}$ gluon coupling is independent of q -flavour [2].

The hypothesis of the interactions invariance under rotations in colour space implies a unique $q\bar{q}$ -gluon coupling constant: G .

(*) Two orthogonal neutral gluons are $\frac{2R\bar{R} - B\bar{B} - W\bar{W}}{\sqrt{6}}$ and $\frac{B\bar{B} - W\bar{W}}{\sqrt{2}}$.

3. THE MASS MATRIX OF MESONS

3.1 The mesonic system

It is the gluon exchange which is responsible of the binding [1] (fig. 2(a)). But besides being exchanged (or reabsorbed by its emitter), the gluon can materialize into a new pair $Q\bar{Q}$ of opposite flavour^(*). At materialization time $Q\bar{Q}$ have all the colour and spin-parity character of a gluon. We have visualized the four main possible evolutions of $q\bar{q}Q\bar{Q}'$ system in fig. 2:

- (a) $Q\bar{Q}$ simply annihilate in a gluon which is absorbed by q or $\bar{q}'$ (fig. 2(b)).
- (b) $q\bar{q}$ annihilate. This possibility is allowed only if $\bar{q}' = q$: transition (or annihilation) of a pair of "hidden-flavour" to another one $Q\bar{Q}$.
- (c) $q\bar{Q}$ and $\bar{q}'Q$ bind into two independent mesons. This pairing is the more likely, as in the majority of the cases (fig. 2(c)) $q\bar{Q}$ and $\bar{q}'Q$ are neutral in colour.
- (d) $q\bar{q}$ and $Q\bar{Q}$ bind into two mesons. This process, which corresponds to a Zweig rule violating decay is more difficult (fig. 2(d)).

If allowed by energy conservation these pairings will end in a two-bodies decay (see [1] for details).

If not allowed, the four particles remain confined and sooner and later returns to a two-particle state via (a) or (b).

We shall concentrate now on the $q\bar{q} \rightarrow Q\bar{Q}$ transition process which is responsible of flavour mixing in neutral mesons.

3.2 The $q\bar{q} \rightarrow Q\bar{Q}$ transition

It is this mechanism which is responsible of $q\bar{q}$ mixing in neutral mesons.

Colour forbids one gluon transitions, as mesons are colour singlets and gluon octets (e.g. $\langle B\bar{B} + W\bar{W} + R\bar{R} | B\bar{B} - W\bar{W} \rangle = 0$).

(*) Symbols Q and $\bar{Q}$ will denote in the following the newly created quark anti-quark pair, in opposition to the original pair $q\bar{q}'$.

Then the transition goes via an intermediate state composed of 2, 3 or more gluons (fig. 3).

C-parity being a good quantum number, and gluon C-parity = -1, the transition requires [7-9]:

$$\begin{aligned} 2,4 \text{ gluons for } C = +1 \text{ mesons } 0^{-+}, 2^{++}, 1^{++}, 0^{++}, \\ 3,5 \text{ gluons for } C = -1 \text{ mesons } 1^{--}, 1^{+-}. \end{aligned}$$

Then the model predicts that $q\bar{q} \rightarrow Q\bar{Q}$ transitions would be difficult for negative C-parity mesons, among them the vector mesons:

- Owing to the flavour-scalar character of gluons, the $q\bar{q} \rightarrow Q\bar{Q}$ transitions are to first order - neglecting higher order diagrams - a flavour scalar term.
- A last remark would be that these transitions are essentially Zweig-rule violating mechanisms (fig. 3) [10].

3.3 The meson-mass matrix

The self-Hamiltonian (or mass matrix) of a $q\bar{q}'$ pair in flavour space, may be seen as the sum of two terms $H = H_1 + H_2$:

- H_1 corresponds to all the diagrams where the old pair $q\bar{q}'$ keeps its identity (does not annihilate).
- H_2 corresponds to processes where $q\bar{q}$ transforms in a new $Q\bar{Q}$ pair.

If all q-masses are equal, it results from the invariance of q-gluon coupling with flavour that H_1 reduces to a scalar E, and H_2 is described by a unique annihilation amplitude a

$$H_2|u\bar{u}\rangle = H_2|d\bar{d}\rangle = H_2|s\bar{s}\rangle = \dots = a \left[|u\bar{u}\rangle + |d\bar{d}\rangle + |s\bar{s}\rangle + \dots \right]$$

The ideal mass-matrix has the simple form

$$H = \begin{vmatrix} E & & & \\ & E & & \\ & & \begin{array}{c|ccc} 0 & & & \\ \hline E+a & a & a & \\ a & E+a & a & \\ a & a & E+a & \end{array} & \\ & & & 0 \end{vmatrix} \begin{matrix} |u\bar{u}\rangle \\ |d\bar{d}\rangle \\ |s\bar{s}\rangle \end{matrix} \quad (8)$$

With three flavours u, d, s , it gives the usual distinction between eight states of mass E and one singlet of mass $E + 3a$

$$3a = M(1) - M(8) . \quad (9)$$

The octet singlet mass splitting is proportional to the annihilation amplitude.

What happens if the quark's mass hierarchy is $M(s) > M(u) \approx M(d)$
(we neglect for the moment the existence of a fourth flavour)?

- Owing to the invariance of q -gluon coupling with flavour, the elements of H_1 reflect in first approximation the mass differences of q and $\bar{q}$. They are related by the Gell-Man/Okubo formula [11].
- A natural hypothesis would be that H_2 is slightly modified by the unequal masses (this is at least true for the strange quark). Introducing a second amplitude c connecting non-strange to strange pairs, one gets for the part of the Hamiltonian connecting the "hidden flavour" pairs

$$H = \begin{vmatrix} E_1 + a & a & c \\ a & E_1 + a & c \\ c & c & E_3 + a \end{vmatrix} \begin{vmatrix} |u\bar{u}\rangle \\ |d\bar{d}\rangle \\ |s\bar{s}\rangle \end{vmatrix} , \quad (10)$$

where $E_1 = E_2 = H_1(u\bar{u})$, $E_3 = H_1(s\bar{s})$.

E_1 and E_3 could be approximate by the use of the Okubo formula^(*), from the masses of charged and strange mesons.

The prediction of the model would be that the masses and flavour composition of the three neutral non-strange mesons depend only of the two amplitudes a and c and that the ratio $c/a \approx 1$.

The experimental information available, at the present moment, lies essentially in the masses of the isoscalar mesons, as few precise measurements have been done on flavour compositions.

(*) For pseudoscalars the formula gives: $E_3 = M^2(s\bar{u}) + M^2(s\bar{d}) - M^2(u\bar{d}) = M^2(K^-) + M^2(K^0) - M^2(\pi^+)$.

What masses should be used [9]? Despite based on ad hoc and not very satisfactory arguments, we shall adopt the conventional use of squared masses (the use of simple masses give difficulties for pseudoscalars and leads to another treatment [7]).

Table I gives the values of E_1 , E_3 and masses for 0^- , 1^- , 2^+ mesons adopting this convention [12].

3.4 Resolution of the Hamiltonian in the three-flavour case

The eigen-value equation is

$$(\lambda - E_1)(\lambda^2 - (E_1 + E_3 + 3a) + (E_1 + 2a)(E_3 + a) - 2c^2) = 0 . \quad (11)$$

The first root $\lambda = E_1$ corresponds to the neutral isovector. The other roots λ' , λ'' correspond to the isoscalar mesons. Knowing the experimental values of λ' , λ'' (table I) one can compute a , c and the eigen-vector (α, β, γ)

$$a = \frac{\lambda' + \lambda'' - (E_1 + E_3)}{3} \quad (12)$$

$$c^2 = \frac{(E_3 + a)(E_1 + 2a) - \lambda'\lambda''}{2}$$

$$\alpha = \frac{c}{\sqrt{2c^2 + (E_1 + 2a - \lambda)^2}}$$

$$\beta = \alpha \quad (13)$$

$$\gamma = \frac{E_1 + 2a - \lambda}{\sqrt{2c^2 + (E_1 + 2a - \lambda)^2}}$$

If the eigen-vector has been measured for one state, it is possible to have an independent estimation of a , c and the second mass λ''

$$a = \frac{2\alpha^2(\lambda' - E_1) - \gamma^2(\lambda' - E_3)}{4\alpha^2 - \gamma^2} ,$$

$$c = \frac{\alpha\gamma(\lambda' + E_1 - 2E_3)}{4\alpha^2 - \gamma^2} , \quad (14)$$

$$\lambda'' = E_1 + 2a - 2c \frac{\alpha}{\gamma} .$$

4. APPLICATION TO PSEUDO-SCALAR (0^{-+}), VECTOR (1^{--}) AND TENSOR (2^{++}) MESONS

4.1 $q\bar{q} \rightarrow Q\bar{Q}$ transition amplitudes

Using eq. (12) for the determination of a and c , one finds in GeV^2 :

	0^{-+}	1^{--}	2^{++}
a	0.242 ± 0.001	0.017 ± 0.005	-0.042 ± 0.024
$ c $	0.205 ± 0.002	0.042 ± 0.066	0.061 ± 0.150

- Pseudoscalars η' , η : a and c are found much more larger than for the two other spin-parities, producing a large η - η' mass-splitting. The ratio $|c/a| = 0.85$ is found close to 1.
- Vector mesons ω , ϕ : a , c are close to 0, which is in qualitative agreement with a difficult transition requiring three gluons.

The very large relative error on c is due to the fact that eq. (12) gives only c^2 and that $c^2 \approx 0$. One begins to be sensible to the error made on E_3 when using the approximation of the Okubo formula. When use is made of a recent measurement of the ω - ϕ mixing angle [13] ($\Delta\beta = 5 \pm 2^\circ$) to determine a and c through eq. (14) one finds that E_3 should be increased of 4 MeV to give the ϕ mass. Then, $a = 0.015 \pm 0.010$ and $c = 0.026 \pm 0.010$.

This determination has the advantage to give the sign of c , which appears positive as for a . Within errors c/a is compatible with 1.

- Tensors mesons f , f' : for these positive C-parity mesons annihilation amplitudes are 2-3 times larger than for $c = -1$ vector mesons. Again error on c is very large. Within errors c/a is found compatible with 1 or $0.85^{(*)}$

4.2 Flavour composition (eigen-vectors)

In absence of a good determination of the ratio c/a for 1^- and 2^+ mesons, we have forced $c = a$ for these two spin parity states. The

(*) a is found negative.

corresponding eigen-vectors given in table II should then be considered as indicative. Changing the value of c around does not affect very much the flavour compositions (if $c = a$ is imposed, one can fit E_3 to obtain the masses λ' , λ'' and compare it to the prediction of Gell-Man/Okubo formula).

0^{-+} appears the only multiplet where H_1 and H_2 are comparable in strength, as can be seen from the behaviour of the ratio of the mean matrix-element of the two operators

$$\frac{\langle H_2 \rangle}{\langle H_1 \rangle} = \frac{a}{\frac{2E_1 + E_3}{3}} = \frac{3a}{2E_1 + E_3} \quad (15)$$

The effect of spin is to increase the energy levels of the $q\bar{q}$ system, then the elements of H_1 . The effect is also to remove q and $\bar{q}$ from each other and to reduce the probability of their annihilation. These two factors - combined to the even-odd C-parity effect - add to produce a drastic fall of the H_2/H_1 ratio. Then for all multiplets, except the pseudoscalars, the transition amplitudes are very small compared to the H_1 diagonal terms.

As soon as $a \ll E_j - E_i$, e.g. the difference of two diagonal terms, the flavours $q_i\bar{q}_i$ and $q_j\bar{q}_j$ mix to a negligible extent. It is the case of the ϕ and f' which are quasi-pure $|\bar{s}s\rangle$ states in opposition to the ω and f which are quasi $1/\sqrt{2}[|u\bar{u}\rangle + |d\bar{d}\rangle]$ states.

The mass-equality of u and d quarks is responsible for physical states being split into isovector ($(|u\bar{u}\rangle - |d\bar{d}\rangle)/\sqrt{2}$) and isoscalar ($(|u\bar{u}\rangle + |d\bar{d}\rangle)/\sqrt{2}$) terms. If u and d masses are slightly different, physical mesons should not be pure isoscalar and isovector. Such a mass-splitting may contribute to the observed ω - ρ interference^(*).

(*) If a begins to be smaller than $E_2 - E_1$, which might be the case for high spin and negative C-parity mesons, physical states might turn to be more $u\bar{u}$ and $d\bar{d}$ pairs than pure isoscalars and isovectors.

4.3 Impact of the fourth charmed quark

The introduction of a fourth flavour leads to add a new line and a new row to Hamiltonian of eq. (10). With evident notations this Hamiltonian becomes

$$H = \begin{vmatrix} E_1 + a & a & c & d \\ a & E_1 + a & c & d \\ c & c & E_3 + a & e \\ d & d & e & E_4 + a \end{vmatrix} \quad (16)$$

The new diagonal term could be speculated to be around $(3 \text{ GeV})^2$ to give the mass of the new mesons. If the recently discovered meson [14] at 1.87 GeV is the charmed pseudoscalar and if Okubo-formula applies to charm, one finds

$$E_4 = 2 \times (1.87)^2 - 0.139^2 = (2.64)^2 \text{ GeV}^2.$$

Anticipating the following results, one can also approximate E_4 by $E_4 = M^2(c\bar{c}) - a$, where $(c\bar{c})$ is the mass of the $c\bar{c}$ state to be found. It is this procedure we have adopted in table III.

Less experimental information is still available to determine the new matrix elements of H_2 . In the incertitude we have imposed: $d = e = a$ and the quarks composition shown in table III should be considered indicative.

One can see that owing to the very large values of $E_4 - E_1$ and $E_4 - E_3$ compared to $q\bar{q} \rightarrow Q\bar{Q}$ transition amplitudes, the recently discovered mesons η_c , ω , p_c do not affect practically the composition of other states, and have very small "normal" quark components (*).

As the mesons cannot decay hadronically (by energy conservation) through their main $c\bar{c}$ component, it is the small admixture of normal quarks flavour which allows them to decay. However, these admixtures are very variable with spin-parity; the pseudoscalar η_c should have much more larger admixtures [9] than the two other states and the η and η' contain a

(*) The separation would be less dramatic with the use of simple masses instead of squared masses.

small, but noticeable $c\bar{c}$ components. On the opposite the extremely narrow hadronic width of the ψ appears a consequence of its negative C-parity. Another consequence of the small transition amplitudes of vector mesons, is the quasi-degeneracy in mass of the ρ and ω , as it can be easily shown that $M^2(\omega) - M^2(\rho) \approx 2a$.

4.4 Impact of more quarks

It was recently suggested [8] that the number of quark flavours may exceed 4. In table IV we have listed what would be the hidden flavour mesons with two new quarks t and b . We have supposed a doublet of equal masses and $E_5 = E_6 \approx (4 \text{ GeV})^2$. What is interesting to notice is that if the new quarks are heavier but not very distinct in mass from the charm quark, their preferential hadronic decay would imply two charmed mesons (the $c\bar{c}$ component is larger than the $u\bar{u}$, $d\bar{d}$ and $s\bar{s}$ components).

5. CONCLUSION

The transition amplitudes between $q\bar{q}$ pairs of different nature are: (a) very small for vector mesons which are of negative C-parity; (b) seem to depend weakly from quark flavours. It is what is expected if these transitions are carried by coloured gluons, scalar of flavour.

This has many consequences. In particular the mass hierarchy of quarks combined with the smallness of many transition amplitudes is responsible for the existence of quasi-pure $s\bar{s}$ and $c\bar{c}$ states.

The transition amplitudes appear very dependent of spin-parity, and as they are linked to a Zweig-rule violating mechanism, one can conclude that the degree of violation of this rule is dependent of spin-parity.

As the mass difference of squared masses of (non-strange) isoscalar and isovector are also linked to the same amplitudes, one can say that these mass-differences are a simple measure of the intensity of Zweig-rule violating mechanisms with spin-parity.

Among the three well established multiplets, only pseudoscalar mesons have strong transition amplitude.

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REFERENCES

- [1] The fast decays of $q\bar{q}$ systems in p-wave, Ch. de la Vaissière,
To be submitted to Nuclear Physics B.
- [2] A simple approach to colour concepts can be found in:
S. Glashow, Quarks with Colour and Flavour, Scientific American,
Oct. 1975.
Y. Nambu, The Confinements of Quarks, Scientific American, Nov. 1975.
- [3] Y. Fujii, Progr. Theor. Phys. (Kyoto) 21 (1959) 232.
- [4] Kobzarev and Okun, Soviet Physics, IETP 14 (1962) 358.
- [5] M. Gell-Mann, Physics Review 125 (1962) 1067
- [6] O.W. Greenberg, Phys. Rev. Letters 13 (1964) 598.
A recent review of above models can be found in:
O.W. Greenberg and C.A. Nelson, Colour Models of Hadrons, University
of Maryland, Tech. Report 77-622.
- [7] A. De Rujula et al., Hadron masses in a Gauge Theory, Phys. Review
D12 (1975) 147.
- [8] H. Harrari, Proceedings of the 1975 International Symposium on lepton
and photon interactions at High Energy (Stanford University).
- [9] H. Fritzsch and P. Minkowski, Nuovo Cimento 30 A (1975) 393.
- [10] G. Zweig TH 412 (1964).
S. Okubo, Phys. Letters 5 (1963).
J. Izuka, Supplement of Progr. Theor. Physics (Kyoto) 37-38 (1966) 21.
- [11] S. Okubo, Progr. Theor. Physics (Kyoto) 27 (1962) 949.
- [12] Particle Data Group, Review of Particle Properties (April 1976 edition),
Review of Modern Physics Vol. 48 No. 2.
- [13] A. Silverman, Proceedings of the 1975 International Symposium on Lepton
and Photon Interactions at High Energy (Stanford University) p. 350.
- [14] G. Goldhaber et al., Phys. Rev. Letters 37 (1976) 255.

TABLE CAPTIONS

Table I	Experimental mass data in MeV for 0^{-+} , 1^{--} , 2^{++} mesons. To be submitted to Nuclear Physics B.
Table II	Isoscalar mesons built from u, d and s quarks. For vector and tensor mesons, c has been imposed equal to a. This requires to shift slightly E_3 from the Okubo value to adjust the meson's masses.
Table III	Impact of the charmed flavour on isoscalar mesons. The value of parameters E_3 , a, c are those of table II. The new matrix elements of H_2 are imposed equal to a. The impact is noticeable only for pseudoscalars.
Table IV	Impact of a hypothetic additional doublet (t and b) of quarks. These masses are supposed equal and not very distinct from the charmed quark (values of other parameters are the same as in tables II and III).

TABLE I

J^{PC}	$\sqrt{E_1} = \sqrt{M^2(u\bar{d})}$	$\sqrt{K^+} = \sqrt{M^2(u\bar{s})}$	$\sqrt{K^0} = \sqrt{M(d\bar{s})}$	$\sqrt{E_3} = \sqrt{M(s\bar{s})}$	$\sqrt{\Lambda^{++}}$	$\sqrt{\Lambda^{+*}}$
0^{-+}	139.57 ± 0.00	493.70 ± 0.04	497.70 ± 0.13	687	548 ± 0.6	957.6 ± 0.3
1^{--}	765.9 ± 2.2	892.24 ± 0.43	896.23 ± 0.36	1007	782.6 ± 0.3	1019.7 ± 0.3
2^{++}	1307.7 ± 1.4	1420.4 ± 2.9	1421.8 ± 1.7	1526	1271.1 ± 1.7	1516.1 ± 3

Table II

	J^{PC}	$\sqrt{E_3}$	$\sqrt{E_3}$ (Okubo)	a	c	$\frac{H_2}{H_1} = \frac{3a}{2E_1 + E_3}$	Mass	Eigen-vectors		
								u $\bar{u}$	d $\bar{d}$	s $\bar{s}$
η η'	0^{-+}	0.687	0.687	0.242	0.205	1.42	0.5488 0.9576	0.5791 0.4058	0.5791 0.4058	-0.5738 0.8190
ω ϕ	1^{--}	1.012	1.007	0.014	0.014	0.018	0.7826 1.0197	0.7064 0.0319	0.7064 0.0319	-0.0451 0.9990
f f'	2^{++}	1.529	1.526	-0.045	-0.045	-0.023	1.271 1.516	0.7041 -0.0655	0.7041 -0.0655	0.0927 0.9957

Table III

J^{PC}	$\sqrt{E_3}$	$\sqrt{E_4}$	a	M	Eigen-vectors			
					$\bar{u}u$	$\bar{d}d$	$\bar{s}s$	$\bar{c}c$
0^{-+}	0.687	2.70	0.242	η	0.5462	0.5842	-0.5630	-0.0203
				η'	0.9453	0.3968	0.3968	0.8255
				η_c	2.750	0.0354	0.0354	0.0374
1^{--}	1.012	3.092	0.014	ω	0.7827	0.7064	0.7064	-0.0450
				ϕ	1.0196	0.0318	0.0318	0.9990
				ψ	3.094	0.0015	0.0015	0.0016
2^{++}	1.529	3.556	0.045	f	1.271	0.7040	0.7040	0.0930
				f'	1.516	-0.0658	-0.0658	0.9957
				P_c	3.550	-0.0040	-0.0040	-0.0043

Table IV

J^{PC}	$\sqrt{E_5} = \sqrt{E_6}$	M	Eigen-vectors					
			$u\bar{u}$	$d\bar{d}$	$s\bar{s}$	$c\bar{c}$	$t\bar{t}$	$b\bar{b}$
0^{-+}	η	0.544	0.5889	0.5889	-0.5530	-0.0203	-0.0095	-0.0095
	η'	0.935	0.3893	0.3893	0.8327	-0.0568	-0.0254	-0.0254
	η_c	2.746	0.0331	0.0331	0.0350	0.9972	-0.0326	-0.0326
	?	3.90	0.	0.	0.	0.	0.7071	-0.7071
	?	3.96	-0.0235	0.235	0.0241	0.0438	0.7058	0.7058
1^{--}	ω	0.7826	0.7064	0.7064	-0.0450	-0.0021	-0.0011	-0.0011
	ϕ	1.0196	0.0318	0.0318	0.9990	-0.0017	-0.0009	-0.0009
	ψ	3.094	0.0015	0.0015	0.0015	1.	-0.0019	-0.0019
	?	4.10	0.	0.	0.	0.	0.7071	0.7071
	?	4.103	0.0012	0.0012	0.0012	0.0026	0.7071	0.7071
2^{++}	f	1.270	0.7040	0.7040	0.0935	0.0061	0.0040	0.0040
	f'	1.516	-0.0661	-0.0661	0.9936	0.0038	0.0024	0.0024
	P_c	3.550	-0.0041	-0.0041	-0.0044	1.	0.0076	0.0076
	?	4.29	-0.0037	-0.0037	-0.0039	-0.0108	0.7070	0.7070
	?	4.30	0.	0.	0.	0.	0.7071	-0.7071

FIGURE CAPTIONS

Fig. 1 Quark's flavour and colour.

Fig. 2 The quark anti-quark system (non-annihilation processes)^(*):
 (a,b) the two particles remain bound in a meson
 (c,d) the materialization of a gluon trigger a decay
 into two independent mesons.

Fig. 3 If q and $\bar{q}'$ are of opposite flavour they can also annihilate
 and transform in another pair $Q\bar{Q}$ of different or identical
 flavour.

(*) The choice of colours, and colour changes in figs 2 and 3 is only for illustration. A meson is a superposition of three colour states, and one has to sum, in exchange, on all possible gluon states.

FLAVOUR	COLOUR		
	BLUE	WHITE	RED
	u (or p) <i>up</i>	●	○
	d (or n) <i>down</i>	●	○
	s (or λ) <i>strange</i>	●	○
	c <i>charmed</i>	●	○

Fig. 1

$q \bar{q}'$ INTERACTIONS

a) GLUON EXCHANGE

b) TEMPORARY MATERIALISATION OF A GLUON

c) natural pairing
 $q \bar{q}' \rightarrow q \bar{q} + \bar{q}' q'$

d) Zweig rule violating pairing
 $q \bar{q}' \rightarrow q \bar{q}' + Q \bar{Q}$

GLUON MATERIALISATION AND MESON DECAY

Fig. 2

$q\bar{q}$ annihilation

Simplest $q\bar{q} \rightarrow Q\bar{Q}$ transition allowed for
 $C = +1$ mesons

Simplest $q\bar{q} \rightarrow Q\bar{Q}$ transition allowed for
 $C = -1$ mesons

Fig. 3