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ASYMPTOTICALLY FREE $SU(2) \times SU(2) \times SU(4)$ MODEL
OF UNIFIED INTERACTION

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An asymptotically free $SU(2) \times SU(2) \times SU(4)$ model of unified interaction is constructed. The gauge coupling constants are taken equal (in the order of magnitude) to the electromagnetic one. The strong interaction and the quark confinement is provided by gluonic mechanism. Some physical consequences of the model are briefly discussed.

The discovery of the spontaneous breakdown of symmetry [1] and the phenomenon of asymptotical freedom in nonabelian gauge theories [2] gives at least in principle the possibility to construct models in quantum field theory which would possess not only a needed physical content but also the due mathematical rigour. The simplest examples of such models for the weak and electromagnetic interaction [3] as well as models for pure strong interaction [4] were repeatedly discussed in the literature. There also exists an example of asymptotically free model of unified interaction [5]. Therefore at present the problem of construction a mathematically selfconsistent model for strong, weak and electromagnetic interaction, i.e. the one which describes the realistic interaction between the elementary particles and is free of the "zero-charge" difficulty [6] may be referred to as in principle solved. The present model of unified interaction also possesses the asymptotic freedom. The gauge group G is now more complicated than in the model of Ref. [5] and is chosen as a direct product of three groups $SU(3) \times SU(2) \times SU(2)$ (analogous gauge group was used firstly in Ref. [7]). Three gauge coupling constants (η^1, η^2, g) are included into the theory, which are, generally speaking, arbitrary as far as the domain of small momentum transfer is concerned, but are interrelated by the condition of asymptotic freedom in the limit of small distances, i.e. in the domain of large momentum transfer. In this domain all the effective coupling constants prove to be of the same order of magnitude: $(\eta^1/g)^2 = 3$; $(\eta^1/\eta^2)^2 = 7$. The magnitude of $(\eta^1)^2$ is taken to be of the order of e^2 . The strong interaction in the region of small momentum transfer arises owing to the infrared divergencies

which are inherent to the theory of massless gauge fields of the $SU(3)$ -subgroup of the coloured gluons. The latter also must provide the confinement of quarks. The choice of the more complicated gauge group enables us to remove some restrictions which were present in the model of Ref.[5] and thus to gain more freedom in handling the physical contents of the model.

The Lagrangian of the model splits into the three structural blocks:

$$\mathcal{L} = \mathcal{L}_g + \mathcal{L}_Y + \mathcal{L}_s \quad (1)$$

$\mathcal{L}_g$ displays the fields involved in the Lagrangian (1) and their interaction with the gauge fields:

$$\begin{aligned} \mathcal{L}_g = & -\frac{1}{4}(\partial_\mu V_\nu^a - \partial_\nu V_\mu^a + g_0 f^{abc} V_\mu^b V_\nu^c)^2 \\ & -\frac{1}{4}(\partial_\mu W_{1,\nu}^a - \partial_\nu W_{1,\mu}^a + \eta_0^L \epsilon^{abc} W_{1,\mu}^b W_{1,\nu}^c) - \frac{1}{4}(\partial_\mu W_{2,\nu}^a - \partial_\nu W_{2,\mu}^a + \eta_0^S \epsilon^{abc} W_{2,\mu}^b W_{2,\nu}^c)^2 \\ & - |(\delta^{ab} \delta^{cd} \partial_\mu - i\eta_0^L \frac{\tau_{ab}^t}{2} \delta^{cd} \partial_\mu W_{1,\mu}^t + i\eta_0^S \frac{\tau_{ab}^t}{2} \delta^{cd} \partial_\mu W_{2,\mu}^t) \Sigma_\ell^d|^2 \\ & - |(\delta^{ab} \delta^{cd} \partial_\mu - i\eta_0^L \frac{\tau_{ab}^t}{2} \delta^{cd} \partial_\mu W_{2,\mu}^t + i\eta_0^S \frac{\tau_{ab}^t}{2} \delta^{cd} \partial_\mu V_\mu^t) M_\ell^d|^2 \\ & - |(\delta^{ab} \partial_\mu + g_0 f^{abc} V_\mu^c) \Phi^b|^2 \\ & + \bar{\Psi}_{e,a}^{(n)} \gamma_\mu [\delta^{ab} \delta^{cd} \partial_\mu + i\eta_0^L \frac{\tau_{ab}^t}{2} \delta^{cd} \partial_\mu W_{2,\mu}^t - i\eta_0^S \frac{\tau_{ab}^t}{2} \delta^{cd} \partial_\mu V_\mu^t] \Psi_{e,a}^{(n)} \quad (2) \\ & + \bar{\Psi}_{l,a}^{(n)} \gamma_\mu [\delta^{ab} \delta^{cd} \partial_\mu + i\eta_0^L \frac{\tau_{ab}^t}{2} \delta^{cd} \partial_\mu W_{1,\mu}^t - i\eta_0^S \frac{\tau_{ab}^t}{2} \delta^{cd} \partial_\mu V_\mu^t] \Psi_{l,a}^{(n)} \\ & + \bar{\psi}^a \gamma_\mu [\delta^{ab} \partial_\mu + \eta_0^L \epsilon^{abc} W_{1,\mu}^b] \psi^b - m_\psi \bar{\psi} \psi \\ & + \bar{\eta}^a \gamma_\mu [\delta^{ab} \delta^{cd} \partial_\mu - i\eta_0^L \frac{\tau_{ab}^t}{2} \delta^{cd} \partial_\mu W_{1,\mu}^t + i\eta_0^S \frac{\tau_{ab}^t}{2} \delta^{cd} \partial_\mu W_{2,\mu}^t] \eta^b \\ & - m_\eta \bar{\eta} \eta + \bar{B}^a \gamma_\mu (\delta^{ab} \partial_\mu + g_0 f^{abc} V_\mu^c) B^b - m_B \bar{B} B \\ & + \bar{Q}_{\alpha\beta} \gamma_\mu [\delta^{ab} \delta^{cd} \partial_\mu + i\eta_0^L \frac{\tau_{ab}^t}{2} \delta^{cd} \partial_\mu W_{2,\mu}^t + i\eta_0^S \frac{\tau_{ab}^t}{2} \delta^{cd} \partial_\mu V_\mu^t] Q^{\delta\delta} - m_Q \bar{Q} Q. \end{aligned}$$

Here $f^{\alpha\beta\gamma}$ are the structural constants of the SU(4) group; $\gamma_\mu\gamma_\nu + \gamma_\nu\gamma_\mu = 2\delta_{\mu\nu}$. $g_{\mu\nu} = (-, +, +, +)$. $Q^{\alpha\beta}$ is the symmetric second rank tensor; $\eta = 1, 2$. With the Lagrangian (2) the model is free of anomalies. $\mathcal{L}_Y$ is the piece of the Lagrangian (1) which is describing the Yukawa-like interaction

$$\mathcal{L}_Y = -h_{ij}^{(a)} \bar{\psi}_{i,a}^{(u)} Z_{i,a}^a \psi_{j,a}^b - (\bar{\psi}_{i,a}^{(c)} Z_{i,a}^d (q_0 \Lambda) \eta_d^b - [f_{ij}^{(c)} \eta_{k,a}^d M_d^b \psi_{k,p}^{a(b)}] + f_{i,a}^{(a)} (\bar{B}_{i,a}^2)^\alpha M_a^\beta \psi_{i,a}^{\alpha(b)}] + i k_0 f^{\alpha\beta\gamma} \bar{L}_i \phi_\beta B_R^\gamma + p_0 \bar{Q}_{L,\mu\beta} (\phi_{i/2}^\beta)^\gamma Q_R^{\alpha\delta} + \text{H.C.}$$

$\mathcal{L}_S$ represents the interaction potential of the scalar fields

$$\begin{aligned} \mathcal{L}_S = & -\frac{\lambda_{\mathbb{Z}}^2}{2} (Z_f^d Z_f^f)^2 - \frac{\delta_{\mathbb{Z}}^2}{2} (Z_f^{+d} Z_f^f) (Z_i^{+a} Z_d^b) \\ & - \frac{\lambda_{\mathbb{M}}^2}{2} (M_\beta^+ M_\beta^f)^2 - \frac{\delta_{\mathbb{M}}^2}{2} (M_\beta^{+d} M_\beta^f) (M_\beta^{+f} M_\beta^f) - \lambda_{\mathbb{Z}\mathbb{M}}^2 (Z_f^{+d} Z_f^f) (M_\beta^{+g} M_\beta^f) \\ & - \delta_{\mathbb{Z}\mathbb{M}}^2 (Z_f^{+d} Z_d^b) (M_\beta^{+f} M_\beta^f) - \frac{\lambda_{\mathbb{F}}^2}{2} (\phi_\beta^+ \phi_\beta^f)^2 - \frac{\delta_{\mathbb{F}}^2}{2} (\phi_\beta^+ D_{\beta\delta}^f \phi_\delta) (\phi_\mu^+ D_{\mu\nu}^f \phi_\nu) \\ & - \lambda_{\mathbb{F}\mathbb{M}}^2 (\phi_\beta^+ \phi_\beta^f) (M_\beta^{+d} M_\beta^f) - \delta_{\mathbb{F}\mathbb{M}}^2 (\phi_\beta^+ D_{\beta\delta}^f \phi_\delta) [M_\mu^{+d} (\frac{\lambda_{\mathbb{F}}^f}{2})_\nu^f M_d^\nu] \\ & - \frac{\lambda_{\mathbb{F}}^2}{4} (\phi_\beta^+ \phi_\beta^f) (\phi_\mu^+ \phi_\mu^f) - \frac{\delta_{\mathbb{F}}^2}{4} (\phi_\beta^+ D_{\beta\delta}^f \phi_\delta) (\phi_\mu^+ D_{\mu\nu}^f \phi_\nu) \\ & - \rho_{\mathbb{F}\mathbb{M}}^2 (\phi_\beta^+ F_{\beta\delta}^f \phi_\delta) [M_\mu^{+d} (\frac{\lambda_{\mathbb{F}}^f}{2})_\nu^f M_d^\nu]. \end{aligned} \quad (4)$$

Here $D_{\beta\delta}^f = d_{\beta\delta}^{f\gamma\gamma}$; $F_{\beta\delta}^f = i f_{\beta\delta}^{f\gamma\gamma}$ and $\lambda_{\mathbb{F}}^f/2$ are generators of the SU(4) group. In eq. (3) $h_{ij}^{(a)} = \delta_{ij}^{(a)} h_j^{(a)}$ and $(q_0 \Lambda) = q_0^{1+\frac{1}{2}\epsilon} + q_0^{\frac{1}{2}} \frac{1-\delta\epsilon}{2}$.

The asymptotic freedom of the model (1) is proved within the set of equations of the renormalization group. In the (one easily calculated by usual methods [8], [9] and "one-loop" approximation these equations are combined into three groups. Here first of all, one has equations for the effective coupling constants of the gauge fields

$$16\pi^2 \dot{\eta}_L = -\frac{1}{3} \eta_L^3, \quad 16\pi^2 \dot{\eta}_R = -\frac{7}{3} \eta_R^3, \quad 16\pi^2 \dot{g} = -g^3 \quad (5)$$

farther the set of equations for the effective coupling constants of the Yukawa-like interaction

$$\begin{aligned}
 16\pi^2 \dot{h}_1 &= 6h_1^3 + 4h_2^2 h_1 + \frac{3}{4}(q_k^2 + q_\pi^2)h_1 + f_1^2 h_1 - [\frac{15}{4}g^2 + \frac{9}{4}(\eta_k^2 + \eta_\pi^2)]h_1, \\
 16\pi^2 \dot{h}_2 &= 6h_2^3 + 4h_1^2 h_2 + \frac{3}{4}(q_k^2 + q_\pi^2)h_2 + \frac{15}{8}f_0^2 h_2 - [\frac{15}{4}g^2 + \frac{9}{4}(\eta_k^2 + \eta_\pi^2)]h_2, \\
 16\pi^2 \dot{q}_k &= \frac{15}{8}q_k^3 + \frac{3}{4}q_k^2 q_\pi + 4(h_1^2 + h_2^2)q_k - [6\eta_k^2 + \frac{9}{4}(\eta_k^2 + \eta_\pi^2)]q_k, \\
 16\pi^2 \dot{q}_\pi &= \frac{15}{8}q_\pi^3 + \frac{3}{4}q_k^2 q_\pi + 4(h_1^2 + h_2^2)q_\pi + 2f_1^2 q_\pi - [6\eta_\pi^2 + \frac{9}{4}(\eta_k^2 + \eta_\pi^2)]q_\pi, \\
 16\pi^2 \dot{f}_1 &= 5f_1^3 + \frac{15}{8}f_0^2 f_1 + (\frac{3}{8}q_k^2 + h_1^2)f_1 - [\frac{9}{4}q_k^2 + \frac{9}{4}(\eta_k^2 + \frac{3}{2}q^2)]f_1, \\
 16\pi^2 \dot{f}_0 &= \frac{53}{16}f_0^3 + 2f_1^2 f_0 + (2k^2 + h_2^2)f_0 - [12g^2 + \frac{9}{4}(\eta_k^2 + \frac{3}{2}q^2)]f_0, \\
 16\pi^2 \dot{k} &= 8k^3 + (\frac{3}{4}p^2 + \frac{1}{2}f_0^2)k - 24g^2 k, \\
 16\pi^2 \dot{p} &= \frac{15}{8}p^3 + 4k^2 p - 27g^2 p, \quad (6)
 \end{aligned}$$

and, at last, the set of equations for the effective constants of the scalar interaction ($h_i^2 = 0$)

$$\begin{aligned}
 16\pi^2 \dot{\lambda}_{\Sigma^+}^2 &= 16\lambda_{\Sigma^+}^4 (\lambda_{\Sigma^+}^2 + \delta_{\Sigma^+}^2) + 6\delta_{\Sigma^+}^4 + 16\lambda_{\Sigma^+ \Sigma^+}^2 (\lambda_{\Sigma^+ \Sigma^+}^2 + \delta_{\Sigma^+ \Sigma^+}^2) + 4[\frac{3}{4}(q_k^2 + q_\pi^2) - \\
 &\quad - \frac{9}{4}(\eta_k^2 + \eta_\pi^2)]\lambda_{\Sigma^+}^2 + [\frac{9}{4}(\eta_k^2 + \eta_\pi^2) + \frac{15}{2}\eta_k^2 \eta_\pi^2] - \frac{1}{4}(q_k^2 + q_\pi^2), \\
 16\pi^2 \dot{\delta}_{\Sigma^+}^2 &= 8[\delta_{\Sigma^+}^2 (\delta_{\Sigma^+}^2 + \frac{3}{2}\lambda_{\Sigma^+}^2) + \delta_{\Sigma^+ \Sigma^+}^2] - 6\eta_k^2 \eta_\pi^2 + 4[\frac{3}{4}(q_k^2 + q_\pi^2) - \frac{9}{4}(\eta_k^2 + \eta_\pi^2)]\delta_{\Sigma^+}^2 - (q_k^2 + q_\pi^2), \\
 16\pi^2 \dot{\lambda}_{\Sigma^+ \Sigma^+}^2 &= 6\lambda_{\Sigma^+ \Sigma^+}^4 (\lambda_{\Sigma^+ \Sigma^+}^2 + \delta_{\Sigma^+ \Sigma^+}^2) + \delta_{\Sigma^+ \Sigma^+}^4 + 8\lambda_{\Sigma^+ \Sigma^+}^2 (\lambda_{\Sigma^+ \Sigma^+}^2 + \delta_{\Sigma^+ \Sigma^+}^2) + (30\lambda_{\Sigma^+ \Sigma^+}^2 - \frac{3}{2}\delta_{\Sigma^+ \Sigma^+}^2) - \\
 &\quad - \rho_{\Sigma^+ \Sigma^+}^4 + 4[(2f_1^2 + \frac{15}{8}f_0^2) - \frac{9}{4}(\eta_k^2 + \frac{3}{2}q^2)]\lambda_{\Sigma^+ \Sigma^+}^2 + \frac{9}{4}[\eta_k^2 + \frac{3}{2}q^2 (q^2 + 4\eta_k^2)] - \frac{1}{16}f_0^4, \\
 16\pi^2 \dot{\delta}_{\Sigma^+ \Sigma^+}^2 &= 12\delta_{\Sigma^+ \Sigma^+}^4 (\delta_{\Sigma^+ \Sigma^+}^2 + \lambda_{\Sigma^+ \Sigma^+}^2) + 4\delta_{\Sigma^+ \Sigma^+}^4 + 4(\rho_{\Sigma^+ \Sigma^+}^4 + \frac{3}{2}\delta_{\Sigma^+ \Sigma^+}^2) + 4[(2f_1^2 + \frac{15}{8}f_0^2) - \frac{9}{4}(\eta_k^2 + \\
 &\quad + \frac{3}{2}q^2)]\delta_{\Sigma^+ \Sigma^+}^2 + \frac{9}{2}q^2 (q^2 + \eta_k^2) - 8f_1^4 - \frac{7}{2}f_0^4, \\
 16\pi^2 \dot{\lambda}_{\Sigma^+ \Sigma^+}^2 &= 4[\lambda_{\Sigma^+ \Sigma^+}^2 + 2(\frac{3}{2}\lambda_{\Sigma^+}^2 + \delta_{\Sigma^+}^2) + 3(\frac{3}{2}\lambda_{\Sigma^+ \Sigma^+}^2 + \delta_{\Sigma^+ \Sigma^+}^2)]\lambda_{\Sigma^+ \Sigma^+}^2 + 2[\delta_{\Sigma^+ \Sigma^+}^2 (2\lambda_{\Sigma^+}^2 + \delta_{\Sigma^+}^2) + (4\lambda_{\Sigma^+ \Sigma^+}^2 \\
 &\quad + \delta_{\Sigma^+ \Sigma^+}^2)]\delta_{\Sigma^+ \Sigma^+}^2 + 2[\frac{3}{4}(q_k^2 + q_\pi^2) - \frac{9}{4}(\eta_k^2 + \eta_\pi^2) + (2f_1^2 + \frac{15}{8}f_0^2) - \frac{9}{4}(\eta_k^2 + \frac{3}{2}q^2)]\lambda_{\Sigma^+ \Sigma^+}^2 + \frac{3}{2}\eta_k^2, \\
 16\pi^2 \dot{\delta}_{\Sigma^+ \Sigma^+}^2 &= 4\delta_{\Sigma^+ \Sigma^+}^4 + [8\lambda_{\Sigma^+ \Sigma^+}^2 + 2(\lambda_{\Sigma^+}^2 + 2\delta_{\Sigma^+}^2) + 2(\lambda_{\Sigma^+ \Sigma^+}^2 + 4\delta_{\Sigma^+ \Sigma^+}^2)]\delta_{\Sigma^+ \Sigma^+}^2 + \\
 &\quad + 2[\frac{3}{4}(q_k^2 + q_\pi^2) - \frac{9}{4}(\eta_k^2 + \eta_\pi^2) + 2(f_1^2 + \frac{15}{8}f_0^2) - \frac{9}{4}(\eta_k^2 + \frac{3}{2}q^2)]\delta_{\Sigma^+ \Sigma^+}^2 - 3q_k^2 f_1^2, \\
 16\pi^2 \dot{\lambda}_{\phi^+}^2 &= 32\lambda_{\phi^+}^4 + 4[\lambda_{\phi^+}^2 + 3(\delta_{\phi^+}^2 + \delta_{\phi^+}^2)]\lambda_{\phi^+}^2 + 2(\delta_{\phi^+}^2 + \frac{9}{4}\delta_{\phi^+}^2) + 16\lambda_{\phi^+ \phi^+}^2 + 3(\delta_{\phi^+}^2 + \delta_{\phi^+}^2)\delta_{\phi^+}^2 \\
 &\quad + 4[(4k^2 + \frac{3}{4}p^2) - 12g^2]\lambda_{\phi^+}^2 + 18g^2 - 8(k^2 + \frac{3}{8}p^2) + \rho_{\phi^+ \phi^+}^4,
 \end{aligned}$$

$$\begin{aligned}
 16\pi^2 \delta_{\phi^*}^2 &= 6\delta_{\phi^*}^4 + 2[2(\delta_{\phi^*}^2 + 3\lambda_{\phi^*}^2) - \sigma_{\phi^*}^2] \delta_{\phi^*}^2 + 2(\delta_{\phi^*}^4 + \rho_{\phi^*}^2) + 4\sigma_{\phi^*}^2 \delta_{\phi^*}^2 + 4[(4k^2 + \\
 &\quad + \frac{3}{2}p^2) - 12g^2] \delta_{\phi^*}^2 + 12g^4 - 8(k^2 + \frac{1}{16}p^2), \\
 16\pi^2 \delta_{\phi^*}^2 &= 15\delta_{\phi^*}^4 + 12(\lambda_{\phi^*}^2 + \delta_{\phi^*}^2) \delta_{\phi^*}^2 + 3(\delta_{\phi^*}^2 + 2\sigma_{\phi^*}^2) \delta_{\phi^*}^2 - 2\rho_{\phi^*}^2 + 4[(4k^2 + \frac{3}{2}p^2) - 12g^2] \delta_{\phi^*}^2 + 12g^4 \\
 16\pi^2 \delta_{\phi^*}^2 &= [\delta_{\phi^*}^2 + 2(\delta_{\phi^*}^2 + 6\lambda_{\phi^*}^2)] \sigma_{\phi^*}^2 + 4(2\delta_{\phi^*}^2 - \delta_{\phi^*}^2) \delta_{\phi^*}^2 - 4\rho_{\phi^*}^2 + 4[(4k^2 + \frac{3}{2}p^2) - 12g^2] \delta_{\phi^*}^2 + 8(k^2 + \frac{1}{16}p^2), \\
 16\pi^2 \lambda_{\phi^*}^2 &= 4\lambda_{\phi^*}^4 + 2[3(3\lambda_{\phi^*}^2 + 2\delta_{\phi^*}^2) + 16\lambda_{\phi^*}^2 + 3\delta_{\phi^*}^2 + \delta_{\phi^*}^2] \lambda_{\phi^*}^2 + 2(\rho_{\phi^*}^2 + \frac{3}{2}\delta_{\phi^*}^2) + \\
 &\quad + 2[(2f_1^2 + \frac{15}{8}f_8^2) - \frac{3}{4}(\eta^2 + \frac{5}{2}g^2) + (4k^2 + \frac{3}{2}p^2) - 12g^2] \lambda_{\phi^*}^2 + 6g^4 - 2k^2 f_8^2, \\
 16\pi^2 \delta_{\phi^*}^2 &= \delta_{\phi^*}^4 + 2[(\lambda_{\phi^*}^2 + 2\delta_{\phi^*}^2) + (\lambda_{\phi^*}^2 + \delta_{\phi^*}^2) + \frac{1}{2}\sigma_{\phi^*}^2 + \frac{3}{2}\delta_{\phi^*}^2 + 4\lambda_{\phi^*}^2] \delta_{\phi^*}^2 + 4\rho_{\phi^*}^2 \\
 &\quad + 2[(2f_1^2 + \frac{15}{8}f_8^2) - \frac{3}{4}(\eta^2 + \frac{5}{2}g^2) + (4k^2 + \frac{3}{2}p^2) - 12g^2] \delta_{\phi^*}^2 + 12g^4 - 4k^2 f_8^2, \\
 16\pi^2 \rho_{\phi^*}^2 &= [2(\lambda_{\phi^*}^2 + 2\delta_{\phi^*}^2) + 2(\lambda_{\phi^*}^2 - \delta_{\phi^*}^2) + 3(\delta_{\phi^*}^2 - \sigma_{\phi^*}^2) + 2(4\lambda_{\phi^*}^2 + 3\delta_{\phi^*}^2)] \rho_{\phi^*}^2 \\
 &\quad + 2[(2f_1^2 + \frac{15}{8}f_8^2) - \frac{3}{4}(\eta^2 + \frac{5}{2}g^2) + (4k^2 + \frac{3}{2}p^2) - 12g^2] \rho_{\phi^*}^2 + 4k^2 f_8^2. \quad (7)
 \end{aligned}$$

The boundary conditions of the ordinary type are meant in every case.

The solutions of the set of equations (6) and (7) will be obtained here in some particular form

$$\omega_i(t) = \eta^i(t) \bar{\omega}_i \quad (8)$$

where $\omega_i(t)$ is one of the effective coupling constants,

$\bar{\omega}_i$ are merely numbers giving the fixed points of the set of equations (6) and (7) modified with the help of eq. (8),

$\eta^i(t)$ is the exact solution of the first equation of the set (5)

$$\eta_L^2(t) = \eta_L^2 [1 + \frac{2}{3}(\eta_L^2/16\pi^2)t]^{-1} \quad (9)$$

Certainly, instead of $\eta^i(t)$ in eq. (8) one might use expressions for $\eta^R(t)$ or $g(t)$ since the effective coupling constants, in accordance with eq. (5), are not independent in the region of the large transferred momentum

$$(\eta^L/g)^2 = 3, \quad (\eta^L/\eta^R)^2 = 7 \quad (10)$$

The investigation of solution of eqs. (6) and (7) in more general form is difficult and not needed here since the asymptotic freedom of the model (1) is completely proved already within eq. (8).

All the sets of fixed points of eqs. (6) are easily obtained analytically. The set of the fixed points suggested below $\bar{h}_i^2=0$, $\bar{q}_L^2=3.996588$, $\bar{q}_K^2=2.324852$, $\bar{f}_\eta^2=0.731385$, $\bar{p}_S^2=0.978321$, $\bar{k}^2=0.579819$, $\bar{p}^2=3.385276$; (11) is more convenient due to some reasons. All the other solutions lead either to unstable potential of scalar interaction or to loss of asymptotic freedom when the set of eq. (7) is solved. It should be noticed that the condition $\bar{h}_i^2=0$ in eq. (11) does not imply switching off the appropriate interaction in Lagrangian (2). Indeed h_i in eq. (2) may change within sufficiently wide limits and the solution $\bar{h}_i^2=0$ in eq. (11) means simply in this case that $h_i(t)$ decreases faster than $\eta^1(t)$ when $t \rightarrow \infty$. This fact leads also to appropriate arbitrariness when the masses of the light quarks are chosen what is undoubtedly convenient from the physical point of view.

The set of fixed points of eqs. (7) when eq. (11) is taken into account is obtained numerically:

$$\begin{aligned} \bar{\lambda}_{Z^4}^2 &= -0.34850, \bar{\delta}_{Z^4}^2 = 1.350490, \bar{\lambda}_{M^4}^2 = -0.176909, \bar{\delta}_{M^4}^2 = \\ &= 0.640883, \bar{\lambda}_{Z^2 M^2}^2 = -0.029269, \bar{\delta}_{Z^2 M^2}^2 = 0.289669, \quad (12) \\ \bar{\lambda}_{\phi^4}^2 &= 0.071325, \bar{\delta}_{\phi^4}^2 = 0.853870, \bar{\delta}_{\phi^4}^2 = -0.416073, \bar{\delta}_{\phi^4}^2 = 0.058079, \\ \bar{\lambda}_{\psi M^2}^2 &= 0.023125, \bar{\delta}_{\psi M^2}^2 = 0.061041, \bar{\rho}_{\psi M^2}^2 = -0.211904. \end{aligned}$$

This set of fixed points satisfy all the requirements of the model. It provides stable form of the potential of the scalar interaction and asymptotic freedom for the needed coupling constants. Other solutions of eqs. (7) which do not satisfy those requirements are not discussed here.

Thus the model possesses the asymptotic freedom on the whole. In the region of the large transferred momentum the coupling constants satisfy eq. (8) wherein the $\bar{U}_i$'s are determined by eqs. (10), (11) and (12). The arbitrariness remains for the gauge coupling constant η_L and the constants h_i . The majority of the fixed points obtained here are the ultraviolet-unstable ones. The importance of such solutions has already been mentioned in papers [3] and [9].

The Higgs mechanism which breaks the initial symmetry of the model by reducing it to the level of $SU(3) \times SU(2) \times U(1)$ and by giving masses to the vector and spinor fields is associated with the nonvanishing vacuum expectation value of the scalar fields

$$Z = \langle Z_i \rangle \Big|_{i=1, \dots, 4} \neq 0, \quad \mu = \langle M_i \rangle \Big|_{i=1, \dots, 4} \neq 0, \quad m = \langle \phi_p \rangle \Big|_{p=15} \neq 0 \quad (13)$$

The mass matrix of the vector fields appears after separating (13) within Lagrangian (2). In the same way the mass matrix of the spinor fields results from analogous transformations of Lagrangian (3). Both the mass matrices are readily diagonalized. The infinitesimal gauge transformation which leaves eq. (13) invariant gives opportunity within the initial $SU(4)$ group to separate the Abel subgroup which corresponds to the electromagnetic field

$$\frac{A_\mu}{e} = \frac{W_\mu^3}{\eta_L} + \frac{W_\mu^3}{\eta_R} + \frac{V_\mu^{15}}{g} ; \quad \frac{1}{e^2} = \frac{1}{\eta_L^2} + \frac{1}{\eta_R^2} + \frac{1}{g^2} \quad (14)$$

and the SU(3) subgroup of the massless coloured gluons mediating the strong interaction. In (14) $\tilde{g} = \sqrt{\frac{3}{2}} g$. The remaining SU(4) vector fields form two triplets of the super-heavy lepton-quark bosons (X-bosons). Their masses are fixed by the choice of m

$$m_X^2 = \frac{4}{3} m^2 g^2 \quad (15)$$

and must be done large enough ($m_X \geq 10^4 \text{ GeV}/c^2$) in order that the unobserved effects of the lepton-baryon interference be removed at low energy.

All the vector mesons responsible for the weak decays of the elementary particles are provided with masses after separating (13) in (2). In accordance with their values these mesons are divided into two groups: i) the light vector mesons

$$\begin{aligned} W_{1,\mu}^\pm &= \frac{1}{\sqrt{2}} (W_{1,\mu}^1 \mp i W_{1,\mu}^2), \quad m_{W_1^\pm}^2 = \frac{1}{2} \eta_L^2 Z^2 ; \\ Z_{1,\mu} &= \left(\frac{\eta_L \sin \varphi}{\sqrt{\eta_L^2 + \eta_R^2}} - \frac{e}{g} \frac{\eta_R \cos \varphi}{\sqrt{\eta_L^2 + \eta_R^2}} \right) W_{1,\mu}^3 - \left(\frac{\eta_R \sin \varphi}{\sqrt{\eta_L^2 + \eta_R^2}} + \frac{e}{g} \frac{\eta_L \cos \varphi}{\sqrt{\eta_L^2 + \eta_R^2}} \right) W_{2,\mu}^3 \\ &+ \cos \varphi \cdot \sqrt{1 - e^2/g^2} V_\mu^{15}, \quad m_{Z_1}^2 = \frac{Z^2}{2(\eta_R^2 + g^2)} \left(\frac{\eta_L \eta_R g}{e} \right)^2 ; \end{aligned} \quad (16)$$

with their masses of the order of $10^2 \text{ GeV}/c^2$ (fixed by choice of Z^2) responsible for the weak decays which are induced mostly by the left current and ii) the heavy vector mesons

$$\begin{aligned} W_{2,\mu}^\pm &= \frac{1}{\sqrt{2}} (W_{2,\mu}^1 \mp i W_{2,\mu}^2), \quad m_{W_2^\pm}^2 = \frac{1}{2} \eta_R^2 \mu^2 ; \\ Z_{2,\mu} &= \left(-\frac{\eta_L \cos \varphi}{\sqrt{\eta_L^2 + \eta_R^2}} - \frac{e}{g} \frac{\eta_R \sin \varphi}{\sqrt{\eta_L^2 + \eta_R^2}} \right) W_{1,\mu}^3 + \left(\frac{\eta_R \cos \varphi}{\sqrt{\eta_L^2 + \eta_R^2}} - \frac{e}{g} \frac{\eta_L \sin \varphi}{\sqrt{\eta_L^2 + \eta_R^2}} \right) W_{2,\mu}^3 \end{aligned} \quad (17)$$

$$+ \sin \varphi \sqrt{1 - e^2/g^2} \cdot V_\mu^{15}, \quad m_{Z_2} \approx \frac{1}{2} \mu^2 (\eta^2 + \tilde{Q}^2)$$

with their mass of the order of $10^3 \text{ GeV}/c^2$ (fixed by choice of μ^2) which induce extra weak decays. The angle φ in eqs. (16) and (17) is determined from diagonalization of the mass matrix for the $W_{L,\mu}^3$, $W_{R,\mu}^3$ - and V_μ^{15} -fields and only leading terms in $Z^2/\mu^2 \ll 1$ and $\mu^2/m^2 \ll 1$ are kept in eqs. (15) (16) and (17) for some masses of the intermediate bosons.

Now a few words must be said about the physical contents of the model. In total, the model contains 48 spinor particles: 13 leptons which are singlets under the SU(3) group of colour (some leptons may be treated as the fourth colour [7] if necessary), and 35 quarks ($9 \times 3 + 2$). All the particles, including neutrinos are 4-component spinors and their assignment on the representations of the gauge group G may be chosen as follows

$$\psi_l = \begin{bmatrix} n_{lc} \\ E_L^+ e^- \\ Z_L^+ X_L \end{bmatrix}, \quad \psi_q = \begin{bmatrix} 2_{lc} \\ \mu^- \\ \mu^+ \end{bmatrix}, \quad \psi = \begin{bmatrix} N_L + L_R \\ U^- \\ (N_L + L_R) \end{bmatrix}, \quad \eta = \begin{bmatrix} L_L + N_R \\ E_L^+ e^- \\ X_L^+ Z_R \end{bmatrix}$$

Here ($l=1,2,3$); $n_{lc} = n_l \cos \theta + \lambda_l \sin \theta$, $\lambda_{lc} = -n_l \sin \theta + \lambda_l \cos \theta$ where θ is the angle of the Gabbibo rotation. The B- and Q-multiplets contain heavy and superheavy leptons and quarks. Masses

of the spinor particles arise after separating (13) within Lagrangian (3) and are partially introduced as bare ones.

Here m_μ and m_η are of the order of the light lepton masses; m_B and m_Q are of the order of $10^3 \text{ GeV}/c^2$ or above this value. The heavy and superheavy leptons and quarks are supplied with large masses [$10^2 \text{ GeV}/c^2$ or above this value] and thus are separated from the light leptons and

quarks determining the low energy physics. The right currents are suppressed also in this range of energy by the factor of $(\eta_L m_{W_L})^2 / (\eta_R m_{W_R})^2 \approx 7 \cdot 10^{-2}$.

Thus no contradictions are evident till now between the predictions of the model and the experimental data. The low energy physics is mainly determined by the usual set of $(p, n, \bar{u}, \bar{c})$ - quarks and the known leptons $(e^-, \mu^-, \nu_e, \nu_\mu)$. The presence here of the third (besides e^- and μ^-) charged U^- -lepton (m_{U^-} is a free parameter of the model) is most likely confirmed by experiment now. The neutrino being 4-component with its mass very small (m_{ν_e}, m_{ν_μ} are of radiational nature here) does not contradict to the totality of the modern observational data and special experiments [10] are needed to establish its true nature. In the high energy region the model is practically vector-like. This fact permits to explain easily the anomalies [11] now observed in the neutrino scattering and also introduces the right currents into the theory in the most natural way. Here $R = \frac{13}{3}$ if the contribution of the U^- -lepton is taken into account only. The farther growth of R is due to the heavy and superheavy quarks and leptons.

To summarise, the present model demonstrates once again (see also Ref. [5]) the possibility to construct, within the class of asymptotically free theories, physically reasonable models of unified interaction, i.e. the models which possess the due mathematical rigour (absence of the "zero-charge" difficulty [6]) and realistic content.

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REFERENCES

- [1] P.Higgs, Phys. Lett. 12 (1964) 132
- [2] D.J.Gross and F.Wilczek, Phys. Rev. Lett. 30 (1973) 1343
H.B.Politzer, Phys. Rev. Lett. 30 (1973) 1346
- [3] H.-P.Chang, Phys. Rev. D 10(1974) 2706
E.S.Fradkin and O.K.Kalashnikov, J.Phys. A8(1975) 1614
- [4] E.S.Fradkin and O.K.Kalashnikov, Phys. Lett. 59B(1975) 159
- [5] E.S.Fradkin and O.K.Kalashnikov, Phys. Lett. 64B (1976) 177
- [6] L.D.Landau and I.Y.Poweramchuk, Dokl. Akad. Nauk USSR
102 (1955) 489
E.S.Fradkin, Zh.Eksp. Teor. Fiz. 28 (1955) 750
- [7] J.C.Pati and Abdus Salam, Phys. Rev. D8 (1973) 1240
Phys. Rev. D 10 (1974) 275
- [8] T.P.Cheng, E.Eichten and L.-P.Li, Phys. Rev. D9(1974)2259
- [9] B.L.Voronov and I.V.Tyutin, Lecture talk given at the
VIII-th Allunion School on Elementary Particle Physics,
Yerevan 1975; Yadernaya Fizika (Sov. Journ. of Nucl.
Phys.) 23 (1976) 664
- [10] B.Pontecorvo, Zh. Eksp. Teor. Fiz 26 (1968) 986
H.Pritzsch and P.Minkowski, Phys. Lett 62B (1976) 72
- [11] A.Benvenuti et al. Phys. Rev. Lett. 36 (1976) 1478;
B.Barish, in Proceedings of the International Neutrino
Conference, Aachen, West Germany, June 1976.

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