QUANTUM ELECTRODYNAMICS TESTS IN MUONIC SYSTEMS

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Abstract

This paper is a discussion on some experimental tests of the vacuum-polarization contribution in quantum electrodynamics (QED).

The possibility of testing -- under particularly favourable conditions -- effects, typical of the QED theory, due to the electronic vacuum polarization, by studying the energy levels of muonic atoms, was first suggested by Koslov, Fitch and Rainwater\(^1\) soon after Fitch and Rainwater had discovered the muonic X-rays\(^2\).

Before commenting on some experimental results obtained so far in muonic atom spectroscopy, let me, for clarification, speak briefly of some important new experimental results lately obtained in the field of experimental QED\(^3\).

One considers, generally, an experimental result as a contribution to the experimental QED if this result can be explained theoretically only by including electromagnetic radiative corrections; these are calculated using the standard QED theory which today is a well-defined mathematical procedure by which one is able to compute electromagnetic processes to any order, provided a certain number of constants are given, in particular the fine structure constant \(\alpha\).

Of course, as is advisable, the value of \(\alpha\) adopted in this case is the one obtained from experiments outside the experimental QED field. The most recent value of \(\alpha\) obtained without QED (\(\alpha_{\text{QED}}\)) comes from the measurements of the proton gyromagnetic ratio at low field and the a.c. Josephson effect: this value is\(^4\)

\[ \alpha_{\text{QED}} = 137.035987 \quad (29) . \]

First, let me mention an important result obtained by Van Dyck, Ekstrom and Dehemelt of the University of Washington, who give a new experimental value for the g-factor anomaly of the electron, \(a_e = (g_e-2)/2\), which is a purely QED quantity (it represents the deviation of the \(g_e\) factor of the electron from its value predicted by the Dirac equation).

The experimental result is\(^5\)

\[ a_e = (1159652410 \pm 200) \times 10^{-12} . \]

This value has been obtained by trapping a magnetic bottle a single electron!

Considering the experimental errors and the uncertainties with which the theoretical value can be computed, the value (2) is in quite good agreement with the value predicted by the QED theory.

It is interesting to know that the uncertainty \(\delta a_{\text{QED}}\) in the experimental value due only to the uncertainty of the value of the fine structure \(\alpha\) [Eq. (1)] is \(\delta a_{\text{QED}} = 250 \times 10^{-12}\). Among the various contributions to the value of the electronic vacuum polarization terms represent about one part in \(10^4\) of \(a_e\); this implies that, with this experiment, given the value (1) one can experimentally check these contributions, assuming everything else known, at most to 0.25\%, a limit already reached now with Eq. (2). It is clear that if a more precise experimental value for \(a_e\) is obtained in the near future, as seems possible\(^6\), then from this, assuming the QED theory one will be able to deduce a more precise value of \(\alpha\).

Another important experimental result is that recently obtained at CERN by Bailey et al.\(^6\) on the g-factor anomaly of the muon \(a_{\mu}\), which is also a purely QED quantity.

The new experimental value is

\[ a_{\mu} = (g_{\mu}-2)/2 = (1165922000 \pm 9000) \times 10^{-12} . \]

(3)

For this experiment, the situation concerning the contributions of the vacuum polarization terms to \(a_{\mu}\) is quite different from that for the electron anomaly \(a_e\); in fact, since in this case the average momentum transfer to the muon is \(m_{\mu}c\), these contributions are now relatively large, so that the uncertainty in the value of \(\alpha_{\text{QED}}\) here does not impose any limitation. However just because of this high momentum transfer, contributions to \(a_{\mu}\) coming from the strong interactions effects begin to have importance, as can be seen from Table 1\(^7\). The contribution to \(a_{\mu}\) coming from strong interaction effects has been computed using the results from experiments performed at the electron intersecting storage rings\(^8\).

### Table 1

The g-factor anomaly of the muon \(a_{\mu}\)

<table>
<thead>
<tr>
<th>(a_{\mu} \times 10^{12})</th>
<th>Some of the contributions to (a_{\mu}) in units of (10^{-12})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exp. value</td>
<td>Theor. value</td>
</tr>
<tr>
<td>Elec. vacuum polarization</td>
<td>Muinic vacuum polarization</td>
</tr>
<tr>
<td>115922000 ± 9000</td>
<td>11592000 ± 12000</td>
</tr>
</tbody>
</table>

One sees here that the limitations imposed for a test of the QED theory come mainly from the uncertainty in the strong interaction contributions; this implies that, at present, the vacuum polarization terms can be considered to be experimentally checked, assuming everything else is known, to a level similar to that obtained by the value (2) of \(a_e\) but at a momentum transfer two hundred times bigger.

One new important fact, that I wish to emphasize, is that to find agreement between experiment and theory it is necessary to take into account, in
this case, also the muonic vacuum polarization, which therefore can be considered tested to a level of at most 25%.

Let us now discuss some of the latest contributions to experimental QED from measurements performed on nucleon-lepton bound systems.

Let me first talk about some new results, recently obtained at the CERN Synchro-cyclotron by a CERN-Pisa University Collaboration, on the value \( S_2 \) of the energy level difference \( S_2 = 2S_{1/2} - 2P_{3/2} \) in the muonic ion \((u^+He)\): Fig. 1 shows the plot of the first lower levels of such a system and Table 2 gives the theoretical predictions for \( S_1 \) and \( S_2 \) according to the analysis of Rinker\(^8\).

\[
\text{Fig. 1 Schema of the lowest energy levels of the (u^+He) \# muonic ion.}
\]

In a quite recent analysis, Borie\(^9\) confirmed the values found by Rinker. She also calculated the contributions due to the two-photon term and to the hadron vacuum polarization term; these contributions were found to be, respectively, 0.44 and 0.05 MeV (they are not included in Table 2).

The new value for \( S_2 \) experimentally found is\(^1\):

\[
S_2^{\text{exp}} = 1527.5 \pm 0.3 \text{ MeV}.
\]  

This value has been obtained by irradiating \((u^+He)^+\) systems, previously prepared in the 2S level, with infrared radiation pulses (20 nsec wide) generated by a suitable dye laser. The value \( (4) \) corresponds to that wavelength \( \lambda \) for which the muonic ion \((u^+He)^+\) has a maximum probability to perform the \( 2S_{1/2} \rightarrow \rightarrow 2P_{3/2} \) transition; the transition is identified by detecting the subsequent prompt \( 2P \rightarrow 1S \) decay by looking at the characteristic 5.2 muonic X-ray emitted essentially in coincidence with the light pulses.

Recently, with new data on electron scattering from \( ^{4}He \), Sick et al.\(^3\) have obtained for the \( ^{4}He \) r.m.s. charge radius the new value

\[
\left( r^2 \right)^{1/2} = (1.674 \pm 0.012) \text{ fm}.
\]  

Using values \( (4) \) and \( (5) \) we see that the difference \( D \) between the theoretical value (taken from Table 2) and the experimental one is

\[
D = 0.2 \pm 4.2 \text{ meV}.
\]  

The uncertainty on \( D \) is mainly due to the uncertainty on the value \( (5) \) and defines for \( D \) a 95% confidence level. Equation \( (6) \) shows that the electronic vacuum polarization contribution to the \( S_2 \) difference is experimentally checked here to about 0.25%. The limit obtained here is very near to the one that it is possible to obtain from the measured value of \( a_0 \) and \( a_1 \) in conditions, however, where the measured quantity is essentially all due to the vacuum polarization effect\(^1\). The average momentum transfer in this case is \( \sigma (m_0 c)/2 \).

In order to have a direct proof that the experimental value \( (4) \) really represents \( S_2 \) and not \( S_1 \) as might be, due to some crazy situation, before quitting the experimental floor, the CERN-Pisa Collaboration also looked for the transition \( S_1 \) of course assuming now the value \( (4) \) for \( S_2 \) and taking for the \( S_2 - S_1 \) difference the fine structure term as given by Table 2. As expected they observed a peak in the right region and, as a preliminary result, they quote the value\(^1\)

\[
S_1^{\text{exp}} = (1381.3 \pm 1) \text{ MeV}.
\]  

Table 2

<table>
<thead>
<tr>
<th>Contributions</th>
<th>( S_1 = 2S_{1/2} + 2P_{3/2} )</th>
<th>( S_2 = 2S_{1/2} + 2P_{3/2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electronic vacuum polarization</td>
<td>Uehling terms ( a(\alpha) )</td>
<td>1666.1</td>
</tr>
<tr>
<td></td>
<td>Källen-Sabry terms ( a(\alpha) )</td>
<td>11.6</td>
</tr>
<tr>
<td>Max. vacuum polarization</td>
<td>( a(\alpha) )</td>
<td>-</td>
</tr>
<tr>
<td>Recoil</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>Nuclear polarization</td>
<td>3.1</td>
<td>3.1</td>
</tr>
<tr>
<td>Fine structure</td>
<td>-</td>
<td>145.6</td>
</tr>
<tr>
<td>Finite size corrections</td>
<td>-5.1-102.0 (( r^2 ))</td>
<td>-5.1-102.0 (( r^2 ))</td>
</tr>
<tr>
<td>Vertex correction</td>
<td>( a(\alpha) )</td>
<td>-10.0</td>
</tr>
<tr>
<td></td>
<td>( a(\alpha)^2 )</td>
<td>-0.2</td>
</tr>
<tr>
<td>Total in meV ( a )</td>
<td>1666.9-102.0 (( r^2 ))</td>
<td>1813.1-102.0 (( r^2 ))</td>
</tr>
<tr>
<td>Total in meV ( b )</td>
<td>1381.7 \pm 4.2</td>
<td>1527.5 \pm 4.2</td>
</tr>
<tr>
<td>Total in Å ( d )</td>
<td>8973.5 \pm 21.8</td>
<td>8118 \pm 21.8</td>
</tr>
</tbody>
</table>

\(^a\) The contribution due to weak interaction is \( 2 \times 10^{-3} \) meV; J. Bernabeu et al., \( \text{CERN TN}-1855 \) (1974).

\(^b\) The theoretical uncertainty is \( 0.1 \) meV and comes mostly from the uncertainty of the nuclear polarization contribution.

\(^c\) \( (r^2) \) taken from Ref. 9; the error given here also takes into account the uncertainty with which \( (r^2) \) is given.

\(^d\) The natural width of \( S_1 \) and \( S_2 \) is \( \Gamma = 8 \) Å.
The error in (7) is relatively large, because of statistics; the Clebsch-Gordan coefficient disfavors the 2S1/2 → 2P3/2 transition. Moreover, given the apparatus as it was built, an absolute calibration line, sufficiently near to the value (7), could not be used.

Value (7) confirms (see Table 2), as was already assumed, that indeed value (4) refers to the 2S1/2 → 2P1/2 transition.

Accurate measurements of the X-rays emitted in transitions between n states in high-Z muonic atoms also provide a very good test of QED corrections to the energy levels; in particular, as already mentioned, of those corrections due to the electronic vacuum polarization. In these cases the momentum transfer is relatively high, since it is (m_e c^2)/n: moreover, in general, for n sufficiently large, effects connected with the nuclear charge radius are negligible.

In the past, a number of measurements were in disagreement with each other and with the theory; now the situation is changed, partly due to the fact that more complete calculations are now available (see article by Mohr1) and partly because more accurate experiments have been performed. In particular, Dablier et al. at SIN and Hargrove et al. at SLAC have provided very accurate measurements of muonic atom transition energies in Pb, Ba, Sr, Ca, etc., in the 150-450 keV energy region. All these measurements are done by measuring directly the energy of the X-ray emitted by means of a solid-state detector. All I can say is that these authors state that both the experimental situation and the theoretical one are now enough under control that it is possible to establish a check of the electronic vacuum polarization corrections to the transition energies to a level of 0.2%. I suppose we will hear directly from these authors at this conference about their interesting results.

Before making some final remarks, let me, also for completeness, say what is the situation about the Lamb shift in the hydrogen atom1. The different contributions to the Lamb shift, as calculated by Mohr1,10) are listed in Table 5: the theoretical value \( \Delta \text{th} \) is 1057.888 ± 0.013 MHz.

\[ \Delta \text{th} = 1057.888 \pm 0.013 \text{ MHz} \]  

where the error is the estimated uncertainty for the uncalculated terms.

I wish to make two remarks here:

i) The value (8) for \( \langle r^2 \rangle^{1/2} \) lately found by Borkowski et al.11) is quite different, within the quoted errors, from the value 0.80 accepted earlier (and for quite a long time): the uncertainty induced in (9) by the uncertainty of (8) is 0.007.

\[ \langle r^2 \rangle^{1/2} = 0.87 \pm 0.02 \text{ fm} \]  

ii) There exists another theoretical estimate of the Lamb shift, due to Erickson, which gives, assuming (8) for R, the value12)

\[ \Delta \text{th} = 1057.939 \pm 0.011 \text{ MHz} \]  

The experimental value obtained by Lundeen and Pipkin13) is

\[ \Delta \text{exp} = 1057.893 \pm 0.020 \text{ MHz} \]  

From the comparison of the value (11) with the theoretical ones, one can say that the electronic vacuum polarization correction to the 2S level of the hydrogen atom is checked (assuming everything else is known) to about 0.3% at least: i.e. one obtains, at present, a limit very similar to the ones obtained above. In this case the average momentum transfer is \( \sqrt{\langle m_e c^2 \rangle}/2 \).

In conclusion, we can say that we see that electronic vacuum polarization contributions can be considered well checked to a level of about 0.2-0.3% and this for a very large variety of momentum transfer: it looks an interesting challenge for the muonic atom spectroscopy to go beyond these results.

A possible line of action is the one followed by a group of experimentalists at SIN, that is to look at some specific transition X-rays for the case of muonic atoms with Z around 12, for which the nuclear finite size contribution is negligible and, also, the electron screening effect (which seems to be one of the limitations for the case of high Z muonic atoms) is minimized. This is one of the reasons for which a high resolution bent crystal spectrometer, now operating in the SIN \( \mu \) channel, was built.

Another way to obtain a better check for the vacuum polarization contributions to the muonic atom energy level is to perform the same type of experiment as the CERN-Pisa Collaboration applied to the muonic helium system, on the neutral muonic atom up: in this case, the correction due to the finite size is relatively less important and therefore the uncertainty on the proton r.m.s. charge radius gives a smaller uncertainty in the theoretical prediction for the 2S – 2P differences.

This is why a group of physicists at SIN are trying to see if it is possible, under some experimental conditions, to maintain up systems in a 2S level, in a low-density gas target, for a sufficiently long time.

### Table 3

<table>
<thead>
<tr>
<th>Correction</th>
<th>Order</th>
<th>Value (MHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Self energy</td>
<td></td>
<td>1085.412</td>
</tr>
<tr>
<td>Vacuum polarization</td>
<td></td>
<td>-24.097</td>
</tr>
<tr>
<td>Fourth order</td>
<td></td>
<td>0.101</td>
</tr>
<tr>
<td>Reduced mass</td>
<td></td>
<td>-1.305</td>
</tr>
<tr>
<td>Relativistic recoil</td>
<td></td>
<td>0.739</td>
</tr>
<tr>
<td>Nuclear size</td>
<td></td>
<td>0.145</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>1057.888</td>
</tr>
</tbody>
</table>
Table 4

<table>
<thead>
<tr>
<th>Contributions</th>
<th>Transitions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$^1S_2 + ^3P_{1/2}$</td>
</tr>
<tr>
<td>Electronic polarization</td>
<td>$a(\omega)$</td>
</tr>
<tr>
<td></td>
<td>$a^*(\omega)$</td>
</tr>
<tr>
<td>Fine structure</td>
<td></td>
</tr>
<tr>
<td>Vertex correction</td>
<td>$a(\omega)$</td>
</tr>
<tr>
<td></td>
<td>$a^*(\omega)$</td>
</tr>
<tr>
<td>Hyperfine structure</td>
<td></td>
</tr>
<tr>
<td>Finite-size correction</td>
<td>$-0.0029$</td>
</tr>
<tr>
<td>Total (in $a(\omega)$)</td>
<td></td>
</tr>
<tr>
<td>Total (in $a^*(\omega)$)</td>
<td></td>
</tr>
<tr>
<td>In $\lambda$</td>
<td></td>
</tr>
</tbody>
</table>

a) Assuming for $(r^2)^{1/2}$ the value (7).

b) Linewidth $\Gamma = 20 \AA$.

The last published values on the expected 2S - 2P energy differences for the muonic atom $^6$Li have been obtained using the old value for the r.m.s. charge radius of the proton $R_1$ in Table 4 are given the new values taking for $R$ the quantity (8).

It is clear, however, that additional experimental information on the proton r.m.s. charge radius would be extremely welcome in order to eliminate any doubt on this very important quantity necessary to describe accurately the lower S levels of the proton-lepton bound systems $ep$ and $np$.

References

4) E. Picasso, Recent experimental tests on QED at low energies, Nova Acta Leopoldina, Suppl. No. 8 (1976) 159.
12) For discussions on a limit set by the value (4) on the muon-hadron anomalous interaction, see, E. Zavattini, On vacuum polarization in muonic atoms and some related topics, Lectures given at the "Ettore Majorana" Centre: Exotic atoms and related topics, Erice, Sicily, 24-30 April, 1977.


