K⁻n AND K⁻p ELASTIC SCATTERING IN K⁻d
COLLISIONS FROM 1.2 TO 2.2 GeV/c

Y. Declais, J. Duchon, M. Louvel, J.-P. Patry and J. Seguinot
Université de Caen, Caen, France

P. Baillon, C. Bricman, M. Ferro-Luzzi, J.-M. Perreau and T. Ypsilantis
CERN, Geneva, Switzerland

GENEVA
1977
Propriété littéraire et scientifique réservée pour tous les pays du monde. Ce document ne peut être reproduit ou traduit en tout ou en partie sans l'autorisation écrite du Directeur général du CERN, titulaire du droit d'auteur. Dans les cas appropriés, et s'il s'agit d'utiliser le document à des fins non commerciales, cette autorisation sera volontiers accordée.

Le CERN ne revendique pas la propriété des inventions brevetables et dessins ou modèles susceptibles de dépôt qui pourraient être décrits dans le présent document; ceux-ci peuvent être librement utilisés par les instituts de recherche, les industriels et autres intéressés. Cependant, le CERN se réserve le droit de s'opposer à toute revendication qu'un usager pourrait faire de la propriété scientifique ou industrielle de toute invention et tout dessin ou modèle décrits dans le présent document.

Literary and scientific copyrights reserved in all countries of the world. This report, or any part of it, may not be reprinted or translated without written permission of the copyright holder, the Director-General of CERN. However, permission will be freely granted for appropriate non-commercial use. If any patentable invention or registrable design is described in the report, CERN makes no claim to property rights in it but offers it for the free use of research institutions, manufacturers and others. CERN, however, may oppose any attempt by a user to claim any proprietary or patent rights in such inventions or designs as may be described in the present document.
K⁻n AND K⁻p ELASTIC SCATTERING IN K⁻d
COLLISIONS FROM 1.2 TO 2.2 GeV/c

Y. Declais*), J. Duchon, M. Louvel, J.-P. Patry and J. Seguinot**)
Université de Caen, Caen, France***)

P. Baillon, C. Bricman†), M. Ferro-Luzzi,
J.-M. Perreau and T. Ypsilantis+++)
CERN, Geneva, Switzerland

GENEVA
1977

*) Presently at LAPP, Annecy, France.
**) Presently at CERN, Geneva, Switzerland.
***) Supported by IN2P3, CNRS, France.
†) Supported by IISN, Bruxelles, Belgium.
+++ Presently at CEN, Saclay, France.
ABSTRACT

This report contains the detailed description of an experiment which has determined the differential cross section of the $K^{-} n \rightarrow K^{-} n$ elastic scattering reaction. The results are 12 angular distributions spanning the $K^{-} n$ c.m. energy interval from $\sim 1.86$ to $\sim 2.32$ GeV. The measurements have been performed at the CERN PS using a beam of negative kaons with momenta from 1.2 to 2.2 GeV/c incident on a liquid deuterium target. By means of an electronic apparatus the process $K^{-} d \rightarrow K^{-} n p_{s}$ was identified and recorded; this process is basically the same as the $K^{-} n$ elastic reaction insofar as the spectator proton $p_{s}$ has low momentum. The elastic reaction was derived from the above process by taking into account the Fermi motion of the target neutron and by introducing the appropriate corrections to compensate for the effects due to the composite nature of the deuteron (double-scattering, final state interaction). These results, constituting the first extensive collection of data on the pure isospin 1 $\bar{K}N$ state, have been used in conjunction with other data in a preliminary partial wave analysis of the $\bar{K}N$ elastic system over the c.m. energy range from 1.84 to 2.23 GeV. Mainly for testing purposes, a similar amount of data has been collected for the $K^{-} p$ elastic reaction also from $K^{-} d$ collisions ($K^{-} d \rightarrow K^{-} p n_{s}$).
## CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>2. EXPERIMENTAL PROCEDURE</td>
<td>4</td>
</tr>
<tr>
<td>2.1 Method</td>
<td>4</td>
</tr>
<tr>
<td>2.2 Set-up</td>
<td>6</td>
</tr>
<tr>
<td>2.3 Beam</td>
<td>9</td>
</tr>
<tr>
<td>2.4 Multiwire proportional chambers</td>
<td>13</td>
</tr>
<tr>
<td>2.5 Neutron detector</td>
<td>18</td>
</tr>
<tr>
<td>2.6 Electronics</td>
<td>23</td>
</tr>
<tr>
<td>3. DATA ANALYSIS</td>
<td>28</td>
</tr>
<tr>
<td>3.1 Geometry</td>
<td>28</td>
</tr>
<tr>
<td>3.2 Momentum analysis</td>
<td>29</td>
</tr>
<tr>
<td>3.3 Neutron detection</td>
<td>32</td>
</tr>
<tr>
<td>3.4 Event reconstruction</td>
<td>34</td>
</tr>
<tr>
<td>4. CROSS SECTIONS</td>
<td>37</td>
</tr>
<tr>
<td>4.1 Acceptance</td>
<td>38</td>
</tr>
<tr>
<td>4.2 Neutron detector efficiency</td>
<td>41</td>
</tr>
<tr>
<td>4.3 Normalization</td>
<td>43</td>
</tr>
<tr>
<td>4.4 Results</td>
<td>46</td>
</tr>
<tr>
<td>5. DEUTERIUM CORRECTIONS</td>
<td>58</td>
</tr>
<tr>
<td>5.1 Introduction</td>
<td>58</td>
</tr>
<tr>
<td>5.2 Formalism</td>
<td>59</td>
</tr>
<tr>
<td>5.2.1 Fermi motion</td>
<td>60</td>
</tr>
<tr>
<td>5.2.2 Interference between graphs</td>
<td>61</td>
</tr>
<tr>
<td>5.2.3 Double scattering and final-state interaction</td>
<td>61</td>
</tr>
<tr>
<td>5.2.4 Spin</td>
<td>62</td>
</tr>
<tr>
<td>5.2.5 Isospin</td>
<td>66</td>
</tr>
<tr>
<td>5.3 Results</td>
<td>66</td>
</tr>
</tbody>
</table>
6. PARTIAL WAVE ANALYSIS
   6.1 Introduction 69
   6.2 Method 69
   6.3 Data 72
   6.4 Results 74

Acknowledgements 84

REFERENCES 85
1. **INTRODUCTION**

The elastic scattering of negative K-mesons on the proton and on the neutron of the deuterium has been measured at six incident momenta equally spaced between 1.2 and 2.2 GeV/c. Differential cross sections over the almost complete angular range have been obtained for \( K^- p \) and \( K^- n \). The aim of the experiment was the measurement of the pure isospin \( I = 1 \) reaction \( K^- n \rightarrow K^- n \). The results for the reaction on proton are a by-product and provide a verification of the assumptions necessary for the analysis of the neutron reaction.

The experiment has been motivated by the need for new data in the current analyses of the \( \bar{K}N \) interaction in the medium-to-low momentum region around 1.5 GeV/c. Data on the \( I = 1 \) channel are particularly useful in disentangling present discrepancies and verifying the claims of the different analyses. Present data of this type are very scarce and have not provided much of a constraint in describing the \( \bar{K}N \) partial waves and their energy dependence. In this connection a brief review of the present state of the art is in order.

The situation concerning the \( \bar{K}N \) partial waves has remained practically unchanged for the last few years. Several analyses have been performed, some of them very ingenious, and new resonances have been proposed on the basis of better or more complete fits to the data. On the other hand, none of these studies has provided a really unambiguous set of results. Only the dominant (and therefore well established) resonances are presently better known than when first proposed. In the region above the \( \bar{K}N \) threshold these are: \( \Lambda(1520, 3/2^-) \), \( \Sigma(1660, 3/2^-) \), \( \Lambda(1670, 1/2^-) \), \( \Lambda(1690, 3/2^-) \), \( \Sigma(1760, 5/2^-) \), \( \Lambda(1815, 5/2^+) \), \( \Sigma(2030, 7/2^+) \) and \( \Lambda(2100, 7/2^-) \). The properties of these resonances are almost all known: mass, width, spin, parity, isospin and branching ratios into two and more bodies. The need for re-evaluation is mainly in the existence and properties of lesser known resonances where the opinions vary greatly between one analysis and the next. For a list of controversial states we refer to the Particle Data Group compilation [1] with the warning that it is now a commonly accepted procedure to include all claims without attempting to discriminate between them. It is our opinion that, although many of the states are probably real, the evidence in their favour (or disfavour) is still quite marginal. This we attribute to the basic poverty of the data bank used in these studies. What is more important, perhaps, is that not much has come in since the early collections in the way of new and different types of data; by this we mean data on reactions other than those obtainable from \( K^- p \) interactions such as differential cross sections of \( K^- n \) and \( \bar{K}^0 p \), polarizations of \( \bar{K}^0 n \), etc.
The major development has instead gone into methods for deriving partial waves from the available data. The most recent work in this direction is briefly summarized below. From a theoretical point of view, the most ambitious attempt is the one by Langbein and Wagner [2]. They tried to deal simultaneously with the three $\bar{K}N$ channels ($\bar{K}N \rightarrow \bar{K}N$, $\bar{K}N \rightarrow \Lambda\pi$, $\bar{K}N \rightarrow \Sigma\pi$) using a multichannel formalism and an energy-independent approach. This is ideally what one aims at and what has been done, with undoubted success, for the $\pi N$ system. Here, unfortunately, the results turned out to be less than satisfactory. The amplitudes are discontinuous in their energy dependence, the known resonances (when not specifically parametrized as such) are hardly recognisable. More seriously, the solutions move from one Barrelet-zero trajectory to another thus violating one of the acknowledged criteria for continuity. Less ambitious attempts have been tried, with more success, by either dropping the "energy-independent" or the "multichannel" feature. The first in this direction has been the energy-dependent multichannel analysis by LMK [3]. The above three channels are treated in a K-matrix approach from threshold to ~ 1.2 GeV/c. The amplitudes are necessarily smooth and so are the Barrelet-zero trajectories. Old resonances are confirmed and a number of new resonances are claimed. Unfortunately, the agreement between data and expectations is not as good as one may wish. In some cases there are serious misrepresentations of channels like $\bar{K}^0\Lambda$ and $\bar{K}^0p$ near the $\Lambda\pi$ threshold. This reduces the confidence on the new resonances claimed and on the correct description of the partial waves. Two more studies should be mentioned, mainly because they make use of data hitherto not available. Thus, the CHM analysis [4] deals with the $\bar{K}N$ channel only but it has the appeal of including a very large amount of new statistics on the $K^-p$ and $\bar{K}^0n$ final states over the region of $\Sigma(2030)$ and $\Lambda(2100)$. An interesting feature of this analysis is that it does not require the marginal resonances put forward by earlier analyses of the same region. On the other hand, not much light is shed on the behaviour of the low-spin background and nothing essentially new is offered concerning the main resonances. One more analysis should be recalled. This is of the multichannel and energy-dependent type and has been performed on data which include new statistics in the region of 1 GeV/c by the RLIC group [5]. The energy range, from ~ 0.3 to ~ 2 GeV/c, is wider than that of any previous study and there are more data here both in the very low energy region around 400 MeV/c [6] and in the central 1 GeV/c region. This analysis, the most complete performed to date, will be used as a basis for the calculations of the deuteron corrections.

It is worth examining the data used in the above analyses. The great majority comes from $K^-p$ interactions in bubble chambers. In particular, this is the main source of information on reactions such as $K^-p \rightarrow \Lambda\pi^0$, $\Sigma^0\pi^0$, $\Xi^+\pi^-$ and $\bar{K}^0n$. There are some counter experiments in which relatively simple
reactions have been measured with good statistics: these are $K^-p + K^-p$
differential cross sections and polarizations, total $K^-p$ and $K^-d$
cross sections, partial $K^-p \rightarrow K^0n$ cross sections. Notice that the total cross
sections on hydrogen and deuterium have been used to derive isospin 0 and 1
total cross sections. Unfortunately, this procedure is not without ambiguities.
The consequence is that not all the experiments give consistent results. In
particular, large discrepancies can be noticed between the results of ref. [7]
and ref. [8] near 400 MeV/c. Also, in the neighbourhood of 800 MeV/c there
is a considerable difference in detailed behaviour between ref. [8] and ref. [9].
Another source of data is the $K^0n$ partial cross section measurement [10]. These
are counter experiments which cover with good statistics and good momentum
resolution the region from $\sim 0.5$ to $\sim 3$ GeV/c. Unfortunately, they are rather
limited in scope and do not provide alone a good handle for the determination of
the individual amplitudes (they only constrain sums of amplitudes). As for data
on reactions from incident channels different from $K^-p$, only a small number of
experiments have tackled the problem. Statistically meagre data exist from
experiments with $K^-$ in deuterium-filled bubble chambers [11]. Very little has
been done with incident $K^0$'s [12]. Notice that the above measurements are the
only ones which give us direct information on the separate $I = 1$ component.

The lack of data on the individual isospin components is at the origin
of most of the discrepancies which can be found between different partial
wave analyses. It is an interesting fact of life that when it comes to
predicting data not used in the fit such as for example the $K^-n$ elastic cross
section, all the analyses differ from one another. The differences can be,
in certain cases, as large as a factor two. It has been the purpose of this
experiment to try to fill this gap by providing an accurate measurement of at
least one such quantity (the $K^-n$ differential cross section), to examine the
above predictions and finally to incorporate these data in a simple one-
channel energy-dependent study of the $\bar{K}N$ partial waves.

The measurements have been performed with the electronic technique by
determining the momentum vector of the scattered $K$ and the direction of the
recoil neutron in the reaction $K^-d \rightarrow K^-n P_s$ (by $P_s$ we mean that we select
protons which to a first approximation do not participate in the interaction,
i.e. they are "spectators"). With a total of six incident momentum settings
(from 1.2 to 2.2 GeV/c in steps of 0.2 GeV/c) we cover the c.m. energy range
of the $K^-n$ system from $\sim 1.8$ to $\sim 2.4$ GeV. The data have been subdivided
in energy bins sufficiently small to provide useful information on the energy
dependence of the reaction, yet sufficiently large to contain meaningful
statistics. The c.m. angular region covers the range of $\cos \theta^*$ from
about -0.9 to about 0.8. The results presented here contain minor improve-
ments, but are essentially the same as those given in ref. [13].
The organization of this article is as follows. The method and the experimental procedure are described in sect. 2. The off-line event reconstruction can be found in sect. 3 and the procedure for calculating the differential cross section in sect. 4. In sect. 5 we shall examine the problems introduced by the use of deuterium as the target; in particular, we evaluate the corrections needed to account for the Fermi motion of the neutron, for the interaction between the two nucleons after scattering, for the effect of double scattering and for the Glauber screening effect. Finally, in sect. 6 we discuss the results of our own partial wave analysis using these and other data.

2. EXPERIMENTAL PROCEDURE

2.1 Method

The purpose of the experiment was to measure the elastic scattering of negative kaons on neutrons. For this we have used a deuterium target and selected out reactions of the type

\[ K^- d \rightarrow K^- n p_s. \]  

(1)

The "spectator" proton \( p_s \) is required to have momentum smaller than that of the neutron and less than 250 MeV/c in any case. The above requirement insures that, to a first approximation, the interaction has taken place on the neutron rather than on the proton. In order to identify reaction (1) we measure the momentum vector of the scattered kaon and the direction of the recoil neutron. The momentum vector of the incident kaon is known from the optics of the beam. If in addition we are sure that no extra charged or neutral particle comes out of the interaction, then we have enough information to perform a zero-constraint calculation of the complete kinematics of reaction (1).

The method consists of sending a suitably separated and tuned beam of negative kaons on a liquid deuterium target. The desired reactions are among those which satisfy the following conditions:

a) at least one charged particle is emitted on one side of the beam axis and in a given angular range subtended by detectors and a magnet;

b) no charged particles or \( \gamma \)-rays are emitted in any other direction around the target.

We then interrogate the detectors of the charged-branch asking for one and only one track. The momentum of this track is reconstructed from its deflection through the magnet. This determines the momentum vector of the scattered kaon. The incident-branch is equipped with MWPC's and provides
the momentum vector of the incident particle. The above two vectors define the position of the interaction. The target is surrounded by a veto-box (coque) approximately a 4π geometry such as to anticoincide all charged particles and γ-rays other than those emitted towards the charged branch. The spectator proton has too little energy to reach the veto-box. We are then left with the neutron identification and measurement. This is done by a series of spark chambers placed along the direction of the neutron. The spark chambers of this neutron-branch are interleaved with converter plates where the neutron can interact and generate charged particles energetic enough to go through one or more plates. From the origin of the secondary tracks and the interaction point in the target one can determine the direction of the neutron.

The data acquisition system is sufficiently fast to allow for the accumulation of other measurements in addition to those corresponding to reaction (1). The neutron-branch has a set of wire chambers which provide the information to reconstruct the trajectory of the proton and identify the reaction

\[ \vec{K}^\uparrow d + \vec{K}^\uparrow p \rightarrow n_s, \]

where \( n_s \) stands for a spectator neutron. The above reaction is analogous to reaction (1) insofar as the scattered-K detection is concerned and does not require the spark chambers of the neutron branch. This provides us with a source of information on the validity of the corrections needed to account for the fact that our target is a bound nucleon inside an atom rather than an isolated stationary particle. The comparison between reactions (2) and those on hydrogen provides a direct verification of the corrections discussed in sect. 5.

Several angular settings of the charged and neutron branches are necessary to cover the full scattering region. This is not a requirement of the method but comes about because of the finite size of the magnet and the detectors. The procedure was to set up a given angular configuration and accumulate data at all desired momentum settings. Then, change the angular setting and repeat the operation. In practice, we achieved a satisfactory coverage of the angular and momentum region with six angular settings and six momenta. This represents a total of 36 distinct runs. From the experimental point of view this implies a close scrutiny on each of the independent normalizations of the cross sections. For these and other reasons we deliberately introduced an appreciable amount of overlap in the various angular settings.

In the procedure described above the measurement of the cross section depends crucially on the neutron detector efficiency. This, in turn, is a function of the interaction rate of the neutron in the converter plates which varies according to the converter material and the neutron energy. All this could
be calculated, but at the expense of introducing a considerable uncertainty. We have preferred to use an experimental - though somewhat circuitous - calibration of the neutron detector efficiency by comparing the event rates for reactions with and without an observed neutron. Due to the Fermi motion of the target, this gives us an efficiency function averaged over the energy range accessible to the neutron recoiling against a scattered particle of fixed momentum and angle. To improve on the statistical significance of such a comparison we have used a large sample of reactions of the type

$$\pi^{-}d + \pi^{-}n \rightarrow p_s$$  \hspace{1cm} (3)

taken specifically for calibration purposes at a fixed incident momentum and all angular settings. The results from the kaon and pion reactions are found to agree within statistics.

2.2 Set-up

The experiment has been performed using the CERN protonsynchrotron. The beam was the partially separated $m_p$ of the South Hall (see sect. 2.3 for a description). Fig. 1 shows the lay-out in the $40^\circ$ position.

![Diagram](image)

**Fig. 1**

Apparatus in the $40^\circ$ position. The symbols are described in the text.
The incident branch has two Cherenkov counters, $C_1$ and $C_2$, used in the threshold mode to identify the kaons among a background consisting mostly of pions (their operational features are discussed in sect. 2.3). The scintillation counters $S_1$ and $S_2$ have size $10 \times 5$ cm$^2$ and 4 cm diameter respectively and thickness 4 mm and 2 mm. They define the beam acceptance and mark the arrival time of the incident particle. The direction of the particle is measured by the two MWPC's $W_1$ and $W_2$ with an overall resolution of $\pm 2$ mrad inclusive of multiple scattering. The characteristics of these chambers are discussed in sect. 2.4 together with the other MWPC's of the apparatus. The iron wall has the purpose of screening the target region and the neutron detector from an appreciable beam-halo.

The target contains liquid deuterium and is made of a cylinder 6 cm in diameter and 25 cm in length with mylar windows at each end. The incoming particle traverses a total mylar thickness of 400 $\mu$ and the outgoing particles 200 $\mu$ of mylar and 2 mm of "Delilith". Fig. 2(a) presents a view of the target and the surrounding apparatus. Around the target sits a veto-box, called "coque" in the figures, made of scintillator-lead sandwiches representing a total of $\sim 3$ radiation lengths. The purpose of the coque is to cover as much

---

**FIG. 2**

Target sections (a) and details of the surrounding veto-box (b).
as possible of the solid angle around the target in the regions not subtended by the outgoing branches. Fig. 2(b) gives a schematic view of this apparatus showing the position and size of the coque apertures, viz.:

a) the entry window, left open for the target-reservoir connections,

b) the exit window, defined by four counters, leaving enough space for the unscattered beam to go through without affecting the veto efficiency of the coque and

c) the side windows which allow the scattered and recoil particles to reach the outgoing branches without being anticoincided (no lead is present along the trajectory of the recoil neutron).

Notice that the open areas are not left unprotected. They are matched by elements outside the coque as shown in fig. 1. \( S_{tr} \) is a scintillator 6 mm thick serving as the "transmission" counter i.e. to reject particles which did not interact in the target and other particles emitted forward. \( S_3, S_5, S_6 \) and \( S_7 \) are scintillators 6 mm thick which detect if a charged particle has been emitted toward the charged- or neutral-branch. \( B_2 \) and \( B_3 \) are lead-scintillator sandwiches whose purpose is to reject the \( \gamma \)-rays which may escape through the side windows in directions other than the outgoing branches. This assembly leaves little solid angle around the target which is not viewed by either a counter or a wire- or spark-chamber.

The charged-branch contains a magnet preceded by three and followed by two MWPC's. The magnet is C-shaped, has a useful field region 50 cm high, 10 cm wide and 100 cm long, and was operated at its maximum unsaturated field strength of 1.5 Tesla. The yoke region is shown shaded in fig. 1; the dotted rectangle shows the limits of the region where the magnetic field is known. The \( S_{4a} \) and \( S_{4b} \) scintillation counters reject particles which traverse the magnet in regions (too near the yoke or too far out) where the field map is poorly known. The \( S_5 \) scintillator at the end of the branch requires that the selected particles traverse the magnet with the approximate momentum required by the elastic kinematics.

The neutron-branch is defined by the counter \( S_7 \), the two MWPC's \( W_8 \) and \( W_9 \), and the neutron detector. The latter is described in sect. 2.5. Notice that this branch measures either the neutron or the proton direction depending on the trigger.

We have taken data under two trigger conditions:

a) the "K\( ^- \)n trigger", which selects the events from reaction (1), defined by

\[
I \cdot (\text{coque} \cdot B_2 \cdot B_3) \cdot S_{tr} \cdot (S_3 \cdot S_4 \cdot S_5) \cdot S_6,
\]

where \( I \) is the incident particle definition, \( I = \frac{\bar{c}_1 \cdot C_2 \cdot S_1 \cdot S_2}{S_7} \).
b) the "$K^-p$ trigger", which selects the events from reaction (2), defined by
\[ I \cdot (c_{\text{coque}} \cdot E_2 \cdot E_3) \cdot S_{tr} \cdot (S_3 \cdot S_4 \cdot S_5) \cdot S_7. \]

The rates for the $K^-n$ and $K^-p$ triggers were low enough (from a fraction of a trigger to a few triggers per burst) to allow the acquisition of both reactions during the same run without loss of time. Details of the electronics are given in sect. 2.6.

2.5 Beam

The $m_7$ beam of the PS is described in detail in ref. [14]. Here we recall its main features. Fig. 3 shows the part of the CERN South Hall traversed by the beam. The latter originates from the internal target 1, a beryllium rod 1 cm long and 1 mm in diameter. The first magnet views the target at an angle of 151 mrad and has an acceptance of 650 μsr. The total beam length is 40 m from source to final image. The separation is

**FIG. 3**

Schematic lay-out of the $m_7$ beam. The symbols represent: Q quadrupoles, BM bending magnets, L collimators, ES electrostatic separator. The shaded area is the concrete shielding. The numbers refer to the PS elements.
achieved in one stage by means of a 10 m long Electrostatic Separator (ES). The optical characteristics of the beam are displayed in fig. 4.

The useful momentum range goes from \( \sim 1 \) to \( \sim 3 \) GeV/c; very few kaons survive below 1 GeV/c, the pions severely outnumber the kaons above 3 GeV/c. A typical separation curve is shown at 1.8 GeV/c in fig. 5. The total number of incident particles, defined by the \( S_1 \cdot S_2 \) coincidence, is given by the upper curve as a function of the current in the separator compensating-magnet. The pion peak is clearly visible. The kaons are buried under the right-hand tail of the pion peak (the \( \pi/K \) ratio at this momentum is \( \sim 5 \)). When the information from the Cherenkov counters is used (\( C_1 \) rejecting pions, \( C_2 \) in coincidence detecting kaons above 1.8 GeV/c), the lower curve shows the kaon peak followed by the antiprotons. The background varies from 4\% at 1.2 GeV/c to less than 1\% at the highest momentum and is mainly due to off-momentum electrons and muons originating from \( \pi \) and \( K \) decays beyond \( C_1 \).

A curve of the above type was measured at each momentum setting and the position of the kaon peak was carefully determined. Fig. 6 shows the intensity of the kaon beam measured at our target as a function of the incident momentum and for a fixed flux of protons on the internal PS target (0.3 \( \times 10^{12} \) protons per pulse). Also shown on this figure is the "incident/K" ratio at each momentum setting obtained from a direct comparison of curves a) and b) of fig. 5 near the K-peak.

The rejection of pions relies crucially on the detection efficiency of the Cherenkov counter \( C_1 \). It can be seen on fig. 7 that the inefficiency is in the order of 0.1\%.

The value of the beam momentum at the centre of the target as determined by the magnet currents was verified by time-of-flight measurements. They agreed to within \( \pm 0.25\% \). A list of the values corresponding to the nominal momenta at which the data have been taken is given in table I. Together with these we list the muon contamination, as a fraction of the K-flux, calculated from \( K_{\mu 2} \)-decays from the middle of \( C_2 \) and beyond, such that the outgoing muon is seen by \( S_2 \). Most of the beam divergence and size comes from multiple scattering in the Cherenkov counters. The latter depends on the momentum setting and the gas pressure in the counter. We estimate that, at the centre of the target and in a plane orthogonal to the axis, the beam has an r.m.s. spread of \( \pm 6 \) mm at 1.2 GeV/c and \( \pm 5 \) mm at 2.2 GeV/c. The corresponding angular spreads are \( \pm 10 \) mrad and \( \pm 8 \) mrad. The momentum spread due to the collimator settings and the internal target size is \( dP/P = \pm 2\% \) throughout the momentum range of the experiment.
Beam optics. Horizontal deflections above the middle line, vertical deflections below (arbitrary units). The symbols are those of fig. 3. The dashed line ES shows the centre of the separator.

FIG. 4

Beam separation curve at an incident momentum of 1.8 GeV/c. The counts are given for an integrated flux of protons on target 1 of ~3 10^{11}. The Cherenkov counters are used in (b) but not in (a). The current in the horizontal scale is that of the compensating magnets at the two ends of the electrostatic separator; this current is linearly related to the spatial separation of the particles at the mass slit L_2.

FIG. 5
FIG. 6
Number of $K^-$ per burst (dashed curve) and number of incident particles per $K^-$ (solid curve) as a function of the incident momentum.

FIG. 7
Cherenkov detection curve i.e. the fraction of beam pions at 1.8 GeV/c giving a signal as a function of the gas pressure. The dots correspond to the actual measurements; the curve is to guide the eye. The arrows indicate the thresholds for detection of the various particles. The counter ($C_1$ of fig. 1) contained ethylene.
2.4 Multiwire proportional chambers

A total of 9 chambers (MWPC) have been used in the experiment. All except one (W₄) have similar characteristics and only vary in size and minor details. Each of them consists of two planes of sensitive wires (the X- and Y-planes) at 90° with respect to one another. The wire spacing is 2 mm and the planes are 1.5 cm apart. The sensitive planes are sandwiched between high-voltage wires, mylar windows and electrostatic shieldings as shown in fig. 8, which applies to the largest of the chambers (W₆ and W₇). The smaller chambers have only one central high-voltage wire plane. The W₄ chamber has only one plane of sensitive wires inclined at 30° with respect to the normal. Size and number of wires are listed for each chamber in table II. The total number of wires is 4 096.

The sensitive wires are made of gold-plated tungsten 20 μm in diameter and stretched at 40 g between "Vetronite" frames. The amount of material represented by each chamber is listed in table III. The gas used is a mixture at atmospheric pressure of 5% propane, 20% isobutane and 75% argon saturated with isopropyl alcohol by means of a bubbling-through procedure at 3°C. The mechanical precision by which the position of each wire is known with respect to the others is about ± 0.2 mm.

**FIG. 8**
Exploded view of the largest of the proportional wire chambers (W₆ and W₇ of fig. 1). The sensitive-wire planes are marked x and y.
The electronics needed for the detection, amplification and storage of
the signals is directly mounted on each chamber [15]. The information is trans-
ferred to the control logic by means of a 32-line data-bus connecting serially
the chambers. A 10-line address-bus controls the read-out flow, scanning
sequentially the various groups. Each wire signal is then coded into a
15-bit word containing the address of the plane, of the group and of the
wire inside the group.

Fig. 9 shows the detection circuit connected to each wire. Four such
circuits are mounted on the same card, together with the memorization circuit
shown in fig. 10.

Detection circuit for one wire. The input impedance (\(R_i\)) is 460 \(\Omega\), protected by a 210 \(\Omega\)
resistance (\(R_a\)) in series and two diodes. The gain of the amplifier-discriminator (a,b,c)
is \(\approx 600\) with a bandwidth of 25 MHz. The detection threshold is 0.35 mV. In the second
part of the circuit the monostable B determines the delay (\(\approx 650\) ns) necessary for the fast
electronics to select the event. The monostable A protects B from the high-frequency
effects of the input signal by vetoing other signals which arrive within 600 ns. The
monostable C generates a standard output signal (30 ns long) synchronous with the backfront
of the B-monostable output. The voltage \(V_T\) allows the adjustment of the time length of B
from a minimum of 250 to a maximum of 650 ns. The "fast" output goes on the "OR" input of
the memory circuit of fig. 10. The "delayed" output goes on the "single wire" input
(writing gate) of the memory circuit of fig. 10. Four of these circuits, together with one
circuit of the type in fig. 10 are mounted on a single printed card. The printed cards
are attached to the chamber frames.
FIG. 10

Memorisation and reading circuits relative to four wires. The standard signal generated by the "delayed" output of fig. 9 enters the "single wire" input where it is put in coincidence ("AND") with the "writing gate" delivered by the fast electronics. The memory part is made by standard flip-flops. The "reading gate" generated by the coding logic transfers the information of the memories to the data bus. This is done 32 wires at a time. An additional output provides an "OR" signal for the four wires. All circuits are in ECL logic.
The reading and coding logic is summarized in fig. 11. Groups and chamber-planes are addressed sequentially at a frequency of 1 MHz. The serial number of the activated wires is coded at a frequency of 20 MHz. Each word thus formed is then sent into an intermediate memory (buffer) of eight stages in series. This allows the decoupling of the reading and coding frequency from the data acquisition frequency set by the CAMAC interface to the HP 2116C on-line computer. For the whole MWPC system the total number of groups is 135 and the average reading time is 150 μs.

The operation of the chambers has been extensively tested in the laboratory prior to their utilization in the experiment [15]. We only review the main results from the various measurements performed with a beam of 1 GeV/c π⁻. Fig. 12 shows the resolution time as a function of the high voltage under a detection efficiency higher than 99%. Sparking begins at a voltage equal to \( V_0 \). Fig. 13 shows the variation of the inefficiency with the high voltage and for different numbers of adjacent wire signals (the particle trajectories were perpendicular to the chamber). The resolution time during the experiment was fixed by the coincidence width to 100 ns. Fig. 14 shows the inefficiency as a function of the counting rate for single-wire signals. The slope of the curve reflects the 1 μs dead-time introduced by the electronics.

**FIG. 11**

Reading and coding logic of the MWPC system from the apparatus in the experimental area to the control room.
FIG. 12
Resolution time of a MWPC as a function of the high voltage on the grid (laboratory test). The dotted curve is to guide the eye.

FIG. 13
Detection efficiency versus high voltage on the grid (laboratory test). The value of n gives the number of wires hit. The direction of the incoming particles was perpendicular to the chamber plane. $V_g$ represents the voltage at which sparking occurs. The lines are to guide the eye.

FIG. 14
Chamber inefficiency as a function of the particle flux incident on the wires (laboratory test). The dotted line is to guide the eye.
2.5 Neutron detector

The neutron detector consists of a series of parallel-plane optical spark chambers sandwiched between plates of material where the neutron interacts and produces charged tracks which are seen in the neighbouring chambers. A plane view of the detector is shown in fig. 1. The chamber planes are perpendicular to the floor and centered with respect to the line (dash-dotted in the figure) corresponding to the chosen average neutron emission angle. There are four sections, each consisting of five chamber moduli and five converter plates. The lateral size of the sections increases with increasing distance from the target so as to match a fixed solid angle from the target centre of \( \approx 120 \, \text{msr} \)\(^(*)\).

![Diagram of neutron detector](image)

**FIG. 15**

Side view of the neutron detector. The neutrons are incident at right angle with respect to the plane of the figure. The lower camera looks directly into the gaps between spark chambers; the upper camera receives the image of the same gaps seen from above via a mirror.

\(^(*)\) The five modules of each section have the same dimensions. The sensitive area is that of a square having 850 mm side in the first section, 1000 mm in the second, 1150 mm in the third and 1300 mm in the fourth.
Fig. 15 shows the detector as seen by the incoming neutron. Two television cameras look at the sparks from orthogonal directions at about 3 m from the axis of the chambers; their signals are digitized, fed into the on-line computer and recorded on tape. This information is then used (off-line) to reconstruct the individual sparks. From these the tracks are formed and their approximate origin is identified. The knowledge of the vertex in the target and of the interaction point in the detector determines the neutron trajectory.

A schematic view of the five spark chambers making up one of the sections is shown in fig. 16. Each chamber is made of three grids: the central grid is connected to the high voltage supply, the other two are at ground potential. The mylar windows, two per chamber, enclose a 30% helium - 70% neon gas - mixture at slightly more than the atmospheric pressure. Two distinct sparks are formed when an ionizing particle goes through a chamber, one at each side of the central grid. In between the chambers are the converter plates, made of iron or opaque polyethylene. In front of the optical surface of each gap, lucite prisms are mounted for sending into the camera the light of the sparks. An electroluminescent plate at the end of each prism illuminates a reference mark used to calibrate the spark-chamber geometry.

![Diagram of spark chambers](image_url)

**Fig. 16**

A section of the spark chambers in one module. The cut is perpendicular to the chambers plane. Between each chamber there is a converter plate (iron or polyethylene). The neutrons come into the module prevalently at 90° with respect to the chambers plane.
The electrical circuit is shown in fig. 17. The trigger signal discharges a single master spark-gap (hydrogen thyratron) which in turn triggers the separate spark-gaps (EGG-GP 17 type) connected with each module. The chambers were operated at 8 kV and had a permanent clearing field of 100 V/cm. In this condition the memory time was \( \approx 1 \mu \text{s} \) and a time of \( \approx 10 \text{ ms} \) was necessary to recharge the capacity bank. A Faraday cage surrounded the whole detector, cameras included, so as to limit the spurious signals induced to the MWPC electronics.

The two television cameras (fig. 18) operate with "plumbicon" tubes. The advantage of these with respect to the more conventional "vidicons" is an increased spatial resolution, a longer image retention and a faster erasing time. The characteristics of the system are discussed in ref. [16]. The image is formed on a 17.1 x 12.8 mm\(^2\) area of the photocathode; this implies a demagnification of 100. The reading of the sparks is performed by the electron beam in a line-scanning mode with the direction of the scan parallel to the chamber plates. With two gaps per chamber, the typical number of video-signals per spark is \( \approx 8 \). Even allowing for a signal loss due to spark intensity reduction, we are still left with a considerable number of signals for each spark. The time-diagram of the television cameras operation is shown in fig. 19. The two cameras are read in parallel and the total time needed for a complete scan (312 lines) is 20 msec. At the end of a line scan the digitisations are transferred into a buffer memory and read into the computer during the following line scan. No additional time is necessary for data taking(*).

Fig. 20 shows a computer-reconstruction of a spark chamber event as recorded by the television cameras and analysed on-line. This on-line analysis was performed on a small sample of events during data acquisition. In this way a regular check was kept on the performance of the chambers and the television cameras. The stability of the electronics made it unnecessary to record the fiducial marks more often than once an hour, thereby reducing by a large amount the number of words per event. The reconstruction precision and the methods for identifying and measuring the recoil neutron are discussed in sect. 3.3.

(*) Typically, with our incident intensities and energies, the spark-chamber trigger rates went from 0.1 to 10 per PS burst.
FIG. 17
Electrical circuit of a spark chamber.

FIG. 18
A cut through a TV camera (plumbicon) showing the details of the electronics arrangement to scan the image of the spark chamber gaps.
FIG. 19

Time diagram showing the various phases of the camera scan of the spark chamber gaps. The inset in the corner shows schematically the neutron counter together with the definition of the "horizontal" and "vertical" directions.

FIG. 20

Computer-generated view of an event in the neutron converter. The fiducial marks are shown as fixed lines along the edges. A charged-particle track is seen crossing the apparatus from one end to the other.
2.6 Electronics

The logic associated with the fast electronics is summarized in fig. 21. The incident particles are defined and counted as \( I = S_1 \cdot S_2 \). They are further separated into pions (\( \pi = I \cdot C_1 \)) and kaons (\( K = I \cdot C_1 \cdot C_2 \)) depending on the desired trigger. The coque signal (\( B_1 \)) is obtained by mixing the ten separate counters of this veto-box (fig. 2). We then require the absence of signals from \( B_1 \), from the two veto-box extensions \( B_2 \) and \( B_3 \) and the transmission counter \( S_{tr} \). This defines a particle which has interacted in the target and is a possible candidate for reactions (1) and (2). The condition for a particle in the charged-branch is then formed (\( S_3 \cdot S_5 \cdot (S_{4a} + S_{4b}) \)) and the appropriate requirements in the neutron-branch are verified (\( S_6 \) for reaction (1), \( S_7 \) for reaction (2)). This generates the trigger signal for the spark chambers, the "writing" signal for the MWPC's and the logic order necessary to start the data acquisition system.

![Diagram](image)

**FIG. 21**

Fast electronics logic. The symbols for the detectors are the same as in fig. 1. INC stands for "incident" particle, INT for "interaction", K for Cherenkov-identified kaon, K'\( n \) and K'\( p \) for reactions (1) and (2) respectively.
The organization of the data acquisition is outlined in fig. 22. All input to the computer goes via CAMAC crates. The computer is a 16K-memory HP 2116 C. The commands for histogram computation, tape initialization, start of run and display of the monitoring samples are entered via teletype.

For a given value of the interaction cross section, above a minimum beam flux, the data acquisition rate is limited only by the time needed for the operations summarized in fig. 22. This depends on the complexity of the event to be recorded; typically, for a clean example of reaction (1), with a one-track neutron interaction in the spark chambers, the time distribution is the one shown in fig. 23. For a basic spill-out time of \( \sim 400 \text{ ms} \) the maximum number of such events which can be recorded during one burst is \( \sim 20 \). Notice that the average amount of information generated by one of these events takes a total of \( \sim 100 \) 16-bit computer words. With 1800 words allocated as buffer in the computer, we can accumulate 18 triggers before having to interrupt the acquisition. In practice there is an average of one neutron conversion every \( \sim 5 \) triggers satisfying the conditions for reaction (1). For events without the spark chambers operation the occupancy time goes down to \( \sim 300 \mu \text{s} \).

This means that the number of \( K^-n \) triggers accumulated during a burst (10 on the average) was only slightly reduced when we allowed the acquisition of \( K^-p \) triggers at the same time.

![Schematic representation of the data acquisition system.](image)

FIG. 22
Schematic representation of the data acquisition system.
Time diagram of the electronics acquisition system. After the detection of an event a "busy" signal is generated which freezes the detection electronics and scalers. The ~690 ns delay time of the monostable is needed to allow the decision making and the propagation of the signals (the control room is ~30 m from the apparatus). The dead-time of the monostable is of the order of 1 μs for a writing gate of 70 ns as reflected by the slope of the plot in fig. 14. The reading time depends on the number of wires hit; typically it was of the order of 300 μs for the K∗ triggers. The spark chambers are triggered ~100 ns after the writing gate of the MWPC's so as to avoid electromagnetic perturbations from the sparks to the MWPC's electronics. The television scan, which is normally in continuous operation, is then stopped and reactivated only after the MWPC's reading has been completed. The typical time to read the television scan is of the order of 20 ms. The computer determines the releasing of the "busy" signal when all the data have been read in; it remains occupied, however, until the data have been transferred to tape. The end of the tape transfer determines the position of the first dotted line on the figure.
TABLE I
Nominal momenta ($P_{\text{nom}}$), momenta at the target centre ($P$),
momentum bite ($\Delta P$) in GeV/c; muon contamination ($\epsilon_{\mu}$) in %.

<table>
<thead>
<tr>
<th>$P_{\text{nom}}$</th>
<th>$P$</th>
<th>$\Delta P$</th>
<th>$\epsilon_{\mu}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.2</td>
<td>1.220</td>
<td>0.024</td>
<td>0.5</td>
</tr>
<tr>
<td>1.4</td>
<td>1.423</td>
<td>0.028</td>
<td>0.5</td>
</tr>
<tr>
<td>1.6</td>
<td>1.619</td>
<td>0.032</td>
<td>0.5</td>
</tr>
<tr>
<td>1.8</td>
<td>1.800</td>
<td>0.036</td>
<td>0.5</td>
</tr>
<tr>
<td>2.0</td>
<td>1.992</td>
<td>0.040</td>
<td>0.5</td>
</tr>
<tr>
<td>2.2</td>
<td>2.167</td>
<td>0.043</td>
<td>0.5</td>
</tr>
</tbody>
</table>

TABLE II
Sensitive area of the MWPC's horizontal ($L_H$) and vertical ($L_V$) size in mm.

<table>
<thead>
<tr>
<th>MWPC</th>
<th>$L_H$</th>
<th>$L_V$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W_1$</td>
<td>192</td>
<td>192</td>
</tr>
<tr>
<td>$W_2$</td>
<td>192</td>
<td>192</td>
</tr>
<tr>
<td>$W_3$</td>
<td>320</td>
<td>256</td>
</tr>
<tr>
<td>$W_{(a)}$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$W_4$</td>
<td>640</td>
<td>320</td>
</tr>
<tr>
<td>$W_6$</td>
<td>1024</td>
<td>640</td>
</tr>
<tr>
<td>$W_7$</td>
<td>1216</td>
<td>832</td>
</tr>
<tr>
<td>$W_8$</td>
<td>318</td>
<td>256</td>
</tr>
<tr>
<td>$W_9$</td>
<td>512</td>
<td>512</td>
</tr>
</tbody>
</table>

(a) 288 wires with 2 mm spacing, inclined 30°
    with respect to the vertical direction.
TABLE III

Composition of the average MWPC. The density is $\rho$, the thickness is $t$, the distance between wires is $d$, the number of radiation lengths is $L/L_0$. The total number of radiation lengths is $\sim 34 \times 10^{-4}$, giving an average multiple scattering angle of $\sim 0.87$ mrad at 1 GeV/c.

<table>
<thead>
<tr>
<th>Element</th>
<th>COMPOSITION</th>
<th>$\rho t$ (mg/cm$^2$)</th>
<th>$L/L_0$ (10$^{-4}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gas</td>
<td>Argon: 5 cm at 0.8 atm</td>
<td>7.12</td>
<td>3.61</td>
</tr>
<tr>
<td></td>
<td>Isobutane (C$<em>4$H$</em>{10}$): 5 cm at 0.2 atm</td>
<td>19.21</td>
<td>4.44</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.56</td>
<td>0.25</td>
</tr>
<tr>
<td></td>
<td></td>
<td>9.20</td>
<td>2.91</td>
</tr>
<tr>
<td>Windows</td>
<td>Mylar (C$_5$H$_4$O$_2$): 2 x 0.1 mm</td>
<td>21.00</td>
<td>16.15</td>
</tr>
<tr>
<td>Cathodes</td>
<td>Copper: 3 grids, 100 $\mu$ diam., $d = 1$ mm</td>
<td>8.00</td>
<td>5.76</td>
</tr>
<tr>
<td>Screens</td>
<td>Iron: 2 grids, 50 $\mu$ diam., $d = 0.5$ mm</td>
<td>0.60</td>
<td>0.88</td>
</tr>
<tr>
<td>Sens. Wires</td>
<td>Tungsten: 2 grids, 20 $\mu$ diam., $d = 2$ mm</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
3. DATA ANALYSIS

3.1 Geometry

The components of the incident branch (scintillators, Cherenkovs and MWPC's) were kept in a fixed position throughout the experiment. The same was true for the target and the veto-box B₁. The elements of the outgoing branches, instead, have been displaced and re-arranged for each of the six angular settings. The centre line of the branches was aligned to the desired angle and the distances between the elements optimized so as to subtend the maximum solid angle compatible with their respective sizes. After each adjustment all the positions and angles were measured optically with respect to the reference marks fixed to the supports of the various elements. This gave the first-approximation determination ("nominal values") for the MWPC coordinates to be used in track reconstruction. Minor, yet significant, alterations of these nominal values were done in the final analysis stage. In fact, during data-taking the external reference marks may vary in relation to the internal wire position; this depends on the frequency and the importance of the dismantling operation occasionally necessary for the chamber maintenance. Also, the temperature of the hall may undergo variations which in turn caused unforeseen changes in the wires position. The definitive coordinate system of the MWPC's was determined a posteriori by a least-square fit of the nominal to the expected coordinates on the basis of a large sample of measured trajectories. Data were taken, for this purpose, without magnetic field and events with only one track in each branch were selected. The three tracks were then used in a least-square fit requiring a common vertex inside the target and three straight and coplanar trajectories.

The result of one such "alignment" is visible in fig. 24. Here we show the difference between measured and fitted coordinates for 1280 tracks in the 40° setting. The measured coordinates are those read-out from the chambers and corrected - with respect to their nominal values - by the least-square method mentioned above. The observed spread is what can be expected from multiple scattering and geometrical resolution.

The position of the neutron detector with respect to the MWPC's was determined independently by using charged tracks. This allows to relate the reference marks of the spark chambers to the position of the MWPC's.

The magnet position was taken as measured for each setting with the conventional optical survey procedure.
3.2 Momentum analysis

The momentum of the particle entering the charged branch is measured by the deflection of its trajectory through the magnet. If we exclude the \( W_4 \) chamber (whose purpose is essentially to reduce the possible ambiguities due to spurious coordinates) we are left with a maximum of 4 sets of x-y coordinates, two sets before and two after the magnet. With these we can, in principle, determine the particle direction before and after the magnet, and therefore the momentum. This simple procedure could not be used because: (a) the magnetic field extends well inside the region of the last two chambers and (b) the value of the field is only approximately constant inside the magnet.

An alternative procedure would be to use the full map of the magnetic field in space to trace the trajectory of the particles. One introduces an assumed initial momentum and finds a "fitted" momentum by requiring agreement between the measured and expected coordinates at the positions where the trajectory intersects the chambers. This operation is, however, costly in terms of computer time. We have resorted to an intermediate, more economical, approach. We first generate simulated particles with known momentum, track them through the magnet, find the corresponding chamber coordinates and finally establish an empirical relationship between these and the momentum. The relationship is then used, for the real events, to determine the momentum knowing the coordinates. The latter operation is straightforward and requires very little computer time. The number of simulated tracks does not need to be as large as the experimental sample. Hence the saving.
A map of the magnetic field was available listing the three components of the field strength at a total of 12,600 points in space. An example of the map, together with the definition of the reference system, is given in fig. 25. The plotted quantity is the component of the field along the z-direction as a function of the distance from the centre (x = 0) and for discrete values of the z-coordinate. The magnet is the CERN C-type Alstom, operated at a controlled constant current of 1700 A resulting in a maximum field along z (near the centre) of \( \sim 1.43 \) Tesla. The measurement grid consists of 100 points at 4 cm intervals in a horizontal plane along x, repeated 21 times at 6 cm intervals along y. This two-dimensional grid is repeated at five z-positions separated by 8 cm intervals.

The simulated tracks were generated (\( \sim 3000 \) of them per set-up) according to an isotropic angular distribution in the laboratory and a uniform momentum spectrum encompassing the range of momenta of each set-up. For each track of given momentum we determine the intersection points with the chambers. This gives us a maximum of eight coordinates (two x and two y before and after the magnet). We then select a suitable subset of five coordinates (four before the magnet and one x after) to represent the missing three coordinates plus the inverse of the momentum via a third-degree polynomial of the following type:

\[
A = a + \sum_{i=1}^{5} b_i x_i + \sum_{i,j=1}^{5} c_{ij} x_i x_j + \sum_{i,j,k=1}^{5} d_{ijk} x_i x_j x_k. \tag{4}
\]

This corresponds to an expansion of the unknown quantity A in a series of 56 terms. We use the simulated tracks to determine the coefficients of eq. (4). For the real events we use these coefficients to calculate, for example, the momentum of the track; the computer time for this calculation is negligible with respect to that taken by the conventional tracking procedure. Typically, the precision of the method gives a \( \pm 1\% \) momentum resolution near 700 MeV/c (which is the central momentum at most angles and incident momenta). This resolution does not improve by reducing the step size of the tracking procedure. The corresponding resolution on the missing coordinates \( x_6, y_6 \) and \( y_7 \) is respectively \( \pm 1, \pm 10 \) and \( \pm 16 \) mm.

The calculation of the missing coordinates allows to reject unwanted tracks such as those which interact or decay along the trajectory. For this purpose the expected and measured coordinates are compared, a \( \chi^2 \) is computed using the above resolutions and the event is rejected if this \( \chi^2 \) is larger than a pre-determined value. The cut-off value was chosen so as to correspond to a 1\% probability. Fig. 26 shows an example of the confidence level distributions obtained from the measured \( \chi^2 \) of real events in one of the worse cases (forward particles at high energy). The separation is usually
better at lower energies and larger angles. The cut-off values were chosen on the basis of the distributions calculated with Monte-Carlo events forced to decay along their trajectories. Using as calibration the distribution for the same events without decay, we estimate that our criteria allow a separation of the unwanted tracks to a level of \(\sim 1\%\).

**FIG. 25**
Vertical component of the magnetic field at three different heights (in cm) as a function of the distance along the scattered particle axis.

**FIG. 26**
Probability distribution of the difference between calculated and measured coordinates of reconstructed tracks. Same histogram in (b) as in (a); expanded scale in (b).
3.3 Neutron detection

The information from the neutron detector is under the form of digital video-signals giving the position of the sparks in each chamber from two views with a $90^\circ$ stereoscopy. The first operation is to identify the individual sparks. As already mentioned in sect. 2.5, there are typically some eight video-signals per spark distributed over successive lines parallel to the chamber plane and covering the gaps between the two high-voltage grids. The signals from each line are examined in succession (there may be more than one spark per chamber) and associated with those from the subsequent line if the distance between them is not larger than a pre-determined value which takes into account spark width and signal fluctuation. There are two gaps per spark chamber and we reconstruct each spark separately (fig. 27). Afterwards we combine the two sparks into a single central digitization which is taken to represent the position of the particle through that chamber.

The same operation is then repeated to combine the individual sparks into straight tracks. Here we must account for the slope of the particle trajectory which, in our case, can be as large as $45^\circ$ with respect to the neutron detector axis. The tracks are built-up in as many segments as there are interruptions along the path (due to chambers inefficiency). The segments are then combined using an extrapolation technique to check that they belong to the same track.

The above operations are performed separately on each view. The two views are then compared to verify that the track segments have been correctly joined and to determine if the event lies in a restricted fiducial volume. Tracks are accepted if they go through at least two chambers and there are no other tracks present for the same trigger.

The precision of the reconstruction is illustrated in fig. 28 using $\pi^- p$ at $40^\circ$ and 1.8 GeV/c. The particles traversing the detector are protons of average 1.2 GeV/c momentum. The histograms show (for each chamber in the horizontal view) the dispersion of the spark position as determined with the above procedure with respect to the track direction as predicted by the MWPC's preceding the spark chambers. From these and similar arguments we conclude that the precision of the vertex reconstruction for a neutron interaction in the middle of the detector is of the order of 1 cm.

The origin of the charged track is extrapolated from the reconstructed trajectory and assumed to be in the middle of the nearest upstream converter. This is then taken to be the interaction point of the neutron.
FIG. 27
Signal generation of the video scan (plumbicon) inside a spark-chamber gap.

FIG. 28
Spark chamber alignment, as verified by straight charged tracks going through the neutron detector and whose direction is determined by the MWPC's preceding the detector. The histograms show the distribution of the sparks with respect to the position expected from the known particle direction.
3.4 Event reconstruction

For an optimum utilization of the computer time we performed the analysis in two stages. First, a complete geometrical and kinematical reconstruction was done using the 150 tapes of the experiment. The results, in the form of momentum vectors, vertex coordinates, flux counts and reject lists, were written onto a small number of summary tapes (DST). The data on the DST's were then used routinely for tests, re-adjustment of fits and calculation of cross sections (this time-saving procedure is essentially the same as customary in bubble chamber data reduction).

The flow-chart of fig. 29 shows the details of the first analysis stage. The procedure is illustrated in fig. 30 where we summarize the main causes of reject. Notice that at this stage we still use loose criteria for these rejects, allowing for more stringent cuts to be performed at the DST stage. We shall discuss in detail these cuts in sect. 4 because they are connected with the normalization procedure for the cross sections.

**FIG. 29**
Flow-diagram of the track reconstruction.
The three steps of the event reconstruction

\[
\begin{align*}
(1) & \quad \text{charged track} \\
(2) & \quad \text{incident track and vertex} \\
(3) & \quad \text{neutral track}
\end{align*}
\]

Reject causes

\[
\begin{align*}
(1) & \quad \text{no reconstruction} \\
(2) & \quad \text{outside fid. vol.} \\
(3) & \quad \text{bad } \chi^2
\end{align*}
\]

\[
\begin{align*}
(4) & \quad \text{no reconstruction} \\
(5) & \quad \text{track distance} \\
(6) & \quad \text{outside target}
\end{align*}
\]

\[
\begin{align*}
(7) & \quad \text{no neutron conversion} \\
(8) & \quad \text{(goes to recoil-less class)} \\
(9) & \quad \text{outside fid. vol.} \\
(10) & \quad \text{kinematics cuts} \\
(11) & \quad \text{MM > nucleon}
\end{align*}
\]

Fig. 30

Causes of event rejection and sequence of reconstruction.

A sample of histograms illustrating the geometrical precision of the reconstruction is given in fig. 31. The distribution of the distance of closest approach between incident and scattered kaon is shown in (a); the arrows indicate the limits of our acceptance. The distributions in (b) and (c) show the interaction vertex along the beam axis; the empty-target events (b) clearly indicate the mylar windows. An enlargement (d) of the full-target histogram shows the precision achievable in the measurement of the target length; the advantage of this measurement in situ is that it includes the low-temperature effects not easily accounted for in the conventional optical measurement in the laboratory.

The computer time (central processor) needed for the procedure described above is of the order of 0.2 sec on a CDC 6600 computer for the events corresponding to the trigger for reaction (1) and of the order of 0.03 sec for events taken with the trigger for reaction (2).
Distance of closest approach between incident and charged particle (a). Position of the interaction vertex along the beam axis for empty (b) and full (c) target. Details of the full-empty distribution in the region of the target edges in (d).
4. CROSS SECTIONS

The events reconstructed and identified so far are at the end of a chain where a series of effects take place all of which must be accounted for if we want to go back to the original features of the reactions under study. Some of these effects are due to the occasional malfunctioning of the apparatus, some to the inherent limitations of our set-up and some to unavoidable physics causes. A detailed discussion of the specific items is presented in the following subsections. Here we only outline the general considerations leading to the absolute value of the cross sections.

The instrumentally related effects arise from the following sources: (a) defaults of the MWPC read-out system, generating wrong recordings or garbled information; (b) inefficiency of the MWPC's or the spark chambers, leading to insufficient information when reconstructing the event; (c) finite sensitive time of the MWPC's, allowing the simultaneous recording of distinct events. All of the above has the effect of a relatively unbiased event loss which can be compensated for by simple accounting (sect. 4.3). Also related to the instrumentation but independent of malfunctioning are: (d) limited (and small) geometrical acceptance of the apparatus, restricting the detection to a fraction of the original number of events; (e) limited detection efficiency of the neutron counter, a momentum dependent effect which leads to a favoured selection of certain types of events. The subsections 4.1 and 4.2 deal respectively with the calculation needed to compensate for points (d) and (e).

On the physics side, the phenomena which need to be understood and accounted for are those related to: (f) particles lost in the incident and scattered branch because of decays or interactions along their path; (g) the presence of spurious particles (mostly muons) among those counted as incident; (h) the emission of $\delta$-rays during particle traversal of the target and their detection by the coque; (i) the interaction inside the coque - and the consequent rejection of the event - of the neutron spectator in the case of reaction (2). These effects can all be either calculated or removed by applying appropriate cuts to the data (sect. 4.3). The most critical physical effect to deal with is the one arising from: (j) the deuterium-related phenomena (Fermi motion, double scattering, etc.) which distort the appearance of the event in comparison to that of the simple $K^-n$ and $\pi^-n$ elastic scattering. A full section (sect. 5) has been devoted to the description of the correction for item (j).

The cross sections have been calculated with the usual procedure of normalising the number of events to the incident flux counted during the data acquisition. Both events and flux were corrected for the effects mentioned above. The correction for points (d) and (j) have been dealt with by introducing a Monte-Carlo simulation whereby the observed events were compared to artificial events generated under known conditions for deuterium effects and apparatus acceptance.
4.1 Acceptance

Events are generated simulating reactions (1) and (2) under the following conditions:

a) the incident particles are created according to a space and angle distribution closely resembling that of the real events;

b) the interaction takes place inside the target with uniform probability along the target axis;

c) the target nucleon has the momentum distribution of the Hulthén wave function:

$$H(P) = \frac{0.0002693 P^2}{[(P^2 + 0.00208849)(P^2 + 0.056169)]^2}$$  \hspace{1cm} (5)

where $P$ is the neutron momentum in the laboratory in GeV/c.

---

**FIG. 32**

Comparison between Monte-Carlo events and real events for the neutron-spectator reaction at 1.2 GeV/c incident momentum and in the 20° position for the scattered kaon.

**FIG. 33**

Comparison between Monte-Carlo events and real events for the neutron-spectator reaction at 2.2 GeV/c incident momentum and in the 20° position for the scattered kaon.
(d) the interaction is elastic, takes place on either one of the nucleons and the angular distribution of the scattered particle (entering the charged branch) is isotropic in the c.m. system;

(e) in order to reproduce as closely as possible the experimental conditions, the charged particle is tracked throughout the magnetic field;

(f) the event configuration satisfies the trigger conditions of the experiment, i.e. we avoid generating events which a priori have no chance of being detected by the apparatus;

(g) the geometrical resolution and the effect of multiple scattering are folded into the simulated event.

All the events thus generated were stored in the same format in which the real events were summarised on the DST. The trigger criteria mentioned in (f) above are looser than those leading to the acceptance of an event at the data analysis stage. This insures that we can apply the same cuts in geometry and kinematics which were chosen for the real events. The advantage, of course, is that the operation of selecting a cut is often repeated; the optimum situation can thus be found with little expenditure of computer time. Once the final cuts were decided, the same were applied to the simulated DSTs. The simulated events were also weighted in order to take into account the neutron conversion probability (sect. 4.2) and the decay probability for the $K^-$ in the spectrometer. The original number of simulated events can be calculated. The preserved fraction gives the overall reduction of the system (detectors, data analysis, fiducial volume, etc.); hence the acceptance correction.

The validity of the above procedure rests upon a correct representation of the events by our Monte-Carlo approach. To see if this is true, we compare quantities which do not depend on the dynamics of the reactions. They should be identical for real and simulated events. In figs 32 and 33 we show some examples of this comparison. The histograms have been obtained, for the two cases, under identical conditions. The missing mass, recoiling against the scattered particle, has been calculated assuming the target nucleon to be at rest. The width of this spectrum reflects the resolution of the measurement plus the Fermi motion of the target. The momentum distribution of the spectator nucleon is artificial in one case, measured in the other. The angular distribution of the spectator polar angle in the laboratory ($\theta_S$) should be isotropic. The azimuthal ($\phi_S$) distribution of the spectator should also in principle be isotropic. The depopulation observable near $\pm 90^\circ$ is due to instrumental deficiencies. Any miscalculation of the scattered momentum is directly reflected into the only completely unmeasured quantity, the spectator
momentum. Because the detected particles are essentially in the horizontal plane and the main source of error is the absolute momentum measurement, events tend to have large horizontal components of the spectator momentum so as to satisfy the momentum-energy constraints. What is important, in our case, is that this effect is equally observable in the real and simulated events. These and other checks make us confident that the event simulation is an adequate representation of the experimental conditions.

The fraction of accepted events with respect to the total is shown in fig. 34 for reaction (1) and fig. 35 for reaction (2), as a function of the cosine of the c.m. scattering angle. Notice that these are acceptance functions averaged over the Fermi motion. In practice we deal with acceptances valid for each combination of incident momentum and c.m. energy. Notice further, that the acceptances of fig. 34 refer to the "visible neutron" reactions and take into account the efficiency of the neutron detectors; the latter is discussed in the next sub-section.

FIG. 34
Acceptance curves at the two extreme momenta of the experiment and for the various set-up angles. In the ordinate the percentage of K n events accepted by the apparatus (i.e. charged and neutral branch); in the abscissa the cosine of the c.m. scattering angle.
4.2 Neutron detector efficiency

In order to determine the fraction of neutrons producing visible tracks in the neutron detector we have used all the triggers for reaction (3) whether the conversion was or was not observed. This reaction was recorded at the same time as reaction (1) so that the efficiency calculation is independent of the occasional malfunctioning of the spark chambers. We use reaction (3) instead of (1) because a larger number of events was accumulated for the former reaction.

Using the measurement of the incident and outgoing $\pi^-$ in reaction (3), we calculate a missing mass assuming that the interaction occurs on a stationary neutron. If the reaction truly comes from (3) with a spectator proton, this missing mass is close to the proton mass. If it comes from an inelastic reaction such as $\pi^-d \rightarrow pn\pi^0$ where the $\pi^0$ has escaped the "coque", the missing mass is larger than that of the proton. By selecting events with a missing mass squared between $0.8$ and $0.9 \text{ GeV}^2$ we obtain a subsample of (3) free of inelastic events.
For each of these events we generate a spectator proton according to an expected distribution calculated using the deuterium wave function. This allows to calculate an associated neutron and to see if it would have been detected by the neutron counter. Thus we generate the momentum distribution that would have been observed if the neutron detector had a 100% efficiency. By comparing this distribution to the one observed for the events of the same sample with a converted neutron we deduce the efficiency of the counter as a function of the neutron momentum.

This procedure was performed separately for each set-up. A third-order polynomial fit was introduced to represent the results. The coefficients of the polynomial were then used to apply the efficiency corrections to the \( K^- \) data. Fig. 36 shows these measurements together with the polynomial fits.

![Graph showing neutron detector efficiency as a function of neutron momentum.](image)

**Fig. 36**
Neutron detector efficiency as a function of the neutron momentum. These results have been obtained using incident \( \pi^- \) as described in the text. The angle of the charged branch is indicated in each case. The converter plates are of iron except at 20° where they are of polyethylene. The curves represent the polynomial fit to the points and have been used at each set-up to calculate the detection efficiency correction for reaction (1).
4.3 Normalization

Apart from the geometrical limitations of the apparatus and the response of the neutron detector, events may be lost because of the following reasons: (a) inefficiency of the MWPC's, (b) \( \chi^2 \) cut in the reconstruction of the scattered particle, (c) secondary interactions along the track of the outgoing particles, (d) decays and (e) \( \delta \)-ray emission either in the target or in the material of the outgoing branches. The above effects can be traced to their source and the appropriate corrections can be calculated using known quantities. For example, the inefficiency of the chambers (sect. 2.4) is known, the interaction cross section can be derived from existing measurements on hydrogen, carbon, copper and other materials, the decay rate and the potential length of each trajectory are also known.

In addition, there is a specific effect due to the neutron spectator which should be corrected for. The spectator neutron having low energy, it has about a 10% probability to be detected by the "coque" by generating a low energy recoil proton. This proton often emits enough light to be seen. We evaluated the probability for the "coque" to see a neutron by calculating for each event the length of scintillator crossed and using known cross sections (ref. [17]) to generate recoil protons. Assuming that a loss of 200 KeV leads on the average to one photo-electron detected by the photomultiplier, we calculate the number of photo-electrons \( N \) emitted by this proton. The discriminator level of the PM being below the one photo-electron level, the probability of not seeing the proton is \( e^{-N} \). Each event was weighted according to this probability. We summarise all the correction factors in table IV.

In order to improve the quality of the data, an upper limit of 80 MeV/c was imposed on the reconstructed spectator momentum. This cut brings the \( K^- n p \) reaction closer to the \( K^- n \) (or \( K^- p \)) elastic reaction and has the following advantage. An 80 MeV/c nucleon has an energy of 3.4 MeV so that within our measurement accuracy, it is impossible for an inelastic event such as \( K^- d \rightarrow K^- p n \pi^0 \) to simulate a quasi-elastic event with a spectator nucleon of lower energy. In this way we insure that all the inelastic events where the \( \pi^0 \) escapes the "coque" are rejected. Events coming from the reaction \( K^- d \rightarrow A \pi^- (p_s) \) can, in particular kinematical configurations, simulate either \( K^- d \rightarrow K^- p (n_s) \) if the \( A \) decays into \( p \pi^- \) with the \( p \) or the \( \pi^- \) simulating the recoil proton and the other particle escaping the detection system or \( K^- d \rightarrow K^- n (p_s) \) if the decay is into \( n \pi^0 \) where the neutron converts in the detector and the \( \pi^0 \) escapes the detection system. The presence of such events is noticeable (in the \( K^- \) missing mass distribution) when the measurement of the outgoing nucleon is not taken into account; with visible neutrons and the above cut they occur at a negligible rate.
At high beam fluxes the memory time of the neutron detector was not short enough to discriminate against neutrons from a near-simultaneous event. The above cut on the spectator proton imposes a relatively well defined direction to the neutron thus rejecting the spurious events.

The above effects have been experimentally verified by a study of the spectator momentum distribution. An example of this distribution (for the very worst case) is shown in fig. 37. The enhancement at low energy agrees with the shape simulated by the acceptance program shown by the curve. In the region above 80 MeV/c we see a deviation from the expected distribution which can be interpreted as the sum of the inelastic and the spurious events occurring at a threshold of \( \sim 100 \text{ MeV/c} \).

It is instructive to examine the flow of rejects through the data analysis chain and the above cuts. Table V summarises the rejection of each stage plus the fraction of events remaining after each stage. The relation between the rejection rates and the values expected on the basis of simple considerations is not always obvious. For example, we expect that the reject in the charged branch reconstruction include those due to decays. However, if we compare the expected decay-rate (straight line) with the \( \chi^2 \)-rejection, we obtain the plot of fig. 38: the rejected fraction as a function of the scattered particle momentum is consistently smaller than the expected value. This is understandable if we recall that the large-angle decays are liable to escape the trigger. The disagreement is less severe at the highest momenta because both parent and decay particles remain inside the branch.

![Diagram](image.png)

**FIG. 37**
Spectator momentum distribution for the reaction \( K^- d \rightarrow K^- n p \) at 2.2 GeV/c with the charged-branch in the 20° position. The curve shows the distribution expected on the basis of the deuterium wave function and the acceptance of the apparatus.
The cross section normalization has been done using the beam count provided by the "K" signals - or "PI" depending on the case - defined in sect. 2.2. The numbers counted were first corrected for the loss of events due to causes unrelated to the trigger type, such as the hardware-logic errors mentioned in sect. 3.4 (column (a) of table V) and the failures of the incident track reconstruction (column (c) of table V) except for those due to multi-track recording. This correction consists in reducing the total flux in the corresponding proportion. A second correction was done to account for the presence of muons in the incident beam due to K-decays after the second Cherenkov counter (fig. 1). The fraction of muons mistakenly labelled kaons has been estimated with a Monte-Carlo procedure and the flux has been reduced accordingly.
Due to the finite memory time of the chambers, when two particles cross the incident arm close enough in time they give rise to unseparable digitizations in each plane of the MWPC. These events were systematically rejected in the analysis. The rate for this type of reject was around 30% for the highest incident flux condition ($\sim 10^6$ particles per burst). On the other hand, the counters of the beam telescope, having a much better time resolution, were able to separate each incident particle; thus the incident flux counting was not affected by this effect. We only have to compensate the flux measurement according to the number of events rejected. This number was evaluated without the veto-box in the trigger because its memory time, although smaller than that of the MWPC's, was not negligible and made this effect trigger-dependent.

4.4 Results

Each reconstructed and accepted event is associated to a well defined value of the incident momentum $P$, the c.m. energy $E^*$ of the $K^-N$ system and of the c.m. angle $\theta^*$ of the $K^-$ in the $K^-N$ system. Due to the Fermi motion, the typical spread of $E^*$ around the value corresponding to a fixed incident momentum on a nucleon at rest is about $\pm 50$ MeV. In order to study better the structures in the $K^-N$ cross section it is our interest to subdivide the results into smaller bins of $E^*$. The choice of the bin size is dictated by the minimum amount of statistics which is still meaningful for each value of the differential cross sections. In practice there is a certain degree of overlap between the energy spectra of adjacent momentum settings; this allows us to average results from at least two different momentum settings when calculating the differential cross section for a given energy bin.

Let us consider the bin of c.m. energy centred at $E^*$ and call $dN(E^*, \theta^*)$ the number of events, corrected for all effects except the geometrical acceptance and the deuterium deformation, counted at a given momentum setting in a bin $d\Omega = 2\pi d\cos \theta^*$ of c.m. solid angle around $\theta^*$. Let $\phi$ be the total flux of incident particles counted for the same momentum setting and corrected for all the effects discussed in the previous subsection. On the basis of these numbers we can write the differential cross section as follows:

$$\frac{d\sigma}{d\Omega} (E^*, \theta^*) = \frac{dN}{d\Omega} (E^*, \theta^*) \cdot \frac{k}{k_A} f_D.$$  \hspace{1cm} (6)

Here we have summarised with $k = 1/(N_A \cdot \rho_D)$ (in our case $k = 980$ mb per interaction and per incident particle) the combined effect of the target length ($l = 25.0$ cm), the deuterium density ($\rho_D = 0.14$ g/cm$^3$) and Avogadro's number ($N_A = 6.022 \cdot 10^{23}$ atoms per mole). The factors $A$ (acceptance) and $f_D$ (deuterium correction) are here shown as separate quantities whereas, in reality, we have obtained them jointly. The different Monte-Carlo procedures described in sect. 4.1 for the acceptance and in sect. 5.3 for the deuterium
effects have been combined in practice into a single operation whereby events were generated as they would appear (i.e. be accepted) in our apparatus under controlled conditions for the deuterium interaction.

Fig. 39 shows the $E^*$ spectra for each nominal momentum $P$ and for each angular setting $\theta$. The values corresponding to the same $E^*$ and $\theta^*$ from different runs were combined by introducing a weighted average. The bins of $E^*$ chosen for the average are shown in fig. 40. The $\cos\theta^*$ binning was taken, in all cases, as 0.1. It should be pointed out that the different resolution in $E^*$ for the various momentum settings can introduce serious distortions in the combined result of the above procedure. Furthermore, averaging the low statistics tail of one distribution with the high statistics portion of another may also create problems. In view of this, we have restricted the combination procedure to values originating from adjacent momenta. This restriction cuts out only a marginal amount of statistics while providing us with more reliable cross sections.

The results are listed in table VI for the $K^-n$ and table VII for the $K^-p$ reactions. In order to give an idea of the relative importance of the more questionable part of the deuterium corrections we list under $\sigma_2$ the differential cross sections corrected with the full deuterium treatment and under $\sigma_1$ those obtained by switching off the double scattering effects (the $D_1$, $D_2$, $D_3$ diagrams in the next section); the latter cross sections take into account the effect of the Fermi motion but ignore anything beyond the "single-scattering" $S_1$ and $S_2$ diagrams.

![Fig. 39](image1)

**Fig. 39**
Effective mass distribution of the $K^-n$ system in the $K^-d + K^-n p_n$ reactions for the different angular positions of the set-up. The incident momenta ($P$) are given in GeV/c.

![Fig. 40](image2)

**Fig. 40**
Effective mass distribution of the $K^-n$ system in $K^-d + K^-n p_n$ and of the $K^-p$ system in $K^-d + K^-p n_n$ reaction. The energy bins adopted for the differential cross sections are shown by the horizontal bars.
The fully corrected data ($\sigma^*_2$) are shown plotted as a function of $\cos \theta^*$ in figs 41 and 42. Also plotted are the predictions from earlier partial wave analyses (more about the latter in sect. 6).

**FIG. 41**

Differential cross sections $K^- n \rightarrow K^- n$ as a function of the cosine of the scattered $K^-$ in the $K^- n$ centre of mass. These data have been derived from the reaction $K^- d \rightarrow K^- n (p_n)$ after correction for the Fermi motion and the other effects (double scattering) occurring in deuterium. The average value of the $K^- n$ effective mass is given for each angular distribution. The dotted and dash-dotted curves represent the differential cross section predicted by the partial wave analyses of CHM [4] and RLIC [5] respectively.
FIG. 42
Same as in fig. 41 but for $K^- p \rightarrow K^- p$ as derived from the reaction $\bar{K} d \rightarrow \bar{K} p (n_s)$. 
TABLE IV

Correction factors for reactions (1) and (2): (a) corrections independent on the scattering angle, (b) corrections dependent on the scattering angle. The values in (b) are those valid at 1.8 GeV/c nominal incident momentum. All the entries are multiplicative factors to be applied on the incident flux.

(a)  

<table>
<thead>
<tr>
<th>Incident momentum (GeV/c)</th>
<th>1.2</th>
<th>1.4</th>
<th>1.6</th>
<th>1.8</th>
<th>2.0</th>
<th>2.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Decays between $\bar{\chi}$ and $S_2$</td>
<td>0.995</td>
<td>0.995</td>
<td>0.995</td>
<td>0.995</td>
<td>0.995</td>
<td>0.995</td>
</tr>
<tr>
<td>Decays between $S_2$ and the target</td>
<td>0.966</td>
<td>0.971</td>
<td>0.974</td>
<td>0.977</td>
<td>0.979</td>
<td>0.981</td>
</tr>
<tr>
<td>Absorption in $S_2$</td>
<td>0.998</td>
<td>0.998</td>
<td>0.998</td>
<td>0.998</td>
<td>0.998</td>
<td>0.998</td>
</tr>
<tr>
<td>Absorption in the target (incident particle)</td>
<td>0.960</td>
<td>0.964</td>
<td>0.964</td>
<td>0.964</td>
<td>0.967</td>
<td>0.967</td>
</tr>
<tr>
<td>Accidentals in $W_1$ and $W_2$</td>
<td>0.952</td>
<td>0.951</td>
<td>0.944</td>
<td>0.907</td>
<td>0.850</td>
<td>0.824</td>
</tr>
</tbody>
</table>

(b)  

<table>
<thead>
<tr>
<th>Charged-branch angle (deg.)</th>
<th>20°</th>
<th>30°</th>
<th>40°</th>
<th>60°</th>
<th>75°</th>
<th>90°</th>
</tr>
</thead>
<tbody>
<tr>
<td>Delta-rays in coque</td>
<td>0.960</td>
<td>0.966</td>
<td>0.969</td>
<td>0.972</td>
<td>0.974</td>
<td>0.974</td>
</tr>
<tr>
<td>Scattered K</td>
<td>Absorption in the target</td>
<td>0.976</td>
<td>0.983</td>
<td>0.984</td>
<td>0.988</td>
<td>0.991</td>
</tr>
<tr>
<td>Absorption in charged-branch</td>
<td>0.978</td>
<td>0.978</td>
<td>0.979</td>
<td>0.980</td>
<td>0.981</td>
<td>0.983</td>
</tr>
<tr>
<td>Delta-rays and inefficiency of MWPC's</td>
<td>0.960</td>
<td>0.958</td>
<td>0.960</td>
<td>0.967</td>
<td>0.884</td>
<td>0.885</td>
</tr>
<tr>
<td>Recoil nucleon</td>
<td>Absorption in the target (for neutrons)</td>
<td>1.007</td>
<td>1.011</td>
<td>1.016</td>
<td>1.025</td>
<td>1.057</td>
</tr>
<tr>
<td>Absorption in the target (for protons)</td>
<td>0.993</td>
<td>0.989</td>
<td>0.984</td>
<td>0.976</td>
<td>0.971</td>
<td>0.965</td>
</tr>
<tr>
<td>Absorption in the neutral-branch (only for protons)</td>
<td>0.977</td>
<td>0.977</td>
<td>0.976</td>
<td>0.975</td>
<td>0.974</td>
<td>0.972</td>
</tr>
</tbody>
</table>
TABLE V

Percentage of events recorded as $K^+d + K^-n$ ($p_T$) triggers rejected at the various stages of the reconstruction: (a) electronic defaults, (b) rejects of the outgoing branches, (c) no incident track, (d) vertex outside target or undefined (e) $\chi^2$ test, (f) missing mass cut, (g) kinematic cuts. The angle of the charged branch is listed under $\theta$ in degree, the nominal incident momentum under $P$ in GeV/c.

<table>
<thead>
<tr>
<th>$P$</th>
<th>$\theta$</th>
<th>(a)</th>
<th>(b)</th>
<th>(c)</th>
<th>(d)</th>
<th>(e)</th>
<th>(f)</th>
<th>(g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.2</td>
<td>20</td>
<td>3.3</td>
<td>97.67</td>
<td>6.5</td>
<td>21.1</td>
<td>20.7</td>
<td>3.8</td>
<td>54.5</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>2.6</td>
<td>96.32</td>
<td>8.9</td>
<td>11.5</td>
<td>10.2</td>
<td>0.0</td>
<td>54.3</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>1.9</td>
<td>95.33</td>
<td>6.8</td>
<td>9.2</td>
<td>21.1</td>
<td>0.0</td>
<td>54.0</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>3.9</td>
<td>94.36</td>
<td>7.0</td>
<td>13.6</td>
<td>19.8</td>
<td>0.0</td>
<td>39.2</td>
</tr>
<tr>
<td></td>
<td>75</td>
<td>4.9</td>
<td>89.24</td>
<td>10.5</td>
<td>7.5</td>
<td>24.0</td>
<td>0.0</td>
<td>49.3</td>
</tr>
<tr>
<td>1.4</td>
<td>20</td>
<td>3.3</td>
<td>96.85</td>
<td>6.3</td>
<td>23.1</td>
<td>14.4</td>
<td>0.8</td>
<td>62.1</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>4.1</td>
<td>94.99</td>
<td>10.2</td>
<td>12.9</td>
<td>12.4</td>
<td>1.1</td>
<td>57.7</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>1.7</td>
<td>93.78</td>
<td>7.3</td>
<td>8.9</td>
<td>18.0</td>
<td>0.0</td>
<td>58.9</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>3.9</td>
<td>91.91</td>
<td>8.9</td>
<td>13.0</td>
<td>17.4</td>
<td>0.0</td>
<td>51.1</td>
</tr>
<tr>
<td></td>
<td>90</td>
<td>5.1</td>
<td>81.60</td>
<td>15.1</td>
<td>8.1</td>
<td>31.4</td>
<td>36.9</td>
<td>55.4</td>
</tr>
<tr>
<td>1.6</td>
<td>20</td>
<td>3.6</td>
<td>96.54</td>
<td>8.8</td>
<td>27.4</td>
<td>19.7</td>
<td>4.3</td>
<td>73.5</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>2.7</td>
<td>97.99</td>
<td>10.0</td>
<td>15.8</td>
<td>11.0</td>
<td>3.1</td>
<td>74.4</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>2.0</td>
<td>88.30</td>
<td>10.4</td>
<td>10.0</td>
<td>16.2</td>
<td>3.6</td>
<td>69.2</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>2.6</td>
<td>88.05</td>
<td>9.1</td>
<td>13.9</td>
<td>17.3</td>
<td>0.0</td>
<td>67.4</td>
</tr>
<tr>
<td></td>
<td>75</td>
<td>5.7</td>
<td>86.17</td>
<td>11.6</td>
<td>9.0</td>
<td>16.1</td>
<td>0.0</td>
<td>53.2</td>
</tr>
<tr>
<td></td>
<td>90</td>
<td>8.1</td>
<td>80.01</td>
<td>15.0</td>
<td>8.9</td>
<td>25.2</td>
<td>36.1</td>
<td>54.0</td>
</tr>
<tr>
<td>1.8</td>
<td>20</td>
<td>4.2</td>
<td>94.60</td>
<td>14.0</td>
<td>27.9</td>
<td>20.7</td>
<td>6.2</td>
<td>78.1</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>4.2</td>
<td>95.78</td>
<td>21.7</td>
<td>17.5</td>
<td>14.7</td>
<td>11.7</td>
<td>69.9</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>2.7</td>
<td>89.76</td>
<td>12.8</td>
<td>9.4</td>
<td>13.8</td>
<td>8.6</td>
<td>67.3</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>3.0</td>
<td>89.16</td>
<td>12.1</td>
<td>15.1</td>
<td>16.3</td>
<td>1.5</td>
<td>63.8</td>
</tr>
<tr>
<td></td>
<td>75</td>
<td>6.9</td>
<td>87.39</td>
<td>13.9</td>
<td>8.6</td>
<td>16.6</td>
<td>0.0</td>
<td>67.6</td>
</tr>
<tr>
<td></td>
<td>90</td>
<td>6.3</td>
<td>83.28</td>
<td>15.5</td>
<td>8.2</td>
<td>26.1</td>
<td>28.6</td>
<td>55.9</td>
</tr>
<tr>
<td>2.0</td>
<td>20</td>
<td>4.0</td>
<td>94.88</td>
<td>13.6</td>
<td>32.9</td>
<td>16.0</td>
<td>7.9</td>
<td>81.7</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>4.9</td>
<td>95.84</td>
<td>20.9</td>
<td>20.6</td>
<td>14.1</td>
<td>19.0</td>
<td>73.4</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>2.0</td>
<td>90.40</td>
<td>20.5</td>
<td>13.4</td>
<td>19.9</td>
<td>8.4</td>
<td>67.5</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>5.3</td>
<td>89.22</td>
<td>19.1</td>
<td>16.1</td>
<td>19.7</td>
<td>7.3</td>
<td>74.0</td>
</tr>
<tr>
<td></td>
<td>75</td>
<td>8.4</td>
<td>87.31</td>
<td>21.5</td>
<td>8.1</td>
<td>23.7</td>
<td>3.2</td>
<td>69.1</td>
</tr>
<tr>
<td></td>
<td>90</td>
<td>5.0</td>
<td>82.10</td>
<td>21.2</td>
<td>7.7</td>
<td>18.4</td>
<td>44.6</td>
<td>58.6</td>
</tr>
<tr>
<td>2.2</td>
<td>20</td>
<td>4.4</td>
<td>94.52</td>
<td>18.1</td>
<td>34.3</td>
<td>16.4</td>
<td>12.6</td>
<td>83.7</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>4.9</td>
<td>94.35</td>
<td>24.7</td>
<td>17.7</td>
<td>15.9</td>
<td>25.8</td>
<td>71.7</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>3.1</td>
<td>88.74</td>
<td>26.7</td>
<td>14.5</td>
<td>19.7</td>
<td>13.7</td>
<td>66.4</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>4.8</td>
<td>86.54</td>
<td>22.8</td>
<td>13.3</td>
<td>16.1</td>
<td>12.1</td>
<td>72.3</td>
</tr>
<tr>
<td></td>
<td>75</td>
<td>7.5</td>
<td>90.41</td>
<td>23.7</td>
<td>9.1</td>
<td>15.0</td>
<td>8.6</td>
<td>77.5</td>
</tr>
<tr>
<td></td>
<td>90</td>
<td>6.8</td>
<td>83.59</td>
<td>29.9</td>
<td>9.0</td>
<td>20.4</td>
<td>5.7</td>
<td>84.8</td>
</tr>
</tbody>
</table>
Differential cross sections for the $K^-n$ elastic scattering as obtained on the basis of the $K^-d \rightarrow K^-n p\bar{p}$ reaction. The $\cos \theta$ column gives the central value of the cosine of the $K^-$ scattering angle in the $K^-n$ center of mass. The number of events used for the calculation in each bin of 0.1 in $\cos \theta$ is listed under $N$. The values under $\sigma_1$ give the differential cross section $d\sigma/d\Omega$ in $\text{mb/sr}$ for $K^-n \rightarrow K^-n$ taking into account the effects of the Fermi motion but not the rescattering phenomena discussed in the text. Under $\sigma_2$ we list the fully corrected $K^-n$ differential cross section with its error; it gives the values as they would be obtained by measuring the elastic scattering of $K^-$ on a stationary and isolated neutron. The centre of the $K^-n$ mass bin is given by $E^*$ in GeV, the corresponding incident $K^-$momentum is given by $P$ in GeV/c.

<table>
<thead>
<tr>
<th>$E^*$ = 1.858</th>
<th>$E^*$ = 1.903</th>
<th>$E^*$ = 1.948</th>
<th>$E^*$ = 1.993</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P$ = 1.138</td>
<td>$P$ = 1.236</td>
<td>$P$ = 1.335</td>
<td>$P$ = 1.435</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\cos \theta$</th>
<th>$N$</th>
<th>$\sigma_1$</th>
<th>$\sigma_2$</th>
<th>$N$</th>
<th>$\sigma_1$</th>
<th>$\sigma_2$</th>
<th>$N$</th>
<th>$\sigma_1$</th>
<th>$\sigma_2$</th>
<th>$N$</th>
<th>$\sigma_1$</th>
<th>$\sigma_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.85</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>4</td>
<td>805</td>
<td>941±470</td>
<td>11</td>
<td>275</td>
<td>308±89</td>
</tr>
<tr>
<td>-0.75</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>2</td>
<td>2203</td>
<td>2455±1417</td>
<td>18</td>
<td>237</td>
<td>252±59</td>
</tr>
<tr>
<td>-0.65</td>
<td>6</td>
<td>473</td>
<td>513±193</td>
<td>5</td>
<td>403</td>
<td>430±175</td>
<td>13</td>
<td>357</td>
<td>380±105</td>
<td>13</td>
<td>565</td>
<td>597±165</td>
</tr>
<tr>
<td>-0.55</td>
<td>10</td>
<td>453</td>
<td>487±154</td>
<td>13</td>
<td>357</td>
<td>380±105</td>
<td>11</td>
<td>421</td>
<td>446±128</td>
<td>13</td>
<td>261</td>
<td>282±78</td>
</tr>
<tr>
<td>-0.45</td>
<td>14</td>
<td>553</td>
<td>597±159</td>
<td>11</td>
<td>421</td>
<td>446±128</td>
<td>11</td>
<td>298</td>
<td>321±82</td>
<td>13</td>
<td>261</td>
<td>282±78</td>
</tr>
<tr>
<td>-0.35</td>
<td>3</td>
<td>341</td>
<td>360±136</td>
<td>15</td>
<td>298</td>
<td>321±82</td>
<td>10</td>
<td>266</td>
<td>291±92</td>
<td>18</td>
<td>220</td>
<td>238±56</td>
</tr>
<tr>
<td>-0.25</td>
<td>5</td>
<td>277</td>
<td>297±133</td>
<td>13</td>
<td>305</td>
<td>332±92</td>
<td>16</td>
<td>226</td>
<td>245±61</td>
<td>16</td>
<td>160</td>
<td>169±41</td>
</tr>
<tr>
<td>-0.15</td>
<td>4</td>
<td>440</td>
<td>480±240</td>
<td>10</td>
<td>250</td>
<td>277±87</td>
<td>16</td>
<td>235</td>
<td>250±62</td>
<td>5</td>
<td>98</td>
<td>99±40</td>
</tr>
<tr>
<td>-0.05</td>
<td>11</td>
<td>420</td>
<td>468±141</td>
<td>1</td>
<td>497</td>
<td>559±595</td>
<td>9</td>
<td>270</td>
<td>282±94</td>
<td>5</td>
<td>119</td>
<td>118±52</td>
</tr>
<tr>
<td>0.05</td>
<td>11</td>
<td>245</td>
<td>274±79</td>
<td>6</td>
<td>297</td>
<td>332±126</td>
<td>5</td>
<td>113</td>
<td>118±52</td>
<td>20</td>
<td>270</td>
<td>269±59</td>
</tr>
<tr>
<td>0.15</td>
<td>13</td>
<td>340</td>
<td>383±102</td>
<td>4</td>
<td>214</td>
<td>235±117</td>
<td>13</td>
<td>226</td>
<td>237±63</td>
<td>6</td>
<td>198</td>
<td>198±74</td>
</tr>
<tr>
<td>0.25</td>
<td>19</td>
<td>286</td>
<td>324±74</td>
<td>20</td>
<td>361</td>
<td>403±90</td>
<td>18</td>
<td>204</td>
<td>219±51</td>
<td>18</td>
<td>272</td>
<td>279±65</td>
</tr>
<tr>
<td>0.35</td>
<td>18</td>
<td>649</td>
<td>743±175</td>
<td>16</td>
<td>463</td>
<td>514±128</td>
<td>26</td>
<td>337</td>
<td>366±71</td>
<td>16</td>
<td>279</td>
<td>292±73</td>
</tr>
<tr>
<td>0.45</td>
<td>45</td>
<td>853</td>
<td>995±228</td>
<td>18</td>
<td>548</td>
<td>622±146</td>
<td>16</td>
<td>552</td>
<td>605±151</td>
<td>21</td>
<td>518</td>
<td>548±116</td>
</tr>
<tr>
<td>0.55</td>
<td>41</td>
<td>1149</td>
<td>1344±207</td>
<td>21</td>
<td>601</td>
<td>699±149</td>
<td>38</td>
<td>706</td>
<td>782±126</td>
<td>20</td>
<td>429</td>
<td>459±102</td>
</tr>
<tr>
<td>0.65</td>
<td>38</td>
<td>1315</td>
<td>1646±263</td>
<td>37</td>
<td>1050</td>
<td>1262±207</td>
<td>41</td>
<td>1083</td>
<td>1240±191</td>
<td>27</td>
<td>726</td>
<td>808±155</td>
</tr>
<tr>
<td>0.75</td>
<td>28</td>
<td>1585</td>
<td>2075±423</td>
<td>20</td>
<td>1086</td>
<td>1381±301</td>
<td>33</td>
<td>1598</td>
<td>1991±341</td>
<td>38</td>
<td>1558</td>
<td>1630±264</td>
</tr>
<tr>
<td>$\cos \theta^* \kappa^{-}$</td>
<td>$E^* = 2.038$</td>
<td>$E^* = 2.083$</td>
<td>$E^* = 2.128$</td>
<td>$E^* = 2.160$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-----------</td>
<td>-------------</td>
<td>-------------</td>
<td>-------------</td>
<td>-------------</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$P = 1.537$</td>
<td>$P = 1.640$</td>
<td>$P = 1.746$</td>
<td>$P = 1.841$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$N$</td>
<td>$\sigma_1$</td>
<td>$\sigma_2$</td>
<td>$N$</td>
<td>$\sigma_1$</td>
<td>$\sigma_2$</td>
<td>$N$</td>
<td>$\sigma_1$</td>
<td>$\sigma_2$</td>
<td>$N$</td>
<td>$\sigma_1$</td>
<td>$\sigma_2$</td>
<td></td>
</tr>
<tr>
<td>-0.85</td>
<td>20</td>
<td>244</td>
<td>268 ± 60</td>
<td>20</td>
<td>115</td>
<td>127 ± 28</td>
<td>15</td>
<td>70</td>
<td>71 ± 18</td>
<td>10</td>
<td>77</td>
<td>76 ± 24</td>
</tr>
<tr>
<td>-0.75</td>
<td>36</td>
<td>240</td>
<td>253 ± 42</td>
<td>31</td>
<td>90</td>
<td>95 ± 16</td>
<td>15</td>
<td>72</td>
<td>77 ± 20</td>
<td>27</td>
<td>81</td>
<td>85 ± 16</td>
</tr>
<tr>
<td>-0.65</td>
<td>21</td>
<td>179</td>
<td>187 ± 40</td>
<td>40</td>
<td>154</td>
<td>162 ± 25</td>
<td>27</td>
<td>134</td>
<td>138 ± 26</td>
<td>36</td>
<td>104</td>
<td>108 ± 17</td>
</tr>
<tr>
<td>-0.55</td>
<td>31</td>
<td>264</td>
<td>277 ± 49</td>
<td>27</td>
<td>180</td>
<td>191 ± 36</td>
<td>33</td>
<td>170</td>
<td>176 ± 30</td>
<td>21</td>
<td>91</td>
<td>94 ± 20</td>
</tr>
<tr>
<td>-0.45</td>
<td>31</td>
<td>281</td>
<td>303 ± 54</td>
<td>30</td>
<td>132</td>
<td>141 ± 25</td>
<td>23</td>
<td>119</td>
<td>124 ± 25</td>
<td>34</td>
<td>154</td>
<td>159 ± 26</td>
</tr>
<tr>
<td>-0.35</td>
<td>22</td>
<td>257</td>
<td>253 ± 54</td>
<td>20</td>
<td>124</td>
<td>131 ± 28</td>
<td>16</td>
<td>108</td>
<td>112 ± 28</td>
<td>16</td>
<td>96</td>
<td>100 ± 25</td>
</tr>
<tr>
<td>-0.25</td>
<td>12</td>
<td>125</td>
<td>120 ± 34</td>
<td>16</td>
<td>143</td>
<td>150 ± 36</td>
<td>15</td>
<td>115</td>
<td>121 ± 31</td>
<td>13</td>
<td>98</td>
<td>103 ± 27</td>
</tr>
<tr>
<td>-0.15</td>
<td>5</td>
<td>116</td>
<td>115 ± 47</td>
<td>5</td>
<td>77</td>
<td>78 ± 31</td>
<td>4</td>
<td>88</td>
<td>93 ± 46</td>
<td>5</td>
<td>97</td>
<td>103 ± 46</td>
</tr>
<tr>
<td>-0.05</td>
<td>13</td>
<td>105</td>
<td>101 ± 27</td>
<td>11</td>
<td>165</td>
<td>152 ± 45</td>
<td>13</td>
<td>99</td>
<td>108 ± 29</td>
<td>6</td>
<td>153</td>
<td>165 ± 62</td>
</tr>
<tr>
<td>0.05</td>
<td>20</td>
<td>124</td>
<td>117 ± 26</td>
<td>6</td>
<td>100</td>
<td>87 ± 33</td>
<td>18</td>
<td>114</td>
<td>120 ± 28</td>
<td>7</td>
<td>92</td>
<td>95 ± 33</td>
</tr>
<tr>
<td>0.15</td>
<td>13</td>
<td>122</td>
<td>116 ± 32</td>
<td>13</td>
<td>99</td>
<td>93 ± 24</td>
<td>10</td>
<td>70</td>
<td>71 ± 22</td>
<td>7</td>
<td>98</td>
<td>99 ± 35</td>
</tr>
<tr>
<td>0.25</td>
<td>10</td>
<td>102</td>
<td>100 ± 31</td>
<td>10</td>
<td>85</td>
<td>84 ± 26</td>
<td>13</td>
<td>76</td>
<td>77 ± 21</td>
<td>18</td>
<td>101</td>
<td>102 ± 24</td>
</tr>
<tr>
<td>0.35</td>
<td>9</td>
<td>75</td>
<td>75 ± 25</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>3</td>
<td>57</td>
<td>58 ± 33</td>
<td>2</td>
<td>91</td>
<td>93 ± 53</td>
</tr>
<tr>
<td>0.45</td>
<td>5</td>
<td>190</td>
<td>193 ± 78</td>
<td>4</td>
<td>108</td>
<td>105 ± 52</td>
<td>44</td>
<td>201</td>
<td>204 ± 102</td>
<td>2</td>
<td>182</td>
<td>184 ± 130</td>
</tr>
<tr>
<td>0.55</td>
<td>13</td>
<td>244</td>
<td>256 ± 71</td>
<td>13</td>
<td>197</td>
<td>208 ± 55</td>
<td>15</td>
<td>227</td>
<td>238 ± 61</td>
<td>1</td>
<td>297</td>
<td>305 ± 215</td>
</tr>
<tr>
<td>0.65</td>
<td>19</td>
<td>309</td>
<td>346 ± 79</td>
<td>20</td>
<td>392</td>
<td>466 ± 104</td>
<td>28</td>
<td>391</td>
<td>441 ± 82</td>
<td>31</td>
<td>601</td>
<td>669 ± 118</td>
</tr>
<tr>
<td>0.75</td>
<td>40</td>
<td>1559</td>
<td>1878 ± 293</td>
<td>27</td>
<td>756</td>
<td>935 ± 176</td>
<td>44</td>
<td>1076</td>
<td>1263 ± 190</td>
<td>50</td>
<td>1089</td>
<td>1245 ± 176</td>
</tr>
<tr>
<td>( \bar{K}_n )</td>
<td>( E^{*} = 2.203 )</td>
<td>( E^* = 2.280 )</td>
<td>( E^* = 2.320 )</td>
<td>( E^* = 2.360 )</td>
<td>( E^* = 2.411 )</td>
<td>( E^* = 2.467 )</td>
<td>( E^* = 2.589 )</td>
<td>( E^* = 2.720 )</td>
<td>( E^* = 2.982 )</td>
<td>( E^* = 3.385 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \bar{K}_n )</td>
<td>( \bar{K}_n )</td>
<td>( \bar{K}_n )</td>
<td>( \bar{K}_n )</td>
<td>( \bar{K}_n )</td>
<td>( \bar{K}_n )</td>
<td>( \bar{K}_n )</td>
<td>( \bar{K}_n )</td>
<td>( \bar{K}_n )</td>
<td>( \bar{K}_n )</td>
<td>( \bar{K}_n )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( N )</td>
<td>( \sigma_1 )</td>
<td>( \sigma_2 )</td>
<td>( \sigma_1 )</td>
<td>( \sigma_2 )</td>
<td>( \sigma_1 )</td>
<td>( \sigma_2 )</td>
<td>( \sigma_1 )</td>
<td>( \sigma_2 )</td>
<td>( \sigma_1 )</td>
<td>( \sigma_2 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.05</td>
<td>2</td>
<td>16</td>
<td>10</td>
<td>4</td>
<td>2</td>
<td>16</td>
<td>10</td>
<td>4</td>
<td>2</td>
<td>16</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.055</td>
<td>2</td>
<td>16</td>
<td>10</td>
<td>4</td>
<td>2</td>
<td>16</td>
<td>10</td>
<td>4</td>
<td>2</td>
<td>16</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.06</td>
<td>2</td>
<td>16</td>
<td>10</td>
<td>4</td>
<td>2</td>
<td>16</td>
<td>10</td>
<td>4</td>
<td>2</td>
<td>16</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.065</td>
<td>2</td>
<td>16</td>
<td>10</td>
<td>4</td>
<td>2</td>
<td>16</td>
<td>10</td>
<td>4</td>
<td>2</td>
<td>16</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.07</td>
<td>2</td>
<td>16</td>
<td>10</td>
<td>4</td>
<td>2</td>
<td>16</td>
<td>10</td>
<td>4</td>
<td>2</td>
<td>16</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.075</td>
<td>2</td>
<td>16</td>
<td>10</td>
<td>4</td>
<td>2</td>
<td>16</td>
<td>10</td>
<td>4</td>
<td>2</td>
<td>16</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.08</td>
<td>2</td>
<td>16</td>
<td>10</td>
<td>4</td>
<td>2</td>
<td>16</td>
<td>10</td>
<td>4</td>
<td>2</td>
<td>16</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.09</td>
<td>2</td>
<td>16</td>
<td>10</td>
<td>4</td>
<td>2</td>
<td>16</td>
<td>10</td>
<td>4</td>
<td>2</td>
<td>16</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td>2</td>
<td>16</td>
<td>10</td>
<td>4</td>
<td>2</td>
<td>16</td>
<td>10</td>
<td>4</td>
<td>2</td>
<td>16</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.15</td>
<td>2</td>
<td>16</td>
<td>10</td>
<td>4</td>
<td>2</td>
<td>16</td>
<td>10</td>
<td>4</td>
<td>2</td>
<td>16</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.25</td>
<td>2</td>
<td>16</td>
<td>10</td>
<td>4</td>
<td>2</td>
<td>16</td>
<td>10</td>
<td>4</td>
<td>2</td>
<td>16</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.55</td>
<td>2</td>
<td>16</td>
<td>10</td>
<td>4</td>
<td>2</td>
<td>16</td>
<td>10</td>
<td>4</td>
<td>2</td>
<td>16</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.75</td>
<td>2</td>
<td>16</td>
<td>10</td>
<td>4</td>
<td>2</td>
<td>16</td>
<td>10</td>
<td>4</td>
<td>2</td>
<td>16</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.85</td>
<td>2</td>
<td>16</td>
<td>10</td>
<td>4</td>
<td>2</td>
<td>16</td>
<td>10</td>
<td>4</td>
<td>2</td>
<td>16</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**TABLE VI.**
TABLE VII

Differential cross sections for the $K^- p$ elastic scattering as obtained on the basis of the $K^- d + K^- p n_n$ reaction. Conventions are the same as in Table VI.

<table>
<thead>
<tr>
<th>$E^*$</th>
<th>$P$</th>
<th>$N$</th>
<th>$\sigma_1$</th>
<th>$\sigma_2$</th>
<th>$N$</th>
<th>$\sigma_1$</th>
<th>$\sigma_2$</th>
<th>$N$</th>
<th>$\sigma_1$</th>
<th>$\sigma_2$</th>
<th>$N$</th>
<th>$\sigma_1$</th>
<th>$\sigma_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1.858$</td>
<td>$1.138$</td>
<td>$-0.85$</td>
<td>$-38$</td>
<td>$511$</td>
<td>$593 \pm 94$</td>
<td>$13$</td>
<td>$164$</td>
<td>$190 \pm 50$</td>
<td>$-45$</td>
<td>$36$</td>
<td>$551$</td>
<td>$585 \pm 97$</td>
<td></td>
</tr>
<tr>
<td>$1.903$</td>
<td>$1.236$</td>
<td>$-0.75$</td>
<td>$-28$</td>
<td>$511$</td>
<td>$593 \pm 94$</td>
<td>$13$</td>
<td>$164$</td>
<td>$190 \pm 50$</td>
<td>$-45$</td>
<td>$36$</td>
<td>$551$</td>
<td>$585 \pm 97$</td>
<td></td>
</tr>
<tr>
<td>$1.948$</td>
<td>$1.335$</td>
<td>$-0.65$</td>
<td>$36$</td>
<td>$551$</td>
<td>$585 \pm 97$</td>
<td>$23$</td>
<td>$243$</td>
<td>$253 \pm 51$</td>
<td>$-35$</td>
<td>$52$</td>
<td>$320$</td>
<td>$326 \pm 45$</td>
<td></td>
</tr>
<tr>
<td>$1.995$</td>
<td>$1.435$</td>
<td>$-0.55$</td>
<td>$32$</td>
<td>$222$</td>
<td>$226 \pm 34$</td>
<td>$13$</td>
<td>$164$</td>
<td>$190 \pm 50$</td>
<td>$-35$</td>
<td>$52$</td>
<td>$320$</td>
<td>$326 \pm 45$</td>
<td></td>
</tr>
<tr>
<td>$-0.45$</td>
<td>$32$</td>
<td>$222$</td>
<td>$226 \pm 34$</td>
<td>$13$</td>
<td>$164$</td>
<td>$190 \pm 50$</td>
<td>$-35$</td>
<td>$52$</td>
<td>$320$</td>
<td>$326 \pm 45$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$-0.35$</td>
<td>$32$</td>
<td>$222$</td>
<td>$226 \pm 34$</td>
<td>$13$</td>
<td>$164$</td>
<td>$190 \pm 50$</td>
<td>$-35$</td>
<td>$52$</td>
<td>$320$</td>
<td>$326 \pm 45$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$-0.25$</td>
<td>$32$</td>
<td>$222$</td>
<td>$226 \pm 34$</td>
<td>$13$</td>
<td>$164$</td>
<td>$190 \pm 50$</td>
<td>$-35$</td>
<td>$52$</td>
<td>$320$</td>
<td>$326 \pm 45$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$-0.15$</td>
<td>$32$</td>
<td>$222$</td>
<td>$226 \pm 34$</td>
<td>$13$</td>
<td>$164$</td>
<td>$190 \pm 50$</td>
<td>$-35$</td>
<td>$52$</td>
<td>$320$</td>
<td>$326 \pm 45$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$-0.05$</td>
<td>$32$</td>
<td>$222$</td>
<td>$226 \pm 34$</td>
<td>$13$</td>
<td>$164$</td>
<td>$190 \pm 50$</td>
<td>$-35$</td>
<td>$52$</td>
<td>$320$</td>
<td>$326 \pm 45$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$0.05$</td>
<td>$32$</td>
<td>$222$</td>
<td>$226 \pm 34$</td>
<td>$13$</td>
<td>$164$</td>
<td>$190 \pm 50$</td>
<td>$-35$</td>
<td>$52$</td>
<td>$320$</td>
<td>$326 \pm 45$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$0.15$</td>
<td>$32$</td>
<td>$222$</td>
<td>$226 \pm 34$</td>
<td>$13$</td>
<td>$164$</td>
<td>$190 \pm 50$</td>
<td>$-35$</td>
<td>$52$</td>
<td>$320$</td>
<td>$326 \pm 45$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$0.25$</td>
<td>$32$</td>
<td>$222$</td>
<td>$226 \pm 34$</td>
<td>$13$</td>
<td>$164$</td>
<td>$190 \pm 50$</td>
<td>$-35$</td>
<td>$52$</td>
<td>$320$</td>
<td>$326 \pm 45$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$0.35$</td>
<td>$32$</td>
<td>$222$</td>
<td>$226 \pm 34$</td>
<td>$13$</td>
<td>$164$</td>
<td>$190 \pm 50$</td>
<td>$-35$</td>
<td>$52$</td>
<td>$320$</td>
<td>$326 \pm 45$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$0.45$</td>
<td>$32$</td>
<td>$222$</td>
<td>$226 \pm 34$</td>
<td>$13$</td>
<td>$164$</td>
<td>$190 \pm 50$</td>
<td>$-35$</td>
<td>$52$</td>
<td>$320$</td>
<td>$326 \pm 45$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$0.55$</td>
<td>$32$</td>
<td>$222$</td>
<td>$226 \pm 34$</td>
<td>$13$</td>
<td>$164$</td>
<td>$190 \pm 50$</td>
<td>$-35$</td>
<td>$52$</td>
<td>$320$</td>
<td>$326 \pm 45$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$0.65$</td>
<td>$32$</td>
<td>$222$</td>
<td>$226 \pm 34$</td>
<td>$13$</td>
<td>$164$</td>
<td>$190 \pm 50$</td>
<td>$-35$</td>
<td>$52$</td>
<td>$320$</td>
<td>$326 \pm 45$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$0.75$</td>
<td>$32$</td>
<td>$222$</td>
<td>$226 \pm 34$</td>
<td>$13$</td>
<td>$164$</td>
<td>$190 \pm 50$</td>
<td>$-35$</td>
<td>$52$</td>
<td>$320$</td>
<td>$326 \pm 45$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Table VII (2)

<table>
<thead>
<tr>
<th>$\kappa_p$</th>
<th>$E^* = 2.038$</th>
<th>$E^* = 2.085$</th>
<th>$E^* = 2.128$</th>
<th>$E^* = 2.168$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$P = 1.537$</td>
<td>$P = 1.640$</td>
<td>$P = 1.746$</td>
<td>$P = 1.841$</td>
</tr>
<tr>
<td>$\cos \theta^*$</td>
<td>$N$ $\sigma_1$ $\sigma_2$</td>
<td>$N$ $\sigma_1$ $\sigma_2$</td>
<td>$N$ $\sigma_1$ $\sigma_2$</td>
<td>$N$ $\sigma_1$ $\sigma_2$</td>
</tr>
<tr>
<td>-0.85</td>
<td>- - -</td>
<td>33 99 129 ± 22</td>
<td>31 60 63 ± 11</td>
<td>- - -</td>
</tr>
<tr>
<td>-0.75</td>
<td>- - -</td>
<td>107 143 168 ± 16</td>
<td>90 91 91 ± 9</td>
<td>55 74 71 ± 9</td>
</tr>
<tr>
<td>-0.65</td>
<td>90 194 216 ± 22</td>
<td>60 94 103 ± 13</td>
<td>66 122 119 ± 14</td>
<td>33 65 63 ± 10</td>
</tr>
<tr>
<td>-0.55</td>
<td>100 231 245 ± 24</td>
<td>77 111 115 ± 13</td>
<td>64 80 79 ± 9</td>
<td>23 46 46 ± 9</td>
</tr>
<tr>
<td>-0.45</td>
<td>50 119 123 ± 17</td>
<td>115 125 129 ± 12</td>
<td>57 88 88 ± 11</td>
<td>34 77 77 ± 13</td>
</tr>
<tr>
<td>-0.35</td>
<td>66 138 141 ± 17</td>
<td>86 165 169 ± 18</td>
<td>59 86 88 ± 11</td>
<td>47 88 90 ± 13</td>
</tr>
<tr>
<td>-0.25</td>
<td>72 142 145 ± 17</td>
<td>86 145 148 ± 15</td>
<td>61 84 86 ± 11</td>
<td>45 78 81 ± 11</td>
</tr>
<tr>
<td>-0.15</td>
<td>43 108 111 ± 16</td>
<td>59 114 117 ± 15</td>
<td>45 98 101 ± 14</td>
<td>40 99 101 ± 16</td>
</tr>
<tr>
<td>-0.05</td>
<td>63 103 106 ± 13</td>
<td>- - -</td>
<td>40 67 68 ± 10</td>
<td>22 74 75 ± 16</td>
</tr>
<tr>
<td>0.05</td>
<td>85 110 113 ± 12</td>
<td>48 84 84 ± 12</td>
<td>76 110 110 ± 12</td>
<td>56 127 128 ± 17</td>
</tr>
<tr>
<td>0.15</td>
<td>62 84 85 ± 10</td>
<td>41 66 66 ± 10</td>
<td>72 95 96 ± 11</td>
<td>59 133 136 ± 17</td>
</tr>
<tr>
<td>0.25</td>
<td>72 59 61 ± 7</td>
<td>77 91 92 ± 10</td>
<td>98 96 96 ± 9</td>
<td>66 107 111 ± 13</td>
</tr>
<tr>
<td>0.35</td>
<td>67 64 67 ± 8</td>
<td>70 77 78 ± 9</td>
<td>72 90 89 ± 10</td>
<td>37 86 90 ± 14</td>
</tr>
<tr>
<td>0.45</td>
<td>55 116 123 ± 16</td>
<td>70 145 149 ± 17</td>
<td>23 90 89 ± 18</td>
<td>16 103 102 ± 25</td>
</tr>
<tr>
<td>0.55</td>
<td>144 207 223 ± 18</td>
<td>147 218 237 ± 19</td>
<td>87 149 151 ± 16</td>
<td>59 143 145 ± 18</td>
</tr>
<tr>
<td>0.65</td>
<td>233 350 394 ± 25</td>
<td>202 339 388 ± 27</td>
<td>163 305 324 ± 25</td>
<td>124 336 356 ± 32</td>
</tr>
<tr>
<td>0.75</td>
<td>205 974 1133 ± 80</td>
<td>183 908 1076 ± 79</td>
<td>270 1029 1144 ± 69</td>
<td>- - -</td>
</tr>
<tr>
<td>k * p</td>
<td>Ω * cos</td>
<td>N</td>
<td>σ₁</td>
<td>σ₂</td>
</tr>
<tr>
<td>-------</td>
<td>--------</td>
<td>----</td>
<td>----</td>
<td>----</td>
</tr>
<tr>
<td>-0.85</td>
<td>-</td>
<td>18</td>
<td>60</td>
<td>67 ± 15</td>
</tr>
<tr>
<td>-0.75</td>
<td>-</td>
<td>30</td>
<td>43</td>
<td>46 ± 8</td>
</tr>
<tr>
<td>-0.65</td>
<td>-</td>
<td>25</td>
<td>57</td>
<td>40 ± 8</td>
</tr>
<tr>
<td>-0.55</td>
<td>-</td>
<td>37</td>
<td>54</td>
<td>59 ± 9</td>
</tr>
<tr>
<td>-0.45</td>
<td>30</td>
<td>107</td>
<td>112 ± 20</td>
<td>34</td>
</tr>
<tr>
<td>-0.35</td>
<td>23</td>
<td>69</td>
<td>72 ± 15</td>
<td>44</td>
</tr>
<tr>
<td>-0.25</td>
<td>18</td>
<td>59</td>
<td>61 ± 14</td>
<td>50</td>
</tr>
<tr>
<td>-0.15</td>
<td>-</td>
<td>20</td>
<td>66</td>
<td>67 ± 14</td>
</tr>
<tr>
<td>-0.05</td>
<td>50</td>
<td>79</td>
<td>80 ± 11</td>
<td>37</td>
</tr>
<tr>
<td>0.05</td>
<td>53</td>
<td>71</td>
<td>73 ± 10</td>
<td>81</td>
</tr>
<tr>
<td>0.15</td>
<td>60</td>
<td>77</td>
<td>82 ± 10</td>
<td>70</td>
</tr>
<tr>
<td>0.25</td>
<td>86</td>
<td>71</td>
<td>76 ± 8</td>
<td>115</td>
</tr>
<tr>
<td>0.35</td>
<td>52</td>
<td>50</td>
<td>55 ± 7</td>
<td>74</td>
</tr>
<tr>
<td>0.45</td>
<td>27</td>
<td>98</td>
<td>107 ± 20</td>
<td>25</td>
</tr>
<tr>
<td>0.55</td>
<td>53</td>
<td>125</td>
<td>132 ± 18</td>
<td>74</td>
</tr>
<tr>
<td>0.65</td>
<td>88</td>
<td>195</td>
<td>209 ± 22</td>
<td>121</td>
</tr>
<tr>
<td>0.75</td>
<td>135</td>
<td>918</td>
<td>1025 ± 87</td>
<td>157</td>
</tr>
</tbody>
</table>
5. DEUTERIUM CORRECTIONS

5.1 Introduction

The reactions under study take place in a deuterium nucleus. This has the consequence that the final states recorded by the experiment cannot be considered as due to a single interaction between incident particle and a well defined type of nucleon. For instance, even if we select out the $K^-n$ events by demanding that they are associated to large values of the neutron momentum and small values of the proton momentum, we are not guaranteed to end up with events with a "spectator" nucleon which had no role in the reaction. What is generally believed to occur is a combination (whose complexity depends to a large extent on the particular kinematical configuration of the final state) of intermediate processes; by selecting "$K^-n p_p$" final states we simply enhance the probability that the interaction has occurred mainly on the neutron. Notice that the incident particle may interact repeatedly before emerging from the nucleus. The nucleons may also interact between each other as a result of the collision. Finally, the interactions take place on nucleons which are not at rest but have a momentum distribution reflecting the deuterium wave function.

Each of the above phenomena must be fully accounted for if we want to isolate the basic interaction between kaon and neutron. In order to do so, unfortunately, we need to know the complete $\bar{K}N$ scattering amplitude - which is just what the experiment is supposed to determine. The problem can be circumvented, in principle, by assuming as a starting value of the unknown $\bar{K}N$ amplitude the value which would have yielded the observed differential cross section in the absence of deuterium effects. This amplitude can then be used to calculate the expected cross section with all the deuterium effect rigorously taken into account. The result can be compared with the measurements and the procedure repeated varying the amplitudes until good agreement is found between measurements and expectations. The difficulty inherent to this otherwise straightforward procedure lies in the fact that each step of the loop requires the equivalent of a solution to the partial wave problem of the $\bar{K}N$ system. This is no easy task if one considers that the analysis must extend over the whole region of momenta kinematically permissible to the reactions: in our case this region goes from the $\bar{K}N$ threshold to a c.m. energy well above 2.2 GeV. Obviously this procedure is not only cumbersome but also extremely costly in terms of computer use.

As a reasonable compromise we have adopted the following simplified procedure. The most reliable $\bar{K}N$ amplitudes available to date have been assumed to represent sufficiently well the real situation to provide at least a good approximation for the deuterium corrections. These we have calculated by deriving first the uncorrected $K^-n$ cross sections (i.e. those corresponding
to kaons on free and stationary neutrons), then the deuterium-distorted "$K^- n \ p^6_S$" cross sections and finally the ratio of the two. This has been taken to represent the (fixed) effect due to deuterium. The measurements were corrected - once and for all - and the "corrected" data were used in a conventional partial wave analysis. At the end, a comparison was performed between the deuterium corrections used in the analysis and those which would have been obtained had we used the final amplitudes as ingredients. The difference was found to be well within the statistical and systematic accuracy of the results.

5.2 Formalism

In order to calculate the interaction in deuterium we have used the tensor analysis and the detailed procedure described at length in an earlier note [18]. A short reminder of the notations and the basis of the approach is provided below. The Feynman diagrams corresponding to our calculation are summarized in fig. 43 together with the usual captions they are referred to. The calculation amounts to a coherent sum of the contributions of the above graphs, taking into account the modifications due to the Fermi motion.

$\begin{align*}
\text{SINGLE - SCATTERING} \\
\begin{array}{c}
\bullet \quad 6 \quad K^- \\
\circ \quad 3 \\
D & S_1
\end{array}
\quad \begin{array}{c}
\bullet \quad 6 \\
\circ \quad 3 \\
D & S_2
\end{array}
\end{align*}$

$\begin{align*}
\text{DOUBLE - SCATTERING} \\
\begin{array}{c}
\bullet \quad 6 \\
\circ \quad 3 \\
D & D_1
\end{array}
\quad \begin{array}{c}
\bullet \quad 6 \\
\circ \quad 3 \\
D & D_2
\end{array}
\end{align*}$

$\begin{align*}
\text{FINAL-STATE INTERACTION} \\
\begin{array}{c}
\bullet \quad 6 \\
\circ \quad 3 \\
D & D_3
\end{array}
\end{align*}$

**FIG. 43**

Feynman diagrams of the three main effects occurring in the $K^- d$ interactions studied in this experiment. The numbers next to each line are used in the text to identify the particles. The symbols ($S_1$ to $D_3$) are assigned as labels of the various graphs.
and to the presence of the spin and isospin of the particles involved. The calculation also includes the same momentum and angle cuts which were applied to the measured reactions; this allows a direct comparison of the results with the measurements. The following subsections describe, somewhat pedagogically, the various steps of the procedure.

5.2.1 Fermi motion

The wave function \( \psi_{k,\lambda} \) chosen to represent the Fermi motion between nucleons labelled \( k \) and \( \lambda \) at the deuterium vertex of the graphs of fig. 43 has the so-called Hulthén form given in eq. (5). The symbols next to the lines in the graphs of fig. 43 will be used as indices identifying the particles. The use of \( \psi_{k,\lambda} \) in the calculation of the differential cross section is illustrated below for the case of single-scattering on neutrons (i.e. the diagram \( S_1 \) alone). This cross section can be written as follows:

\[
\frac{d\sigma}{d\Omega_{2,6}} = \frac{P_{\lambda,6}^2 M_{2,6}}{|q_1|^2 E_3} \left| H_{1,3} \right|^2 T_{1,2},
\]

(7)

where \( q_1 \) and \( q_3 \) are the laboratory momenta of the incident and spectator particle respectively, \( E_3 \) is the total laboratory energy of the spectator, \( M_{2,6} \) is the c.m. energy of the 2-6 system (outgoing kaon and scattered neutron), \( P_{\lambda,6} \) is the momentum modulus and \( d\Omega_{2,6} \) the element of solid angle in the c.m. system of particles 2 and 6, \( T_{1,2} \) is the element of the scattering matrix representing the \( K^-n \) interaction on a free neutron at a c.m. energy equal to \( M_{2,6} \) and at a \( t \)-value \( t = |q'_1 - q'_2|^2 \).

It can be seen that the factor \( (P_{\lambda,6}^2 M_{2,6})/|q_1|^2 E_3 \) in eq. (8) reduces to what is sometimes referred to as the "flux correction factor" discussed for example in sect. 3.2 of ref. [11]. In this reference the factor appears as

\[
\left[ \frac{vE}{v_k E_k} \right] K \left[ \frac{E}{E_k} \right] n,
\]

(8)

where \( v \) and \( E \) are respectively the velocity and total energy of the kaon or the neutron depending on the subscript \( K \) or \( n \); the subscript \( k \) indicates that the quantities are in the laboratory system whereas the quantities without subscript are in the reference system where the neutron is at rest.

Notice that eq. (8) reduces to the familiar form

\[
\frac{d\sigma}{d\Omega_{2,6}} = |T_{1,2}|^2
\]

in the absence of Fermi motion \( (P_{\lambda,6}^2 M_{2,6} = q_1 E_3 \text{ and } \int H_{1,3}^2 d^3q_3 = 1) \).
5.2.2 Interference between graphs

Take, for example, the graphs $S_1$ and $S_2$ and add them coherently. This produces interference effects between single-scattering on the neutron and on the proton. The differential cross section of the $K^-$ p final state can be written

$$\frac{d\sigma}{d\Omega_{2,6} d^3q_3} = \frac{P_{2,6}}{|q_1| E_3 M_{2,6}} |M_{2,6} H_{1,3} T_{1,2} + M_{5,6} H_{1,2} T_{1,3}|^2 =$$

$$= \frac{P_{2,6}}{|q_1| E_3} \left\{ M_{2,6} H_{1,3}^2 T_{1,2}^2 + 2 H_{1,3} H_{1,2} M_{3,6} \text{Re} T_{1,2}^* T_{1,3} + \frac{M_{3,6}^2 H_{1,2}^2}{M_{2,6}^2} |T_{1,3}|^2 \right\}.$$  \hspace{1cm} (10)

The contribution of $T_{1,3}$ in eq. (10) can be minimized with respect to that of $T_{1,2}$ by means of experimental cuts on the selected reactions. $H(q)$ is a function sharply peaked below $q \sim 80 \text{ MeV/c}$ and very small for $q$ above $\sim 300 \text{ MeV/c}$. Thus an upper limit of 250 MeV/c on the "spectator" momentum ($q_3$ in this case) together with an appropriate choice of the scattering angle of the $K^-$ system (2-6 in our case) such that $q_2$ is always larger than 400 MeV/c has the effect that the $S_2$ graph can be neglected in the above calculations. This reduces eq. (10) to eq. (8).

5.2.3 Double scattering and final-state interaction

These effects are represented by the graphs $D_1$, $D_2$, and $D_3$. If they are introduced in the calculation of the cross section, beside $S_1$ and $S_2$, the result is

$$(\text{contributing diagram}) \hspace{1cm} (S_1)$$

$$+ \left\{ \frac{i}{4\pi E_4} \left[ \frac{M_{2,6} M_{2,6}}{|q_3 + q_2|} d\phi_4^{6,2} \right] \right\} \hspace{1cm} (D_1)$$

$$+ \left\{ \frac{M_{2,5} M_{2,6} M_{2,3}}{|q_3 + q_2|} d\phi_4^{6,3} \right\} \hspace{1cm} (D_2)$$

$$+ \left\{ \frac{M_{6,5} M_{2,3} M_{2,5}}{|q_3 + q_2|} d\phi_4^{3,2} \right\} H_{1,4} \left| q \right| d\phi_4^{2} \hspace{1cm} (D_3)$$
The variables chosen for the integration over the 1-4-5 loop of the D-diagrams are the four components of $q_4$. It can be shown [18] that the singularities associated to the propagators of particles 4 and 5 can be replaced by two delta-functions imposing on-shell masses for these particles. The delta-functions disappear when integrating particle 4 over its energy and polar angle (with respect to the direction defined by $\vec{q}_4 + \vec{q}_5$). The remaining integration, shown in eq. (11), is over the modulus of $\vec{q}_4$ and the azimuthal angle $\phi_4$ (the azimuth is around the direction of $\vec{q}_4 + \vec{q}_5$ as defined by the final state particles specified in the suffix).

5.2.4 Spin

Spin effects of nucleons present in the reaction will be taken into account by summing over all possible spins of internal nucleon lines and summing the amplitude squared for external nucleon lines since we do not measure the polarization of the outgoing nucleons.

In order to do so, the spin of the nucleons will be defined in one of their rest frames which will be determined by a succession of Lorentz transformations from the laboratory system (in most cases this is a simple boost along their momentum). In this way the summation will have to be done only on two-component spinors.

Let us treat first the $\bar{K}N$ scattering vertex. The vertex amplitude $T_{if}$ can be represented in any system and in particular in the laboratory system by:

$$T_{if} = U_f^\dagger \left[ A_{if} \delta + B_{if} \gamma (\vec{q}_i + \vec{q}_f) \right] U_i,$$

(12)

where $\vec{q}_i$ and $\vec{q}_f$ are the four-momenta of the initial and final kaon respectively and $\gamma$ stands for the usual $\gamma$ matrices with the sign convention

$$\gamma q = \gamma_0 q_0 - \gamma_1 q_1 - \gamma_2 q_2 - \gamma_3 q_3.$$

(13)

$U_i$ and $U_f$ are four-component spinors defined from the two-component nucleon spinor $S$ in their rest frame through the boost $L$ associated to each nucleon

$$L = \frac{m + \gamma q q_0}{\sqrt{2m(m+E)}}.$$

Here $m$, $q$ and $E$ are the mass, momentum and energy of the considered nucleon in the laboratory system. We thus have $U = LS$ (here $S$ can be considered as a four-component spinor with third and fourth component equal to zero).
Let us deal now with the NN vertex. The rest frame of each nucleon will be defined by a Lorentz transformation which goes to the NN c.m. from the laboratory system followed by a Lorentz boost along each nucleon momentum going from the NN c.m. to the nucleon rest frame. In the case of the deuterium vertex, the first transformation is an identity. In this way it is possible to show that the nucleon spin can be mixed with the orbital spin to form a definite NN total angular momentum state with a given parity using ordinary Clebsch-Gordan coefficients as in the non-relativistic case. In particular, considering the S and D wave amplitudes of the deuterium, it is now straightforward to construct a 2 x 2 tensor for each magnetic moment of the deuterium state $H^{M,i,j}$ where $i$ and $j$ can assume only two values.

In the same way we can use known NN phase shifts at the NN vertex to construct a 2 x 2 x 2 x 2 tensor $T_{r,n}^{t,m}$ where $t, r, n, m$ can take only two values related to the two spin states of each nucleon. The tensor will give the vertex amplitude of each spin state.

The next operation consists in replacing terms in eq. (11): $T_{1,2}^{13}$ goes into $F_{M,k,\ell}^{13}$ $(S_1)$, $T_{1,3}^{12}$ $H_{1,2}^{1,1}$ into $F_{M,k,\ell}^{13}$ $(S_2)$, $T_{1,3}^{14}$ $H_{1,4}^{1,1}$ into $F_{M,k,\ell}^{13}$ $(D_1)$, $T_{1,2}^{1,5}$ $H_{1,4}^{1,1}$ into $F_{M,k,\ell}^{13}$ $(D_2)$, $T_{1,5}^{1,4,5}$ $H_{1,4}^{1,1}$ into $F_{M,k,\ell}^{13}$ $(D_3)$.

The functions $F_{M,k,\ell}$ are internal-spin-summed amplitudes which depend only on the magnetic component $M$ of the deuteron spin ($M = -1, 0, 1$) and on the spin states of the external nucleons ($k = 1, 2$ and $\ell = 1, 2$). They are calculated as follows:

\[
F_{M,k,\ell}^{13} (S_1) = \left( \frac{k}{L_2,\nu} \right)^{-1} \left[ A_{1,2}^\nu_\mu + B_{1,2}^\nu_\mu (\vec{q}_1 + \vec{q}_6) \right] L_{1,1,1}^{iM,i,\ell} \tag{14}
\]
\[
F_{M,k,\ell}^{13} (D_1) = \left( \frac{k}{L_5,\lambda} \right)^{-1} \left[ A_{1,3}^\lambda_\mu + B_{1,3}^\lambda_\mu (\vec{q}_1 + \vec{q}_5) \right] L_{1,1,1}^{iM,i,\ell} \cdot \left( \frac{L_2,\rho}{L_2,\nu} \right)^{-1} \left[ A_{4,2}^\rho_\nu + B_{4,2}^\rho_\nu (\vec{q}_5 + \vec{q}_6) \right] L_{4,1,1}^{iM,j,j} \tag{15}
\]
\[
F_{M,k,\ell}^{13} (D_3) = \left( \frac{k}{L_5,\rho} \right)^{-1} \left[ L_{2,\nu}^{\ell} \right] \left[ L^{\mu}_{\lambda}(via \ 4,5) \right] L^{\lambda}_{\rho}(via \ 4,5) T_{r,n}^{t,m} (4,5), r, m \cdot \left( \frac{L^m_{\nu}}{L^m_{\lambda}} \right)^{-1} \left[ A_{1,5}^\ell_\nu + B_{1,5}^\ell_\nu (\vec{q}_1 + \vec{q}_6) \right] \cdot \left[ L^\mu_{1,1} L^{\mu}_{4,1} H_{1,1,1}^{iM,i,j} \right] \tag{16}
\]

The amplitudes relative to $S_2$ and $D_2$ are analogous to those for $S_1$ and $D_1$ with the appropriate change of indices. The symbol $L_{a,b}^c$ represents a Lorentz
boost which transforms the spin and momentum of particle a from its own rest frame to the laboratory system; b and c are indices identifying the spin state of the particle in the laboratory and in its own rest frame respectively. Roman indices go from 1 to 2, Greek indices from 1 to 4. When specifically indicated (e.g. via 4,5) the Lorentz transformation may consist of a sequence of boosts transforming first the particle from its rest frame to the centre of mass of a specified system, then from this frame to the laboratory. The symbol \( H_{a,b}^{M,i,k} \) represents the tensor of the deuteron spin with magnetic component M, with nucleons a and b in spin states i and k respectively.

The second operation consists in adding the internal-spin-added amplitudes, integrated over the loops as in eq. (11), squaring the result and adding now over the indices of the external nucleon states

\[
\frac{d\sigma}{d\Omega_{2,6} d^3q_5} = \frac{P_{2,6}}{3|q_1| E_3 M_{2,6}} \sum_{M,k,l} \left[ M_{2,6} \frac{F_{M,k,l}(S_1)}{M_{1,6}} + M_{3,6} \frac{F_{M,k,l}(S_2)}{M_{1,6}} + \right.
\]

\[
\frac{1}{4\pi F_4} \left[ \frac{M_{3,5} M_{2,6}}{|q_3 + q_2|} F_{M,k,l}(D_1) d\phi_4^{6,2} + \right.
\]

\[
+ \frac{M_{2,5} M_{3,6}}{|q_6 + q_5|} F_{M,k,l}(D_2) d\phi_4^{6,3} + \right.
\]

\[
\left. + \frac{M_{6,5} M_{2,3}}{|q_3 + q_2|} F_{M,k,l}(D_3) d\phi_4^{6,2} \right] |q_4|^2 |dq_4|^2.
\]

(17)

For the purpose of our calculation it is convenient to re-write the \( \bar{K}N \) scattering amplitude of eq. (12) under the familiar form expressed in the \( \bar{K}N \) center of mass

\[
T_{if} = S_{fi}^T (f + ig \sigma \cdot \bar{n}) S_{if}^T.
\]

(18)

Here the 2-component spinors \( S_f^\dagger \) and \( S_i \) refer to the spin state of the initial and final nucleon respectively, \( \sigma \) and \( \bar{n} \) are the Pauli matrices and the normal to the scattering plane respectively, \( f \) and \( g \) are the customary non-spin-flip and spin-flip amplitudes. The latter can in turn be expressed as a function of the partial-wave amplitudes [19]. These are the quantities available for the description of the \( \bar{K}N \) interaction and must be introduced into formulae (14)-(16) in order to perform the calculation. The relation between eqs (12) and (18) can be found by applying a Lorentz boost over the incident and final nucleons so as to bring them from their own rest frame to the \( \bar{K}N \) c.m. system. After this we are left with only two components of the spinor \( U \) of eq. (12).
A comparison between eq. (12) transformed by the above boost and eq. (18) gives the following relations:

\[
\begin{align*}
A &= \frac{m}{w} \left[ \frac{w + m}{E + m} f + \left( \frac{w + m}{E + m} \cos \theta^* + \frac{w - m}{E - m} \frac{g}{\sin \theta^*} \right) \right] \\
B &= \frac{m}{w} \left[ \frac{1}{E + m} f + \left( \frac{1}{E + m} \cos \theta^* - \frac{1}{E - m} \frac{g}{\sin \theta^*} \right) \right]
\end{align*}
\]  

(19)

E, m, and \( \theta^* \) refer to the nucleon in the \( \bar{K}N \) c.m. system and are respectively its energy, mass and scattering angle; \( w \) is the total c.m. energy.

Similarly, the amplitude for NN scattering (only the \( l=0 \) part needs to be considered in deuterium) can be expressed in terms of phase-shifts relative to the singlet and triplet initial spin states. The final state can be expanded as usual into a sum of states having definite orbital angular momentum \( (\lambda) \), magnetic component of the total spin \( (s) \) and magnetic component of the total angular momentum \( (t) \). Only odd values of \( \lambda \) are contributed by the singlet state, only even values by the triplet state. The expansion is

\[
|F\rangle = \sum_{\lambda (\text{odd})} A_{\lambda} |\lambda,0\rangle \otimes |s\rangle + \sum_{\lambda (\text{even})} B_{\lambda,s,t} |\lambda,s-t\rangle \otimes |T,s\rangle.
\]  

(20)

The first term comes from the initial singlet and the second from the initial triplet state. The \( |s\rangle \) and \( |T,s\rangle \) eigenfunctions refer respectively to the singlet and triplet wave function. The coefficients \( A \) and \( B \) are known functions of the nucleon-nucleon phase shifts via Clebsch-Gordan coefficients.

Finally, we can express the matrix elements \( T_{l,m}^{t,n} \) of eq. (16) via \( A \) and \( B \). The result is

\[
\begin{align*}
T_{ij}^{11} &= \sum_{t=-1,0,1} \sum_{\lambda = 0,2,4} \sum_{k=1,t} B_{\lambda,k,1,t}^i Y_{\lambda}^{t-1}(\cos \theta, \psi) \\
T_{ij}^{22} &= \sum_{t=1,0,1} \sum_{\lambda = 0,2,4} \sum_{k=-1,t} B_{\lambda,k,-1,t}^i Y_{\lambda}^{t+1}(\cos \theta, \psi) \\
T_{ij}^{12} &= \frac{1}{\sqrt{2}} \sum_{t=-1,0,2} \sum_{\lambda = 0,2,4} \sum_{k=0,t} B_{\lambda,k,0,t}^i Y_{\lambda}^t(\cos \theta, \psi) + \frac{1}{\sqrt{2}} \sum_{\lambda = 1,3,5} A_{\lambda}^{ij} Y_{\lambda}^0(\cos \theta, \psi) \\
T_{ij}^{21} &= \frac{1}{\sqrt{2}} \sum_{t=1,0,1} \sum_{\lambda = 0,2,4} \sum_{k=0,t} B_{\lambda,k,0,t}^i Y_{\lambda}^t(\cos \theta, \psi) - \frac{1}{\sqrt{2}} \sum_{\lambda = 1,3,5} A_{\lambda}^{ij} Y_{\lambda}^0(\cos \theta, \psi)
\end{align*}
\]  

(21)
In the above formulae we have omitted the index (4,5) referring to the particle system. The lower indices refer to the initial state, the upper indices to the final state. The angles $\theta$ and $\psi$ are respectively the polar and azimuthal angle in the centre of mass of the reaction.

5.2.5 Isospin

The final states of the graphs in fig.43 can be reached via different charge configurations of the intermediate particles. As in the case of ordinary spin we sum over the intermediate states allowed by isospin conservation. If we introduce amplitudes relative to specific charge states, we can express the general amplitudes of the previous formulae as a function of the allowed charge combinations. The substitutions are as follows:

<table>
<thead>
<tr>
<th>in graph</th>
<th>replace</th>
<th>with</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_1$</td>
<td>$T_{1,2}$</td>
<td>$T_{1,2} (K^-n + K^-n)$</td>
</tr>
<tr>
<td>$S_2$</td>
<td>$T_{1,3}$</td>
<td>$T_{1,3} (K^-p + K^-p)$</td>
</tr>
<tr>
<td>$D_1$</td>
<td>$T_{4,2} T_{1,3}$</td>
<td>$T_{4,2} (K^-n + K^-n) \cdot T_{1,3} (K^-p + K^-p)$</td>
</tr>
<tr>
<td>$D_2$</td>
<td>$T_{4,3} T_{1,2}$</td>
<td>$T_{4,3} (K^-p + K^-p) \cdot T_{1,2} (K^-n + K^-n) - T_{4,3} (K^0 n + K^-p) \cdot T_{1,2} (K^-p + K^0 n)$</td>
</tr>
<tr>
<td>$D_3$</td>
<td>$T_{4,5} T_{1,5}$</td>
<td>$T_{4,5} (NN + NN, I=0) \cdot [T_{1,5} (K^-p + K^-p) + T_{1,5} (K^-n + K^-n)]$</td>
</tr>
</tbody>
</table>

5.3 Results

A computer program was written incorporating the model outlined above and based on the following Monte-Carlo procedure. First, incident momenta were produced according to the probability distribution of the experimental momentum spectra. Next, the spectator momentum vector was chosen following the Hulthén distribution of eq. (5) and isotropically in the laboratory. Finally, the c.m. angle of the outgoing $K^-$ in the $K^-n$-ucleon system ($K^-p$ for $K^-p n_s$ events, $K^-n$ for $K^-p p_s$ events) was chosen with a uniform cosine distribution between the experimental limits. Cuts on the spectator momentum and recoil neutron were applied according to the same criteria adopted for the data (sect. 4.3).

The event being thus kinematically defined, the next step was the calculation of the various dynamical contributions enumerated in the previous section. The NN amplitudes were those of ref. [20]; these amplitudes being relatively well known, we have not invested much time in trying other choices. The choice of the $\bar{K}N$ amplitudes was instead more delicate. The available sets of $\bar{K}N$ amplitudes are considerably dissimilar among each
other. Furthermore, most sets extend over a limited and usually small momentum region. Finally, the different sets either do not join at all at the common momenta or, when they do, are discontinuous. As the conclusion of an exhaustive investigation on the effects produced by the different choices, we have arrived to the following concoction which presently appears as the best compromise. We have taken the RLIC amplitudes of ref. [5] from 0.3 to 1.8 GeV/c and the CDFS amplitudes of ref. [21] from 1.8 to 2.5 GeV/c; outside this range we have taken constant amplitudes equal to those valid at the two extremes. The practical advantage and inherent reliability of the RLIC amplitudes over the other choices is that they span the widest momentum interval thereby providing a continuous and internally consistent evaluation of the re-scattering effects. It is important to remember that internal c.m. energies down to the \( \bar{\eta}N \) threshold enter these calculations; thus \( \bar{\eta}N \) configurations with low effective mass play an important role because of the large cross sections involved. Their effect is particularly noticeable in the region of backward \( K^- \) angles. In fact, having used amplitudes which are only valid as far down as 0.3 GeV/c (for the incident \( K^- \) momentum) impairs the calculation for scattering angles whose cosine is smaller than -0.9. As it happens, these angles are below the lowest angles reached by the experiment. On the other hand, the effect of the high-mass configuration on the correction factors is notably less important than that of the low masses. It is for this reason that we feel reasonably confident that the unphysical assumption of maintaining constant amplitudes from 2.5 GeV/c onwards has a negligible effect on the (already small) correction factors for which these configurations are responsible. In particular, we have verified that this choice provides correction factors which are practically undistinguishable from those generated under other - possibly more realistic - situations. Finally, it should be stressed that the RLIC and CDFS amplitudes are far from continuous at 1.8 GeV/c. In this respect we can only invoke that: (a) in their respective regions of validity each set reproduces satisfactorily the available data hence presumably also most of the deuterium effects and (b) we do not observe a serious discontinuity in the values of the correction factors when passing from one region to the other.

The integrals of sect. 5.2 were evaluated by means of a numerical Monte-Carlo procedure. The events were then put together and the differential cross sections calculated according to the prescription described in sect. 4.4 integrating over all spectator momenta and sub-dividing the data over the same bins of c.m. energy and angle used for the experimental data. The result is the differential cross section expected for the \( K^-d \to K^-n \, p_6 \) reaction for a specific and known form of the elementary \( \bar{\eta}N \) interaction. The same operation (more straightforward this time because all deuterium effects except the Fermi motion are now switched off) has been repeated to
give the expected differential cross section for the $K^-n \rightarrow K^-n$ reaction. The ratio of the two provides the factor by which we multiply our measured cross section to obtain the "corrected" value corresponding to the isolated $K^-n$ interaction. A similar procedure was followed to obtain the correction factors for the $K^-d \rightarrow K^-p n_s$ reaction.

The above factors are shown in fig. 44. Here we plot the values of $f_D$ as a function of the cosine of the $K^-$ angle in the $K^-n$ c.m. These are the correction coefficients $f_D$ of eq. (7) in sect. 4.3 used in tables VI and VII to pass from the columns marked $\sigma_1$ (uncorrected values) to the columns marked $\sigma_2$ (corrected values).

Deuterium correction factor ($f_D$ of eq. (7)) as a function of the cosine of the $K^-$ angle in the c.m. of the final $K^-n$ system. The numbers above the graphs refer to all the histograms in the column beneath; they represent the central value (in GeV) of the c.m. energy bin of the $K^-n$ system. The numbers in larger size are the incident momentum settings; they refer to the histograms in the same row. Graphs as the left-hand side are for the $K^-p$ cross sections, those on the right-hand side are for the $K^-n$ cross sections.
6. PARTIAL WAVE ANALYSIS

6.1 Introduction

The data obtained in the experiment, corrected for the deuterium effects as described in the previous section, were introduced together with data from other experiments in a partial wave analysis program. The purpose of the analysis was to determine a set of amplitudes which, within the simplifying assumptions reported below, would correctly describe all the presently available data on the $\bar{K}N$ system in the region from 1 to 2 GeV/c. Only a limited goal was pursued because we did not tackle an extensive search of all the possible solutions compatible with the data, nor did we try other possible amplitude parametrizations which may have better described the data over our region. Only the minimum number of resonances was introduced and only the simplest energy dependence was assigned to the background amplitudes. It must be pointed out that the present study is the first ever done on the $\bar{K}N$ system using differential cross sections of the pure $I = 1$ state. All earlier analyses dealt only with data in various mixtures of $I = 0$ and $I = 1$ states. It is not obvious a priori that the results of such analyses lead to a unique determination of the separate isospin states. In fact, as shown below, we find that our background amplitudes are different from those of the earlier analyses. On the other hand, this is not the case for the resonances; their constraint on the data is sufficiently strong and unambiguous to allow a determination of the amplitude without the need of separate isospin channels.

The procedure followed is essentially the one adopted by the CHS group in their analyses of the region from 400 to 1200 MeV/c [22]. To this basic procedure we added the larger flexibility provided by the method of a Legendre polynomial expansion of the background amplitudes as used for example in the recent CHM analysis [4]. The fit was performed over the coefficients of the usual Legendre polynomial expansion of the differential cross sections for the $K^-p$ and $\bar{K}^0n$ reactions; however, we used directly the $K^-\pi$ differential cross sections and the $K^-p$ polarizations when dealing with these data.

The subsections below describe the formalism, the data employed and the results.

6.2 Method

The differential cross section for $\bar{K}N$ elastic scattering can be generally written [19]

$$\frac{d\sigma}{dn} = |f|^2 + |g|^2,$$

(22)
where the reference system is the Český Nucleon centre of mass and \( f \) and \( g \) are the customary non-spin-flip and spin-flip scattering amplitudes respectively. These amplitudes can be expanded in a Legendre and first-associated Legendre series of the partial waves \( T \) as follows:

\[
f = \frac{1}{k} \sum_{\ell=0}^{N} \left[ (\ell+1)T^\ell_+ + \ell T^\ell_- \right] P^\ell_+, \tag{23}
\]

\[
g = \frac{i}{k} \sum_{\ell=1}^{N} \left[ T^\ell_+ - T^\ell_- \right] P^1_+. \tag{24}
\]

Here \( T^\ell_\pm \) corresponds to \( J = \ell \pm 1/2 \) where \( \ell \) and \( J \) are respectively the orbital and total angular momentum of the system and \( k = P_{\text{cm}}/\hbar c \) is the c.m. momentum of the Český Nucleon system in \( \text{fm}^{-1} \). In the tables given below we shall use the spectroscopic notation with the symbols \( S, P, D \) etc. to denote partial waves with \( \ell = 0, 1, 2, \) etc. and subscripts giving the isospin and twice the value of \( J \).

Similarly, the polarization times the differential cross section can be written

\[
\hat{p} \frac{d\sigma}{d\Omega} = 2 \text{Re} \left( \hat{g} f^* \right) \n, \tag{25}
\]

where the direction of the polarization vector \( \hat{p} \) is along the normal to the production plane \( \hat{n} \).

The real and imaginary part of the forward scattering amplitude \( (f(0)) \) are experimentally accessible and we have used them in the analysis. Values of the former derive from dispersion relation fits to direct measurements, values of the latter follow from the measured total cross sections \( \sigma \) via the optical theorem. In our notations and units (mb for \( \sigma \) and \( \text{fm}^{-1} \) for \( k \)) the optical theorem has the form

\[
\sigma = 40 \pi k^{-2} \text{Im} f(0). \tag{26}
\]

The isospin decomposition of the reactions considered is as follows:

\[
\begin{align*}
T_{K^-p} &= \frac{1}{2} T_0 + \frac{1}{2} T_1 \\
T_{K^0n} &= \frac{1}{2} T_0 - \frac{1}{2} T_1 \\
T_{K^-n} &= T_1.
\end{align*} \quad \tag{27}
\]

The comparison between data and partial wave amplitudes can be done either directly - expressing \( d\sigma/d\Omega \) and \( P d\sigma/d\Omega \) at a given value of \( \cos \theta^* \) as
a function of the amplitudes - or indirectly via the coefficients of the expansions

$$\frac{d\sigma}{d\Omega} = k^{-2} \sum_{n=0}^{N} A_n P_n(\cos\theta^*)$$

(28)

$$P \frac{d\sigma}{d\Omega} = k^{-2} \sum_{n=1}^{N} B_n P_n(\cos\theta^*).$$

(29)

The reason for introducing the latter expansions is essentially one of convenience because it amounts to reducing a large number of data points to a more manageable set of numbers. The latter are related to the partial waves via simple relations, (see for example [19]). The disadvantage of using $A_n$ and $B_n$ of (28) and (29) instead of $d\sigma/d\Omega$ and $P \frac{d\sigma}{d\Omega}$ is of course that one does not know which is the order of the expansion. There are also cases where the differential cross sections have been measured only over a limited range of $\cos\theta^*$. This is the case, for example, of our $K^-n$ data. To attempt an expansion of the type of (28) on such data may well give a complete misrepresentation of the complete distributions. In our analysis we have used a combination of $A_n$ coefficients, $d\sigma/d\Omega$ and $P \frac{d\sigma}{d\Omega}$ as described in the following sub-section. This has allowed us to keep the computer requirements to a minimum while retaining a good degree of accuracy in the representation of the data.

The amplitude parametrization was chosen to be:

a) **For the background waves:**

$$T_b = \sum_{n=0}^{N} a_n P_n(y)$$

(30)

with

$$y = \frac{2p - (p_2 + p_1)}{p_2 - p_1}, -1 \leq y \leq 1,$$

(31)

where $p$ is the laboratory momentum of the incident kaon and $p_1$ and $p_2$ are respectively the lowest and highest value of $p$ in the interval considered. In our analysis $p_1$ was taken equal to 1.0 GeV/c and $p_2$ equal to 2.1 GeV/c. For the high-$J$ amplitudes, the contribution of which is likely to be unimportant below a certain energy, we have left $p_1$ free as a parameter of the fit; we shall refer to $p_1$ when free as $p_{\text{min}}$ in the table of parameters listed at the end of the section.

b) **For the resonant waves:**

$$T_r = \frac{x}{\varepsilon-1}$$

(32)
with
\[ \varepsilon = 2 \frac{E-E_\text{r}}{\Gamma}. \]  

Here \( \varepsilon \) is the resonance elasticity defined as \( \Gamma_{\text{KN}}/\Gamma \) with \( \Gamma_{\text{KN}} \) and \( \Gamma \) being respectively the partial width of the elastic channel and the total width of the resonance. The widths are dependent on the energy \( E \) via the centrifugal barrier factor \( B_k \)

\[ \Gamma(E) = \Gamma(E_\text{r}) B_k(E)/B_k(E_\text{r}) \]  

\[ B_k(E) = \frac{k}{E_\text{r}} \left( \frac{k^2}{k^2 + k_0^2} \right)^{\frac{k}{2}}. \]  

Eq. (35) is the Glashow-Rosenfeld parametrization of the barrier; \( k_0 \) is a constant related to the radius of interaction, taken equal to 1.7738 fm\(^{-1} \); \( E_\text{r} \) is the resonant energy.

c) For the superposition of a resonant and background wave with the same quantum numbers:

\[ T = T_b + T_r e^{i\phi}. \]  

The analysis program performs:

(i) the calculation of the expected physical quantities on the basis of an input set of amplitudes,

(ii) the comparison of these expectations with the measured values by means of an overall chi-square \( (\chi^2) \),

(iii) the variation of the input set so as to minimize the \( \chi^2 \).

The minimization routine was MINROS [23] and was typically used with 134 free parameters and 2090 data points. The computer space was 60 k words of 7600 CDC central processor, one minimization cycle taking approximately 5 sec.

6.3 Data

As a first step in the use of our results for the study of the \( \bar{K}N \) interaction we have performed a partial wave analysis only covering the region from 1.1 to 2.0 GeV/c (c.m. energy from 1.84 to 2.23 GeV). This avoids extending the parametrization of the amplitudes over too large a range; furthermore it permits a direct comparison between the previous results and ours. Left out from this study are therefore the results from the uppermost incident momentum of our experiment; they will be incorporated in a later analysis.
The other data used in this analysis are those previously examined by the CHM study [4]; the references to the specific experiments can be found in the latter work. A brief accounting of the number and type of data is given below. We have used:

(a) coefficients \( A_n \) of the Legendre polynomial expansion of the \( K^-p \) elastic differential cross sections used in the reduced form \( A_n/A_0 \) plus \( A_0 \) separately; they represent a total of 674 data points at 79 different incident momenta;

(b) the same type of data as in (a) but for the charge exchange reaction \( K^-p \rightarrow K^0 n \); they represent a total of 527 data points at 61 different incident momenta;

(c) polarization \( P(\cos^\theta) \) of the \( K^-p \) elastic reaction at a total of 26 different incident momenta; the total number of data points is 496;

(d) differential cross sections \( d\sigma/d\Omega(\cos^\theta) \) of the \( K^-n \) elastic reaction, all of them from the present experiment; the number of c.m. energy bins is 9 and the total number of data points 145;

(e) total \( K^-p \) cross sections \( \sigma_{K^-p} \) and total \( \bar{K}N \) cross sections in isospin 1 \( \sigma_1 \); 97 points for \( \sigma_{K^-p} \), 97 for \( \sigma_1 \); here as in [4] we have ignored the isospin 0 total cross section data not because we have reason to be biased against them but because they add an unnecessary, and perhaps dangerous, redundancy to the fit;

(f) values of the ratio \( \alpha(K^-p) \) of the real and imaginary part of the \( K^-p \) elastic forward scattering amplitude; these 9 data points come from a dispersion relation fit to the actual measurements [24];

(g) values of the ratio \( \alpha(K^-n) \) of the real and imaginary part of the \( K^-n \) elastic forward scattering amplitude; these 9 data points which, similarly to the \( \alpha(K^-p) \) are not the direct result of measurements but come from a dispersion relation fit [25], were not used by the CHM analysis;

(h) partial cross sections \( A_0 \) coefficients alone) for the charge exchange reaction; a total of 33 data points.

The distribution of the data over the momentum interval is shown in fig. 45; it is the same as in ref. [4] with the addition of the \( K^-n \) data.
\[ \begin{align*}
K^0 n &\rightarrow K^- n \\
\alpha(K^0 n) \\
K^- p &\rightarrow K^0 p \\
K^0 p &\rightarrow K^- n \\
\sigma_{\text{CEX}} \\
\sigma_{\text{fot}} \\
\sigma_f \\
\alpha(K^- p) \\
P(K^- p)
\end{align*} \]

\textbf{FIG. 45}

Data distribution over the momentum range of the partial wave analysis described in the text. This is the same data set as in fig. 4 of ref. [4] augmented by the \( K^- n \rightarrow K^- n \) and \( \alpha(K^0 n) \) data of the upper two lines.

6.4 Results

The starting amplitudes in our search for the best fit to the data were those arrived at by the CHM analysis. Their predictions for the \( K^- n \) differential cross sections are shown as dotted curves in the histograms of fig. 41.

Our best fit was obtained - in the manner described in sect. 6.2 - by varying the parameters shown in table VIII and keeping those in brackets fixed at the CHM values. We end up with a \( \chi^2 \) of 2667 with a number of free parameters equal to 130; the average \( \chi^2 \) for data point is 1.3.

The result of the fit together with the fitted data is shown in figs 46 to 50 for all the channels except the polarization. For the latter we have plotted in fig. 51 the coefficients \( B_n \) of the expansion in associated
Legendre polynomials of the $K^-p$ polarization times the differential cross section. Notice that the fit of the polarization was not performed using these coefficients but directly the measurement of the polarization as a function of the angle.

The great majority of the data are reproduced quite well by our fit. The only exception is for the predicted values of the separate isospin total cross sections which, as shown by fig. 50, disagree qualitatively with the data. This disagreement is also present in the results of the CHM analysis and in those of later analyses such as that of ref. [5]. It may imply an inadequacy of our parametrization as well as a faulty extraction of the isospin components from the original $K^-p$ and $K^-d$ total cross sections. The agreement of the $K^-n$ data with our fit is satisfactory. In fact, apart from the latter, all the other data fit as well as in the CHM analysis.

The amplitudes corresponding to the best fit are those plotted in fig. 52 and listed in table VIII. The main differences with respect to the CHM analysis can be summarized as follows:

(i) there is no need for an $F_{15}$ resonance (the $\Sigma(1915)$ of old); we list the fitted parameters only to show that the data do not require the narrow state claimed by many analyses;
(ii) the $F_{07}$ state, claimed by some analyses, is not rejected by the data and can be considered mildly supported;
(iii) the $F_{17}$ resonance (known as $\Sigma(2030)$) has a somewhat larger width than known at present.

As for the background amplitudes, we observe that, lacking a better investigation of the parametrization, the uncertainty of the procedure justifies for the moment the discrepancies observable between our set and those of the previous analyses.
FIG. 46
Differential cross sections $K^- n \rightarrow K^- n$ as a function of the cosine of the scattered $K^-$ in the $K^- n$ centre of mass (same as in fig. 41). The curves represent the fit of the partial wave analysis described in the text.
Coefficients of the Legendre polynomial expansion of eq. (28) for the differential cross sections $K^-p \rightarrow K^-p$ as a function of the incident $K^-$ momentum. The data are from a variety of experiments listed in ref. [4]. The curves represent the fit of the partial wave analysis described in the text.
FIG. 48
Same as in fig. 47 but pertaining to the reaction $K^- p \rightarrow K^0 n$. See ref. [4] for references.
Values of $\alpha(K^- p)$ and $\alpha(K^- n)$ from refs [24,25] respectively, as a function of the incident $K^-$ momentum. The values plotted are from a dispersion relation fit to the measurements; the error bars represent an estimate of the uncertainty of the fit. The curves represent the fit of the partial wave analysis described in the text.
Total $K^-p$ cross sections ($\sigma_{K^-p}$) and total $\bar{K}N$ cross sections in isospin 0 ($\sigma_0$) and 1 ($\sigma_1$) as a function of the incident $K^-$ momentum. Only $\sigma_{K^-p}$ and $\sigma_1$ have been used in the analysis. The curves represent the fit to these data and the prediction for the $\sigma_0$ data from the partial wave analysis described in the text. The references are given in [4].
Coefficients of the associated Legendre polynomial expansion of eq. (29) for the polarization times the differential cross section of the reaction $K^- p \rightarrow K^- p$ as a function of the incident $K^-$ momentum. The references to the data are given in [4]. The curves represent the predictions of the partial wave analysis described in the text; the data fitted by the analysis are the polarization as a function of $\cos^2 \theta$, not the data shown in this figure.
Argand diagrams of the partial waves obtained in the analysis described in the text. The vertical scales go from 0 to 1, the horizontal from -0.5 to 0.5. The arrows show the direction of increasing momentum, from 1.1 to 2.0 GeV/c.
Parameters of the best fit. The resonances are described by the mass $M$ (in GeV), the width at resonance $\Gamma$ (in GeV), the elasticity $x$ and the phase $\phi$ (in rad) of eq. (36). The background is described by the real and imaginary parts of the coefficients $a_n$ of the expansion (30) and the parameter $p_{\text{min}}$ (in GeV/c). Values in brackets have been kept fixed.

<table>
<thead>
<tr>
<th>(a) Resonances</th>
<th>$M$</th>
<th>$\Gamma$</th>
<th>$x$</th>
<th>$\phi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>P03</td>
<td>[1.889]</td>
<td>[0.127]</td>
<td>[0.23]</td>
<td>[0.90]</td>
</tr>
<tr>
<td>D15</td>
<td>[1.765]</td>
<td>[0.120]</td>
<td>[0.41]</td>
<td>[0.00]</td>
</tr>
<tr>
<td>F05</td>
<td>1.830</td>
<td>0.082</td>
<td>0.51</td>
<td>-0.08</td>
</tr>
<tr>
<td>F15</td>
<td>2.437</td>
<td>2.212</td>
<td>0.03</td>
<td>-0.38</td>
</tr>
<tr>
<td>F07</td>
<td>2.117</td>
<td>0.167</td>
<td>0.05</td>
<td>-0.03</td>
</tr>
<tr>
<td>F17</td>
<td>[2.020]</td>
<td>0.260</td>
<td>0.15</td>
<td>[0.00]</td>
</tr>
<tr>
<td>G07</td>
<td>2.094</td>
<td>0.250</td>
<td>0.29</td>
<td>[0.00]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(b) Background</th>
<th>$\text{Re } a_1$</th>
<th>$\text{Im } a_1$</th>
<th>$\text{Re } a_2$</th>
<th>$\text{Im } a_2$</th>
<th>$\text{Re } a_3$</th>
<th>$\text{Im } a_3$</th>
<th>$p_{\text{min}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>S01</td>
<td>0.045</td>
<td>0.664</td>
<td>0.190</td>
<td>-0.137</td>
<td>-0.151</td>
<td>0.083</td>
<td>-</td>
</tr>
<tr>
<td>S11</td>
<td>0.034</td>
<td>0.499</td>
<td>-0.075</td>
<td>-0.183</td>
<td>0.221</td>
<td>0.003</td>
<td>-</td>
</tr>
<tr>
<td>P01</td>
<td>-0.005</td>
<td>0.527</td>
<td>0.234</td>
<td>-0.155</td>
<td>0.151</td>
<td>0.177</td>
<td>-</td>
</tr>
<tr>
<td>P11</td>
<td>0.099</td>
<td>0.368</td>
<td>0.135</td>
<td>0.111</td>
<td>0.065</td>
<td>-0.033</td>
<td>-</td>
</tr>
<tr>
<td>P03</td>
<td>0.160</td>
<td>0.301</td>
<td>-0.171</td>
<td>0.140</td>
<td>-0.027</td>
<td>-0.007</td>
<td>-</td>
</tr>
<tr>
<td>P13</td>
<td>0.041</td>
<td>0.211</td>
<td>-0.067</td>
<td>0.056</td>
<td>-0.019</td>
<td>-0.078</td>
<td>-</td>
</tr>
<tr>
<td>D03</td>
<td>-0.048</td>
<td>0.126</td>
<td>-0.056</td>
<td>0.133</td>
<td>-0.015</td>
<td>0.047</td>
<td>-</td>
</tr>
<tr>
<td>D13</td>
<td>0.095</td>
<td>0.122</td>
<td>0.111</td>
<td>0.089</td>
<td>-0.086</td>
<td>0.017</td>
<td>-</td>
</tr>
<tr>
<td>D05</td>
<td>-</td>
<td>-</td>
<td>0.012</td>
<td>0.042</td>
<td>-0.077</td>
<td>-0.066</td>
<td>-</td>
</tr>
<tr>
<td>D15</td>
<td>-</td>
<td>-</td>
<td>0.042</td>
<td>0.052</td>
<td>-0.035</td>
<td>-0.027</td>
<td>1.337</td>
</tr>
<tr>
<td>F05</td>
<td>-</td>
<td>-</td>
<td>-0.052</td>
<td>0.243</td>
<td>-0.040</td>
<td>-0.028</td>
<td>1.308</td>
</tr>
<tr>
<td>F15</td>
<td>-</td>
<td>-</td>
<td>-0.036</td>
<td>0.152</td>
<td>0.015</td>
<td>0.062</td>
<td>1.003</td>
</tr>
<tr>
<td>F07</td>
<td>-</td>
<td>-</td>
<td>-0.012</td>
<td>0.004</td>
<td>-0.056</td>
<td>0.005</td>
<td>1.253</td>
</tr>
<tr>
<td>F17</td>
<td>-</td>
<td>-</td>
<td>0.032</td>
<td>0.031</td>
<td>-0.022</td>
<td>0.002</td>
<td>1.088</td>
</tr>
<tr>
<td>G07</td>
<td>-</td>
<td>-</td>
<td>0.064</td>
<td>-0.003</td>
<td>0.027</td>
<td>0.003</td>
<td>1.038</td>
</tr>
<tr>
<td>G17</td>
<td>-</td>
<td>-</td>
<td>0.018</td>
<td>0.003</td>
<td>-0.007</td>
<td>0.000</td>
<td>[1.600]</td>
</tr>
<tr>
<td>G09</td>
<td>-</td>
<td>-</td>
<td>0.016</td>
<td>-0.005</td>
<td>0.026</td>
<td>-0.023</td>
<td>1.426</td>
</tr>
<tr>
<td>G19</td>
<td>-</td>
<td>-</td>
<td>-0.016</td>
<td>0.056</td>
<td>-0.021</td>
<td>0.000</td>
<td>1.848</td>
</tr>
<tr>
<td>H09</td>
<td>-</td>
<td>-</td>
<td>0.004</td>
<td>0.008</td>
<td>-0.013</td>
<td>-0.001</td>
<td>[1.000]</td>
</tr>
<tr>
<td>H19</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>H011</td>
<td>-</td>
<td>-</td>
<td>-0.001</td>
<td>-0.001</td>
<td>-0.011</td>
<td>-0.004</td>
<td>1.162</td>
</tr>
<tr>
<td>H111</td>
<td>-</td>
<td>-</td>
<td>-0.022</td>
<td>0.005</td>
<td>-0.003</td>
<td>-0.005</td>
<td>1.374</td>
</tr>
</tbody>
</table>
Acknowledgements

As in all large-scale high-energy experiments the number of people who in one way or another contributed to the preparation, performance and analysis of this experiment is impressively large. We list them below whilst taking the occasion of conveying to them our sincerest thanks.

Professor Ch. Peyrou has allowed us to do a counter experiment in spite of our formal attachment to his bubble chamber division. He also supported us with the necessary financial and moral backing. Similarly, the Caen group owes its existence and support to Professor M. Scherer of the Caen University and most of its funding to CNRS and IN2P3.

The beam was built by G. Petrucci and Michele Ferro-Luzzi and performed to everybody’s satisfaction. G. Mazzone and his team constructed the liquid deuterium target which stood well the large usage and the frequent emptying and refilling routines to which it was subjected. The PS floor support by J. Geibel and his team has been prompt and efficient. The extensive aligning and re-aligning chores required for the many spectrometer settings of our experiment were carefully and ungrudgingly performed by the PS surveyors under the direction of J. Leault.

No experiment can go very far without expert technical help. Ours came from C. Detraz and A. Magouriots who supplied a vast amount of useful work. The electronics was assembled in the TC electronics laboratory under the direction of G. Amato. In particular, the design of the chambers read-out system was mainly contributed by E. Chesi, the Plumbicon system by G. Amato; Di Torre did most of the cabling. The workshop of the Physics Department of the Caen University was responsible for the spark chamber system of the neutron detector.

In the early stages of the experiment we benefited from contributions by Drs R.D. Tripp, Ph. Eberhard, P. Litchfield and P. Jenni.

Finally, a considerable amount of clerical help - from statistical analysis to histogram plotting - has come from the TC scanning personnel under the direction of Mme M. Poncin-Fournier; in particular we should mention the help of K. Protoulis and R. Chacornac. The large quantity of drawings appearing in this report have been produced (albeit repeatedly) in the TC drawing office by M. Bellettieri, C. Plumettaz and C. Rigoni. Finally, Muriel King had the tiring task of typing the multifarious versions of this paper.

To all the above, again, our thanks.
REFERENCES

REFERENCES (Cont'd)


