RADIO FREQUENCY BEAM SPREADER

by

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Summary

A new injector for the CERN PS might require means for increasing the vertical emittance of the injected beam in order to fill the synchrotron acceptance. A method previously proposed, called a beam "spreader" or "shaker", uses RF deflecting fields to sweep the beam in the vertical direction. Various methods of applying this idea are examined, together with their relationship to the debunching problem.

The fundamental dynamical limitations imposed by time-varying deflection are discussed and it is shown that, in practical cases, these limitations are not of serious consequence.

The main RF requirements are examined and found to be fairly easy to satisfy.
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1. Introduction

When injecting a proton beam from a linear accelerator into a synchrotron the situation can arise that, even at high intensities, the linac beam emittance is appreciably smaller than the synchrotron acceptance. Since the space charge limit in the synchrotron is roughly proportional to the injected beam vertical emittance, it might be necessary to increase artificially the linac vertical emittance in order to raise the space charge limit to a sufficiently high value.

Taylor (1965) has proposed a method of increasing the vertical emittance without degrading the beam in the horizontal transverse phase plane, by applying a radio-frequency vertical deflecting field of suitable amplitude and frequency, produced in a cavity or waveguide. Although this device has previously been called a "beam shaker" the present writer has a slight preference for the term "beam spreader" as conveying the impression of a rather less random process. Also, the name "beam spreader" was used by Jones (1961) in connection with a similar scheme applied to longitudinal phase space.

The present 50 MeV linac of the CERN PS has a vertical emittance of around 30 $\pi \times 10^{-6}$ m radian for 95% of a 60 mA injected beam. Making allowances for closed orbit deviations one can consider the vertical acceptance of the PS to be completely filled under these conditions. This would no longer be true for either of the proposed new injection schemes. In the case of a 200 MeV linac we would expect no appreciable beam blow-up above 50 MeV since there appears to be little, if any, above 10 MeV in the present linac. Consequently, the vertical emittance would scale with $(\beta y)^{-1}$ and would be reduced by a factor of about 2 in going from 50 to 200 MeV. The beam from a 200 MeV linac would therefore occupy only 50% of the PS vertical acceptance. For the TART scheme using the existing 50 MeV linac there would be a factor of 4 difference, since the TART rings would have a vertical acceptance of about $140 \pi \times 10^{-6}$ m radian.
Thus, for either injection scheme, the need might arise to increase the vertical emittance of the linac beam by a factor of, at maximum, either 2 or 4. On the other hand it is not by any means certain at the present time that this facility is necessary or, if so, to what degree. Furthermore it is clear that, for filling the ISR, there would be no interest in blowing up the beam, quite the contrary in fact. Consequently, an RF beam spreader must be adjustable in amplitude from zero to maximum and should not in any way influence the properties of the beam when not in use.

The action of one kind of beam spreader or shaker is illustrated in Fig. 1. Here it is assumed that the spreader RF is synchronised on a harmonic of linac RF; this question is discussed in the next section. Figure 1 is intended merely to illustrate the principle; the phase plane contours shown are somewhat arbitrary and the bunch duration of 0.1 nano-second is approximate, and appropriate to a 200 MeV rather than to a 50 MeV linac.

2. Synchronism with Linac

The beam leaving a linac is rather tightly bunched, lying typically within an RF phase interval of 5-20 degrees. Due to the energy spread the bunch width increases as the beam drifts further from the linac, but even at 50 MeV the drift distance necessary for the bunch to spread over 2π or more is normally in excess of a hundred metres. Consequently, the beam spreader must act on beam bunches which occupy a fairly small fraction of one linac RF period. One can imagine the following distinct situations:

(i) The beam spreader frequency is lower than linac frequency. In this case all the particles in one bunch are deflected by about the same amount but different bunches are deflected differently. This does not seem to be a very desirable situation, since it would take longer in the synchrotron for the RF structure of the beam spreader to disappear than for the basic linac bunch structure.

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(ii) The beam spreader frequency is high enough that the linac bunch width covers at least one cycle of spreader RF. This would have the advantage that the spreader RF could be free-running and unsynchronised. However, in most practical cases the frequency would have to be so high that the spreader cavity would have insufficient aperture for the linac beam. This possibility will, however, be examined quantitatively in a later section.

(iii) The frequency of the beam spreader is chosen such that the bunch occupies somewhat less than half an RF period of spreader frequency. With this arrangement it is necessary to operate the spreader on a harmonic of linac RF and to lock the phase, otherwise the particle distribution in the vertical phase plane will vary randomly from bunch to bunch. The choice of linac harmonic will depend on the bunch length and therefore in the position along the beam where the spreader cavity is placed. A high frequency reduces the required RF power, but may of course be limited by aperture or other requirements.

It appears that method (iii) offers the best possibilities and the greatest flexibility. Although somewhat more complicated than a free-running RF system, the extra electronics required for harmonic operation and phase lock is fairly modest in cost and conventional in design. The synchronous beam spreader has several specific advantages. Firstly, since there is no random element in its operation, the vertical phase plane particle distribution should be the same for each bunch. This coherence might make diagnosis of beam behaviour easier than with incoherent operation. From Fig. 1 one can deduce that, by a suitable choice of frequency, the synchronous spreader can improve the uniformity of vertical phase plane density, since the tails of the bunch are located towards the peaks of the sine wave, where the rate of deflection is lower.

Finally, the synchronous beam spreader could in principle be used for measuring and displaying the linac bunch shape. This possibility is subject to some fundamental restrictions which will be discussed later in this report.
3. Bunch Width

In the previous section it was mentioned that the linac bunch duration is too small to permit operation of an asynchronous beam spreader. This statement requires some qualification and is examined in this section.

The bunch leaving the linac has typically a duration of around $10^{-10}$ second. If at least one RF period of beam spreader has to occur during this time the frequency would have to be at least $10^4$ MHz, corresponding to a wavelength of around 3 cm. A deflecting structure for this frequency could hardly have a full aperture greater than about 1.8 cm, which would not be adequate to pass the beam from a normal proton linac. Consequently, we can reject the idea of an asynchronous spreader placed immediately after the linac.

There remains, however, the possibility of placing an asynchronous beam spreader further down the beam channel towards the synchrotron where the linac beam has been partly debunched. If at such a position the phase spread has increased by a certain factor, then the RF wavelength of the spreader, and hence its aperture, can also be increased by the same factor. The shortest wavelength which could permit a reasonable aperture for the beam would be in S-band ($\lambda \approx 10$ cm), which would require a factor of 3.5 or more increase in phase spread of the beam.

In fact, such an increase in phase spread is of the same order as that normally required for reducing the energy spread of a linac beam by a debuncher, hence one could envisage placing the RF beam spreader cavity in the neighbourhood of the normal debuncher cavity and running the spreader asynchronously. This method has, however, certain drawbacks which will become evident in later sections of this report. To investigate this further we must first look quantitatively into the debunching process.

3.1 Debunching Relationships

The debuncher was first proposed by Johnsen (1955), who gave an approximate theory which is quite adequate for debunching factors of, say, 3 or more. We shall here use a slightly different representation to cover other cases.
Consider the longitudinal phase plane \((w, \phi)\) at the end of the linac and assume that the beam bunch is contained within the elliptical contour of Fig. 2 (a). \(w\) is the total energy and \(\phi\) the phase measured in units of linac frequency. We further assume that the centre of the ellipse corresponds to the phase stable particle which we use as origin for the phase plane. Then, after drifting a distance \(L\) beyond the linac, the beam will occupy the area shown in Fig. 2 (b); in particular the point \(P\) of (a) will have moved a distance \(\delta\phi\) to \(P'\) in (b). This distance \(\delta\phi\) is related to the velocity difference \(\delta(\beta\gamma)\) between particle \(P\) and the phase stable particle by:

\[
\delta\phi = -\frac{2\pi L}{\lambda_0 \beta} \delta\left(\frac{1}{\beta}\right) \quad (3.1)
\]

where \(\lambda_0\) is the free space wavelength of the linac RF. Also, if \(\delta W\) (= \(\Delta W\) for the particular particle \(P\)) is the corresponding energy difference, it can be shown that:

\[
\delta W = -m_0 c^2 \beta^3 \gamma^3 \delta\left(\frac{1}{\beta}\right) \quad (3.2)
\]

where \(\gamma = (1 - \beta^2)^{-1/2}\) and \(m_0\) is the rest mass.

Hence

\[
\delta\phi = \frac{2\pi L}{\lambda_0} \frac{1}{m_0 c^2 \beta^3 \gamma^3} \cdot \delta W \quad (3.3)
\]

From the geometrical properties of an ellipse it can be shown that, in the notation of Fig. 2 (b), the total phase width \(\Delta\phi'\) of the bunch is given by

\[
(\Delta\phi')^2 = (\delta\phi)^2 + (\Delta\phi)^2 \quad (3.4)
\]

From (3.3) and (3.4), with \(\delta W = \Delta W\), we then obtain
\[
\frac{\Delta W'}{\Delta W} = \frac{1}{\sqrt{1 + \left\{ \frac{2\pi L}{\lambda m c^2 \beta^3 \gamma} \cdot \frac{\Delta W}{\Delta \phi} \right\}^2}}
\] (3.5)

It is convenient to make the substitution:

\[
\tan \theta = \frac{2\pi L}{\lambda m c^2 \beta^3 \gamma} \cdot \frac{\Delta W}{\Delta \phi}
\] (3.6)

and then:

\[
\frac{\Delta W'}{\Delta W} = \cos \theta \quad \text{(debunching factor)}^{-1}
\] (3.7)

Similarly one can show that

\[
\frac{\Delta \phi'}{\Delta \phi} = \sec \theta \quad \text{(debunching factor)}
\] (3.8)

which, of course, follows from the invariance of \((\Delta W \cdot \Delta \phi)\).

After the drift space \(L\) the beam passes through the debuncher cavity which is so phased that the RF accelerating force is zero at the instant when the centre of the bunch passes the gap and is increasing with time. The RF amplitude is chosen such that at phase \(\Delta \phi'\) from the zero crossing the gap voltage is \(\Delta U\) (Fig. 2(b)). The beam bunch will then be transformed as in Fig. 2 (c). Using the above notation it can then be shown that:

\[
\frac{\Delta U}{\Delta W} = \sin \theta
\] (3.9)

The above treatment assumes linearity of the process, which amounts to saying that \(\Delta \phi'\) must remain within the "linear" region of a sine wave. For \(10^0\%\) non-linearity or less this requires \(\Delta \phi'\) not to exceed \(\pi/4\) which is normally the case in practical situations.
We can relate $\Delta U$ to the peak RF voltage of the debuncher cavity $V_o$. Assuming strict linearity we have:

$$\Delta U = V_o \Delta \varphi'$$  \hspace{1cm} (3.10)

If $\Delta \varphi'$ were sufficiently small there might be some advantage in using a debuncher operating on a harmonic of $h$ times linac frequency, in which case (3.10) would become:

$$\Delta U = h V_o \Delta \varphi'$$  \hspace{1cm} (3.11)

From (3.8), (3.9) and (3.10) we have:

$$\frac{V_o}{(\Delta M/\Delta \varphi)} = \sin \Theta \cos \Theta$$  \hspace{1cm} (3.12)

In Fig. 3, (3.7) and (3.12) are plotted as a function of (3.6). The graph shows how, for a given linac, the reduction in energy spread, and the RF voltage, vary with increasing drift length from linac to debuncher cavity. For $\tan \Theta = 1$ the debuncher voltage, (for a given $\Delta W/\Delta \varphi$), is a maximum, and the corresponding debunching factor is $\sqrt{2}$. The value of $L$ for $\tan \Theta = 1$ in (3.6) is characteristic of a given linac and may be called $L_o$. Then for ratios of $L/L_o$ greater than about 3, $L/L_o$ is a good approximation to the debunching factor.

We can calculate $L_o$ for a 50 MeV and for a 200 MeV linac with certain assumptions. We take $\Delta W/\Delta \varphi = 2.15$ MeV per radian at 50 MeV, which corresponds theoretically to a stable phase angle ($\varphi_s$) of 30° and an energy gain of 1.72 MeV/m for the CPS linac. We assume further that there is no blow-up above 50 MeV in a 200 MeV linac so that the $(\beta \gamma)^{3/4}$ scaling law applies if energy gain and $\varphi_s$ remain constant. We then obtain (with $\lambda_o = 1.48$ m):

$$\begin{align*}
[ L_o ]_{50 \text{ MeV}} & = 3.72 \text{ m} \\
[ L_o ]_{200 \text{ MeV}} & = 12.94 \text{ m}
\end{align*}$$  \hspace{1cm} (5.13)
3.2 Debunching Factor

The choice of debunching factor for a new injector will depend on many considerations which cannot be examined in detail here. However, we can expect that a factor of at least 4-5 will be desirable because of the following general arguments.

For the case of a 200 MeV linac, the increase in energy spread scales with $(\beta \gamma)^{3/4}$ whereas the PS bucket height increases as $\beta \gamma^{3/2}$ (well below transition). One might therefore conclude that a smaller debunching factor would suffice for 200 MeV injection than the present theoretical factor of about 4.5 at 50 MeV$^x$. However, the increased $\beta$ foreseen in the PS improvement programme can only be switched in at a certain time after injection when, amongst other things, the new RF buckets for the higher $\beta$ are large enough to contain the trapped beam. Due to filamentation, the phase area occupied by the beam at this time is determined by the trapping area at injection; the smaller this area, the earlier can $\beta$ be increased, other conditions permitting. Consequently, to maintain a good trapping efficiency with small buckets at injection one would be encouraged to choose a reasonably large debunching factor. Furthermore, if one believes in the ultimate possibility of overcoming the longitudinal space charge problem at transition, the smaller trapping area at injection might give a higher final longitudinal phase space density with consequent benefits to the Intersecting Storage Rings.

Similar arguments apply to the case of 50 MeV injection into TART. Even with quasi-adiabatic trapping, the trapping area can be reduced by reducing the energy spread of the injected beam for the same trapping efficiency. In addition to the other benefits mentioned, the transfer from TART to the PS is facilitated by a reduction in bunch area.

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$x$) The currently measured debunching factor is near to 2. The discrepancy may be due to the optimum operating conditions of the linac, arrived at by adjusting a number of parameters experimentally, differing appreciably from those conventionally assumed in the theory. This is consistent with the tendency of the measured energy spread (without debunching) to be rather smaller than theory would indicate.
4. The Effects of Time Varying Deflection

There exists a fundamental property of dynamical systems which manifests itself if motion is coupled in any way between two or more co-ordinate directions. Courant and Snyder (1958) refer to this property as the symplectic condition, and discuss it with reference to weak coupling between vertical and radial betatron motion in a synchrotron. The symplectic condition applies to any dynamical system whose equations of motion can be derived from a Hamiltonian, and is thus of very wide generality. It can be deduced in a straightforward manner by taking variations of the action integral (Fowler and Good, 1960), leading to the so-called Poincaré invariants. Warner (1965) has discussed the implications of this property in the measurement of linac bunch widths.

For the purpose of this report, the symplectic condition can be expressed as a special case of a Poincaré invariant:

\[ \Delta W = \frac{dp_z}{dt} \cdot \Delta z \]  

(4.1)

where \( \frac{dp_z}{dt} \) is the rate of gain of transverse momentum of a particle; \( \Delta z \) and \( \Delta W \) are respectively the transverse position and energy gain of the particle relative to some reference particle. (4.1) thus states that a time-varying deflection applied to a particle beam of non-zero cross-section results in an increased energy spread of the beam.

As a numerical example we can consider a 50 MeV linac beam with a bunch width of 0.2 nsec. A kinetic energy of 50 MeV corresponds to a momentum of 310 MeV/c; if we deflect the bunch at the rate of \( \pm 1 \) milliradian in \( \pm 0.1 \) nsec the rate of change of transverse momentum \( \frac{dp_z}{dt} \) is 3.1 MeV/c per nanosecond. This corresponds to an energy gain of 10.3 MeV/m, so that a beam of transverse dimensions \( \pm 1 \) cm will receive an extra energy spread of \( \pm 103 \) keV. This approximates to increasing by about 50% the existing energy spread of a 50 MeV linac and must obviously be taken seriously. Let us examine in detail what happens with a synchronous beam spreader.
4.1 Synchronous Beam Spreader Close to Linac

We assume the parameters used as an example in the previous paragraph, since they are sufficiently close to reality to be meaningful. In addition to assuming a bunch width of ± 0.1 nsec (corresponding to Δφ = ± 7° at 202.5 MHz) we take the normal linac energy spread to be ± 200 keV. Then the (W, ϕ) phase plane at the output end of the linac may be represented as in Fig. 4 (a). The ellipse labelled as z = 0 represents either the envelope of the whole undeflected beam or that part of the deflected beam which passes close to the axis of the deflector. The other ellipses correspond to parts of the beam passing at ± Δz from the axis.

Let us now suppose that the beam spreader is followed by a drift space and a normal debuncher. Then Fig. 4 (a) transformed to a position immediately after the debunching cavity will appear as (b). One sees that, after debunching, the energy spread produced by the time-varying deflection has been converted into an extra phase spread but that the energy reduction factor of the debuncher is the same as in the absence of deflection. For this situation to be true, it is evident that the increased phase spread ± Δϕ" must lie within the substantially linear range of the debuncher waveform, and this clearly limits the debunching factor to a lower value than in the case of no RF deflection. For example, suppose the linac energy spread to be ± 200 keV, the bunch width ± 7° and the linear range of the debuncher waveform ± π/4. Then without beam spreader the debunching factor could be as high as 6.4, but with a beam spreader introducing a further ± 100 keV of energy spread the debunching factor could not exceed about 4.3 without encroaching into the non-linear region.

Nevertheless, the situation is not too bad. Firstly, a debunching factor of 4.3 is quite useful in itself and one might not feel too concerned about increasing it much above this value. Secondly, if one really needed a larger debunching factor without introducing too much non-linearity it would be possible, if a little complicated, to add one or more harmonic bunching cavities to extend the linear range. Thirdly, since the extremities of the phase spread in Fig. 4 (b) correspond, not only to the transverse limits of the beam, but also to the energy spread limits, the phase space
density in these regions is likely to be very much lower than the average; substantial non-linearity would then have little effect on trapping efficiency. It seems then that this type of beam spreader could be used without serious difficulties.

It should perhaps be pointed out that, although the numerical values used in the above examples approximate to those believed to apply to the present CPS linac, no attempt has been made to be strictly consistent with measured data. This is partly because knowledge of bunch width is still very approximate at present, and partly because the effects discussed depend strongly on the amount of beam spreading (if any) that will be required, an amount which is quite uncertain at the present time.

4.2 Synchronous Beam Spreader Remote from Linac

From the discussion in the previous section it is easy to demonstrate the properties of a beam spreader located, for example, immediately after the debuncher cavity. Figure 4 (c) shows the corresponding situation. The same time-varying deflection as in the previous example produces the same increase in energy spread, but now it has to be compared with the reduced energy spread provided by the debuncher. It is evident that such an arrangement partly or wholly removes the useful property of the debuncher, without any compensating advantage except, possibly, a reduction in the transverse acceptance required in the beam channel between linac and beam spreader.

The unfavourable situation created by this arrangement reminds us that in section 4.1 we assumed that the beam spreader was located immediately after the linac, where the longitudinal phase space ellipse is in principal axes. Now in section 3.1 we defined a characteristic length $L_o$ corresponding to a factor $\sqrt{2}$ in bunch width increase. From (3.13), $L_o$ for 50 MeV is 3.72 m, and an RF beam spreader placed at this distance after the linac would have the effect of reducing the useful debunching factor by $\sqrt{2}$. It is thus important that the spreader be located as close as possible to the linac if one is to achieve the minimum injected energy spread.
4.3 Asynchronous Beam Spreader Remote from Linac

It is obvious that the criticisms of 4.2 apply equally well to the asynchronous case, the only difference being the academic one that there is now no coherence in the distribution of particles in the longitudinal phase plane. It therefore seems unnecessary to discuss this case further.

4.4 Implications for Bunch Width Measurement

This subject has been discussed in some detail by Warner (1965). Here we shall just make a few comments relating to the use of the beam spreader for bunch width measurements.

Firstly, (4.1) shows that, if we wish to display the bunch shape by sweeping the beam across a slit we have to accept some loss of information about energy in a finite beam. However, this does not necessarily preclude us from making useful measurements with such a device. For example, we could agree to make separate measurements of bunch width and energy spread, the latter with the spreader switched off. This involves the hope that linac conditions remain unchanged between the two sets of measurements. Alternatively, one could measure bunch width and energy spread simultaneously, correcting the latter with (4.1). A further possibility would be to work with a small transverse beam emittance and make successive measurements of different parts of the beam.

The general implication of this limitation is that one can in principle measure any quantity to any desired accuracy provided sufficient time is devoted to the measurement. The practical consequence is, of course, that the apparatus and conditions must remain sufficiently stable over the measuring period.

5. RF Requirements for a Beam Spreader

In previous sections we have examined the theoretical properties of a beam spreader and the relationship with the requirements of debunching. From this it seems clear that the synchronous spreader located close to the
linac would have the minimum undesirable effects on the beam dynamics. Such a system might permit the use of frequencies as high as 3000 MHz (S-band), though linac bunch length may impose a lower frequency, 1200 MHz (L-band) for example, where there would be the advantage of more spare aperture in the deflector. We now examine spreader RF requirements for both a 50 MeV linac injecting into TART and for a 200 MeV linac injecting into the CPS.

5.1 Transverse Momentum Impulse

We assume a linac vertical emittance of $30 \pi \times 10^{-6}$ m.rad at 50 MeV ($\beta \gamma = 0.3307$), and that the 200 MeV linac ($\beta \gamma = 0.6869$) would have the emittance scaled by $\beta \gamma$, so that for the same vertical size of beam, the maximum transverse momentum $p_z$ would be the same in each case. Assuming a beam half-height of 15 mm (to pass comfortably through an S-band deflector if necessary), the beam half-divergence would be 2 milliradian. Now the momentum of a beam is $\beta p_0 m$, which for 50 MeV kinetic energy is about 310 MeV/c, so that 2 milliradian half-divergence corresponds to 620 keV/c transverse momentum. In order to double the vertical emittance of the beam we must apply therefore a peak transverse impulse of 620 keV/c.

For a deflector of length $l$, the momentum imparted by an equivalent deflecting field $E$ is:

$$\Delta p_z = \frac{eEl}{\beta c} \quad (5.1)$$

Now if $\omega_s$ is the angular frequency of the spreader RF and $\Delta t$ the half bunch width, the peak deflecting field is:

$$E_0 = \frac{E}{\sin (\omega_s \Delta t)} \quad (5.2)$$

$$= \frac{c\beta \Delta p_z}{el \sin (\omega_s \Delta t)} \quad (5.3)$$
Putting $\Delta t = 0.1$ n.sec, $\omega = 2\pi \times 1215$ MHz (the 6th harmonic of linac frequency), $\beta = 0.314$, $\Delta p_Z = 620$ keV/c and assuming a deflector 1 metre in length, we obtain:

$$E_0 = 282 \text{ kV m}^{-1}$$  \hspace{1cm} (5.4)

5.2 RF Power

The peak RF field given by (5.4) is quite modest and well below any reasonable breakdown limit. It would therefore be logical to operate the deflecting cavity or waveguide in a standing-wave mode. If one were to use a disc-loaded waveguide with the $E_{11}/H_{11}$ hybrid mode, one would then have, with standing-wave operation, considerable freedom in choosing the aperture, since the group velocity would be non-critical above a certain value.

In view of the preceding remarks it seems safe to assume a value of $5 \text{ M} \Omega \text{ m}^{-1}$ for the shunt impedance of such a waveguide in L-band, where we define shunt impedance as $E_0^2/P$, ($P$ = power loss per metre). This leads to:

$$P = 16 \text{ kW m}^{-1}$$  \hspace{1cm} (5.5)

from (5.4) and hence a total power of 16 kW. This is quite a small amount of power for pulsed operation, and one might even consider reducing the cavity length and increasing the power level.

We have so far assumed that the linac vertical emittance would have to be doubled for injection into TART. In the worst case a factor of four might be necessary, requiring twice the field we have assumed and hence 4 times the power, or 64 kW. This is still quite an acceptable figure.

For a 200 MeV linac injecting into the PS an emittance of 2 would be the maximum required. Since in (5.3), $\Delta p_Z$ is invariant, the factor in $E_0$ which is of importance is $\beta$, which increases by about 1.8 between 50 and 200 MeV. Thus the maximum RF power requirements would be almost the same for the 50 and 200 MeV cases, i.e. around 64 kW.
5.3 Non-linearities

In a deflecting waveguide of the type discussed, when used for high-energy RF separators, the phase velocity is arranged to be close to c. Then under these conditions the deflecting wave mode is fundamentally aberration-free over the whole aperture of the waveguide. For the beam spreader the phase velocity will have to be matched to the particle velocity βc and the deflecting mode will have some aberrations. The magnitude of these has not yet been calculated, but the fact of using only a small part of the aperture of the waveguide will reduce their effect.

5.4 Phasing

There should be no difficulty with the phasing of the RF beam spreader to the linac. The tolerances demanded will certainly be less strict than those required for an RF particle separator where the problem has already been solved for more difficult conditions (Bramham 1966 a, 1966 b).

6. Conclusions

There appears to be no serious difficulty in the design and construction of an RF beam spreader to increase the vertical emittance of a linac beam, either at 50 or at 200 MeV. Although a fundamental dynamical property imposes some limitations on what can be done, things can be so arranged that the required emittance increase is obtained without appreciably degrading the beam in other respects.

The detailed design of such a device is closely linked with the properties of the linac beam and in particular with longitudinal phase space parameters and debunching requirements. Any future studies should include a check on the magnitude of deflection aberrations in a wave with phase velocity substantially below c.

The RF engineering requirements are straightforward and fall easily in the range of known techniques; they are practically identical for both a 50 MeV linac injecting into TART and for a 200 MeV linac injecting into the PS.
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References

Bramham, P. (1966 a), AR/Int. FSep/65-9
Bramham, P. (1966 b), CERN 66-8
Courant E.D. and Snyder H.S. (1958, Annals of Physics 3 1
Fowler T.K. and Good W.M. (1960), Nucl. Inst. and Meth. 7 245
Goldstein H. (1953), Classical Mechanics
Johnsen K. (1955), CERN-PS/KJ 29
Jones E. (1961), AR/Int. SR/61-5
Taylor C.S. (1965), Private communication

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Fig. 1

a) Linac beam bunch

b) Spreader deflection

\[ \Delta z' \]

\[ t \]

\[ \approx 0.1 \text{ nsec} \]

\[ 7.2^\circ \text{ at } 202.5 \text{ MHz} \]

\[ 100^\circ \text{ at } 2835 \text{ MHz} \]

c) Vertical phase plane—undeflected beam

d) Deflected beam

Best fitting ellipse

Envelope of deflected beam over 1 RF period
Fig. 2
Fig. 3

The graph shows the functions $\cos \theta$ and $\sin \theta \cos \theta$ against $\tan \theta$. The $y$-axis represents the values of $\cos \theta$ and $\sin \theta \cos \theta$, ranging from 0 to 1.0. The $x$-axis represents $\tan \theta$, ranging from 0 to 10.