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HADRODYNAMICS WITH THE ELUSIVE QUARKS

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INTRODUCTION

Today it has become almost inconceivable to talk about hadrons and their interactions without thinking about their so-called constituents: the quarks. Their number, as we are witnessing, is having an ominous tendency to increase under the effect of the exciting discoveries of new particles and unexpected lepton yields in e^+e^- and ν -nucleon collisions. However, I believe that after the experimental situation will more or less settle the usefulness of the quark notion will definitely be preserved even after some possibly inevitable proliferation.

In these lectures I will take a very cautious attitude as far as the quark quantum numbers, by not committing myself to any particular of the numerous quark schemes which are currently being proposed. As a matter of fact, I will even disregard the spin degree of freedom and deal with scalar quarks, even though, obviously, nobody doubts that the quark spin should be taken to be $1/2$. As it should be clear from the title of these lectures my attention shall be totally concentrated on trying to determine whether there is some way to describe (understand?) the dynamics of the strong interaction in terms of this so far eminently elusive object: the quark. Any realistic calculation, however, will have to take properly into account the "complications" played by spin and internal quantum numbers; but this will be outside the scope of these lectures. In fact, I think

that we shall have come already a long way if we will be able to deal with some sort of comfort with the dynamics of a neutral spinless and confined quark.

Anybody who has even a minimal familiarity with what is going on in high energy physics today certainly knows that the notion of "Quark Confinement" is in. Even though at first sight this notion appears inevitable in view of the many unsuccessful searches so far performed; after some thought it seems to me that such compulsiveness is not really so obvious. I think that it all stems from our natural inclination to introduce quarks by visualizing them as the usual fields that, after 40 years of Quantum Field Theory (QFT), we have more or less learnt how to handle without tears. But this either conscious or subconscious connection between quarks and, say, electrons may well be at the very root of the paradox that we are trying to solve (?) with the idea of confinement.

Whereas there is no evidence, in fact quite the contrary seems to hold, that general QFT may find it impossible to describe the hadronic particles and their interactions, that this should also apply to the quarks may in fact be an unwarranted extrapolation of old ideas into a completely new domain of physical phenomena. The history of physics abounds of examples where paradoxes could be removed only by changing our way of thinking in a fundamental way. I have particularly in mind the example of special relativity and the futile attempts to fix things up by way of the ether.

It would be wrong, however, to overemphasize this point, at least at this time, where it is not clear that the well established principles of Lagrangian QFT cannot lead us to the long sought theory of hadrons and their interactions. This is the hope of Ken Wilson, as we have learnt from his lectures at this school [1], which is shared by a great many of my colleagues. I cannot but wish them good luck, if that is the right way they will find out sooner or later. Personally I am much less confident in this approach, and the developments of the Massive Quark Model (MQM) which I exposed last year at this school [2] show that it is possible to go pretty far on the way of constructing a consistent and realistic des-

cription of hadronic interactions by starting from some simple but unconventional way to deal with the quark degree of freedom - on the other hand it cannot be excluded that these ideas will be derived from a Lagrangian QFT of coloured quarks.

Be as it may, the main achievement of the MQM has been to provide a realistic framework for the kind of physics where the parton model has found its applications. This gives us confidence that the deep inelastic phenomena and the processes at large transverse momentum can really teach us how to construct a complete and powerful theory of the strong interactions starting from some generalization as well as simplification of the MQM ideas. These lectures will concentrate on an attempt of this type which appears to be extremely promising [3].

This is a summary of these lectures: Section 1 contains a brief review of the MQM ideas which will be important for further developments; in Section 2 a simple space time description is proposed for hadronic states; further developments of this description in relation to some basic MQM properties are contained in Section 3. Section 4 deals with the general strategy of a perturbative approach to hadronic interactions, and a number of calculations are presented in Section 5. The structure of hadronic final states and some ideas about hadron-hadron scattering are the subject of the final Section 6.

1. THE MASSIVE QUARK MODEL (MQM) - A BRIEF REVIEW

This section contains a brief review of the main ideas of the MQM, a full exposition of which can be found in last year's Erice book [2].

The MQM revolves around a number of ideas stemming from a realistic picture of hadrons in terms of quarks. They can be summarized as follows:

- a. Quarks are the carriers of the space-time and quantum number characters of hadronic matter

This means that all properties of hadrons and their interactions

can be traced back to the behaviour of their constituents, i.e. the quarks. Provisionally they can be visualized as quantum fields, but

- b. no quanta are associated with the quark fields.

This is a statement about the unobservability of quarks which so far seems to be strongly supported by experiments. It obviously endows the quark field with very peculiar characteristics. One very important peculiarity is that Green's functions constructed out of these fields do not possess singularities in their quark legs. In fact, if present, these singularities should have to be interpreted as physical states enjoying normal propagation properties in space-time, which would contradict experimental evidence. As for the quark Green's functions:

- c. Quark Green's functions are universally peaked at small quark masses and Regge behaved at high energy.

These two properties of the Green's functions are crucial in making contact with some basic features of hadronic interactions.

As for universal peaking at small mass, it is suggested from the precocity of both the Regge and scaling behaviours. In such description, in fact, it is the "effective" quark mass (i.e. the values at which the quark Green's functions peak) which controls the onset of the mentioned behaviours. Regge behaviour at high energy, on the other hand, is strongly suggested for quark Green's functions by the analogous behaviour in hadron-hadron scattering. In order to make this connection tighter, however, we must supply a scheme to construct a theory of hadronic interactions in terms of quark Green's functions. This is done in the following way:

- d. One can construct the hadronic amplitudes in terms of quark Green's functions following a systematic diagrammatical expansion.

The elementary building blocks of our diagrammatic expansion are listed below:

- i) Wave functions: They describe a given hadronic state

Fig. 1: The meson wave-functions

in terms of its quark degrees of freedom. In Fig. 1 we have drawn a mesonic ($\bar{q}q$) state. In conventional QFT the analogous object would be a Bethe-Salpeter wave-function. However, on the basis of point b.) above, the analytic structure of quark wave functions in momentum space is radically different.

- ii) Elementary scattering amplitudes: They comprise the $\bar{q}q$ and qqq Green's functions (Fig. 2),

Fig. 2: The $\bar{q}q$ and the qqq Green's functions

and describe scattering among quarks through their rearrangement in "allowed" hadronic states. In fact, the singularities of these Green's functions in the triality-zero channels

do correspond to observed hadronic states.

iii) Irreducible vertex functions: They describe the coupling of three meson-like ($q\bar{q}$) systems V_6 and of two baryon-like (qqq) and one meson-like systems V_8 (see Fig. 3). The possibility

Fig. 3: The irreducible vertex functions V_6 and V_8

of associating them with small parameters (λ_6, λ_8) is suggested by several facts of hadronic phenomenology like the very success of the naive quark model, the absence of exotic states and the relatively small resonance widths. It is this fact that gives a diagrammatic expansion based on the previous elements the possibility of being a serious candidate for a complete theory of hadrons and their interactions. In fact we can, using the above building blocks, expand the hadronic S-matrix in powers of the two coupling constants λ_6 and λ_8 and enforce desirable properties like: causality, Lorentz-invariance and unitarity in a perturbative fashion. We shall get back to this point later (Section 4).

iv) Weak and e.m. currents: This is the last element that we have to introduce in order to be able to describe the weak and e.m. interactions of hadrons. This is done according to the diagram in Fig. 4, which represents the two-quark irreducible current vertex.

Fig. 4: The weak and e.m. two-quark irreducible vertices.

This concludes the brief summary of the central ideas of the MQM. By following them one can go pretty far in describing multiple aspects of the physics of both deep inelastic and large p_T processes; as the reader can ascertain by reading, for example, the mentioned 1974 Erice Lectures.

However, the MQM formulated so far is not really a complete theory of hadronic interactions. What is clearly lacking is a precise description of the spectrum and of the related problems of constructing the hadronic wave functions and the elementary scattering amplitudes. This will be attempted in the following sections.

2. A SPACE-TIME DESCRIPTION OF QUARKS AND HADRONS. THE MESON WAVE FUNCTIONS^(*)

The MQM has led us to develop a picture of the elusive hadronic constituents, the quarks, as localized "lumps" of energy, momentum, charge, etc., within the extended hadron states they comprise. The elusiveness of quarks can be introduced by the requirement that quarks span only finite regions in the 4-dimensional space of their relative coordinate. A further constraint which one ought to impose to this picture is scaling in deep inelastic scattering. Light cone and parton physics seems to suggest that the hadronic constituents propagate at short distances

(*) I am following closely Ref. [3].

as free particles with no or little interactions. One is thus led to a class of "primitive" hadronic states which are the simplest zero-triality combinations of quarks (mesonic: $q\bar{q}$; baryonic: qqq). They are confined to finite space-time domains, inside which they move freely. We may call these states, whose structure will be studied in detail in this section, "allowed quark orbits" (*). One then envisages scattering and decay processes as transitions from a set of allowed orbits to another set of allowed orbits, their probability amplitude being essentially given by a space-time overlap between initial and final configurations. This last point shall be taken up in Section 4.

Let us now study the simplest zero-triality system: the mesonic ($q\bar{q}$) states. Throughout these lectures only scalar quarks shall be considered in order to illustrate the whole approach in the simplest possible situation. The extension to the realistic case where quarks have spin $\frac{1}{2}$ and carry **SU(3)** quantum numbers is carried out elsewhere [5].

A. Equations of motion

Let us then consider the $q\bar{q}$ wave function (w.f.) $\psi(p; x_1, x_2)$ depicted in Fig. 1. Translational invariance gives us immediately

$$\psi(p; x_1, x_2) = e^{ipR} \psi(p, x) \quad (2.1)$$

where $R = \frac{1}{2}(x_1 + x_2)$ and $x = x_1 - x_2$.

We must now impose on this w.f. the requirement that quarks are permanently confined. The simplest and admittedly quite crude way of achieving this is to set

$$\psi(p, x) = 0 \quad \text{for } x \notin R(p) \quad (2.2),$$

where $R(p)$ is a certain finite space-time domain which must in principle depend on the 4-momentum p of the hadron we wish to describe (**).

(*) These states will be frequently called "bags". They have in fact some superficial similarity with the MIT-Bag [4].

(**) This is required, for instance, by Lorentz invariance.

The next question to answer is to determine how quarks will propagate in the "bag" region: $x \in R(p)$. As mentioned above, the guess is that for small x both quarks obey free field equations. We should, however, expect some deviation from this behaviour when x is near the boundary $R(p)$. We choose for the time being to neglect this "surface" effects and we shall try to take them into account when their presence cannot be consistently ignored^(*). We shall then impose on $\psi(p; x_1, x_2)$ the two conditions

$$\begin{aligned} -\square_1 \psi(p; x_1, x_2) &= m^2 \psi(p; x_1, x_2) \\ -\square_2 \psi(p; x_1, x_2) &= m^2 \psi(p; x_1, x_2) \end{aligned} \quad (2.3)$$

for $x \in R(p)$; m is a parameter which can be called the "quark mass". It should be clearly kept in mind that m describes quark propagation inside $R(p)$, and has nothing to do with the notion of observable mass, which in this description can be taken as infinity [2]. By employing the translational invariant wave function (2.1), the eqs.(2.3) become

$$\begin{aligned} \left(\frac{p}{2} - \frac{1}{i} \frac{\partial}{\partial x}\right)^2 \psi(p, x) &= m^2 \psi(p, x) \\ \left(\frac{p}{2} + \frac{1}{i} \frac{\partial}{\partial x}\right)^2 \psi(p, x) &= m^2 \psi(p, x) \end{aligned} \quad (2.4)$$

for $x \in R(p)$. These equations can be cast in the following form

$$p \cdot \frac{\partial}{\partial x} \psi(p, x) = 0 \quad (2.5a)$$

$$\left(\frac{M^2}{4} - \square_x\right) \psi(p, x) = m^2 \psi(p, x) \quad (2.5b).$$

We shall now look at eqs.(2.5) in the rest frame $p \equiv (M, \vec{0})$. It is immediate to see that in this frame space and time get decoupled. In order to obtain compatibility with the boundary, $R(M, \vec{0})$ should also split into a "time-bag" and a "space-bag". Thus we have $[\psi(M, \vec{0}; t, \vec{x}) \equiv \psi_M(t, \vec{x})]$

(*) Surface effects on the meson spectrum will be briefly discussed in the next section.

$$\frac{\partial}{\partial t} \psi_M(t, \vec{x}) = 0 \quad (2.6a)$$

and

$$\left(\frac{M^2}{4} + \vec{\nabla}^2 - m^2\right) \psi_M(t, \vec{x}) = 0 \quad (2.6b)$$

for $|t| \leq R_t(M)$ and $|\vec{x}| \leq R_s(M)$. (2.6a) is quite important because it stipulates that in the rest frame the w.f. is t-independent, thus washing out the dynamics of the relative time coordinate which has always been a source of troubles in relativistic Bethe-Salpeter (BS) approaches^(*).

In order to avoid divergences in combining wave functions together, which we shall have to do later, it is necessary that the w.f.'s be continuous at the boundary. Thus we must expect the existence of a "skin" region in time where (2.6a) cannot hold and should be modified by a "potential" term giving a $\psi_M(t)$ of the type shown in Fig. 5. This kind of surface effect, however, is expected to play some role only at low energy

Fig. 5: A possible shape of the time.w.f.

As for (2.6b), it will provide us with a spectrum once we impose the continuity of $\psi_M(t, \vec{x})$ for $|\vec{x}| = R_s(M)$.

(*) A discussion of the problem of ghosts in BS-equation can be found, for example, in Ref. [6].

We have then for the w.f.'s of the basic $q\bar{q}$ states:

$$\psi_M(t, \vec{x}) = \psi_M(t) \varphi_{M, n\ell}(\vec{x}) \quad (2.7)$$

where

$$\varphi_{M, n\ell}(\vec{x}) = N_{n\ell} j_\ell \left(\sqrt{\frac{M_{n\ell}^2 - m^2}{4}} |\vec{x}| \right) Y_\ell^m(\theta, \varphi) \quad (2.8).$$

$N_{n\ell}$ is a normalization factor to be determined later, $j_\ell(z)$ is the spherical Bessel function of order ℓ and Y_ℓ^m is a normalized spherical harmonic. The continuity condition implies the spectrum equation

$$(M_{n\ell}^2 - 4m^2)^{1/2} = \frac{2\beta_{n\ell}}{R_s(M_{n\ell})} \quad (2.9)$$

where $\beta_{n,\ell}$ is the n^{th} positive zero of the Bessel function $j_\ell(z)$; asymptotically

$$\beta_{n\ell} \xrightarrow{n \text{ large}} \pi \left(n + \frac{1}{2} \right) \quad (2.10).$$

The spectrum eq.(2.9) will give us definite predictions only after the function $R_s(M)$ has been specified. We shall discuss possible criteria to restrict the form of $R_s(M)$ in a short while.

B. An intermediate normalization

It will be useful to obtain a formula for $N_{n\ell}$ in (2.8) from the condition

$$\int d^4x |\psi(p, x)|^2 = 1,$$

even though the normalization related to the correct charge of the states will lead to a somewhat different requirement, as shall be discussed in Sections 4 and 5. A simple exercise yields:

$$|N_{n\ell}| = \frac{2}{\sqrt{\pi}} \frac{\beta_{n\ell}^{1/2}}{|J_{\ell+3/2}(\beta_{n\ell})|} \frac{1}{[2R_\ell(M)R_s^3(M)]^{1/2}} \quad (2.11).$$

3. MOMENTUM SPACE ANALYSIS. THE MQM CRITERION AND THE SPECTRUM

A. The w.f. in momentum space

We shall now compute the Fourier transform of (2.7), which will provide us with an expression for the w.f. in momentum space (Fig. 6).

Fig. 6: The momentum space wave function

We have

$$\psi_M(k) = \int d^4x e^{ikx} \psi_M(t, \vec{x}) \quad (3.1).$$

By performing the 4-dimensional integration we obtain in a straightforward way

$$\begin{aligned} \psi_M(k) = & (2\pi)^2 \frac{2}{\pi} \frac{R_s(M)^2}{R_t(M)} \frac{\sin k_0 R_t(M)}{k_0} \times \\ & \times \frac{\beta_{\eta\ell}}{\beta_{\eta\ell}^2 - \vec{k}^2 R_s(M)^2} \int_{\ell} (R_s(M) |\vec{k}|) Y_{\ell}^m(\theta_k, \varphi_k) \end{aligned} \quad (3.2).$$

In order to appreciate the basic properties of $\psi_M(k)$, let us first express the variables k_0 , and $\vec{k}$ in terms of the "quark masses"

$$m_{1,2}^2 = \left(\frac{P}{2} \pm k \right)^2.$$

We have in the rest frame:

$$k_0 = \frac{1}{2M} (m_1^2 - m_2^2) \quad (3.3)$$

and

$$|\vec{k}|^2 - \frac{M^2}{4} + m^2 = -\frac{1}{2}(m_1^2 + m_2^2) + \frac{1}{4M^2}(m_1^2 - m_2^2)^2. \quad (3.4)$$

We can immediately see that (3.2) is an entire function of the variables m_1^2 and m_2^2 . This is one of the basic properties of the MQM (see Section 2) and a general feature of permanently confined quarks. Thus it should not come as a surprise that we get it explicitly here.

The behaviour of the w.f. (3.2) as a function of $m_1^2 - m_2^2$ is reported in Fig. 7. We see clearly that the width of the distribution is

$$\Delta(m_1^2 - m_2^2) \simeq \frac{2\pi M}{R_t(M)} \quad (3.5);$$

and analysing the dependence on the variable $m_1^2 - m_2^2$ in a similar way

Fig. 7: The w.f. behaviour as a function of $m_1^2 - m_2^2$.

we get

$$\Delta(m_1^2 + m_2^2) \simeq \frac{2\pi M}{R_s(t)} \quad (3.6).$$

B. The MQM Criterion and the Spectrum

As mentioned at high energy the MQM can recover all the desirable aspects of scaling provided the behaviour in the "quark masses" of Green's function is universal, i.e. it does not depend on the particular state at high mass. Thus we are led to the very important constraints

$$R_t(M) \xrightarrow{M \text{ large}} R^2 M \quad (3.7)$$

$$R_s(M) \xrightarrow{M \text{ large}} R^2 M \quad (3.8)$$

where R^2 is a parameter carrying the dimensions of the square of a length. Taking the spectrum eq.(2.9) into account, we can interpolate (3.7) and (3.8) with the simple formula

$$R_s^2(M_{n\ell}) = R_t^2(M_{n\ell}) = R_{n\ell}^2 = R_0^2 (\beta_{n\ell} + \lambda) \quad (3.9)$$

where λ is a free parameter and R_0^2 is related to R^2 through the relation $\frac{R_0^2}{2} = R^2$. By use of (3.9), the spectrum equation becomes

$$M_{n\ell}^2 - 4m^2 = \frac{4\beta_{n\ell}^2}{R_0^2 (\beta_{n\ell} + \lambda)} \quad (3.10).$$

The Regge trajectories (3.10) are given in Fig. 8 for the following values of the parameters:

$$R_0^2 = 1.5 \pi \text{ GeV}^{-2} \quad \text{and} \quad \lambda = 2.$$

Fig. 8: A Chew-Frautschi plot for the meson spectrum.

The mass spectrum consists of a set of Regge trajectories approximately linear and parallel,^(*) with a strong resemblance to actual physics. The only unphysical feature is the high mass for the lowest state ($n=1, l=0$). A way to remedy this pitfall is to take into account the energy stored in the "skin" of the time-bag which we have neglected. It is easy to see that this contribution subtracts from the kinetic energy of the spatial motion; and calling this term $\langle k_o^2 \rangle_{nl}$ we would have instead of (3.10)

$$M_{ne}^2 - 4m^2 = \frac{4}{R_o^2} \frac{\beta_{ne}^2}{\beta_{ne} + \lambda} - 4 \langle k_o^2 \rangle_{ne}. \quad (3.11)$$

The actual value of $\langle k_o^2 \rangle_{nl}$ is difficult to determine unless one has some extra information^(**). We only emphasize that this term should be there in general and that its relative contribution should become less and less important with the increase of the mass of the mesonic system, being associated with the surface of an expanding bag. Thus, for simplicity, we shall take $\langle k_o^2 \rangle_{nl}$ to be independent on nl and given by

(*) Notice also the absence of odd daughters.

(**) In the spin 1/2 case this would be chiral symmetry and the special role played by pseudoscalar mesons as Goldstone bosons.

$$\langle k_0^2 \rangle_{nl} \simeq \frac{4}{R_0^2} \frac{\beta_{10}^2}{\beta_{10} + \lambda} \quad (3.12),$$

so that the trajectories in Fig. 8 get rigidly shifted towards the left by an amount such that the lowest state has a mass $4 m^2$.

It is quite interesting to note that with the parameters $R_0^2 = 1.5 \pi \text{ GeV}^{-2}$ and $\lambda = 2$ the radius of the lowest state turns out to be

$$R_{10} = R_0 (\pi + \lambda)^{1/2} \simeq 1 F \quad (3.13)$$

which keeps nice contact with reality. Thus we see an intimate connection between realistic Regge trajectories and quite reasonable spatial extensions of hadronic objects.

C. Decoupling the Higher Angular Momentum States - The Firesausage

Our nicely looking linear and parallel Regge trajectories, we have seen, are a direct consequence of the eqs.(3.7) and (3.8), which stipulate that the radii of the space and time "bags" increase linearly with the mass M of the hadronic state.

A little reflection, however, shows that in a geometrical picture of interactions, such as the one we want to develop here, this leads immediately to disaster if we do not legislate away those states for which

$$l \geq l_{\max} = \frac{1}{2} R_{\perp} M, \quad (3.14)$$

where $R_{\perp}$ is some fixed length of the order of 1F.

This restriction is absolutely crucial to ensure a finite range (or at most logarithmically increasing with energy) for the strong interactions.

In fact, if (3.14) were violated, at high energy (E) two hadrons with impact parameter $b \sim R^2 E$ would be able to interact by making a transition into an "allowed quark orbit" with angular momentum $l = R^2 E^2$ which would be sitting on the leading Regge trajectory. (3.14), on the other hand, makes sure that only those collisions take place which have a finite impact parameter. Any realistic model must embody such cut-off and, as it is well known, a particularly interesting example of this is the Veneziano model where the states with $l \geq l_{\max}$ are essentially decoupled in two-body scattering. A similar situation seems to occur in the MIT bag model [4]; by use of the virial theorem one gets for large mass

$$\frac{M}{2} \sim BV \simeq \frac{\pi}{2} B R_1^2 R_1 \quad (3.15),$$

where the classical volume spanned by the quarks has been taken as a cylinder of radius $\frac{1}{2} R_1$ and height $2R_1$. Thus we see that the only way R can increase linearly with M is to have a constant "transverse dimension" R_1 .

(3.14) gives us the possibility of picturing high mass mesonic states in a very suggestive way. At very high mass the spectrum equation becomes:

$$M_{n\ell}^2 \rightarrow \frac{4\pi}{R_0^2} (n + \ell/2) \quad (3.16),$$

which implies a set of $(\frac{1}{2} \max)^2$ degenerate states (counting obviously the different polarizations of the spinning systems) where $l_{\max}$ is given by (3.14). Out of those states, characterized by n , l and m , we can construct, by a unitary transformation, an equal number of coherent states characterized by the $(\frac{1}{2} \max)^2$ directions determined by subdividing half spatial hemisphere (2π) into $(\frac{1}{2} \max)^2$ equal portions of solid angle. Given one such direction, which we chose as the z -axis, the highly excited $q\bar{q}$ -orbit spans the spatial region in Fig. 9

Fig. 9: The highly excited $q\bar{q}$ state (Firesausage)

We shall give such structure the suggestive name of "Firesausage". It represents the general and universal structure of the mesonic states which get produced in all kinds of high energy collisions: from hadronic scattering at low transverse momenta to deep inelastic physics and to hadronic scattering at large $p_{\perp}$.

D. The $q\bar{q}$ Wave Function in any Frame

The expression (3.2) we have calculated for the meson wave function only holds in the $q\bar{q}$ rest frame, i.e. for $p \equiv (M, \vec{0})$. In order to have a wave function in any moving frame we need only consider a Lorentz boost in a particular direction which we choose as the z-axis. In this way we obtain $\psi(p, k)$ by substituting in (3.2) for K_0 its expression (3.3), for $|\vec{k}|$ (3.4); for θ_k

$$\bar{\theta}_k = \arccos \left(\frac{Ek_3 - pk_0}{M|\vec{k}|} \right)$$

while φ_k remains the same.

In performing calculations a convenient form for wave functions with mass $\gtrsim 2$ GeV is

$$\psi_{n_l}(p, k) \simeq (2\pi)^2 \frac{2\pi}{R^2} \delta_{R^2(\frac{m_1^2+m_2^2}{2}-m^2)} \delta_{R^2(\frac{m_1^2-m_2^2}{2})} Y_{\ell}^{\hbar}(\bar{\theta}_k, \varphi_k) \quad (3.17)$$

where $M^2 \simeq \frac{2\pi}{R^2} (n + l/2)$, and $\delta_{R^2}(z)$ is a δ -like function whose prototype is

$$\delta_{R^2}(z) = \frac{1}{\pi} \frac{\sin R^2 z}{z}$$

Two noticeable properties of $\delta_{R^2}(z)$ are:

$$\int_{-\infty}^{+\infty} dz \delta_{R^2}(z) = 1 \quad (3.18)$$

and

$$\delta_{R^2}(0) = \frac{R^2}{\pi} \quad (3.19).$$

In the limit $R^2 \rightarrow \infty$, $\delta_{R^2}(z)$ obviously tends to a Dirac δ -function. For $M \lesssim 2 \text{ GeV}$ (3.17) becomes a much too imprecise representation and we should rather use (3.2).

4. A GEOMETRICAL DESCRIPTION OF HADRONS AND THEIR INTERACTIONS. THE GRAPHICAL RULES.

In the preceding two sections we have studied a set of simple hadronic states in terms of their quark-like constituents. We now proceed to construct a scheme of couplings among such states.

We shall follow very closely the MQM ideas, and especially point d.) in the first section.

If we forget for the time being about baryons, the coupling scheme amounts to a diagrammatic expansion of the S-matrix which completely parallels that of a $\lambda \phi^3$ theory, provided we make the following identifications (see Fig. 10):

- (i) ϕ -propagator $\leftrightarrow$ $q\bar{q}$ Green's function G ;
- (ii) ϕ^3 -vertex $\leftrightarrow$ irreducible six point vertex V_6 .

Fig. 10: Dictionary leading from $\lambda\phi^3$ to the MQM expansion

To any given order in λ , which should then be a relevant expansion parameter, we can draw the diagrams of $\lambda\phi^3$ and then proceed to substitute the φ -propagators with the $q\bar{q}$ -Green's function and the ϕ^3 -vertex with the irreducible vertex V_6 . Crossing and unitarity will then be implemented order by order in λ , provided both G and V_6 are crossing symmetric.

Thus the whole coupling scheme boils down to determining the structure of G and V_6 and the rules governing their juxtaposition.

A. The $q\bar{q}$ -Green's Function

The Green's function G embodies the space-time propagation properties of a $q\bar{q}$ (qq) system. The simple bag states constructed in Sections 2 and 3 are the obvious candidates to describe such propagation properties. We shall therefore write the crossing symmetric Green's function as

$$G = G_s + G_t \quad (4.1)$$

where G_s is built up out of bag states in the s-channel, and G_t out of bag states in the t-channel (see Fig. 11).

Fig. 11: Construction of the Green's function G

A discussion of the main properties of G is deferred to the next section.

B. The Irreducible Vertex V_6

As already emphasized, in determining the form of V_6 we shall be guided by a geometrical view of hadronic interactions. Let us look at the coupling of three bag states (Fig. 12).

Fig. 12: The coupling of 3 bag states

V_6 gives the probability amplitude for the quarks of one bag, say p_1 , to disappear and to reappear as the constituents of a two bag system, p_2 and p_3 . According to the idea that quarks exist only in small space-time domains associated with zero-triality hadronic states, this amplitude should vanish whenever the space-time regions spanned by the three bags do not overlap. This is achieved if V_6 is given by:

$$V_6(x_1 \dots y_3) = \mu^6 \delta(x_1 - x_2) \delta(y_2 - y_3) \delta(x_3 - y_1) + \text{permutations} \quad (4.2)$$

where μ is a mass parameter, which has been introduced here for dimensional reasons, playing the role of a coupling constant. According to (4.2) the momentum-space diagrams describing the three-bag coupling are depicted in Fig. 13.

Fig. 13: The three-bag coupling diagrams

The calculational rules for such diagrams are given in the next paragraph.

C. Graphical Rules

A general quark diagram can be reduced to a network of quark lines joining quark Green's functions and wave functions in such a way that each quark loop consists of no more and no less than three quark lines. The rules to evaluate any such diagram are the following:

- (i) each incoming bag state of momentum p_i and relative quark momentum k_i is introduced through the wave function

$\psi(p_i, k_i)$, whose explicit expression is given by (3.2) times a suitable normalization factor yet to be determined;

- (ii) each outgoing bag state of momentum p_f and relative quark momentum k_f is given by $\psi^*(p_f, k_f)$;

(iii) each "exchanged" bag state (see Fig. 14) of momentum p_e

Fig. 14: An exchanged "bag" state

and relative quark momentum k_e is given by the $(i)^l \psi(p_e, k_e)$ where l is the angular momentum of the state;

(iv) for each quark line we must multiply by μ^2 ;

(v) four-momentum at each two-quark hadron vertex must be conserved;

(vi) for each loop we have an integration $\int \frac{d^4\ell}{(2\pi)^4}$;

(vii) $q\bar{q}$ Green's functions are totally connected and their discontinuities correspond to physical hadron states;

(viii) any weak and e.m. current is represented by a point-like coupling to a $q\bar{q}$ Green's function (see Fig. 15); the two-quark line loop is calculated without multiplying by the factor μ^2 for each quark line.

Fig. 15: The current $q\bar{q}$ coupling and its $\lambda\phi^3$ analog

Equipped with these rules we can construct amplitudes having any degree of complexity. As remarked, the viability of this approach rests on the possibility of treating μ as a perturbative parameter. We shall

check later that this is indeed the case.

5. MISCELLANEOUS CALCULATIONS

We shall now use the coupling strategy just discussed to perform a few particularly important calculations.

A. The Current Vertex

The current vertex structure determines the normalization of the wave functions, and consequently that of the $q\bar{q}$ Green's functions. To lowest order in μ^2 we must compute the diagram in Fig. 16.

Fig. 16: The lowest order current-particle p vertex

A very interesting and appealing feature of this diagram is that it naturally contains vector meson dominance through the vector meson poles appearing in the Green's function G . Keeping only the ρ intermediate state^(*) the normalized wave function turns out to be

$$\psi_M^{\text{Norm}}(k) \simeq \left(\frac{2mp}{\mu^3} \right)^{1/2} \psi_M(k) \quad (5.1)$$

where $\psi_M(k)$ is given by (3.2).

(*) The following paragraph will, in fact, show that the high mass vector meson contributions are strongly suppressed.

B. The Three-bag Vertex

This calculation is particularly important because it gives us the possibility to determine the decay properties of the $q\bar{q}$ -firesausages and the consequent structure of final states. Following the graphical rules of the preceding section the diagrams in Fig. 13 are easily calculated as

$$\begin{aligned} \mathcal{M}(1 \rightarrow 2+3) = (\mu^2)^3 \int \frac{d^4 k}{(2\pi)^4} \psi_1(p_1, k) \psi_2^*(p_2, \frac{p_3}{2} + k) \psi_3^*(\frac{p_3}{2}, \frac{p_2}{2} + k) \\ + (2 \leftrightarrow 3) \end{aligned} \quad (5.2)$$

On substituting (3.17) for the wave functions we have:

$$\begin{aligned} \mathcal{M}(1 \rightarrow 2+3) = (2m_p \mu)^{3/2} (2\pi)^2 \left(\frac{2\pi}{R^2}\right)^3 \int dk \delta_{R^2}(m_1 k_0) \times \\ \times \delta_{R^2}\left(k^2 - \frac{m_1^2}{4}\right) \delta_{R^2}\left[\frac{m_2^2 + m_3^2 - m_1^2}{4} - m_1 |\vec{p}| \cos \theta_{p\vec{k}}\right] Y_{\ell_1}^{m_1} Y_{\ell_2}^{m_2} Y_{\ell_3}^{m_3} \\ + (2 \leftrightarrow 3) \end{aligned} \quad (5.3)$$

where

$$|\vec{p}| = \frac{1}{2m_1} [(m_1^2 - m_2^2 - m_3^2)^2 - 4m_2^2 m_3^2]^{1/2}$$

is the decay momentum. An analysis of kinematics shows that the peaking of wave functions at small quark masses requires that in the rest frame of p_1 the decay momentum $\vec{p}$ is in the direction of the initial quark relative momentum $\vec{k}$. Thus calling $h_{2,3}$ the helicities of $p_{2,3}$, and integrating the two first δ_{R^2} functions in (5.3) we get

$$\begin{aligned} \mathcal{M}(1 \rightarrow 2+3) \simeq 2 (2m_p \mu)^{3/2} (2\pi)^2 \left(\frac{\pi}{R^2}\right)^3 [(2\ell_2+1)(2\ell_3+1)]^{1/2} \\ \delta_{h_2} \delta_{h_3} Y_{\ell_1}^{m_1}(\theta, \varphi) \int_{-1}^{+1} dz \delta_{R^2}(a-bz)^4 \end{aligned} \quad (5.4)$$

where θ, φ are the polar coordinates of the decay momentum $\vec{p}$, and

$$a = \frac{1}{4} (m_1^2 - m_2^2 - m_3^2) \quad (5.5)$$

$$b = \frac{1}{4} [(m_1^2 - m_2^2 - m_3^2)^2 - 4m_2^2 m_3^2]^{\frac{1}{2}}$$

The z -integral in (5.4) is crucial in determining the decay properties of the excited bag p_1 . It is not difficult to see that the matrix element strongly favors the decay of the highly excited state 1 into a light meson (say the π) and another high mass state. Setting for simplicity $m_1^2 = m^2$, $m_2 = 0$ and $m_3^2 = m'^2$ we have

$$\begin{aligned} \int_{R^2} (m^2, m'^2) &= \left(\frac{\pi}{R^2}\right)^3 \int_{-1}^{+1} dz \delta_{R^2}(a-bz)^4 \\ &= \frac{4}{\pi} \frac{1}{m^2 - m'^2} \int_0^{\frac{R^2}{2}(m^2 - m'^2)} d\xi \frac{\sin^4 \xi}{\xi^4} \end{aligned} \quad (5.6)$$

where we have explicitly substituted the prototype expression $\frac{1}{\pi} \frac{\sin R^2 z}{z}$ for the $\delta_{R^2}(z)$ function.

Introducing the variable $x = \frac{m'^2}{m^2}$, a plot of (5.6) for $m^2 = 10, 20$, and 30 GeV^2 and for $R^2 \simeq \pi \text{ GeV}^{-2}$ is in Fig. 17.

Fig. 17: The function $f_{R2}(m^2, m'^2)$ for 3 different values of the decaying mass

In order to get a feeling about the magnitude of μ^2 we compute the width for the decay of the state with $\ell = 1$ (which we may call ρ , with mass m_ρ) into two $\ell = 0$ states (π , with mass m_π). Setting $\Gamma(\rho \rightarrow 2\pi) \simeq 150 \text{ Mev}$, we estimate $\mu \sim 300 \text{ Mev}$, a quite small value when compared with a typical hadronic mass.

The results of this paragraph will constitute the starting point for the analysis of final states in hadronic collisions, which will be performed in Section 6.

C. The Green's Function G_s at High Energy

We shall now compute the high energy behaviour of the G_s contribution to the $q\bar{q}$ (qq) Green's function (see Fig. 11). Writing down

$$G_s^{(0)} = \sum_{n\ell} \frac{1}{s - m_{n\ell}^2} \psi_{n\ell}(p, k) \psi_{n\ell}^*(p, k) \quad (5.7)$$

$G_s^{(0)}$ exhibits an infinite number of poles in the s -channel, corresponding to stable states. The decay mechanism just discussed will shift these poles onto the second sheet by an amount proportional to the quantity $M_{n\ell} \Gamma_{n\ell}$. Anticipating a result of Section 6, we have asymptotically

$$M_{n\ell} \Gamma_{n\ell} \rightarrow \lambda \frac{2\pi}{R^2} \quad (5.8)$$

where λ is a number of order 1 independent of the particular state under consideration. To take the width into account we shall proceed as follows:

We take the asymptotic expression (3.17) for $\psi_{n\ell}(p, k)$ and write:

$$G_s \simeq \delta_{R^2} \left(\frac{m_1^2 + m_2^2}{2} - m^2 \right) \delta_{R^2} \left(\frac{m_1'^2 + m_2'^2}{2} - m^2 \right) \delta_{R^2} \left(\frac{m_1^2 - m_2^2}{2} \right) \delta_{R^2} \left(\frac{m_1'^2 - m_2'^2}{2} \right) G_s(s, t) \quad (5.9)$$

where we expand in partial waves

$$G_s(s, t) = \sum_{\ell=0}^{\infty} (2\ell+1) a_{\ell}(s) P_{\ell}(\cos \theta_s) \quad (5.10)$$

and

$$a_{\ell}(s) = (2\pi)^3 \frac{4m_f}{\mu^3} \left(\frac{1}{R^2} \right)^2 \sum_{n_0}^{\infty} \frac{1}{s - m_{n_0}^2 + i(MP)_{n_0}} \quad (5.11).$$

In (5.11) $n_0 \simeq 2/\pi R^2/R_1^2 \ell^2$ is the minimum n compatible with the condition (3.14). The meaning of (5.11) should now be clear:

G_s propagates the bag states independently. By making use of (5.8) and

the asymptotic expression

$$m_{n\ell}^2 \rightarrow \frac{2\pi}{R^2} (n + \ell/2)$$

(5.11) becomes:

$$Q_\ell(s) \approx (2\pi)^3 \frac{4m_p}{\mu^3} \left(\frac{\pi}{R^2}\right)^2 \sum_{n=0}^{\infty} \frac{1}{s - \frac{2\pi}{R^2}(n + \ell/2) + i\lambda \frac{2\pi}{R^2}} \quad (5.12)$$

It is immediate to check that (5.12) does not converge, its imaginary part, however, does converge and we easily obtain the asymptotic result:

$$\text{Im} Q_\ell(s) \rightarrow (2\pi)^3 \frac{4m_p}{\mu^3} \left(\frac{\pi}{R^2}\right)^2 \frac{R^2}{2} \theta\left(s - \frac{4\ell^2}{R_1^2}\right).$$

Performing the partial wave sum, we obtain:

$$\text{Im} G_s(s, t) \rightarrow (2\pi)^3 \frac{4m_p}{\mu^3} \frac{\pi}{2R^2} \sum_{\ell=0}^{\frac{1}{2}R_1\sqrt{s}} (2\ell+1) P_\ell(\cos\theta_s) \quad (5.13)$$

and using the result:

$$\sum_{\ell=0}^{\frac{1}{2}R_1\sqrt{s}} (2\ell+1) P_\ell(\cos\theta_s) \rightarrow \frac{sR_1^2}{2} \frac{J_1(R_1\sqrt{-t})}{R_1\sqrt{-t}} \quad (5.14)$$

we get

$$\text{Im} G_s(s, t) \rightarrow (2\pi)^3 \frac{4m_p}{\mu^3} \frac{\pi}{4R^2} R_1^2 s \frac{J_1(R_1\sqrt{-t})}{R_1\sqrt{-t}} \quad (5.15)$$

By putting (5.15) in a subtracted dispersion relation, we finally obtain

$$G_s(s, t) \rightarrow (2\pi)^3 \frac{4m_p}{\mu^3} \frac{\pi}{4R^2} \frac{R_1^2 s}{\pi} \left[\log(-s + i\epsilon) + c \right]. \quad (5.16)$$

(5.16) is very good news indeed. It provides for the existence of the "primeval Pomeron" [2] which in the MQM framework plays a fundamental role in ensuring the validity of scaling and Current Algebra. The structure of $G_s(s,t)$ in the J-plane is thus a fixed pole at $J = 1$. The t-dependence of (5.7) is typical of a grey disc of diameter $R_\perp$.

D. The G_t Contribution and the Regge States

As discussed in Section 4 crossing requires the Green's function G to receive a contribution from the bag states in the t-channel. Proceeding as in the previous paragraph we have

$$G_t \simeq \delta_{R^2} \left(\frac{m_1^2 + m_1'^2}{2} - m^2 \right) \delta_{R^2} \left(\frac{m_2^2 + m_2'^2}{2} - m^2 \right) \delta_{R^2} \left(\frac{m_1^2 - m_1'^2}{2} \right) \delta_{R^2} \left(\frac{m_2^2 - m_2'^2}{2} \right) G_t(s,t) \quad (5.17)$$

where

$$G_t(s,t) = \frac{4m_P}{\mu^3} (2\pi)^3 \left(\frac{\pi}{R^2} \right)^2 \sum_{n\ell} \frac{(2\ell+1) P_\ell(-\cos\theta_t)}{t - m_{n\ell}^2} \quad (5.18)$$

where $\cos \theta_t = 1 + 2s/t - 4m^2$. Notice the minus sign in front of $\cos \theta_t$ in (5.1); it arises from the rule (iii) in Section 4.

We want to determine the behaviour of (5.18) when s is large. We make a Sommerfeld-Watson transform

$$\sum_{n\ell} (2\ell+1) P_\ell(-\cos\theta_t) \frac{1}{t - m_{n\ell}^2} = \frac{1}{2i} \sum_n \int_C \frac{d\ell (2\ell+1)}{\sin \pi \ell} P_\ell(\cos\theta_t) \frac{1}{t - m_n^2(\ell)} \quad (5.19)$$

where C is the usual contour encircling clockwise the positive real axis. Defining the trajectory function $\alpha_n(t)$ as the solution of

$$t = m_n^2(\alpha(t))$$

we get

$$G_t(s,t) = \frac{4m_P}{\mu^3} (2\pi)^3 \left(\frac{\pi}{R^2} \right)^2 \sum_{n=1}^{\infty} \frac{\pi \alpha_n'(t) (2\alpha_n(t)+1)}{\sin \pi \alpha_n(t)} P_{\alpha_n(t)} \left(1 + \frac{2s}{t - 4m^2} \right) \quad (5.20)$$

Using the asymptotic expansion of the Legendre function $P_{\alpha_n(t)}(x)$, we have the behaviour:

(5.21)

$$G_t(s,t) \xrightarrow{s \text{ large}} \frac{4mp}{\mu^3} (2\pi)^3 \left(\frac{\pi}{R^2}\right)^2 \frac{\pi \alpha_1(t)}{\sin \pi \alpha_1(t)} (2\alpha_1(t)+1) \frac{\Gamma(1+2\alpha_1(t))}{\Gamma(1+\alpha_1(t))^2} \left(\frac{-s}{4m^2}\right)^{\alpha_1(t)}$$

Thus $G_t(s,t)$ exhibits at high energy Regge behaviour, which is controlled by a set of Regge trajectories $\alpha_n(t)$ which correspond precisely to the bag states. These trajectories are exactly exchange degenerate, a property which is crucial in order to enforce the absence of physical states [discontinuities for $s < 0$], in a channel of non-zero triality. This constitutes a very important check of self-consistency of the whole approach. As a consequence (5.21) has only a right-hand cut corresponding to zero triality states. The question is now how to interpret such "Regge states". As we have seen these states arise by virtue of crossing and should be regarded as genuine physical states of a character different from simple bag states. Their spectrum is continuous and the typical logarithmic Regge shrinkage shows that their "transverse extension" increases logarithmically with energy. This suggests that the discontinuity of $G_t(s,t)$ corresponds to hadrons in a configuration of a multiperipheral kind.

A very important distinction between bag and Regge states is that the former build up diffractive scattering, while the latter generate normal Regge pole behaviour.

6. THE STRUCTURE OF FINAL STATES. SOME IDEAS ABOUT HADRON-HADRON SCATTERING

In this last section we shall explore a little further the structure of final states emerging from the decay of a "firesausage". We shall then conclude with a brief analysis of some interesting aspects of hadron-hadron scattering.

A. The Linear Decay Chain

The results obtained in Section 5 are very important for reaching an understanding of the structure of final states in hadronic collisions. Due to the very substantial enhancement of the decay configuration $M \rightarrow M' + \pi^{(*)}$ with M' close in mass to M , to a good approximation the final states evolve through a linear decay chain as depicted in Fig. 18.

Fig. 18: The linear decay chain of an excited $q\bar{q}$ system

Such picture implies that a $q\bar{q}$ system, moving on a highly excited orbit, gets de-excited through the emission of pions while cascading down through a series of states of decreasing mass. This resembles very much the cascade decay of an excited atom through photon emission. The final states produced by this cascade will then be composed of a number of pions distributed in rapidity and transverse momentum in a characteristic way which shall be described shortly.

If we want to compare this picture with the usual quark-parton description, we notice the quite different meaning the notion of " $q\bar{q}$ sea" acquires here. Such sea is represented by a large number of low momentum pions and arises as the product of a violent collision which brings the initial hadronic system in a highly excited state.

Taking up the atomic physics analogy again, it does not appear any more useful to speak about the " $q\bar{q}$ sea" as a property of the initial low

(*) Actually other low mass bags like k , ρ , etc., do not give totally negligible contributions, and should be also considered.

mass hadrons, than to try to include the "photon sea" in the wave function of the atomic ground state.

The linear decay chain problem can be solved by means of the integral equations reproduced in Fig. 19.

Fig. 19: The integral equations governing the decay of highly excited $q\bar{q}$ states

Here I only summarize the results:

(i) if we call $\Gamma(M)$ the width of an excited state of Mass M , asymptotically we get

$$M\Gamma(M) \rightarrow \lambda \frac{2\pi}{R^2} \quad (6.1)$$

where λ can be explicitly computed in terms of the parameters μ and R^2 , and in this scalar model turns out to be of order unity (3), (7). This property of λ implies that asymptotically the Green's function is smooth due to the overlap of neighbouring resonances. The spacing between neighbouring resonances, in fact, is given by

$$2M\Delta M \rightarrow \frac{2\pi}{R^2};$$

(ii) The one-particle yield exhibits Feynman scaling;

(iii) for small x the inclusive distribution shows a plateau,

thus giving rise to a multiplicity increasing logarithmically with s ,

$$\langle n \rangle \approx c \log s \quad (6.2),$$

for the charged multiplicity we calculate $c_{ch} \approx 1$;

(iv) there is no correlation between impact parameter and rapidity of secondary pions, and the transverse momentum is cut-off, i.e.

$$\langle k_{\perp}^2 \rangle \approx \frac{\pi}{R_{\pi}^2} \approx .1 \text{ GeV}^2, \quad (6.3)$$

(v) the π 's are emitted independently, thus following a Poisson distribution.

It is almost superfluous to emphasize that these results look quite close to what is going on in hadronic final states.

B. Hadronic Scattering at High Energies

We are not yet in a position to perform a realistic calculation of high energy scattering due to our lack of knowledge about baryonic states. However, we can try and see whether the theory developed in Section 4 can reproduce the correct qualitative features of high energy hadron-hadron scattering.

The lowest order diagram in the perturbative approach is the "Born" diagram in Fig. 20.

Fig. 20: Hadron-hadron scattering diagrams to lowest order

It is easy to see that both diagrams become irrelevant at high energy. Diagram (a) controls resonance formation at low energy but at high energy is suppressed by kinematics and it behaves like $\frac{1}{s}$. Diagram (b) yields a real contribution in the s-channel and at high energy, and small t is dominated by π -exchange. Thus in lowest order we can only describe low energy resonance formation and high energy π -exchange. In order to get the diagrams at high energy we must go to 4th order in λ . The relevant diagrams appear in Fig. 21

Fig. 21: The 4th order contribution to 2-body scattering

A slightly involved analysis shows that at high energy the complicated diagram in Fig. 21 becomes proportional to the one appearing in Fig. 22. One should remember, however, that such diagram is not a legal, quark diagram as it violates the graphical rules set up in Section 4.C.

Fig. 22: The "illegal" diagram equivalent to the one in Fig. 21

This diagram is calculated very easily by means of Sudakov techniques and yields at high energy a purely imaginary amplitude increasing like s .

This is the "Pomeron", which at this order appears as a fixed pole at $J = 1$. The secondary pions emitted through firesausage decay obey scaling, and their density in rapidity is twice (notice the two firesausages) the value in a single firesausage decay. Whereas a fixed pole at $J = 1$ can be perfectly tolerated in $q\bar{q}$ -scattering (no two-body unitarity is present at the quark level), such a J -plane structure is incompatible with unitarity at hadronic level. A straightforward way to implement it is to sum the "multiperipheral diagrams" of the type in Fig. 23.

Fig. 23: Implementing t-channel unitarity

This calculation was done sometime ago in the framework of the MQM by Craigie and myself [8], and these are the results:

- (i) The $J = 1$ fixed pole gets shifted up by approximately 10%

$$\alpha_{\mathbb{P}}(0) = 1 + \epsilon \quad (6.4)$$

$$\epsilon \simeq .06$$

- (ii) It acquires a slope, which was computed as

$$\alpha'_{\mathbb{P}}(0) \simeq .3 \text{ GeV}^{-2} \quad (6.5)$$

Again, it is almost superfluous to stress that (6.4) and (6.5) are in good shape as far as the ISR data are concerned. As a matter of principle, however, (6.4) violates the Froissart bound, and one should analyse the various absorption corrections which must bring the cross section to finally obey the Froissart bound. The smallness of ϵ , on the other hand, phenomenologically postpones to very high energy the need to consider such corrections^(*).

EPILOGUE

What has been presented in these lectures is an attempt to develop the MQM into a tool as effective as possible to describe hadrodynamics. Except for the general formulation of the approach in Section 4, the attention has been concentrated on the dynamics of unrealistic scalar and neutral quarks. This was done in order to be able to determine as quickly as possible the viability of the whole approach. I believe that abundant evidence has piled up that the direction followed here is worth pursuing.

Some preliminary results on spinning and charged quarks appear to strengthen the case for the approach advocated in these lectures [5].

(*) Some interesting investigations have been recently carried out by Amati, Caneschi and Jengo, CERN preprints (1975), on the very high energy renormalization of an input pole for $J > 1$.

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REFERENCES

- [1] K. Wilson: 1975 Erice Lectures;
- [2] G. Preparata: A Possible Way to Look at Hadrons: The MQM.
Lectures given at the XII. Course of the International School
of Subnuclear Physics "E. Majorana", A. Zichichi ed. (1974);
- [3] G. Preparata and N.S. Craigie: A Space-time Description of
Quarks and Hadrons. CERN TH.2038 (to appear in Nuclear
Phys. B), (1975);
- [4] A. Chodos, R.L. Jaffe, K. Johnson, C.B. Thorn and
V.F. Weisskopf: Phys. Rev. D9, 3471 (1974);
- [5] G. Preparata: (to be published);
- [6] P. Menotti: in Subnuclear Phenomena. Proceedings of the
1969 Erice Summer School, A. Zichichi ed., Academic
Press (1971);
- [7] N.S. Craigie and G. Preparata: The Structure of Final States
in a Space-time Description of Quark and Extended Hadrons.
CERN TH.2056 (to appear in Nuclear Phys. B), (1975);
- [8] N.S. Craigie and G. Preparata: Phys. Letters 52B, 84 (1974).