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31 JUL. 1979

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ACADEMY OF SCIENCES OF THE USSR
P. N. LEBEDEV PHYSICAL INSTITUTE

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I.E.Tamm Department of Theoretical Physics

High energy and cosmic rays physics

Preprint No 80

PHASE TRANSITION IN QUARK-GLUON
PLASMA

O.K.Kalashnikov and V.V.Klimev

Moscow, 1979

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CM-P00067539

At present owing to the discovery of the asymptotic freedom [1] quantum chromodynamics (QCD) with finite temperatures and densities attracted attention of a number of authors [2]. As a result of these investigations a quite plausible idea has been formed within QCD about the possibility of a phase transition in quark-gluon plasma that leads to the gas of ordinary hadrons. Following this idea we are presenting in this paper a number of constructive arguments in favour of the existence of such a phase transition and are specifying its domain of existence. Our conclusions are based on investigations of the thermodynamical potential of this system.

The partition function which defines the thermodynamical properties of the quark-gluon plasma within QCD is presented here through the function ^{cal}integral of the usual form

$$Z = N \int d\psi d\bar{\psi} dV \exp(+S) \quad (1)$$

where S is the known expression for the Euclidean QCD action

$$S = \int d^4x \int_0^{\beta} dx_4 \mathcal{L}(\psi, \bar{\psi}; V); \quad (2)$$

$$\mathcal{L} = -\frac{1}{4}(G_{\mu\nu}^a)^2 + \frac{1}{2\alpha}(\partial_\mu V_\mu^a)^2 + \bar{C}^a(\partial_\mu \delta^{ab} + g f^{acb} V_\mu^c)\partial_\mu C^b + \sum_{f=1}^{N_f} \bar{\psi} [\gamma_\mu(\partial_\mu - i g \frac{\lambda}{2} V_\mu^a) + m_f] \psi - \mu \sum_{f=1}^{N_f} \bar{\psi} \psi.$$

The designations used in eq. (2) are usual: $G_{\mu\nu}^a = \partial_\mu V_\nu^a - \partial_\nu V_\mu^a + g f^{abc} V_\mu^b V_\nu^c$ is the Yang-Mills field strength; N_f is the number of "flavours"; μ is a chemical potential to the baryon charge density; $\lambda/2$ are generators of $SU(3)$ group.

For the thermodynamic potential

$$\Omega = -T \ln Z \quad (3)$$

the calculations are carried out by the usual methods [3], following at the first stage the ordinary perturbation theory. In doing so within Eq. (2), we may easily obtain that the first non-trivial approximation is described here by the following graphs

$$\frac{\Omega}{V}^{(1)} = \frac{1}{12} \text{ (gluon loop) } + \frac{1}{8} \text{ (quark loop) } - \frac{1}{2} \text{ (gluon tadpole) } - \frac{1}{2} \text{ (quark tadpole) } \quad (4)$$

for the first three of which the Yang-Mills fields are responsible, and the last one takes into account the contribution of quarks.

It is found that the result of the calculation of these graphs is gauge-invariant, and when the quark masses being neglected, it has the following form

$$\frac{\Omega}{V} = -\frac{8\pi^2 T^4}{45} - 6N_f T \int \frac{d^3k}{(2\pi)^3} \ln \left[(1 + e^{-\beta(k-\mu)}) (1 + e^{-\beta(k+\mu)}) \right] + \\ + \frac{g^2 T^4}{8} + 16N_f g^2 \left[\frac{T^2}{12} \int \frac{d^3k}{(2\pi)^3} \frac{\eta_k}{2k} + \frac{1}{2} \left(\int \frac{d^3k}{(2\pi)^3} \frac{\eta_k}{2k} \right)^2 \right] \quad (5)$$

where $\eta_k = (\exp(\beta(k-\mu)) + 1)^{-1} + (\exp(\beta(k+\mu)) + 1)^{-1}$.

However, it is essential that expression (5) can be specified here both in high-temperature and in high-density regions. This is possible because this theory is asymptotically free and the renormalization group equations for an effective charge hold in these regions. In practice doing here within approximation (4), after having expanded expression (5) in the small parameter which corresponds to

these regions, we obtain the final result replacing in the latter g by $g_{eff}(T)$ at high temperatures or by $g_{eff}(\mu)$ at high densities.

We shall begin with the high temperature region. Here $\mu/T \ll 1$ and expression (5) expanded in this parameter should be represented as follows.

$$\begin{aligned} \Phi = & -\frac{g^2 T^4}{45} - N_f \left[\frac{T^2 \mu^2}{2} + \frac{7}{60} T^4 \right] \\ & + g^2 \left[\frac{T^4}{8} + N_f \left(\frac{5}{72} T^4 + \frac{T^2 \mu^2}{24} \right) \right] \end{aligned} \quad (6)$$

Account taken of the leading contributions from higher order diagrams by the renormalization group method results in the fact that the coupling constant in Eq. (6) should be replaced by an effective one according to the ansatz presented below

$$\frac{g^2}{4\pi} \rightarrow \frac{g^2(T)}{4\pi} = \frac{1}{1 + 6.6 \ln T/S_T} \quad (7)$$

In Eq. (7) $S_T = \left(\frac{1}{2} - \frac{1}{6} \right) \frac{1}{\beta}$ - is the coefficient in the β -function calculated by usual formulas [4].

S_T is a normalization point.

Thus, using (7) and introducing g_{eff} , expression (6) for the thermodynamic potential of quark-gluon plasma in a high-temperature region should be finally transformed here to the following form

$$-p = \Phi = -\frac{g^2 T^4}{45} - N_f \left[\frac{T^2 \mu^2}{2} + \frac{7}{60} T^4 \right] \quad (8)$$

$$+ 3\pi^2 d_{\text{eff}}(T) \left[\frac{T^4}{6} + N_f \left(\frac{5T^4}{72} + \frac{T^2 \mu^2}{4\pi^2} \right) \right]$$

In eq. (8) p is pressure, d_{eff} is an effective expansion parameter for $SU(N)_{\text{group}}$ [5]

$$d_{\text{eff}}(T) = \frac{1}{4\pi} \cdot \frac{1}{\pi} \cdot \frac{N^2 - 1}{2N} \quad (9)$$

the introduction of which into (8), where $N = 3$, is very essential for a correct interpretation of the results obtained.

Now, according to (8), we see that, as temperature decreases, on the curve $\mu_c = \mu_c(T)$ in the vicinity of the point T_0 the pressure becomes zero and, after all, as temperature goes on next decreasing, the pressure becomes negative. This fact means, in our opinion, that quark-gluon plasma becomes unstable under such conditions; and a phase transition of quark-gluon plasma into a hadron phase must take place. In the vicinity of this point we find, following eq. (8), that the curve $\mu_c = \mu_c(T)$ is determined by the simple condition

$$\mu_c^2 = \frac{15\pi^4}{N_f} \frac{(1 + \frac{5}{12} N_f)^2}{7 + N_f} [d_{\text{eff}}(T) - d_{\text{eff}}(T_0)]^{(10)}$$

where $d_{\text{eff}}(T_0) = \frac{1}{3} \left(\frac{1}{18} + \frac{5N_f}{48} \right) \left(1 + \frac{5}{12} N_f \right)^{-1}$. Under $N_f = 3$ we have $d_{\text{eff}}(T_0) \approx 0.47$ and

$$\mu_c(T) \approx 4.3 \sqrt{T_0(T_0 - T)} \quad (11)$$

The point T_0 is found according to (7) at $d_{\text{eff}}(T_0) \approx 0.47$. Here $T_0 \approx 0.94 S_{\pi}$. To estimate T_0 one can use $S_{\pi} \approx 300 \text{ Mev}$ [6]

and obtain $T_0 \approx 280$ Mev under this condition.

Such like situation arises also in a high-density region. However the renormalization group should be used here for scaling with respect to μ , and the leading terms of the expansion in the parameter $T/\mu \ll 1$ should be left in expression (5). The result thus obtained is as follows

$$\frac{\mathcal{Q}}{V} = -p = -N_f \left[\frac{\mu^4}{4\pi^2} + \frac{\mu^2 T^2}{2} \right] \left(1 - \frac{2}{3} \alpha_{\text{eff}}(\mu) \right) \quad (12)$$

where $\alpha_{\text{eff}}(\mu) = \frac{4}{3\pi} (1 + 8.0 \ln \mu/S_0)^{-1}$

According to Eq. (12) we see that at $\alpha_{\text{eff}} = 2/3$ the pressure becomes negative again, i.e. at lower densities the system proves to be unstable and a first-order phase transition to the hadron matter takes place. Here

$$n_0 = -\frac{1}{3} \frac{1}{V} \frac{\partial \mathcal{Q}}{\partial \mu} = N_f \frac{3\mu_c^3(0)}{16\pi^2} \quad (13)$$

For the estimation one may consider that $S_0 \approx 300$ Mev also. Then $n_0 \approx 0.1 \left(\frac{n}{\text{fm}^3} \right)$ what is of the order of the nuclear ($n_n \sim 0.17 \text{ n}/(\text{fm}^3)$). From Eq. (12) we also see that taking account of the first surviving terms in the expansion of expression (5) in T/μ does not lead to an alteration of μ_c . By our estimates it is most probable that the deviation of the curve $\mu_c(T)$ from $\mu_c(0)$ will be exponentially small in the vicinity of $T=0$.

In the region of intermediate temperatures and densities in all probability a gradual decrease of $\mu_c(T)$ up to zero takes place at the point $T=T_0$, i.e. as temperature increases the phase transition proceeds at lower

baryon densities since this density increase effectively due to quark pair production.

Thus, the curve of $\mu_c = \mu_c(T)$ will likely look here as follows

Fig. 1

In doing so we preserve the asymptotical behaviour of $\mu_c(T)$ at extreme points as it has been found before and approximate the curve $\mu_c(T)$ in rest range by the one determined implicitly from the simple equation

$$\frac{\mu_c(T)}{\mu_c(0)} = \ln \frac{T_0}{T} \frac{\mu_c(T)}{\mu_c(0)} \quad (14)$$

Here in order that Eq. (11) be satisfied it is necessary to put in Eq. (14) $\mu_c(0)/T_0 \approx 2.6$. On the other hand we have found that $\mu_c(0)/T_0 \approx 0.8 S_D/S_T$, i.e. the above estimation, where $S_D = S_T$ are rather relative if, of course, one trusts the functional form of the chosen curve. In any case it seems to us that there is no particular evidence in favour of the fact that $S_D = S_T$ and, in all probability, they must be actually different. Verification of $\frac{S_D}{S_T} \approx 3$ may in our opinion be of interest by itself.

Thus we have found that under extreme conditions (viz. in the way rare plasma with the temperatures about $T_c \approx 280$ Mev or in the cold plasma with the density of the order of that of the nuclei $n_c \approx 0.1 \text{ fm}^{-3}$) the quark-gluon plasma must undergo the first-order phase transition resulting in its condensation into hadronic matter. In these domains our calculations pretend to be sufficiently rigorous, although the effective expansion parameter is still rather large ($d_{eff} \approx 0.47$ at $T \sim T_c$ and $d_{eff} \approx 0.67$ at $\mu \sim \mu_c$). We believe, nevertheless, that the phase transition domain as pointed here is accurate and the estimates found may be used as starting points for further calculations. We have also carried out the preliminary calculations of the radiative corrections to the thermodynamic potential whose appearance is due to the summation of the leading infra-red divergencies in the higher approximations. Moreover we have found here that this does not change qualitatively the results obtained by us. As for the kinetics of the phase transition the situation is quite unclear yet and one may think that the true phase transition, due to the thermodynamical fluctuations, occurs somewhat earlier, the collapse being prevented.

In conclusion the authors wish to express their gratitude to Prof. E.S.Fradkin and Prof.D.A.Kirzhnits for useful advices and fruitful discussions.

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Т - 10213

Подписано в печать 15 мая 1979 года

Заказ № 240. Тираж 100 экз. л/л 0,45

Отпечатано в Редакционно-издательском
отделе ФИАН СССР

Москва, В-312, Ленинский проспект, 53