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Relativistic Glauber Amplitudes for Elastic and Inelastic
Scattering of Electrons and Positrons by Hydrogen Atoms and
Hydrogen-like Ions

by

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Abstract

A relativistic spin-dependent amplitude for elastic and inelastic scattering of electrons and positrons by hydrogen atoms and hydrogen-like ions is derived in the straight-line Glauber approximation, without exchange terms. Relativistic effects within the target have not been included. The results are given in closed form in terms of hypergeometric functions. Numerical results for the excitation to the $n=2$ levels are compared with existing experimental data.

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1. Introduction

Since it was first introduced in atomic physics by Franco [1] the Glauber approximation has had a wide variety of applications to electron-atom collisions. The reader is referred to the articles by Gerjuoy and Thomas [2], and Byron and Joachain [3], and to the recent and detailed review by Chan et al. [4] for references to and discussions of such calculations.

In this paper we are concerned with scattering at high projectile energies, where exchange effects are unimportant but relativistic corrections to the Coulomb-scattering amplitude may become appreciable. In order to obtain such corrections we use a Glauber formalism previously employed by one of the authors [5] for elastic scattering, using the relativistic small-angle amplitudes for the scattering of the projectile on the individual scatterers in the composite target as input, rather than the potential between the projectile and the scatterers. Relativistic effects within the target have not been included in the present calculations, i.e. we have used non-relativistic electron wave functions. We limit ourselves to excitation of the n_s and n_p levels of hydrogen atoms or hydrogen-like ions.

The inelastic non-relativistic ($1s \rightarrow n\ell$) Glauber amplitudes for the one-electron system have already been calculated by Thomas and Franco [6], who expressed the results in terms of Meijer's G -functions. When the target is atomic hydrogen these amplitudes can be reduced to those obtained

by Thomas and Gerjuoy [7], the amplitudes are there written as sums of hypergeometric functions. We find below that even for the relativistic case the $(1s \rightarrow ns, np)$ amplitudes can be expressed as sums of hypergeometric functions, for scattering on hydrogen-like ions as well as on hydrogen atoms. The same conclusion was drawn in Ref. [5] for the elastic case.

2. Relativistic scattering amplitudes for a one-electron atom

Following Ref. [5] we write the operator for scattering of the incoming electron or positron (charge $\underline{Z}_1 e$) on the one-electron target (nuclear charge $\underline{Z}_2 e$) as

$$\begin{aligned} \mathcal{F}(\underline{q}) = & -(2\pi)^{-1} i k \int d^2 \underline{b} \exp(i \underline{q} \cdot \underline{b}) d^3 r \psi_f^*(\underline{r}) \\ & \times \frac{1}{2} [S(\eta, \underline{b}) S(-\eta_1, \underline{b}') + S(-\eta_1, \underline{b}') S(\eta, \underline{b})] \psi_i(\underline{r}). \end{aligned} \quad (1)$$

The charge-velocity parameters are $\eta = \underline{Z}_1 \underline{Z}_2 \alpha / \beta$ (projectile-nucleus scattering) and $\eta_1 = \underline{Z}_1 \alpha / \beta$ (projectile-electron scattering), where $\alpha = 1/137$ and β is the projectile velocity. We use units for which $\hbar = c = 1$ throughout. The impact parameter $\underline{b}$ and momentum transfer $\underline{q} = \underline{k}_i - \underline{k}_f$ are located in the plane perpendicular to the $\underline{z}$ direction $\underline{k}_i + \underline{k}_f$, in which $\underline{k}_i$ and $\underline{k}_f$ are the initial and final projectile momenta (we take $|\underline{k}_i| = |\underline{k}_f| = k$). The initial and final electron wave functions are ψ_i and ψ_f , and $\underline{b}' = \underline{b} - \underline{s}$, the electron coordinate being $\underline{r} = (\underline{s}, \underline{z})$.

The operator $\underline{S}$ appearing in Eq. (1) is the spin-dependent phase-shift function for relativistic Glauber scattering in a Coulomb field and can be written

$$S(\eta, \underline{b}) = S_0(\eta, \underline{b}) + (\underline{\sigma} \times \underline{e}_z) \cdot \underline{\hat{b}} S_1(\eta, \underline{b}). \quad (2)$$

The spin operator is here that of the projectile, and the various contributions to $\underline{S}_0$ and $\underline{S}_1$ are given in Eq. (11) of Ref. [5].

Our procedure for calculating the scattering operator (1) is in principle the same as the one used previously^a [5] for the elastic case. The electron $\underline{z}$ coordinate is integrated out first, and the remaining integrals over $\underline{b}$ and $\underline{s}$ lead to expressions that are similar to but more complicated than the ones encountered for elastic scattering. In analogy with Ref. [5] we find it convenient to write the resulting scattering operator as

$$\begin{aligned} \mathcal{F}(\underline{q}) &= c(\eta, \underline{x}) \exp[i\phi(\eta_1)] C(\eta_1) [\Gamma(\zeta) + iG(\zeta) \underline{\sigma} \cdot \underline{n}] \exp[-im\phi_q] \\ c(\eta, \underline{x}) &= -2k^{-1} \eta [\Gamma(1+i\eta)/\Gamma(1-i\eta)] (\tfrac{1}{2}x)^{-2i\eta} x^{-2}, \\ \exp[i\phi(\eta_1)] &= \zeta^{i\eta_1} [\Gamma(1-i\eta_1)/\Gamma(1+i\eta_1)] (\tfrac{1}{2}x)^{2i\eta_1}, \\ C(\eta_1) &= |\Gamma(1 \pm i\eta_1)|^2, \quad x = q/k. \end{aligned} \quad (3)$$

The new variable is $\zeta = \lambda \underline{q}^{-2}$, $\lambda = (1 + \underline{n}^{-1})/\underline{a}$, where $\underline{a}$ is the Bohr radius corresponding to the nuclear charge Z_2 , $\underline{n}$ is the principal quantum number of the final electron state, and the unit vector $\underline{n}$ is the normal to the scattering plane.

The factors in front are the Rutherford amplitude $\underline{c}(\eta, \underline{x})$ for pure projectile-nucleus scattering, a phase factor $\exp[i\phi(\eta)]$, and a Coulomb reduction factor $\underline{C}(\eta_1)$. Furthermore, the angle ϕ_q indicates the direction of $\underline{q}$ relative to an arbitrary $\underline{x}$ - axis, $\underline{m}$ being the magnetic quantum number of the final electron state.

Consistency with the input amplitudes for relativistic two-body Coulomb scattering requires that we drop all terms that are of higher order than unity in the fine-structure constant and of higher order than two in the momentum transfer. This corresponds to writing the reduced amplitudes of Eq. (3) as

$$\begin{aligned} \underline{F} &= \underline{F}_0 + \underline{F}_1, \quad \underline{G} = \underline{G}_0 + \underline{G}_1 \\ \underline{F}_0 &= \underline{F}_0^{00} + \underline{F}_0^{02} + \underline{F}_0^{11} + \underline{F}_0^{12}, \quad \underline{F}_1 = \underline{F}_1^{02}, \\ \underline{G}_0 &= \underline{G}_0^{01} + \underline{G}_0^{12}, \quad \underline{G}_1 = \underline{G}_1^{01} + \underline{G}_1^{12}, \end{aligned} \quad (4)$$

where the upper indices indicate the power of $\underline{\alpha}$ and the power of $\underline{x}$, respectively. The lower index is characteristic of the projectile spin, in that $\underline{F}_0$ describes a transition without spin flip, $\underline{G}_0$ and $\underline{G}_1$ a transition with spin flip in the projectile-nucleus and projectile-electron scattering, respectively, and $\underline{F}_1$ a transition with double spin flip.

3. The (1s>ns) transitions

For a final s state of principal quantum number n the result of the z integration can be written

$$\int_{-\infty}^{\infty} dz \psi_f^*(\underline{r}) \psi_i(\underline{r}) = (2/\pi) a^{-3} n^{-\frac{3}{2}} \times \sum_{j=0}^{n-1} (-n+1)_j [(2)_j j!]^{-1} [2/(na)]^j (-\partial/\partial \lambda)^{j+1} K_0(\lambda s), \quad (5)$$

where $(x)_j$ is Pochhammer's symbol, and K_0 is the zeroth-order modified Bessel function of the second kind.

After the integration over s and b the amplitudes can then be expressed in terms of functions

$$I_{KLMN}^{ns}(\zeta) = X_{KLMN}^{ns}(\zeta) \sum_{j=0}^{n-1} \omega_j^{ns} \zeta^{\frac{1}{2}j} (\zeta^{\frac{1}{2}} \partial/\partial \zeta)^{j+1} \times \{ \zeta^{\frac{1}{2}N-1+i\eta_1} {}_3F_2(M+\frac{1}{2}N+i\eta_1, 1-\mu+i\eta, 1-\nu+i\eta; \frac{1}{2}N+i\eta_1, M+1; -\zeta) \}, \quad (6)$$

where ${}_3F_2$ is a generalized hypergeometric function and

$$\mu = \frac{1}{2}(K+L-N), \quad \nu = \frac{1}{2}(K-L-N),$$

$$\omega_j^{ns} = (-n+1)_j [(2)_j j!]^{-1} [-4/(n+1)]^j,$$

$$X_{KLMN}^{ns}(\zeta) = -in^{\frac{3}{2}} (\frac{1}{2}x)^K X_{KLMN}^n(\zeta),$$

$$X_{KLMN}^n(\zeta) = 2^3 (n+1)^{-3} (M!\eta)^{-1} \zeta^{\frac{3}{2}-i\eta_1} \times [\Gamma(1-\frac{1}{2}N-i\eta_1)/\Gamma(1-i\eta)]^2 [\Gamma(M+\frac{1}{2}N+i\eta_1)/\Gamma(\frac{1}{2}N+i\eta_1)] \times [\Gamma(1-\nu+i\eta)/\Gamma(1+i\eta)] [\Gamma(1-i\eta)/\Gamma(\mu-i\eta)]. \quad (7)$$

For the (1sng) transition the amplitudes are then

$$F_0^{00} = I_{0000}^{ns},$$

$$F_0^{02} = -\xi(\eta^2 I_{2000}^{ns} + \eta_1^2 I_{2002}^{ns}),$$

$$F_0^{11} = \frac{1}{2}\pi i \beta^2 [\eta^2 \gamma(\eta) I_{1000}^{ns} + \eta_1^2 \gamma(-\eta_1) I_{1001}^{ns}],$$

$$F_0^{12} = \frac{1}{2}\pi \beta^2 (\eta^3 I_{2000}^{ns} - \eta_1^3 I_{2002}^{ns}),$$

$$F_1^{02} = \eta \eta_1 \xi^2 (I_{2002}^{ns} - I_{2012}^{ns}),$$

$$G_0^{01} = -i \eta \xi I_{1100}^{ns},$$

$$G_0^{12} = \frac{1}{2}\pi \beta^2 \eta [i \eta (\frac{1}{2} - i \eta) \gamma(\eta) I_{2100}^{ns} + \xi \eta_1^2 \gamma(-\eta_1) I_{2101}^{ns}],$$

$$G_1^{01} = i \eta_1 \xi (I_{1102}^{ns} - I_{1112}^{ns}),$$

$$G_1^{12} = -\frac{1}{2}\pi \beta^2 \eta_1 [\xi \eta^2 \gamma(\eta) (I_{2102}^{ns} - I_{2112}^{ns})$$

$$- i \eta_1 (\frac{1}{2} + i \eta_1) \gamma(-\eta_1) (I_{2103}^{ns} - I_{2112}^{ns})],$$

$$\gamma(\eta) = [\Gamma(1+i\eta)/\Gamma(1-i\eta)] [\Gamma(\frac{1}{2}-i\eta)/\Gamma(\frac{1}{2}+i\eta)],$$

$$\xi = \epsilon^{-1} (\epsilon - 1), \quad (8)$$

where $\epsilon = E/m_e$ is the energy of the projectile in terms of its rest mass.

For elastic scattering (n=1) these results coincide

with those given in Ref. [5]. The non-relativistic scattering amplitude is represented by the F_0^{00} term, which for $n \neq 1$ is in agreement with the result of Thomas and Franco [6].

It should be noted that the amplitude calculated by these authors is not applicable to elastic scattering. For scattering on hydrogen ($n_1 = n$) the non-relativistic term agrees with the result of Thomas and Gerjuoy [7]; note, however, that our n is defined to be positive for positron scattering and thus has the opposite sign of the corresponding quantity in Ref. [7].

4. The ($1s \rightarrow np$) transitions

For a final p state of principal quantum number $m = \pm 1$ the result of the $\underline{z}$ integration is

$$\int_{-\infty}^{\infty} dz \psi_f^+(\underline{r}) \psi_i(\underline{r}) = \pi^{-1} (2/3)^{1/2} a^{-1} n^{-3} [(n+1)!/(n-2)!]^{1/2} \exp(-im\phi_s) \\ \times \sum_{j=0}^{n-2} (-n+2)_j [(4)_j j!]^{-1} [2/(na)]^j (-\partial/\partial \lambda)^{j+1} K_0(\lambda s), \quad (9)$$

where ϕ_s indicates the direction of $\underline{s}$ relative to the arbitrary $\underline{x}$ axis. For $m=0$ the integral is zero due to the odd parity of the final state, and the corresponding scattering operator therefore vanishes.

As expected, the integration over $\underline{s}$ and $\underline{b}$ is somewhat more complicated than for final s states. Instead of the

quantities (6) it now becomes natural to introduce two types of functions, viz.

$$\begin{aligned}
 I_{KLMN}^{np}(\zeta) &= X_{KLMN}^{np}(\zeta) \sum_{j=0}^{n-2} \omega_j^{np} (\zeta^{\frac{1}{2}} \partial / \partial \zeta)^{j+1} \\
 &\times \{ \zeta^{\frac{1}{2}N-1+i\eta_1} {}_3F_2(M+\frac{1}{2}N+i\eta_1, 1-\mu+i\eta, 1-\nu+i\eta; \frac{1}{2}N+i\eta_1, M+1; -\zeta) \}, \\
 J_{KLMN}^{np}(\zeta) &= Y_{KLMN}^{np}(\zeta) \sum_{j=0}^{n-2} \omega_j^{np} (\zeta^{\frac{1}{2}} \partial / \partial \zeta)^{j+1} \\
 &\times \{ \zeta^{\frac{1}{2}N-2+i\eta_1} [(M+1) {}_3F_2(M+\frac{1}{2}N+i\eta_1, -\mu+i\eta, -\nu+i\eta; \frac{1}{2}N-1+i\eta_1, M+1; -\zeta) \\
 &- (M+\frac{1}{2}N+i\eta_1) {}_3F_2(M+\frac{1}{2}N+1+i\eta_1, -\mu+i\eta, -\nu+i\eta; \frac{1}{2}N-1+i\eta_1, M+1; -\zeta)] \}, \\
 &\hspace{15em} (10)
 \end{aligned}$$

where μ and ν are defined in Eq. (7) and

$$\begin{aligned}
 \omega_j^{np} &= (-n+2)_j [(4)_j j!]^{-1} [-4/(n+1)]^j, \\
 X_{KLMN}^{np}(\zeta) &= 6^{-\frac{1}{2}} [(n+1)!/(n-2)!]^{\frac{1}{2}} (ak)^{-1-K} [2n\zeta^{\frac{1}{2}}/(n+1)]^{-K} X_{KLMN}^n, \\
 Y_{KLMN}^{np}(\zeta) &= (\frac{1}{2}N-1+i\eta_1) [(-\nu+i\eta)(-\mu+i\eta)]^{-1} X_{KLMN}^{np}(\zeta). \hspace{5em} (11)
 \end{aligned}$$

For $\underline{m}=\pm 1$ the $(1\underline{s}\rightarrow\underline{n}p)$ amplitudes can then be written

$$\begin{aligned}
 F_0^{00} &= I_{-1110}^{np}, \\
 F_0^{02} &= -\xi(\eta^2 I_{1110}^{np} + \eta_1^2 I_{1112}^{np}), \\
 F_0^{11} &= \frac{1}{2}\pi i \beta^2 [\eta^2 \gamma(\eta) I_{0110}^{np} + \eta_1^2 \gamma(-\eta_1) I_{0111}^{np}],
 \end{aligned}$$

$$F_0^{12} = \frac{1}{2}\pi\beta^2 (\eta \begin{smallmatrix} 3 \\ 1110 \end{smallmatrix} I_{1110}^{np} - \eta \begin{smallmatrix} 3 \\ 1112 \end{smallmatrix} I_{1112}^{np}),$$

$$F_1^{02} = \eta\eta_1 \xi^2 \left[\begin{smallmatrix} 2 \\ 1112 \end{smallmatrix} I_{1112}^{np} - \frac{1}{2} (\begin{smallmatrix} 2 \\ 1122 \end{smallmatrix} I_{1122}^{np} + \begin{smallmatrix} 2 \\ 1102 \end{smallmatrix} J_{1102}^{np}) \right],$$

$$G_0^{01} = \frac{1}{2}i\eta\xi (\begin{smallmatrix} 2 \\ 0010 \end{smallmatrix} I_{0010}^{np} - \begin{smallmatrix} 2 \\ 0210 \end{smallmatrix} I_{0210}^{np}),$$

$$G_0^{12} = -\frac{1}{4}\pi\beta^2 \eta \left[i\eta(\frac{1}{2}-i\eta)\gamma(\eta) (\begin{smallmatrix} 2 \\ 1010 \end{smallmatrix} I_{1010}^{np} - \begin{smallmatrix} 2 \\ 1210 \end{smallmatrix} I_{1210}^{np}) \right. \\ \left. + \xi\eta_1^2 \gamma(-\eta_1) (\begin{smallmatrix} 2 \\ 1011 \end{smallmatrix} I_{1011}^{np} - \begin{smallmatrix} 2 \\ 1211 \end{smallmatrix} I_{1211}^{np}) \right],$$

$$G_1^{01} = -\frac{1}{2}i\eta_1 \xi \left[\begin{smallmatrix} 2 \\ 0012 \end{smallmatrix} I_{0012}^{np} - \begin{smallmatrix} 2 \\ 0212 \end{smallmatrix} I_{0212}^{np} - \frac{1}{2} (\begin{smallmatrix} 2 \\ 0022 \end{smallmatrix} I_{0022}^{np} + \begin{smallmatrix} 2 \\ 0002 \end{smallmatrix} J_{0002}^{np} - \begin{smallmatrix} 2 \\ 0222 \end{smallmatrix} I_{0222}^{np} - \begin{smallmatrix} 2 \\ 0202 \end{smallmatrix} J_{0202}^{np}) \right],$$

$$G_1^{12} = \frac{1}{4}\pi\beta^2 \eta_1 \left\{ \xi\eta_1^2 \gamma(\eta) \left[\begin{smallmatrix} 2 \\ 1012 \end{smallmatrix} I_{1012}^{np} - \begin{smallmatrix} 2 \\ 1212 \end{smallmatrix} I_{1212}^{np} \right. \right.$$

$$\left. - \frac{1}{2} (\begin{smallmatrix} 2 \\ 1022 \end{smallmatrix} I_{1022}^{np} + \begin{smallmatrix} 2 \\ 1002 \end{smallmatrix} J_{1002}^{np} - \begin{smallmatrix} 2 \\ 1222 \end{smallmatrix} I_{1222}^{np} - \begin{smallmatrix} 2 \\ 1202 \end{smallmatrix} J_{1202}^{np}) \right]$$

$$- i\eta_1 (\frac{1}{2} + i\eta_1) \gamma(-\eta_1) \left[\begin{smallmatrix} 2 \\ 1013 \end{smallmatrix} I_{1013}^{np} - \begin{smallmatrix} 2 \\ 1213 \end{smallmatrix} I_{1213}^{np} \right.$$

$$\left. - \frac{1}{2} (\begin{smallmatrix} 2 \\ 1023 \end{smallmatrix} I_{1023}^{np} + \begin{smallmatrix} 2 \\ 1003 \end{smallmatrix} J_{1003}^{np} - \begin{smallmatrix} 2 \\ 1223 \end{smallmatrix} I_{1223}^{np} - \begin{smallmatrix} 2 \\ 1203 \end{smallmatrix} J_{1203}^{np}) \right] \}. \quad (12)$$

Again, the $\underline{F}_0^{00}$ term is in agreement with the Thomas and Franco [6], and Thomas and Gerjuoy [7] amplitudes.

5. Excitation of the $\underline{n}=2$ levels

In practical applications of the results presented in Sect. 4-5 it is useful to express the amplitudes in terms of the ordinary hypergeometric functions ${}_2F_1$, in particular because of the convenient transformation properties of these functions. As an important example we now look at excitation of the $\underline{n}=2$ $\underline{s}$ and $\underline{p}$ levels.

5.1. The (1s+2s) transition

For $\underline{n}=2$ the required functions (6) are

$$\begin{aligned} I_{KLON}^{2s}(\zeta) &= a_{KLN}(\zeta) A_{KLN}(\zeta), \\ I_{KL1N}^{2s}(\zeta) &= a_{KLN}(\zeta) B_{KLN}(\zeta), \end{aligned} \quad (13)$$

where

$$\begin{aligned} a_{KLN}(\zeta) &= 2^{\frac{3}{2}} (2/3)^4 \left[\Gamma(1 - \frac{1}{2}N - i\eta_1) / \Gamma(1 - i\eta_1) \right]^2 \\ &\times \left[\Gamma(-i\eta) / \Gamma(\mu - i\eta) \right] \left[\Gamma(1 - \nu + i\eta) / \Gamma(1 + i\eta) \right] (\frac{1}{2}x)^K \zeta^{\frac{1}{2}N}, \\ A_{KLN}(\zeta) &= (\frac{1}{2}N + i\eta_1) \left[(1 - \frac{1}{2}N - i\eta_1) f_{01} + 2yz\zeta f_{12} \right] - \frac{1}{2}(y)_2 (z)_2 \zeta^2 f_{23}, \\ B_{KLN}(\zeta) &= (\frac{1}{2}N + i\eta_1) \left[(\frac{1}{2}N + i\eta_1) (1 - \frac{1}{2}N - i\eta_1) f_{02} \right. \\ &\quad \left. + \frac{3}{2}(\frac{1}{2}N + \frac{1}{3} + i\eta_1) yz\zeta f_{13} \right] \\ &\quad - \frac{1}{2}(\frac{1}{2}N + \frac{2}{3} + i\eta_1) (y)_2 (z)_2 \zeta^2 f_{24} + \frac{1}{24}(y)_3 (z)_3 \zeta^3 f_{35}, \end{aligned}$$

$$y = 1-u+i\eta, \quad z = 1-v+i\eta, \quad f_{mn} = {}_2F_1(y+m, z+m; n; -\zeta). \quad (14)$$

By using well-known relations for contiguous hypergeometric functions we could write the expressions both for A_{KLN} and B_{KLN} as two hypergeometric functions multiplied by rational functions of η, η_1 and ζ . The expressions are, however, more tractable as they stand.

Writing the amplitudes of Eq.(4) as

$$\begin{aligned} F_j^{k\ell}(\zeta) &= 2^{\frac{3}{2}}(2/3)^4 \hat{F}_j^{k\ell}(\zeta), \\ G_j^{k\ell}(\zeta) &= 2^{\frac{3}{2}}(2/3)^4 \hat{G}_j^{k\ell}(\zeta), \end{aligned} \quad (15)$$

We then have

$$\hat{F}_0^{00} = A_{000},$$

$$\hat{F}_0^{02} = -\frac{1}{4}\xi x^2 (A_{200} - \zeta A_{202}),$$

$$\hat{F}_0^{11} = -\frac{1}{4}\pi\beta^2 \eta x^2 [A_{100} - i(\eta_1/\eta) \operatorname{tgh}(\pi\eta_1) \zeta^{\frac{1}{2}} A_{101}],$$

$$\hat{F}_0^{12} = \frac{1}{8}\pi\beta^2 \eta x^2 [A_{200} + (\eta_1/\eta) \zeta A_{202}],$$

$$\hat{F}_1^{02} = -\frac{1}{4}\xi^2 (\eta/\eta_1) x^2 \zeta (A_{202} - B_{202}),$$

$$\hat{G}_0^{01} = \frac{1}{2}\xi x A_{110},$$

$$\hat{G}_0^{12} = -\frac{1}{8}\pi\beta^2 \eta x^2 [A_{210} - i\xi(\eta_1/\eta) \operatorname{tgh}(\pi\eta_1) \zeta^{\frac{1}{2}} A_{211}],$$

$$\begin{aligned}\hat{G}_1^{01} &= -\frac{1}{2}i\xi[(1+i\eta)/\eta_1]x\zeta(A_{112}-B_{112}), \\ \hat{G}_1^{12} &= \frac{1}{8}\pi i\beta^2 x^2 \zeta\{\xi(\eta/\eta_1)(\frac{1}{2}+i\eta)(A_{212}-B_{212}) \\ &\quad +\eta_1[(1+i\eta)/(\frac{1}{2}+i\eta_1)]\tanh(\pi\eta_1)\zeta^{\frac{1}{2}}(A_{213}-B_{213})\}. \quad (16)\end{aligned}$$

On the basis of these expressions one can easily check that in the limit that the target electron is absent, i.e. η_1, ζ and $\zeta/\eta_1 \rightarrow 0$, the excitation amplitudes vanish. It is also seen that as $q \rightarrow 0$ ($\zeta \gg 1$) the contributions to the scattering operator (3) from the amplitudes $\underline{F}_j^{kl}$ behave as $\ln q$, and that the $\underline{G}_j$ terms show the even stronger q^{-1} divergence. In fact, in the Glauber approximation this behaviour is characteristic of all ($1s \rightarrow ns$) amplitudes.

For large momentum transfers ($\zeta \ll 1$) the contribution to the cross section from the $\underline{F}_0^{00}$ term shows the q^{-4} dependence characteristic of point-charge Coulomb scattering, and the behaviour of the other terms relative to this can be inferred from Eq.(16) ($\zeta \rightarrow 0$ and $\underline{A}_{KLN}, \underline{B}_{KLN} \rightarrow \text{constant}$).

In the region of intermediate momentum transfers ($\zeta \sim 1$) the contributions to the cross section behave roughly as q^{-12} , q^{-16} , q^{-10} and q^{-18} for $\underline{F}_0, \underline{F}_1, \underline{G}_0$ and $\underline{G}_1$, respectively. In this region the $\underline{F}_0$ contribution therefore has the same q dependence as that which follows from the Born approximation [8].

5.2. The (1s→2p) transitions

The functions (10) appearing in the $\underline{n}=2$ p-state amplitudes for magnetic quantum number $\underline{m}=\pm 1$ are

$$\begin{aligned} I_{KL1N}^{2p}(\zeta) &= b_{KL1N}(\zeta) C_{KLN}(\zeta), \\ I_{KL2N}^{2p}(\zeta) &= b_{KL2N}(\zeta) E_{KLN}(\zeta), \\ J_{KLON}^{2p}(\zeta) &= b_{KLON}(\zeta) \zeta^{-1} F_{KLN}(\zeta), \end{aligned} \quad (17)$$

where

$$\begin{aligned} b_{KLON}(\zeta) &= (3/4) 2^{-\frac{1}{2}} i (ak)^{-1} (M!)^{-1} [\Gamma(M+\frac{1}{2}N+i\eta_1)/\Gamma(\frac{1}{2}N+i\eta_1)] \\ &\quad a_{KLN}(\zeta), \\ C_{KLN}(\zeta) &= (1-\frac{1}{2}N-i\eta_1) f_{02} + yz \zeta f_{13} - \frac{1}{6} (\frac{1}{2}N+i\eta_1)^{-1} (y)_2 (z)_2 \zeta^2 f_{24}, \\ E_{KLN}(\zeta) &= (1-\frac{1}{2}N-i\eta_1) f_{03} + yz \zeta f_{14} - \frac{1}{4} (\frac{1}{2}N+i\eta_1)^{-1} (y)_2 (z)_2 \zeta^2 f_{25} \\ &\quad + [60(\frac{1}{2}N+i\eta_1)(1+\frac{1}{2}N+i\eta_1)]^{-1} (y)_3 (z)_3 \zeta^3 f_{36}, \\ F_{KLN}(\zeta) &= (\frac{1}{2}N-1+i\eta_1)^2 (\frac{1}{2}N-2+i\eta_1) [(y-1)(z-1)]^{-1} f_{-11} \\ &\quad + [(\frac{1}{2}N-2+i\eta_1) - 3(\frac{1}{2}N+i\eta_1)(\frac{1}{2}N-\frac{4}{3}+i\eta_1)] \zeta f_{02} \\ &\quad + \frac{3}{2} (\frac{1}{2}N-\frac{1}{3}+i\eta_1) yz \zeta^2 f_{13} - \frac{1}{6} (y)_2 (z)_2 \zeta^3 f_{24}, \end{aligned} \quad (18)$$

$\underline{y}$, $\underline{z}$ and $\underline{f}_{mn}$ being defined as in Eq.(14), and $\underline{a}_{KLN}(\zeta)$ as

in Eq.(13).

For the (1s→2p) transitions it is convenient to write the amplitudes (4) as

$$\begin{aligned} F_j^{k\ell}(\zeta) &= -2(2/3) \zeta^{\frac{1}{2}} \tilde{F}_j^{k\ell}(\zeta), \\ G_j^{k\ell}(\zeta) &= -2(2/3) \zeta^{\frac{1}{2}} \tilde{G}_j^{k\ell}(\zeta), \end{aligned} \quad (19)$$

and to use the definition

$$D_{KLN}(\zeta) = \frac{1}{2} E_{KLN}(\zeta) + [(1 + \frac{1}{2}N + i\eta_1)(\frac{1}{2}N + i\eta_1)\zeta]^{-1} F_{KLN}(\zeta). \quad (20)$$

We obtain

$$\begin{aligned} \tilde{F}_0^{00} &= \eta_1 (1 + i\eta) C_{-110}, \\ \tilde{F}_0^{02} &= -\frac{1}{4} i \xi x^2 [\eta \eta_1 C_{110} + (1 + i\eta)(1 + i\eta_1) \zeta C_{112}], \\ \tilde{F}_0^{11} &= -\frac{1}{4} \pi \beta^2 \eta_1 x [\eta (\frac{1}{2} + i\eta) C_{010} - (\frac{1}{2} + i\eta_1)(1 + i\eta) \operatorname{tgh}(\pi \eta_1) \zeta^{\frac{1}{2}} C_{011}], \\ \tilde{F}_0^{12} &= \frac{1}{8} \pi i \beta^2 \eta_1 x^2 [\eta^2 C_{110} - (1 + i\eta)(1 + i\eta_1) \zeta C_{112}], \\ \tilde{F}_1^{02} &= \frac{1}{8} i \xi^2 (\eta/\eta_1)(1 + i\eta)(1 + i\eta_1) x^2 \zeta [2C_{112} - (2 + i\eta_1) D_{112}], \\ \tilde{G}_0^{01} &= \frac{1}{4} \xi \eta_1 x [i\eta C_{000} + (1 + i\eta) C_{020}], \\ \tilde{G}_0^{12} &= \frac{1}{16} \pi \beta^2 x^2 \{ \eta \eta_1 [(\frac{1}{2} + i\eta_1) C_{100} - (\frac{1}{2} + i\eta) C_{120}] \\ &\quad + \xi \eta_1 (\frac{1}{2} + i\eta_1) \operatorname{tgh}(\pi \eta_1) \zeta^{\frac{1}{2}} [i\eta C_{101} + (1 + i\eta) C_{121}] \}, \end{aligned}$$

$$\begin{aligned}
 \tilde{G}_1^{01} &= -\frac{1}{8}\xi[(1+i\eta_1)/\eta_1](1+i\eta)\times\zeta\{2[(1+i\eta)C_{002}+(2+i\eta)C_{022}] \\
 &\quad -(2+i\eta_1)[(1+i\eta)D_{002}+(2+i\eta)D_{022}]\}, \\
 \tilde{G}_1^{12} &= \frac{1}{32}\pi\beta^2 \times \zeta(\xi(\eta/\eta_1)(\frac{1}{2}+i\eta)(1+i\eta_1)\{2[(\frac{1}{2}+i\eta)C_{102}-(\frac{3}{2}+i\eta)C_{122}] \\
 &\quad -(2+i\eta_1)[(\frac{1}{2}+i\eta)D_{102}+(\frac{3}{2}+i\eta)D_{122}]\} \\
 &\quad +(1+i\eta)\eta_1[(\frac{3}{2}+i\eta_1)/(\frac{1}{2}+i\eta_1)]\tanh(\pi\eta_1)\zeta^{\frac{1}{2}}\{2[(1+i\eta)C_{103} \\
 &\quad +(2+i\eta)C_{123}]-(\frac{5}{2}+i\eta_1)[(1+i\eta)D_{103}+(2+i\eta)D_{123}]\}).
 \end{aligned}
 \tag{21}$$

As in the previous case of the s-state transition the expressions for the amplitudes are presented in such a way that it is relatively easy to discuss their behaviour for small ($\zeta \gg 1$), large ($\zeta \gg 1$) and intermediate ($\zeta \sim 1$) momentum transfers. It is seen that for small momentum transfers the $\underline{F}_j^{k\ell}$ contributions to the scattering operator (3) now behave as $\underline{q}^{-1}$, and that the corresponding $\underline{G}_j^{k\ell}$ terms show a $\underline{q}^{-2}$ dependence. For large momentum transfers the leading $\underline{F}_0^{00}$ term gives a $\underline{q}^{-6}$ behaviour of the cross section, where the extra factor $\underline{q}^{-2}$ relative to the point-charge dependence comes from the electron wave function. For intermediate momentum transfers the $\underline{F}_0$ term in the cross section behaves as $\underline{q}^{-14}$, again as for the Born approximation [8], and the $\underline{F}_1, \underline{G}_0$ and $\underline{G}_1$ terms have a $\underline{q}^{-18}, \underline{q}^{-12}$ and $\underline{q}^{-20}$ dependence, respectively.

6. Numerical results and conclusions

As far as separate measurements of the $(1\underline{s} \rightarrow 2\underline{s})$ and $(1\underline{s} \rightarrow 2\underline{p})$ excitation cross sections are concerned the only experiments known to us are those by Williams [9] for atomic hydrogen. The projectile energy in these experiments ($\leq 20\text{eV}$) is, however, well below the region of interest for comparison with Glauber calculations. Summed excitation cross sections for the $n=2$ levels in hydrogen have been obtained by Williams and Willis [10] for energies up to 680eV . A comparison between theory and experiment at 680eV is shown in Fig.1. Since at this energy the relativistic effects are quite unimportant our theoretical curves are, in effect, the same as those which would follow from the calculations of Ghosh and Sil [11] and Tai et al. [12].

To observe the relativistic effects in the hydrogen excitation cross section one will need projectile energies at least in the 100 keV region, as can be seen from Fig.2 for $2\underline{p}$ excitation. In this and the subsequent figure we have removed a factor proportional to q^{-6} from the cross section, in order to display the asymptotic behaviour of the non-relativistic result. The difference between the relativistic and non-relativistic cross sections is here largely accounted for by the spin-flip contribution to the scattering against the target electron (the G_1 term in Eq.(4)), and the $\underline{e}^+$ and $\underline{e}^-$ cross sections are essentially the same.

A distinct $\underline{e}^+ - \underline{e}^-$ difference appears, however, in the high-energy cross section for scattering on highly ionized

atoms, as indicated in Fig. 3 for $Z = 20$ and 500keV projectiles. The difference between the e^+ and e^- cross sections is here due to the charge dependence of the relativistic input amplitudes for point-charge scattering.

A similar $e^+ - e^-$ effect appears in the $(1s \rightarrow 2s)$ excitation cross section, as shown in Fig. 4 for the same nuclear charge and projectile energy as in Fig. 3. In displaying the results it is again convenient to remove the q -dependent factor which characterizes the asymptotic behaviour of the non-relativistic cross section, this time q^{-4} . Also in the $(1s \rightarrow 2s)$ case the e^+ and e^- cross sections are essentially the same for low nuclear charges, e.g. for atomic hydrogen. But in this case it is the spin-flip scattering against the nucleus (the G_0 term of Eq. (4)) that is the main contributor to the relativistic correction term, especially at large angles. This is shown in Fig. 5 for positron scattering on He^+ at 1 MeV.

Generally speaking, relativistic effects in the excitation cross section are of importance for momentum transfers for which $a \cdot q \gtrsim 10^2$. It should be remembered, however, that in this region the cross section is actually very small. Let it also be said once more that relativistic effects within the target have not been included in the present calculation. For light targets such effects are small in any case, but their influence on the scattering cross section could become quite appreciable as the nuclear charge increases.

As far as the general $(n\ell m \rightarrow n'\ell' m')$ transition is concerned Thomas and Gerjuoy [7] presented the selection rule that such a transition is forbidden when $\ell + \ell' + |m| + |m'|$ is an odd integer. We should like to point out that our

relativistic Glauber calculation adds no new terms that will violate this rule, which is a consequence of parity conservation.

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Captions for figures

Fig. 1. Differential cross sections for inelastic $\underline{e}^+$ or $\underline{e}^-$ scattering on atomic hydrogen at 680 eV. Curve (a): The $(1\underline{s} \rightarrow 2\underline{s})$ excitation. Curve (b): The $(1\underline{s} \rightarrow 2\underline{p}_{\pm})$ excitation. Curve (c): The total $(1\underline{s} \rightarrow 2\underline{s}, \underline{p})$ excitation. The experimental $(1\underline{s} \rightarrow 2\underline{s}, \underline{p})$ cross section is from Ref. [10] .

Fig. 2. Differential cross sections (multiplied by $\sin^6(\frac{1}{2}\theta)$) for the $(1\underline{s} \rightarrow 2\underline{p}_{\pm})$ excitation of atomic hydrogen by 0.1 MeV $\underline{e}^+$ or $\underline{e}^-$. Curve (a): Non-relativistic. Curve (b): Relativistic.

Fig. 3. Differential $(1\underline{s} \rightarrow 2\underline{p}_{\pm})$ excitation cross sections (multiplied by $\sin^6(\frac{1}{2}\theta)$) for $\underline{Z}_2=20$ and 0.5 MeV $\underline{e}^+$ and $\underline{e}^-$. Curve (a): Non-relativistic. Curve (b): Relativistic $\underline{e}^-$. Curve (c): Relativistic $\underline{e}^+$.

Fig. 4. Differential $(1\underline{s} \rightarrow 2\underline{s})$ excitation cross sections (multiplied by $\sin^4(\frac{1}{2}\theta)$) for $\underline{Z}_2=20$ and 0.5 MeV $\underline{e}^+$ and $\underline{e}^-$. Curve (a): Non-relativistic. Curve (b): Relativistic $\underline{e}^-$. Curve (c): Relativistic $\underline{e}^+$.

Fig. 5. Differential $(1\underline{s} \rightarrow 2\underline{s})$ excitation cross sections (multiplied by $\sin^4(\frac{1}{2}\theta)$) for $\underline{Z}_2=2$ and 1 MeV $\underline{e}^+$. Curve (a): Non-relativistic. Curve (b): Relativistic. Curve (c): Spin-flip against the nucleus only.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5