Beam stability in the SPL-Proton Driver accumulator for a Neutrino Factory at CERN

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Longitudinal and transverse instabilities in the isochronous accumulator for the CERN proton driver are studied, in order to find cures and set limits to the machine impedance.
Beam Stability in the SPL-Proton Driver Accumulator for a Neutrino Factory at CERN

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Keywords: proton driver, microwave instability, impedance, isochronous ring

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INTRODUCTION

The Proton Driver proposal [1] for a Neutrino-Factory based at CERN foresees the use of the Superconducting Proton Linac (SPL) upgraded to high power, to accelerate protons (H⁻ beam) to 5 GeV with a repetition rate of 50 Hz, for a total beam power of 4 MW. In order to achieve the required time structure at the target, the beam is stored in an accumulator for about 400 μs and then sent to a compressor where a longitudinal phase space rotation takes place. The accumulator and compressor designs for the baseline option, which assumes the total beam intensity of 10¹⁴ protons per burst distributed in 6 bunches, are well documented in Refs. [2, 3, 4].

The main characteristic of the accumulator is that it will be isochronous. This choice has the advantage that the longitudinal motion during the accumulation will be frozen and the energy spread will be kept as small as possible to achieve the required 2 ns bunch length after the compression. The drawback is that transverse and longitudinal fast instabilities may be an issue, since the damping mechanism provided by synchrotron motion is not available. Collective-effects studies are therefore important in order to identify possible instability sources, find cures for them and/or set limits on the machine impedance budget.

This paper presents the results of the beam stability studies in the accumulator ring for the baseline option of 6-bunch accumulation. It will also report preliminary investigation for the alternative 3-bunch accumulation scenario proposed in [5].

THE ACCUMULATOR

Table 1 summarizes the accumulator parameters, which have been defined in order to match the SPL-beam input characteristics to the compressor requirements, for realizing the output time structure at the target [2]. In particular, the bunch length and energy spread have been determined by taking into account the dynamics in the phase-rotation compression, while the transverse emittance has been defined by evaluating the competing issues of injection-foil heating, aperture and space charge. The longitudinal bunch profile is realized by adjusting the chopping of the linac micro-bunches and it is flat with smooth edges, with the linear density going to zero in 10 ns, to avoid longitudinal space-charge issues.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Symbol</th>
<th>Value</th>
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<tr>
<td>Relativistic γ</td>
<td>γ</td>
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<tr>
<td>Number of bunches</td>
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</tr>
<tr>
<td>Number of particles per bunch</td>
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<td>εx, εy</td>
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<td>Total bunch length (ns)</td>
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<td>RMS momentum spread</td>
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<td>Circumference (m)</td>
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<td>Average β function (m)</td>
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<td>α₀</td>
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<tr>
<td>Nominal tune</td>
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<td>Natural chromaticity</td>
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<td>RF Voltage</td>
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<td>2nd-order momentum compact</td>
<td>α₁</td>
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<tr>
<td>Beam pipe half-height (mm)</td>
<td>b</td>
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</table>

BEAM STABILITY STUDIES

The impedance of an accelerator is usually determined by three main contributions: resistive wall, narrow-band resonators and a broad-band resonator that takes into account discontinuities of the beam pipe and several non-resonating objects (e.g., pick-ups, kickers). In our study the narrow-band impedance will be neglected, since RF cavities, which represent the major contribution to it,
are not required in the isochronous accumulator design and the impedance of other objects is assumed to be minimized by a careful design. Space charge issues have been addressed in [2] and are under control.

The electron-cloud-related transverse instability is not considered to be an issue, since no significant electron build-up is expected. The bunches are long compared to the electron oscillation frequency and their longitudinal profile is flat, therefore neither multi-bunch resonant build-up nor “trailing-edge” multipacting [6] is a concern. Eventually, surface coating and conditioning techniques, presently under strong development at CERN [7], can be applied to reduce the electron secondary emission yield of the vacuum chamber.

The (mainly) single-bunch collective effects are studied by analytical estimation and by numerical simulations with the multiparticle tracking code HEADTAIL, developed and maintained at CERN [8, 9]. In HEADTAIL, the impedance is assumed to be localized at a few positions around the ring. The bunch is sliced longitudinally and, at the impedance location, each slice leaves a wake-field behind and gets a kick by the field generated by the preceding slices. The bunch is then transferred to the next point via a transport matrix. The effects of chromaticity, octupole detuning, space-charge and longitudinal motion are also modeled in the code.

In the following analysis, the most pessimistic scenario is considered. The beam is assumed to circulate at full intensity for the whole storage time, while in reality during the first 400 µs the accumulation process takes place and the intensity increases from zero to its maximum. Figure 1 shows the total beam intensity evolution. The last bunch is extracted at \( t = 460 \mu s \), therefore it stays at its full intensity for only 60 µs.

\[ Z(\omega) = \text{Sgn}(\omega) + j \frac{R}{b^2} \sqrt{\frac{2}{\sigma_c e_0 |\omega|}}. \]

The coupling of the impedance with the bunch power-spectrum gives rise to a complex frequency shift of beam mode \( m \) [11, 12],

\[ \Delta \omega_m = \frac{1}{(|m| + 1) 2 m_0 \gamma Q \omega_b \tau_b c} \sum_{k=-\infty}^{\infty} Z(\omega_k) h_m(\omega_k), \]

valid for zero chromaticity, whose imaginary part is related to the instability rise time \( \tau = -[|m| \Delta \omega_m]^{-1} \), \( I_0 = N_0 e / \tau_b \) is the beam current, \( Q \) is the horizontal (vertical) tune, \( m_0 \) is the proton rest mass and \( \omega_k \) is the revolution angular frequency. The bunch spectrum of mode \( m \) is:

\[ h_m(\omega) = (|m| + 1)^2 \left( \frac{\omega}{\omega_b} \right)^2 \left[ \frac{1 + (-1)^{|m|} \cos(\omega \tau_b)}{\omega^2} \right]. \]

The actual spectrum is a line spectrum within the envelope \( h_m(\omega) \). For a single bunch, the frequencies \( \omega_k = (k + Q) \omega_b \) occur for any integer \(-\infty < k < \infty\).

Figure 2 shows the real part of the resistive-wall impedance for the proposed accumulator stainless-steel beam pipe and the bunch power-spectrum envelope for head–tail modes \( |m| = 0, 1, 2 \).

From Eq. 2, it is possible to determine whether the bunch is unstable and for which mode. In the limit of zero chromaticity \( (\omega_b = 0) \), the contribution from the spectrum line closest to the origin dominates; in particular it is stabilizing for a tune just above an integer and destabilizing below. In our case, with \( Q_\parallel = 7.77 \) and zero chromaticity, the mode \( m = 0 \) (rigid bunch) is unstable, but the risetime \( \tau = 8.2 \text{ ms} \) is long compared to the 460 µs accumulation time. In the vertical plane, for \( Q_\perp = 6.67 \), the risetime is even longer and is about 14 ms.

**Broadband resonator transverse impedance**

The short-range transverse wake-field in a ring can be modeled with a broad-band resonator which takes into account the contribution from beam-pipe discontinuities and non-resonating objects (e.g., pick-ups, kickers). The broad-band impedance expression is

\[ Z_{\perp}(\omega) = \frac{c}{\omega} \frac{R_{b1}}{1 + i Q \frac{\omega_0}{\omega} \omega_b}. \]

**Resistive wall transverse impedance**

A single bunch interacting with a metal beam pipe of conductivity \( \sigma_c \) leaves behind a transverse dipole wake field. In the “classical” regime, that is for \((Z_0 \sigma_c b)^{-1/3} b \ll |z| \ll b(Z_0 \sigma_c b)\), where \( b \) is the beam-pipe half-size and \( Z_0 = 377 \Omega \) is the free-space impedance, the expression for the resistive wall impedance is [10, 11]:

\[ Z(\omega) = \text{Sgn}(\omega) + j \frac{R}{b^2} \sqrt{\frac{2}{\sigma_c e_0 |\omega|}}. \]
where $R_{s,1}$, in $\Omega/m^2$, is the shunt impedance, $Q_R$ is the quality factor, typically set to $Q_R = 1$ for the broad-band resonator, and $f_R = \omega_R/(2\pi)$ is the resonator frequency. As it is $\omega_R \sim c/b$, where $c$ is the velocity of light and $b$ is the beam-pipe half-size, the resonator frequency is assumed to be $f_R = 1\, \text{GHz}$. Concerning the shunt impedance, we will refer to the quantity $R = R_{s}/m$, where $m = 0$, $m = 1$, $m = 2$, and $\sigma_{x}$ is short compared to the total time the beam is stored in the ring (assuming the maximum bunch intensity from the beginning of the accumulation).

The fast transverse instability can be cured by introducing some betatron tune spread, e.g., by chromaticity or by amplitude-dependent tune shifts. From an approximate stability criterion it follows that the necessary tune spread is $\Delta Q > \sqrt{2} |\Delta \omega_m| / \omega_0 \sim 0.025$.

**Effect of chromaticity**

The first way to generate some tune spread in the beam, is to introduce a tune dependence on the particle momentum offset via the chromaticity $Q' = (Q - Q_0)/\langle \Delta p/p \rangle$ where $Q_0$ is the nominal betatron tune for a particle with zero energy offset and $\Delta p/p$ is the particle momentum deviation. Figure 3 shows the simulation results in terms of horizontal beam-size evolution versus time for various values of chromaticity and a broadband impedance of 1 M$\Omega$/m. Since the ring is running exactly at transition, only the absolute value of chromaticity matters. The strongest instability occurs for a total chromaticity compensation ($Q' = 0$), for which the instability risetime is $\tau \sim 28\, \mu s$. For $|Q'| = 7$ the instability is less severe and it is completely cured for $|Q'| = 10$, thus for a normalized chromaticity $\xi = Q'/Q_0 \sim 1.3$. In principle, keeping the ring natural chromaticity ($Q'_0 = -8.4$, as from Table 1) would almost be enough to cure the instability. Similar values are found for the vertical plane.

A scan has been made over the values of transverse impedance ($Z_t = 1 \rightarrow 3$ M$\Omega$/m). Figure 4 summarizes the simulation results and shows the inverse of the instability risetime as a function of chromaticity, normalized by the impedance value. From the plot it is possible to see that the chromaticity which is necessary to cure the instability is proportional to the impedance value, i.e., to kill the instability (using only chromaticity) a chromaticity $|Q'| \geq 10Z_t$ is needed. Moreover, the simulations confirm that the risetime is inversely proportional to the impedance, for a given value of $(Q'/Z_t)$.

**Effect of detuning with amplitude**

An alternative or additional way to achieve a betatron tune spread is the use of octupoles, which produce a detuning proportional to the single-particle amplitude.

In particular, in the code HEADTAIL, octupoles are modeled by assigning to each particle a tune shift equal to $\Delta Q_x = Q'_x a_x + Q''_x a^2_x$, and $\Delta Q_y = Q'_y a_y + Q''_y a^2_y$, where $a_x(y)$ is the single-particle horizontal (vertical) action.

![FIGURE 2](image)

Real part of the resistive-wall impedance for the proposed accumulator beam pipe: $b \sim 50\, \text{mm}$, $\sigma = 10^6\, \Omega^{-1}\text{m}^{-1}$ (stainless steel).

![FIGURE 3](image)

Simulations with broad-band resonator impedance ($1\, \text{M}\Omega$/m, $Q_R = 1$, $f_R = 1\, \text{GHz}$): horizontal beam size evolution versus time, for various values of chromaticity.
LONGITUDINAL STABILITY

Concerning the longitudinal plane, since there are no RF-cavities, the major contribution comes from the injection and extraction kickers. In order to study the kicker impedance, it is possible to use the broad-band model and make a parametric study on the shunt impedance, resonator frequency and quality factor, to set the maximum allowed impedance in the machine.

Since the accumulator is isochronous (η = 0), according to the Boussard criterion \([15]\) the threshold would be zero. However, since in the longitudinal plane everything is frozen, the instability would take an infinite time to develop. It is therefore necessary to take into account the second order terms for η and in particular,

$$\frac{dz}{dt} = \beta c \left[ -\eta_0 \frac{dp}{p} - \left( a_0 a_1 + \frac{3\beta^2}{2\gamma^2} - \eta_0 \right) \frac{dp^2}{p} \right]. \quad (6)$$

The formula is described in e.g. \([16]\) and has recently been implemented in HEADTAIL. For the isochronous accumulator design, \(a_0 a_1 = 0.117\) (for natural chromaticity) \([17]\), while the term \(\frac{3\beta^2}{2\gamma^2} = 0.0365\). The fourth term is of course zero.

The so called microwave instability manifests itself as an increase of momentum spread (and longitudinal emittance) and in bunch profile deformation with a modulation at frequency \(f_K\).

Figure 6 shows the longitudinal emittance growth in time for various values of broad-band impedance, for \(Q_R = 1\), which is the most severe case. The resonator frequency is assumed again to be \(f_R = 1\) GHz. The longitudinal impedance should be smaller than \(Z_{sh} = 5\, \Omega\). This value corresponds to \(Z_s/n = 5\, \Omega\), with \(n = \omega_R/\omega_0\), which is easily achieved in modern machines.

\(a_x = (x^2 + x^2 \beta_x^2)/\beta_x\). With two octupole families, powered independently, one can control \(Q''_{x,y}\). The skew term \(Q''_{x,y}\) in general depends on the lattice or it could be trimmed by an additional octupole family.

A scan is made on the values of \(Q''_{x,y}\) necessary to damp the fast instability. Simulations are done for \(Q''_y = 0\). Adding some extra uncorrelated detuning will induce additional damping.

Figure 5 shows the horizontal beam size and position evolution in time, for a case with octupoles. The instability develops up to a certain amplitude, and then the detuning become effective and stops it. By increasing the octupole strength it is possible to completely kill the instability. In particular, one needs \(Q''_{x,y} = 1200\) for a positive octupole polarity or \(Q''_{x,y} \approx 2200\) for the opposite sign. Assuming a beam size of \(\sigma_x \sim 8\, \mm\), this corresponds to a tune spread of \(\Delta \phi \sim 0.015\).

\(\frac{dz}{dt} = \beta c \left[ -\eta_0 \frac{dp}{p} - \left( a_0 a_1 + \frac{3\beta^2}{2\gamma^2} - \eta_0 \right) \frac{dp^2}{p} \right]. \quad (6)\)

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3–BUNCH OPTION

Stability studies have also been done for the alternative proton driver scenario, which assumes 3 bunches in the accumulator, with twice the intensity per bunch. Beam and machine parameters are summarized in [5].

The natural chromaticity ($Q' \sim -10$) can almost cure the fast transverse instability for a broad-band impedance of $1 \text{ M} \Omega/m$, as shown in Fig. 7, for the 3-bunch option as well. Indeed, even if we have twice as much intensity per bunch, the machine dimensions change and in particular the betatron focusing is stronger ($\beta_{x,\text{ave}} = 8 \text{ m}$).

A scan over the broad-band longitudinal shunt impedance shows that the maximum allowed impedance is $Z_l = 2k\Omega$, corresponding to $Z_l/n = 3.2\Omega$, which again is a reasonable value.

CONCLUSIONS

From the point of view of beam stability, the 6-bunch option for the Proton Driver shows no show-stoppers.

Space-charge considerations have been addressed in [2] and have guided the definition of transverse emittance and bunch length.

Concerning the machine impedance, the narrow-band component can be neglected due to the absence of RF-cavities in the ring, and the resistive-wall wake-field is not an issue, since the rise time is long compared to the accumulation time.

In the transverse plane, a reasonable value of $1 \text{ M} \Omega/m$ has been assumed for the macroparticle simulations. It is shown that chromaticity ($\xi \sim -1.3$) and/or detuning with amplitude induced by octupoles can damp the fast-rising instability. The tune spread necessary to cure it with either method is of the order of $\Delta Q \sim 0.02$, as one can find with approximate analytical estimation, which is a reasonable value from the point of view of the tune footprint in the resonance diagram. Tune spread and rise times scale with the impedance value.

The electron-cloud-related instability as well should not be an issue, due to the long and flat bunch profiles.

In the longitudinal plane, a maximum value of $Z_l/n = 5\Omega$ can be tolerated for the broad-band resonator impedance, with an error bar which depends on the value of the resonant frequency. A longitudinal impedance on the order of a few $\Omega$ can be anyway easily achieved in modern machines.

Preliminary results for the 3-bunch accumulator scenario indicate that this option seems feasible as well for what concerns fast instabilities in the transverse and longitudinal planes.

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REFERENCES