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P. N. LEBEDEV PHYSICAL INSTITUTE

I.E. Tamm Department of Theoretical Physics

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O.K. Kalashnikov and V.V. Klimov

INFRARED BEHAVIOR OF THE POLARIZATION OPERATOR
IN SCALAR ELECTRODYNAMICS AT FINITE TEMPERATURE

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Infrared behavior of the polarization operator in massless scalar electrodynamics is investigated. We find, that to all orders of the perturbation theory transversal photons are not screened by the thermal fluctuations while longitudinal ones are.

At present time, detailed investigation of infrared behavior of polarization operator in gauge theories at non-zero temperature associated with quark liberation (or slavery) is rather important. It was suggested [1] that thermal fluctuations in Yang-Mills theory lead to the screening of both transversal and longitudinal gluons and thus to quark liberation.

In the present paper the polarization operator of the massless scalar electrodynamics is investigated in detail. Here we obtain to any order of perturbation theory, that the transversal electromagnetic modes are not screened while longitudinal ones are. To our opinion such situation takes place also in the Yang-Mills theory.

The Lagrangian of the theory under consideration is as follows:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 + \frac{1}{2\alpha}(\partial_\mu A_\mu)^2 + |\partial_\mu - ieA_\mu\phi|^2 - \frac{1}{4}(\phi^*\phi)^2 + \mu[\phi^*\partial_4\phi - \phi\partial_4\phi^* - 2ieA_4\phi^*\phi] + \mu^2\phi^*\phi \quad (1)$$

Here $F_{\mu\nu}$ is the electromagnetic field strength tensor, α is the gauge-fixing parameter, μ is the chemical potential of the charged particles, which enters into (1) not only linearly, but quadratically what may be seen if one performs correct transition from Hamilton's approach to Lagrange's one. In (1) we have also $0 \leq \kappa_4 \leq \beta = 1/T$ that leads here to quantization of the fourth component of euclidean momentum [2]:

$$p_4 = \frac{2\pi n}{\beta}; \quad n = 0, \pm 1, \pm 2, \dots \quad (2)$$

The set of equations for Green's functions within (1) can be

obtained by usual methods [2] and although the momentum space is discrete some general relations, connected with the gauge invariance remain valid in quantum statistics. For example, the polarization operator in scalar electrodynamics at nonzero temperature does not depend on the gauge. What is the most important is the fact that the transversality of polarization operator

$$K_\mu \Pi_{\mu\nu} = 0 \quad (3)$$

retains for nonzero temperature as well.

In the case of nonzero temperature it follows from (3) that the polarization operator is now determined by two scalar functions A and Π_{44} [2,3]

$$\Pi_{ij} = (\delta_{ij} - \frac{K_i K_j}{K^2}) A + \frac{K_i K_j}{K^2} \Pi_{44};$$

$$\Pi_{i4} = \Pi_{4i} = - \frac{K_i K_4}{K^2} \Pi_{44} \quad i, j = 1, 2, 3. \quad (4)$$

and the photon propagator has the following form (we use the transversal gauge):

$$D_{ij} = (\delta_{ij} - \frac{K_i K_j}{K^2}) \frac{1}{K^2 + A} + \frac{K_i K_j}{K^2} \frac{K_4^2}{K^2} D_{44};$$

$$D_{i4} = D_{4i} = - \frac{K_i K_4}{K^2} D_{44}; \quad D_{44} = \frac{K^2}{K^4} \frac{1}{K^2 + \Pi_{44}} \quad (5)$$

From (5) one can easily see, that the asymptotic behavior of A and Π_{44} in the limit $K_4 \neq 0; \vec{K} \rightarrow 0$ (infrared limit) is of great interest because it is this limit that determines the screening of transversal and longitudinal waves.

The expression for $\Pi_{\mu\nu}$ is given here by the four diagrams:

$$\Pi = - \text{diagram 1} - \text{diagram 2} + 2 \text{diagram 3} + \text{diagram 4} \quad (6)$$

where the solid line and vertices stand for their exact expressions.

The goal of the present paper is to prove within (6) a number of exact statements about asymptotic behavior of in quantum statistics and to confirm them by direct calculations.

Here, first of all, proceeding from (6) and the Ward identities one can show that the exact relation takes place for infrared asymptotic behavior of Π_{44}

$$\Pi_{44}(k_4=0; \vec{k} \rightarrow 0) = e^2 \frac{\partial N}{\partial \mu} = - e^2 \frac{\partial^2 \Omega}{\partial \mu^2} \quad (7)$$

where N is the density of the charged particles and Ω is the thermodynamical potential of the theory under consideration. The proof of (7) is known (see, for example, [2]) and therefore we omit it from here.

It is of great importance that for infrared asymptotic behavior of A for present theory we can also prove the exact relation:

$$A(k_4=0; \vec{k} \rightarrow 0) = 0 \quad (8)$$

which gives us a basis to insist that the infrared behavior of such theory is essentially different for transversal and longitudinal photons.

To prove (8) in any order of perturbation theory it is

sufficient, firstly, to find the self-energy operator of charged particles:

$$\Sigma = - \text{cloud} + \frac{1}{2} \text{cloud} + \text{circle} + \dots \quad (9)$$

and to show that the latter does not lead to infrared singularity of the propagator of the charged particles. That is enough to perform here the "one-loop" calculation only

because it shows that due to thermal fluctuations

$\Sigma^{(2)}(k_4=0; \vec{k} \rightarrow 0)$ is really nonzero:

$$G(k_4=0; \vec{k} \rightarrow 0) = \frac{1}{\Sigma^{(2)}(0)} = \frac{12}{T^2(3e^2 + N)} \quad (10)$$

and what is more this result is gauge-independent. Under this fact we have the conclusion that singularities of (6) in the limit $k_4=0$, $\vec{k} \rightarrow 0$ may only come from the exact vertices with zero photon momentum. However the explicit consideration of any skeleton diagrams for these vertices indicates not only the absence of divergencies in the limit $k_4=0; \vec{k} \rightarrow 0$ but also the independence of this limit of the direction along which $\vec{k}$ goes to zero. Now, combining the existence of the limit for Π_{ij} at $k_4=0; \vec{k} \rightarrow 0$ and the fact, that according to (4) Π_{ij} looks as follows

$$\Pi_{ij} = \left(\delta_{ij} - \frac{k_i k_j}{k^2} \right) A(k_4=0; \vec{k}) \quad (11)$$

we obtain that $A(k_4=0; \vec{k} \rightarrow 0)$ should be equal to zero.

Thus we have the general proof of the properties (7) and (8) and it is rather desirable now to trace their ex-

cution within the direct approximate calculations as well.

First of all the interesting are the "one-loop" calculations, wherein the polarization operator is represented by the two simple diagrams:

$$\Pi^{(2)} = - \text{diagram 1} + \text{diagram 2} \quad (12)$$

whose expressions should be found using the free propagators of the charged particles and free vertices. Here calculation is easily performed (with the use of vacuum counterterms for the renormalization) and leads to the simple result:

$$A(k_4=0; \vec{k} \rightarrow 0) = \frac{e^2 |\vec{k}| T}{16} + \frac{e^2 \vec{k}^2}{24\pi^2} \ln \frac{g}{T}; \quad (13)$$

$$\Pi_{44}(k_4=0; \vec{k} \rightarrow 0) = \frac{e^2 T^2}{3}$$

which agrees with (7) and (8) (S is the subtraction point).

As for verification of (7) and (8) within the fourth order of perturbation theory it is directly impossible because of the presence of infrared divergencies within the following diagrams

$$\text{diagram 3} \quad \text{diagram 4} \quad (14)$$

where $\bullet$ denotes the self-energy $\Sigma^{(2)}$. However, this difficulty is overcome easily here when the summation of the leading infrared divergencies has been performed. For example, in the case of nonzero external momentum the expression for the correlation part of $\Pi_{\mu\nu}$ may be presented as:

$$\Pi^{(corr)} = -2 \text{ (diagram with double line loop)} + \text{ (diagram with double line tadpole)} \quad (15)$$

where the double line implies the use of the propagator with the self-energy $\Sigma^{(2)}(0)$ taken in the denominator. Calculating (15) we obtain the gauge-invariant transversal result:

$$\begin{aligned} \Pi_{\mu\nu}^{(corr)} &= \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) \left[- \frac{e^2 T^2}{4\pi} \sqrt{e^2 + \frac{\lambda}{3}} \right]; \\ A^{(corr)} &= - \frac{e^2 T^2}{4\pi} \sqrt{e^2 + \frac{\lambda}{3}}; \\ \Pi_{44}^{(corr)} &= \frac{\vec{k}^2}{k^2} A^{(corr)}; \end{aligned} \quad (16)$$

which presents here the nonanalytical part of (15) only. However the expressions (16) are not valid in the region of very small momentum $|\vec{k}| \sim gT$, because in this case some other diagrams are important. Nevertheless, here it is also possible to sum up all important diagrams, of such kind

again:

$$\begin{aligned} A &= \frac{1}{2} \text{Sp} \left\{ - \text{ (diagram with double line loop)} + \text{ (diagram with double line tadpole)} \right\}; \\ \Pi_{44} &= \left\{ - \text{ (diagram with double line loop)} + \text{ (diagram with double line tadpole)} \right\}; \end{aligned} \quad (17)$$

when the limit $k_4 = 0, \vec{k} \rightarrow 0$ is calculated.

Now, the result after simple algebra within (17) we obtain as follows

$$\begin{aligned} A(k_4=0; \vec{k} \rightarrow 0) &= \frac{e^2 \vec{k}^2}{12\pi^2} \left\{ \frac{\pi}{\sqrt{e^2 + \lambda/3}} + \frac{1}{2} \ln \frac{5}{T} \right\}; \quad (18) \\ \Pi_{44}(k_4=0; \vec{k} \rightarrow 0) &= \frac{e^2 T^2}{3} - \frac{e^2 T^2}{4\pi} \sqrt{e^2 + \frac{\lambda}{3}}; \end{aligned}$$

where the "one-loop" expression is added.

The more detailed investigation of the small K region will be published later. From (18) it is seen, that the correlation part of $\Pi_{\mu\nu}$ agrees with (18) exactly. To check (7) the correlation part of thermodynamical potential must be calculated. For small chemical potential ($\mu \ll T$) we find here the following result

$$\Omega^{(corr)} = -\frac{T^4}{12\pi} \left\{ \frac{1}{4} \left(e^2 + \frac{4}{3} \right)^{\frac{3}{2}} + \frac{e^3}{9\sqrt{3}} \right\} + \frac{\mu^2 T^2}{8\pi} \sqrt{e^2 + \frac{4}{3}} \quad (19)$$

underwhich, after differentiating we come to (7) directly.

Thus, we obtain that in massless scalar electrodynamics the account of thermal fluctuations in any order of perturbation theory does not lead to the screening of transversal electromagnetic modes unlike the longitudinal one. The explicit calculation of the polarization operator in the "one-loop" approximation and in the next order, which comes from the summing of the leading infrared divergencies, is in full agreement with the results of the general proof.

Now, analogous investigation for the Yang-Mills theory at nonzero temperature is in progress.

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