A TWIN-QUADRUPOLE INTEGRAL RESONANCE EXTRACTION SYSTEM

by

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1. Analysis of the Present Situation

Resonant particle extraction from synchrotrons is obtained by exploiting basically non-linear processes, in which the resonant growth of the betatron amplitudes is produced by non-linear perturbations. The only role of linear perturbations is to tune the betatron frequency to the resonant value.

At integral resonance extraction, one faces the presence of spurious linear resonances, in addition to the non-linear resonance which actually produces the beam spill-out. In present integral resonance extraction systems (1), the betatron frequency is tuned by means of a single quadrupole lens. This method allows the spurious resonances to be tamed, their presence being only revealed by the high sensitivity of the stable area in the phase-plane to the position of the equilibrium orbit at the quadrupole. In fact, the particles are spilt-out by varying this position in a controlled way.

Since a momentum spread of the particles produces a spread of their equilibrium orbits, the system is highly chromatic. A momentum spread may cause a pre-ejection loss of particles on the vacuum chamber walls (2) (3), because the integral resonance greatly amplifies any spread of equilibrium orbits. Moreover, a ripple in any magnetic field acting on the beam produces an intensity fluctuation of the extracted beam, due to perturbations of the equilibrium orbits, and limits the maximum spill-time which can be attained.

For practical beam emittances and momentum spreads, this high chromaticity causes the orbit displacement for extracting a monochromatic sample of the beam to be, in general, considerably smaller than the whole spread of equilibrium orbits. Hence particles are extracted at successive times mainly according to their momentum, rather than their betatron amplitude, giving a longer spill than for a monochromatic beam without having to retard the displacement of the orbit, thus without increasing beam intensity fluctuations due to magnetic field ripples.

In conclusion, a large momentum spread lengthens the spill-time,
decreases the influence of magnetic field ripples, speeds up the
debugging of the beam and provides Landau damping for coherent
beam instabilities, already detectable at present beam intensities
(1). On the other hand, a small momentum spread is necessary in
order to avoid pre-ejection losses. In any single-quadrupole extraction
system, there is little possibility to optimize the para-
eters, since a rigid relation exists between particle momentum
and phase-plane configuration.

2. The Twin-Quadrupole System

Two quadrupoles, half a betatron wavelength apart, can equally
well tune the betatron frequency. If their strengths are equal, no
overall distortion is given to equilibrium orbits which have equal
displacements from their centres, because the deflection produced
by the first quadrupole is cancelled by the second one. An extraction
system based on the above quadrupole configuration, originally
suggested by M.G.N. Hine, is thus completely achromatic. Some un-
desired chromatic effects of the single quadrupole system are sup-
pressed, but inevitably particles of different momenta are ex-
tracted simultaneously. Hence long spill-times may be difficult
to achieve.

Since the above system and the single-quadrupole system have dis-
advantages which are mutually opposite in nature, this paper
examines a more general system, characterized by varying the ratio
of quadrupole strengths in the above system. We denote by \( K_1 \) the
normalized (4) strength of the upstream quadrupole LL and by \( x_1 \) the
displacement of the unperturbed (i.e. when the equipment for the
resonant extraction is switched off) equilibrium orbit from the
centre of LL. Similarly for L2. It will be shown that the chroma-
ticity of this extraction system is determined by the dimensionless
parameter:

\[
k = \frac{(K_2 - K_1)}{(K_2 + K_1)}
\]  

(1)

In the normalized (4) phase-plane shown in Fig. 1, a trajectory
is represented, which enters LL on the unperturbed equilibrium orbit
(point \( L_\text{lin} \)). At the exit of L2 this trajectory has the slope
\( K_2 x_2 - K_1 x_1 \), which is zero if the system is achromatic. The same deflection
with respect to the equilibrium orbit is obtained if these two lenses are replaced by a single lens L of strength $k_L$, placed at the same location as L2, and if the displacement of the equilibrium orbit from the centre of L is taken to be:

$$x_L = \frac{x_2 - x_1}{2} + k \frac{x_2 + x_1}{2}$$  \hspace{1cm} (2)

so that $k_L x_L = k_2 x_2 - K_1 x_1$.

Although Fig. 1 refers to a particular trajectory, it can be shown that the above substitution is generally valid. Therefore, one can directly apply the theory of single-quadrupole extraction systems (5) (6) (7) in order to determine the phase-plane topology downstream of L. As in the single-quadrupole case, the particles can be split out over the stability limits by making use of the almost linear dependence of the stable area on $x_L$.

Here we only have to examine the expression of $x_L$ given by Eq. (2). The effect of symmetric displacements (equal at the two quadrupoles) of the unperturbed equilibrium orbit is determined by the chromaticity parameter $k$, whereas the effect of antisymmetric displacements (equal and opposite at the two quadrupoles) is independent of $k$.

Let us assume that the equilibrium orbit displacements at the two quadrupoles are made up of two variable components, namely a displacement $x_\alpha (p-p_0)$, which results from a momentum deviation and is equal at the two quadrupoles, and a displacement given by an equilibrium orbit bump of amplitude $\alpha$ and opposite phase $\pm \gamma$ at the two quadrupoles. The expression of $x_L$ is then:

$$x_L = a \sin \gamma + k \alpha (\frac{p-p_0}{p_0})$$  \hspace{1cm} (3)

The beam can be extracted by varying $a$ or $p-p_0$. Equilibrium orbit distortions do not show up in the above expression: one is free to imagine that $x_L$ is referred to the value which gives zero stable area, whatever this is. More general cases, such as having different values of the normalized momentum compaction function $\alpha_p$ or different bumps at the two quadrupoles, are a straightforward extension of the one considered here.
The essential features of the single-quadrupole system can be derived from the simple and efficient model proposed by Hereward (5), where one has a single sextupole lens, diametrically opposite to the quadrupole. In the twin-quadrupole case, one adds a second quadrupole half a betatron wavelength upstream to the first one. In order to investigate the effect of varying the distance between the two quadrupoles, the quadrupoles are given equal and opposite displacements of phase angles ± \(\epsilon\) with respect to their ideal position, while to preserve the total Q-shift their strength is suitably adjusted. A calculation, based on the methods of Ref. (7), shows that in the first order of approximation in \(\epsilon\) one obtains for \(x_L\) in the expression for the fixed-points:

\[
x_L \approx \frac{x_2-x_1}{2} + k \left[ 1 - \epsilon \tan Q(1-k^2) \right] \frac{x_2+x_1}{2},
\]

whereas all the other quantities remain unchanged. This shows that the achromatic properties of a system having \(K_1 = K_2\) are not altered in a first approximation.

3. The Twin-Quadrupole System versus the Single-Quadrupole System

Starting from the single-quadrupole case \((k=1)\), we analyze the behaviour of the twin-quadrupole system when the chromatic parameter \(k\) is decreased. In practice, this is done by introducing a second quadrupole half a betatron wavelength upstream to the first one and by decreasing the strength of this by a corresponding amount, to preserve the Q-shift.

Pre-ejection losses impose an upper limit to the effective momentum spread, \(k\Delta p/p\). In the twin-quadrupole system one can then have a momentum spread, \(\Delta p/p, 1/k\) times larger than in the single-quadrupole case. Thus the debunching time may be reduced by a factor \(k\), the threshold for coherent longitudinal instabilities increased by a factor \(1/k^2\) (8) and the threshold for high frequency coherent transverse instabilities increased by a factor \(1/k\) (9). These higher thresholds could be particularly attractive for future high intensity accelerators or improvement programs of present day machines. Since
one can have the same $k\Delta p/p$ as in the single-quadrupole case, but
with a $1/k$ times larger momentum spread, the influence of symmetric
components of fluctuations in the radial position of one beam, caused
by magnetic field ripples, can be reduced proportionally to $k$, whereas
the influence of antisymmetric components remains constant. Long
spill-times are undoubtedly made easier.

However, if $k\Delta p/p$ is kept constant and $k$ is made too small, an
upper limit for the acceptable momentum spread is eventually en-
countered. Further reduction in $k$ implies a reduction of $k\Delta p/p$
and hence of the total effective displacement $\Delta x_L$ necessary to extract
all the particles. Thus antisymmetric orbit fluctuations start
giving more troubles than in the single-quadrupole case, whereas
symmetric fluctuations still have a decreasing influence, because
of the coefficient $k$ which they carry in the expression of $x_L$. For
a certain value of $k$, depending on the nature of the magnetic field
ripple, this limits the spill-time as much as in the single-quadrup-
ole case.

For very small values of $k$ the stable area is practically
insensitive to momentum. For extraction one must use an equilibrium
orbit bump which contains an antisymmetric component at the two
quadrupoles. If $k=0$, (equal quadrupole strengths) no momentum
spread effects are detectable, but the total $\Delta x_L$ to extract all the
particles is uniquely determined by the spread in their betatron
amplitudes. Thus antisymmetric fluctuations in the beam position
have rather serious effects. This set-up seems suitable only when
short spill-times are required.

4. Concluding Remarks

The superiority of the twin-quadrupole system over conventional
integral resonance extraction systems is due to its widely variable
chromaticity. The performance of the extraction can be substantially
improved by varying the chromaticity and exploiting the advantages
that a low chromaticity may offer, such as for example a larger
tolerable momentum spread.
A comparison between this improved integral resonance extraction and third-order extraction for practical accelerator designs would be valuable and of special interest for the next generation of high energy accelerators, due to the severe requirements which are imposed to the performance of their extraction system.

References

(1) Y. Baconnier, O. Barbalat and D. Dekkers. Paper submitted to 7th Int. Conf. on High Energy Accelerators, Yerevan (1969).

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Figure Caption

Combined effect of two quadrupole lenses half a betatron wavelength apart.
Fig. 1