SPIN-PARITY ANALYSIS OF THE DECAY DISTRIBUTIONS
OF UNNATURAL PARITY MESONS

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ABSTRACT

We describe a method of spin-parity analysis of the full angular distribution arising from the two-stage decays of mesonic states. We show, in terms of a simple model for diffractive meson production, how it may be used to explore the unnatural parity meson spectrum. As typical examples we discuss the procedure for partial wave analyses of the decay distributions resulting from the reactions $K^-p \rightarrow (\Lambda\bar{p})p$ and $Kp \rightarrow (\phi K)p$.

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1. INTRODUCTION

The advent of hadron beams of momenta in the region of 100 GeV/c affords the opportunity to explore the spectrum of mesons with unnatural parity. In experiments at lower energies these mesons are difficult to establish^{*)}. Most detailed information that is available is for natural parity states from di-meson ($\pi\pi$, $K\pi$, $K\bar{K}$) production processes. The situation is reversed at high energies for those meson production processes where natural parity exchange dominates, since then only unnatural parity mesons can be produced in the favoured zero-helicity states. Moreover, by choosing a suitable exclusive channel a relatively clean, sharp spin-parity analysis is possible. The processes we have in mind are the two-body decay modes of states R

$$\begin{array}{c} a + b \rightarrow R + c \\ \quad \quad \quad \downarrow \\ \quad \quad \quad R_1 + R_2 \end{array}$$

where the decay products are either directly observed or are well-defined narrow states whose spin properties are revealed by their subsequent decay. Typical examples are

$$K^- p \rightarrow (\Lambda \bar{p}) p \quad (A)$$

$$K^- p \rightarrow (\phi K) p \quad (B)$$

where the observed decays $\Lambda \rightarrow p\pi^-$ and $\phi \rightarrow K^+K^-$ act as polarization analysers of the parent particle. These reactions are readily amenable to observation with all decay products being charged. Indeed, many spectrometer data are already available. The ϕ meson is a particularly favourable resonance to use as it is narrow and dominates K^+K^- production.

In sections 2 and 3 we describe the general formalism for performing a partial wave amplitude analysis of the full decay angular distributions observed in such reactions. In general we find that a model-independent analysis is not possible. In section 4 we introduce physically motivated simplifications that enable an amplitude analysis to be undertaken for high-energy diffractive meson production. We

*) So far, most information has come from three-body decay channels, such as $\pi\pi\pi$, $K\pi\pi$, with their well-known complications for spin-parity analysis; see, for example, Hansen et al.¹⁾.

use the reactions $K^-p \rightarrow (\Lambda\bar{p})p$ and $Kp \rightarrow (\phi K)p$ to illustrate the procedure. In the following paper²⁾ the unnatural parity meson spectrum is explored by performing such a spin-parity analysis on 50 GeV/c data obtained for the reactions $K^-p \rightarrow (\Lambda\bar{p})p$ and $K^+p \rightarrow (\bar{\Lambda}p)p$.

2. EXPERIMENTAL OBSERVABLES

To be specific, we consider the process $a + b \rightarrow R + c$ with the two-body decay $R \rightarrow R_1 + R_2$ where R_2 is observed, but R_1 is a well-defined narrow state of spin j which subsequently decays: $R_1 \rightarrow R_3 + R_4$ (see fig. 1). We take particles a and R_4 to be spinless. We suppose parity is conserved in the R decay, but not necessarily in the R_1 decay. The objective is to determine the spin-parities (J^π) of R in terms of the angular distribution, W , of the decay products. We take the moments, H , of this distribution, for a given R_1R_2 mass interval, ΔM_R , and a given range, Δt , of momentum transfer for $a + b \rightarrow R + c$, as the experimentally measurable quantities

$$H(LM\ell m) = \iint W(\Omega, \Omega_1) D_{Mm}^L(\Omega) D_{m0}^\ell(\Omega_1) d\Omega d\Omega_1 \quad (1)$$

with $\ell \leq 2j$. For clarity we have omitted the labels showing the dependence of $H(LM\ell m)$ and $W(\Omega, \Omega_1)$ on the production variables s , t , and M_R . We define $D(\Omega) = D(\phi, \theta, 0)$, where the angles $\Omega = (\theta, \phi)$ describe the direction of R_1 in the rest frame of R , with reference to axes defined such that the z axis is along the direction of particle a and the y axis is normal to the production plane. This choice is the t -channel helicity frame. Similarly, the angles $\Omega_1 = (\theta_1, \phi_1)$ describe the direction of R_3 in the R_1 rest frame, with reference to axes defined such that z_1 is along R_1 and y_1 is along $\vec{z} \times \vec{z}_1$ in the R rest frame. These angles are shown in fig. 2. The inverse of eq. (1) reads

$$W(\Omega, \Omega_1) = \sum_{LM\ell m} \left(\frac{2L+1}{4\pi} \right) \left(\frac{2\ell+1}{4\pi} \right) H(LM\ell m) D_{Mm}^{L*}(\Omega) D_{m0}^{\ell*}(\Omega_1) . \quad (2)$$

We choose the normalization so that

$$\int W d\Omega d\Omega_1 = H(0000) = N ,$$

where N is the number of events in a specified R_1R_2 mass interval, integrated over a given range of momentum transfer.

Not all of the moments $H(LM\ell m)$ are independent. Firstly, to ensure the angular distribution is real, $W = W^*$, we require

$$H(LM\ell m) = (-1)^M H(L-M\ell-m)^* . \quad (3)$$

Secondly from parity conservation in the production $a + b \rightarrow R + c$ and the decay $R \rightarrow R_1 + R_2$ we find

$$H(LM\ell m) = (-1)^\ell H(LM\ell m)^* . \quad (4)$$

The independent observables can be taken as $H(LM\ell m)$ with $M \geq 0$. From eq. (4) we see that H is real if ℓ is even and pure imaginary if ℓ is odd. If parity was also conserved in the subsequent $R_1 \rightarrow R_3 + R_4$ decay then ℓ would necessarily be even and the moments real.

3. SPIN-PARITY ANALYSIS

The angular distribution may be expressed as bilinear combinations of helicity amplitudes, T^A , for producing R in spin-parity states $A \equiv J^\pi$. If we define the particle helicities by

$$\begin{aligned} a + b(\lambda_b) &\rightarrow R(A) + c(\lambda_c) \\ (\Omega) &\downarrow R_1(\lambda) + R_2(\mu) \\ (\Omega_1) &\downarrow R_3(\nu) + R_4 , \end{aligned}$$

then the unpolarized distribution is given by^{*)}

$$W = \sum_{\nu\mu\lambda_c\lambda_b} \left| \sum_{A\lambda} T^A \right|^2 , \quad (5)$$

where T^A is a function of the particle momenta and helicities. Assuming factorization of the production and the decays we may write

$$T^A = \langle \Omega_1, \nu | t | \lambda \rangle \langle \Omega, \lambda \mu | F | A, \Lambda \rangle H_{\Lambda\lambda_c\lambda_b}^A(s, t, M_R) , \quad (6)$$

where H^A is the helicity amplitude for producing a state R of mass M_R , spin-parity $A \equiv J^\pi$ and helicity Λ ; s and t are the Mandelstam variables for the production process. To derive an expression for the moments it is necessary to transform

*) For the examples we shall consider particles b and c have spin $1/2$ and so the omission of the spin averaging factor in eq. (5) means that we will sum only over the independent helicity amplitudes.

the decay matrix elements to the angular momentum basis. For the decay $R \rightarrow R_1 + R_2$ we have

$$\langle \Omega, \lambda \mu | F | A, \Lambda \rangle = \sqrt{\frac{2J+1}{4\pi}} D_{\Lambda\alpha}^{J*}(\Omega) F_{\lambda\mu}^A, \quad (7)$$

with $\alpha \equiv \lambda - \mu$ and where $F_{\lambda\mu}^A$ are the helicity couplings describing the decay $R \rightarrow R_1 + R_2$ from a spin-parity state $A = J^\pi$. Parity conservation in this decay implies

$$F_{\lambda\mu}^A = \eta \eta_1 \eta_2 (-1)^{J-j_1-j_2} F_{-\lambda-\mu}^A, \quad (8)$$

where j_i and η_i are the spin and parity of R_i . Similarly the matrix element describing the subsequent decay of R_1 may be written

$$\langle \Omega_1, \nu | t | \lambda \rangle = \sqrt{\frac{2j+1}{4\pi}} D_{\lambda\nu}^{j*}(\Omega_1) t_\nu, \quad (9)$$

where $j \equiv j_1$ is the spin of R_1 .

To obtain the moments we substitute eqs. (7) and (9) into eqs. (6) and (5), use the Clebsch-Gordan coupling relation for the D^J 's and the D^j 's, and compare with eq. (2). We find

$$H(LM\ell m) = \frac{2(-1)^\ell s_\ell}{(2\ell+1)(2L+1)} \sum_{A, A'} \sqrt{(2J+1)(2J'+1)} T_M^L(AA') S_{\ell m}^L(AA'), \quad (10)$$

where

$$T_M^L(AA') = \sum_{\Lambda, \Lambda'} \rho_{\Lambda\Lambda'}^{AA'} \langle J-\Lambda J'\Lambda' | LM \rangle (-1)^\Lambda \quad (11)$$

$$S_{\ell m}^L(AA') = \sum_{\lambda\lambda', \mu} F_{\lambda\mu}^A F_{\lambda'\mu}^{A'*} (-1)^{\mu+\epsilon} \langle J-\alpha J'\alpha' | Lm \rangle \langle j-\lambda j\lambda' | \ell m \rangle \quad (12)$$

$$s_\ell = \sum_\nu |t_\nu|^2 (-1)^{-\nu-\epsilon} \langle j-\nu j\nu | \ell 0 \rangle \quad (13)$$

with $\alpha \equiv \lambda - \mu$ and $\epsilon = 0$ or $1/2$ according to whether (R_1, R_2, R_3) are all bosons or fermions; recall R and R_4 are assumed to be bosons.

The above discussion is similar to that given for an analysis of $\pi p \rightarrow (\omega\pi)p$ data³⁾. Earlier spin-parity analyses⁴⁾ of $\pi\omega$ (and $K\omega$) production data did not attempt a complete amplitude analysis of the type that we describe below.

3.1 Production amplitudes

The statistical tensors T_M^L and the density matrix ρ are alternative ways of describing the production of, and the interference between, the spin-parity states of R

$$\rho_{\Lambda\Lambda'}^{AA'} = \sum_{\lambda_b, \lambda_c} H_{\Lambda\lambda_c}^A H_{\Lambda'\lambda_b}^{A'*} \quad (14)$$

Parity conservation in the production process requires

$$\rho_{\Lambda\Lambda'}^{AA'} = \sigma\sigma'(-1)^{\Lambda-\Lambda'} \rho_{-\Lambda-\Lambda'}^{AA'} \quad (15)$$

where $\sigma \equiv \eta(-1)^J$ is the naturality of R.

From eq. (15) it follows that ρ can be written in block diagonal form if, instead of the helicity basis for R, we use eigenstates of the operator $(Pe^{-i\pi J_y})$ for reflections in the production plane. This is discussed, for example, by Chung and Trueman⁵⁾, who go on to derive a general parametrization of the density matrix. In the applications we wish to consider, such a model-independent approach is not tractable. However, if we have spin coherence at the lower vertex of the production reaction $ab \rightarrow Rc$ of fig. 1, we may write eq. (14) in the form

$$\rho_{\Lambda\Lambda'}^{AA'} = \bar{H}_{\Lambda}^A \bar{H}_{\Lambda'}^{A'*} \quad (16)$$

This is a physically reasonable assumption for our processes. For example, it is valid if states R are produced by a common exchange mechanism, or if helicity non-flip, say, dominates at the lower vertex. We shall see this allows a full spin-parity analysis.

As mentioned above, it is convenient to transform from the helicity to the reflectivity basis, and to use the combinations

$$\begin{aligned} N_{\Lambda}^A &= C_{\Lambda} \left[\bar{H}_{\Lambda}^A - \sigma(-1)^{\Lambda} \bar{H}_{-\Lambda}^A \right] \\ U_{\Lambda}^A &= C_{\Lambda} \left[\bar{H}_{\Lambda}^A + \sigma(-1)^{\Lambda} \bar{H}_{-\Lambda}^A \right], \end{aligned} \quad (17)$$

where σ is the naturality of R, and where $C_{\Lambda} = 1/2$ or $1/\sqrt{2}$ according to $\Lambda = 0$ or $\Lambda \neq 0$, respectively. Only reflectivity amplitudes with $\Lambda \geq 0$ are independent; for example

$$N_{-\Lambda}^A = -\sigma(-1)^{\Lambda} N_{\Lambda}^A \quad (18)$$

The use of these amplitudes has two advantages. Firstly using eq. (15) it follows that no U-N interference terms occur in the density matrix

$$\rho_{\Lambda\Lambda}^{AA'} = \left(N_{\Lambda}^A N_{\Lambda'}^{A'*} + U_{\Lambda}^A U_{\Lambda'}^{A'*} \right) / 4C_{\Lambda} C_{\Lambda'} . \quad (19)$$

Secondly for $\Lambda = 0$ the amplitudes N describe R production, $a + b \rightarrow R + c$, via natural parity exchange (NPE) in the t channel; the U describe production by unnatural parity exchange (UPE). For $\Lambda \neq 0$ this result is true to leading order in $1/s$. Here we have taken the beam to be pseudoscalar, $\eta_a = -1$.

An important observation, relevant to the experiments at SPS energies, is that N_0^A is only non-zero if $\sigma = -1$, see eq. (18). That is, zero-helicity R production by natural parity exchange (which is anticipated to be dominant) selects out unnatural parity R states.

3.2 Symmetry relations⁺

From the Hermitian character of ρ it follows that

$$T_M^L(A'A) = (-1)^M T_{-M}^{L*}(AA') , \quad (20)$$

and parity conservation in production, eq. (15), requires

$$T_M^L(AA') = \eta\eta'(-1)^{L+M} T_{-M}^L(AA') . \quad (21)$$

The factor, $S_{\ell m}^L$, arising from the decay $R \rightarrow R_1 + R_2$ satisfies the following symmetry requirements. Using eq. (8) we find parity conservation leads to the condition

$$S_{\ell m}^L(AA') = \eta\eta'(-1)^{\ell+L} S_{\ell -m}^L(AA') , \quad (22)$$

whereas from symmetry under interchange of A and A' we have

$$S_{\ell m}^L(A'A) = S_{\ell -m}^{L*}(AA') . \quad (23)$$

We note that the symmetry relations for the moments $H(LM\ell m)$, eqs. (3) and (4), can be obtained by combining the above relations for T_M^L and $S_{\ell m}^L$.

Relation (22) means it is convenient to form the linear combinations

$$H^{\pm}(LM\ell m) = H(LM\ell m) \pm H(LM\ell -m) = \left[1 \pm \eta\eta'(-1)^{\ell+L} \right] H(LM\ell m) , \quad (24)$$

⁺) See also Ref. 3.

so that contributions to the moments, $H^\pm$, are divided depending on the relative parity $\eta\eta' = \pm 1$. The independent observables are $H^\pm(LM\ell m)$ with $M \geq 0$ and $m \geq 0$ with $H^\pm$ real (pure imaginary) if ℓ is even (odd), see eqs. (3) and (4). For $M = 0$ there are further constraints. In this case eq. (3) requires H^+ to be real and H^- to be pure imaginary, which in turn gives

$$\begin{aligned} H^+(L0\ell m) &\equiv 0 & \text{for odd } \ell \\ H^-(L0\ell m) &\equiv 0 & \text{for even } \ell . \end{aligned} \quad (25)$$

3.3 Inclusion of the decay amplitudes

First we consider the factor s_ℓ of eq. (13), arising from the final decay $R_1 \rightarrow R_3 + R_4$. In cases of practical interest this factor is known. For example, for $\Lambda \rightarrow p\pi^-$

$$s_0 = 1/\sqrt{2} , \quad s_1 = -\alpha_\Lambda/\sqrt{2} , \quad (26)$$

where we have used the normalization of the Λ -decay helicity couplings, t_ν ,

$$|t_+|^2 + |t_-|^2 = 1$$

and the definition of the asymmetry parameter

$$\alpha_\Lambda \equiv |t_+|^2 - |t_-|^2 .$$

In our second example the final decay is $\phi \rightarrow K^+K^-$. We have ($|t_0|^2 \equiv 1$)

$$s_\ell = \langle 1010 | \ell 0 \rangle \quad (27)$$

and so, for this parity-conserving decay, ℓ can only be 0 or 2.

The unknown decay amplitudes are the helicity couplings, $F_{\lambda\mu}^A$, describing the decay $R \rightarrow R_1 + R_2$ for each of the various spin-parity ($A \equiv J^\pi$) states of R that are produced. Now, with the assumption of spin coherence at the lower vertex of fig. 1 [cf. eq. (16)], we see the moments are bilinear combinations of amplitudes where each amplitude is factorized into a production amplitude, N_Λ^A or U_Λ^A of eq. (17), and a decay amplitude $F_{\lambda\mu}^A$. That is, eq. (10) is of the form

$$H(LM\ell m) = \sum \dots \left(N_\Lambda^{AA'} F_{\lambda\mu}^A \right) \left(N_{\Lambda'}^{A'A} F_{\lambda'\mu'}^{A'} \right)^* + \dots \left(U_\Lambda^{AA'} F_{\lambda\mu}^A \right) \left(U_{\Lambda'}^{A'A} F_{\lambda'\mu'}^{A'} \right)^* , \quad (28)$$

where the summation is over $A, A', \Lambda, \Lambda', \lambda, \lambda', \mu$ and where $\dots$ denote known coefficients given by eqs. (10)-(13). The data therefore determine only the products, e.g. $N_\Lambda^{AA'} F_{\lambda\mu}^A$, not both amplitudes separately.

The general expression, eq. (28), is the basis of the partial wave amplitude analysis. It is instructive to see, for our two examples, which decay helicity couplings, $F_{\lambda\mu}^A$, contribute to the various observable moments. This we present in table 1. Note that the $\ell = 1$ moments of example A have been identified with the Λ polarization distributions; the exact relations are derived in the Appendix, see eqs. (A2).

4. SIMPLIFICATIONS FOR DIFFRACTIVE PRODUCTION

It is clear that the system of equations (10), or equivalently eqs. (28), cannot be solved in a completely model-independent manner, as in general there are more unknown amplitude magnitudes and phases than experimentally observable moments. This is true even after truncation at a maximum spin $J_{\max}$ and a maximum helicity $\Lambda_{\max}$ of the produced system R. Recall that for each J there are, in general, states of both natural and unnatural parity, each with their characteristic couplings to the observed decay channel. Fortunately, for diffractive production we can make physically reasonable assumptions that greatly simplify the analysis.

First experimental data for the forward production of resonant states in reactions such as $K^-p \rightarrow (\Lambda\bar{p})p$ and $Kp \rightarrow (\phi K)p$ show that the $M = 0$ moments dominate, which in turn implies the dominance of helicity $\Lambda = 0$ amplitudes⁺⁾ . On the other hand, high-energy diffractive production proceeds essentially by NPE and so, as discussed in section 3.1, this selects only those states, R, with unnatural parity; i.e. from a pseudoscalar beam states with $J^\pi = 0^-, 1^+, 2^-, \dots$ are preferentially produced⁺⁺⁾. Note that the omission of the relatively small $\Lambda = 1$ amplitudes should be a good approximation as they enter quadratically into the $M = 0$ moments. That is "linear" terms of the form $N_0^A N_1^{A' *}$ do not occur in $H(L0\ell m)$.

+) This could be anticipated, as from angular momentum considerations the production amplitudes $\sim |t|^{\Lambda/2}$.

++) This is the basis of the so-called Gribov-Morrison rule⁶⁾ for diffractive meson production. If the decay channel is such that only natural parity states are allowed, for example $Kp \rightarrow (K\pi)p$, then NPE dominance at SPS energies implies the $\Lambda = 1$ NPE amplitudes, N_1^A , are dominant. Recall $N_0^A \equiv 0$ in this case. These expectations are confirmed by the dominance of the $M = 0$ and $M = 2$ moments of the experimental data⁷⁾.

with the experimental data. In this case $P_x = P_z = 0$ as discussed in the Appendix. An example of an analysis based on such a model is presented in the following paper²⁾.

As a refinement of the model, we can release the restriction to $\Lambda = 0$ and admit helicity $\Lambda = 1$ amplitudes. Now both unnatural and natural parity production must be considered, although the latter occurs dominantly in even-spin states (see table 2). Such contributions manifest themselves mainly by interference with the relatively large $\Lambda = 0$ amplitudes, and contribute to $M = 1$ moments in the form

$$H(L1\ell m) \sim N_1^A N_0^{A'*}.$$

The possible contributions are tabulated in the second half of table 3. As for $M = 0$, we predict $H(L100) = 0$ for L odd. The longitudinal polarization, $P_z \sim H(L110)$, may indicate the relative importance of $\Lambda = 1$ natural parity production. $H^\pm(L111)$ receive contributions for all L .

The $M = 2$ moments are expected to be even smaller and serve to constrain the magnitudes of the $\Lambda = 1$ amplitudes:

$$H(L2\ell m) \sim N_1^A N_1^{A'*}.$$

4.2 Model analysis of $Kp \rightarrow (\phi K)p$

For the $R \rightarrow \phi K$ decay the helicity and LS couplings are different. It is advantageous to use the latter couplings, $g_{L'}^A$, if the partial wave series is to be truncated. The subscript S is omitted, as $S = 1$. A prime is added to the ϕK orbital angular momentum, L' , to avoid confusion with the L occurring in $H(LM\ell m)$. The relation between helicity and $L'S$ coupling is¹¹⁾

$$F_\lambda^A = \sum_{L'} (-1)^{\lambda+1} \langle J-\lambda 1 \lambda | L' 0 \rangle g_{L'}^A. \quad (30)$$

If R has natural parity, then parity conservation in the decay gives $L' = J$, and the decay is described by a single coupling g_J^A or, equivalently, F_1^A (recall $F_0^A \equiv 0$). On the other hand, for unnatural parity states we have $L' = J \pm 1$; there are two independent couplings $g_{J^\pm}^A$ or, alternatively, $F_{0,1}^A$. For a narrow resonant state we can use time reversal invariance to show that the couplings are real¹²⁾.

A second important simplification is due to the generalization of C-parity conservation⁺⁾ to diffractive processes, which proceed dominantly by $C = +1$, $SU(3)$ -singlet exchange. If we apply this conservation at the upper vertex of our process $ab \rightarrow Rc$ of fig. 1 where a is, for example, a kaon, then only states R with generalized $C = +1$ can be produced.

These two approximations facilitate a spin-parity analysis of the produced states R , which parallels the complementary partial wave analyses⁹⁾ of $\pi\pi$, $K\pi$... production. Moreover, if the statistics of the data permit, they provide a framework for a more refined analysis. For illustration we consider, in turn, our two sample processes.

4.1 Model analysis of $Kp \rightarrow (\bar{\Lambda}p)p$

Conservation of generalized C-parity leads to a selection rule for the decay amplitudes of the $\bar{\Lambda}p$ system. A $\bar{p}p$ state produced by an incident π by $C = 1$ exchange has $C(\bar{p}p) = 1$, while from the decay $C(\bar{p}p) = (-1)^{\ell+S}$, where ℓ is the orbital angular momentum and S the spin configuration. Therefore $\ell + S$ must be even. On the other hand, unnatural parity states have $\ell = J$ from parity conservation in decay, giving $J + S$ even, while natural parity states have $\ell = J \pm 1$ and $S = 1$, giving J even. The discussion can be generalized, for $SU(3)$ -singlet exchange, to apply to $\bar{\Lambda}p$ production by an incident K . This leads to the selection of decay helicity amplitudes listed in table 2¹⁰⁾. In particular, each unnatural parity state then decays via a single amplitude, just as is the case for a decay into two spinless particles. A partial wave analysis is therefore possible.

In fact a simple model, which embodies both of the approximations introduced above, predicts that the only non-zero moments, $H(LM\ell m)$, are the $M = 0$ moments listed in the first part of table 3. The assumptions can therefore be checked by comparing the model predictions

$$\begin{aligned} H(L000) &= 0 & \text{for } L \text{ odd} \\ H^-(L011) &= 0 & \text{for } L \text{ even} , \end{aligned} \tag{29}$$

+) This is sometimes called R-parity invariance, see, for example, Carruthers⁸⁾. This is a natural extension of G-parity invariance to incorporate strange particles.

Suppose we now apply the simple diffractive model. That is, we assume the dominance of the NPE helicity-zero production amplitudes, N_0^A . Then only unnatural parity states can be produced. Moreover, if we apply generalized C parity then we are restricted to states belonging to $J^{PC} = 0^{-+}, 1^{++}, 2^{-+} \dots$ multiplets. The observable moments $H(L, 0, m)$ can be expressed as the sum of bilinear combinations of the form $\text{Re}(f_L^{A'} f_L^{A''*})$, where each $f_L^{A'}$ is the product of a production and a decay amplitude, $N_0^{A'} g_L^{A'}$. The terms divide depending on whether the factor $\eta' \eta'' (-1)^L = \pm 1$. The $M = 0$ moments with even (odd) L receive contributions from terms with $J' + J''$ even (odd), which in turn implies even (odd) $L' + L''$. Again a straightforward amplitude analysis is possible. As the ϕK mass increases, terms with increasing L' should be successively introduced.

If the $M = 1, 2$ moments of the data are significantly non-zero then N_1^A amplitudes, corresponding to both natural and unnatural parity production, can be introduced. Although such a refined analysis is possible in principle, in practice it will require very high statistics data.

5. CONCLUSIONS

We have expressed the full angular distribution arising from the two-stage decay of a system R in terms of amplitudes. Each amplitude is the product of a production amplitude ($ab \rightarrow Rc$) and a decay amplitude ($R \rightarrow R_1 R_2$). This general expression serves as the basis of a spin-parity analysis of the superposition of decaying states R .

We have emphasized how experimental data for particular two-body decay channels are appropriate to explore the unnatural parity meson spectrum. As typical examples we discussed

$$\begin{aligned} K^- p &\rightarrow (\Lambda \bar{p}) p \\ K p &\rightarrow (\phi K) p, \end{aligned}$$

for which high-statistics, high-energy, spectrometer data are already available^{2,10,13}. The analysis complements, and to some extent parallels, the studies of $\pi\pi$, $K\pi$, $K\bar{K}$ production which have been used to probe the natural parity meson spectrum.

Even though we have a two-stage decay we have described how, within terms of a simple model for diffractive production, a complete partial wave analysis of the decay channel is possible. Moreover, depending on the available data, the amplitude analysis can be performed at various levels of complexity. To explore the possible unnatural parity resonance structure of the decay channel a physically reasonable first approximation is to consider just the helicity-zero production amplitudes. Alternatively, if the data permit, a more detailed analysis involving helicity-one production of both parities, together with helicity-zero, can be performed.

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Table 1

The decay helicity amplitudes contributing to the double moments $H(LM\ell m)$

Example A: $K^-p \rightarrow (\Lambda\bar{p})p$

Moment	Meaning (for $M=0$)	Contributions from	$\eta\eta'(-1)^L$	Decay amplitudes
$H(LM00)$	W	$\text{Re } (T_{M00}^{LSL})$	1	} $F_{++}^A F_{++}^{A'*}, F_{+-}^A F_{+-}^{A'*}$
$H(LM10)$	P_z	$\text{Im } (T_{M10}^{LSL})$	-1	
$H^+(LM11)$	P_x	$\text{Im } (T_{M1+}^{LSL})$	-1	} $F_{++}^A F_{+-}^{A'}, F_{+-}^A F_{++}^{A'*}$
$H^-(LM11)$	P_y	$\text{Im } (T_{M1-}^{LSL})$	1	

Example B: $Kp \rightarrow (\phi K)p$

Moment	Contributions from	$\eta\eta'(1)^L$	Decay amplitudes
$H(LM00)$	$\text{Re } (T_{M00}^{LSL})$	1	} $F_1^A F_1^{A'*}, F_0^A F_0^{A'*}$
$H(LM20)$	$\text{Re } (T_{M20}^{LSL})$	1	
$H(LM21)$	$\text{Re } (T_{M21}^{LSL})$	± 1	$F_1^A F_0^{A'}, F_0^A F_1^{A'*}$
$H(LM22)$	$\text{Re } (T_{M22}^{LSL})$	± 1	$F_1^A F_1^{A'*}$

Note that for a natural parity (ϕK) state $F_0^A \equiv 0$.

Table 2

The decay amplitudes
selected by generalized C-parity conservation

Naturality σ	Spin J	Decay amplitude
-1	even	F_{++}
-1	odd	F_{+-}
+1	even	F_{++}, F_{+-}

Table 3

The dominant contributions to the $M = 0$ and $M = 1$ moments,
 $H(LMlm)$, for $Kp \rightarrow (\Lambda\bar{p})p$. $\sigma \equiv \eta(-1)^J$ and $A \equiv J\eta$.
Interfering states A and A' contribute only to either even L
or odd L moments according to the value of $\eta\eta'(-1)^L$.

M = 0 moments

J	σ	J'	σ'	F^A	$F^{A'}$	$\eta\eta'(-1)^L$	W	P_y
even	-1	even	-1	F_{++}^A	$F_{++}^{A'}$	1	even L	-
odd	-1	odd	-1	F_{+-}^A	$F_{+-}^{A'}$	1	even L	-
even	-1	odd	-1	F_{++}^A	$F_{+-}^{A'}$	1	-	odd L
odd	-1	even	-1	F_{+-}^A	$F_{++}^{A'}$	1	-	odd L

M = 1 moments

J	σ	J'	σ'	F^A	$F^{A'}$	$\eta\eta'(-1)^L$	W	P_z
even	-1	even	-1	F_{++}^A	$F_{++}^{A'}$	1	even L	-
even	+1	even	-1	F_{++}^A	$F_{++}^{A'}$	-1	-	even L
odd	-1	odd	-1	F_{+-}^A	$F_{+-}^{A'}$	$(-1)^L$	even L	odd L
even	+1	odd	-1	F_{+-}^A	$F_{+-}^{A'}$	$(-1)^L$	even L	odd L

REFERENCES

- 1) J.D. Hansen, G.T. Jones, G. Otter and G. Rudolph, Nucl. Phys. B81 (1974) 403.
Proc. Daresbury Study Weekend No. 8 (Eds. J.B. Dainton and A.J.G. Hey), DL/R 34,
Feb. 1975.
- 2) W.E. Cleland et al., following paper.
- 3) S.U. Chung et al., Phys. Rev. D11 (1975) 2426.
- 4) V. Chaloupka et al., Phys. Lett. 51B (1974) 407.
S.U. Chung et al., Phys. Lett. 51B (1974) 413.
S.U. Chung et al., Phys. Lett. 47B (1973) 526.
- 5) S.U. Chung and T.L. Trueman, Phys. Rev. D11 (1975) 633.
- 6) D.R.O. Morrison, Phys. Rev. 165 (1968) 1699.
- 7) R. Baldi et al., Nucl. Phys. B134 (1978) 365.
A.D. Martin et al., Nucl. Phys. B134 (1978) 392.
Geneva-Lausanne spectrometer, experiment WA10, 50 GeV/c data, to be published.
- 8) P. Carruthers, Introduction to unitary symmetry (Interscience Division, Wiley,
NY, 1966).
- 9) P. Estabrooks et al., Proc. Tallahassee Conference on π - π scattering, AIP
Conf. Proc. No. 13, 1973 (AIP, NY, 1973), p. 37.
B. Hyams et al., Nucl. Phys. B64 (1973) 134.
P. Estabrooks et al., Nucl. Phys. B133 (1978) 490.
- 10) W.E. Cleland et al., Phys. Lett. 89B (1980) 290.
- 11) A.D. Martin and T.D. Spearman, Elementary particle theory (North-Holland,
Amsterdam, 1970).
- 12) S.U. Chung, CERN 71-8 (1971).
- 13) Geneva-Lausanne Collaboration, experiment WA10, 50 GeV/c data. Birmingham-
Glasgow-CERN Collaboration (Omega spectrometer), experiment WA48, 13 GeV/c
data.

APPENDIX

MEASUREMENT OF THE Λ POLARIZATION

We consider the production and sequential decay of mesonic states R^- in the reaction

$$K^- p \rightarrow R^- p, \quad R^- \rightarrow \Lambda \bar{p}, \quad \Lambda \rightarrow p \pi^-.$$

As the Λ -decay violates parity, we can measure its polarization from the decay asymmetry.

For a fixed set of production variables s , t , and M_R , we have characterized the decay $R^- \rightarrow \Lambda \bar{p}$ by angles θ , ϕ , and the subsequent decay $\Lambda \rightarrow p \pi^-$ by θ_1 , ϕ_1 , in the t -channel helicity frame (see fig. 2). The full angular distribution is then written as

$$W(\Omega, \Omega_1) = \frac{W(\Omega)}{4\pi} [1 + \alpha_\Lambda \vec{P}(\Omega) \cdot \hat{q}] \quad (A1)$$

where $W(\Omega)$ is the R -decay distribution integrated over the Λ decay, α_Λ is the Λ -decay asymmetry parameter, and $\hat{q}$ is a unit vector in the direction of the p from the Λ decay. The Λ polarization is specified by the real vector $\vec{P}$, which is a function of Ω .

Comparing eq. (A1) with the general expansion of the angular distribution in terms of double moments, eq. (2), we find the following relations between Λ polarization $\vec{P}$ and double moments $H(LM\ell m)$:

$$\begin{aligned} W(\Omega) &= \sum_{LM} \frac{2L+1}{4\pi} H(LM00) D_{M0}^{L*}(\Omega) \\ P_x(\Omega) \cdot W(\Omega) &= -\frac{3}{\sqrt{2}} \frac{1}{\alpha} \sum_{LM} \frac{2L+1}{4\pi} \left[H(LM11) D_{M1}^{L*}(\Omega) - H(LM1-1) D_{M-1}^{L*}(\Omega) \right] \\ P_y(\Omega) \cdot W(\Omega) &= -\frac{3i}{\sqrt{2}} \frac{1}{\alpha} \sum_{LM} \frac{2L+1}{4\pi} \left[H(LM11) D_{M1}^{L*}(\Omega) + H(LM1-1) D_{M-1}^{L*}(\Omega) \right] \\ P_z(\Omega) \cdot W(\Omega) &= \frac{3}{\alpha} \sum_{LM} \frac{2L+1}{4\pi} H(LM10) D_{M0}^{L*}(\Omega). \end{aligned} \quad (A2)$$

If states with helicity $\Lambda = 0$ dominate, only $M = 0$ moments are present. In this case, the transverse polarization components P_x and P_y are characterized by the combinations of moments $H^\pm(L011) \equiv H(L011) \pm H(L01-1)$ discussed in section 3.2. In addition, the symmetry relation eq. (4) implies $P_x = P_z = 0$, and the only possible non-zero polarization component is P_y , measured by $H(L011) - H(L01-1)$.

CP conservation has an interesting consequence for the sign of the polarization moments, since it implies $\alpha_\Lambda = -\alpha_{\bar{\Lambda}}$. Comparing $\Lambda\bar{p}$ production by incident K^- with $\bar{\Lambda}p$ production by K^+ , we see from eq. (A1) that all polarization moments must change sign when going from the K^- to the K^+ initiated reaction. Here we have assumed even-signature (Pomeron) exchange, implying $\vec{P}_\Lambda = \vec{P}_{\bar{\Lambda}}$.

Figure captions

Fig. 1 : Production and decay of states R , showing the notation used for the helicities of the decay products. We take particle R_4 to have spin zero.

Fig. 2 : Reference frames for the two-body decay $R \rightarrow R_1 + R_2$ and the subsequent decay $R_1 \rightarrow R_3 + R_4$. We show the t -channel helicity frame for R produced by the reaction $a + b \rightarrow R + c$.