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PIONIC MODES OF EXCITATION IN NUCLEI *)

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ABSTRACT

These lectures review recent developments in the understanding of nuclear excitations carrying the quantum numbers of a pion. The first part considers the excitation and propagation of $\Delta(1232)$ states in a nuclear environment, as seen in pion-nucleus elastic scattering and in photon-induced processes. The second part discusses high-momentum transfer properties of low-energy pion-like nuclear states and their relation to the problem of pion condensation.

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PIONIC MODES OF EXCITATION IN NUCLEI

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1. Introduction

Pionic modes of excitation in nuclear systems have received considerable experimental and theoretical interest in recent years. Vast amounts of pion-nucleus scattering data ^{/1/} have been produced at the meson factories, the analysis of which has provided information about the "optical" branches of the pion-nuclear excitation spectrum, where the propagation of $\Delta(1232)$ isobars and their interaction with the nuclear environment is one of the outstanding issues ^{/2-4/}, and will be one of the subjects of these lectures. Another part will also describe photonuclear processes in the 3.3 resonance region ^{/5,6/}.

The low frequency, or "acoustic" branches of the pion spectrum, those related to low-lying particle-hole excitations carrying the quantum numbers of a pion, have likewise been subject of various investigations. The interest there is motivated by the possibility of a phase transition of nuclear matter into a pion condensate at sufficiently high density ^{/7-9/}. Most of the conclusions or expectations derived from the possible existence of a pion condensate rest very strongly on our (limited) knowledge of spin-isospin dependent correlations operating in pionic modes. It is therefore an important task to learn as much as possible about this interaction from "standard" experiments, both at low and intermediate energies. Of particular interest are the high momentum transfer ($q \sim 2-3 m_\pi$) properties of these correlations, because this is the domain where the tensor force from one-pion exchange, the driving interaction to form a pion condensate, is strongest.

Even if the threshold for pion condensation is far from being reached at normal nuclear densities, certain precritical effects might appear as indicators of the proximity of the pion condensation threshold. Ways to observe such pionic opalescence phenomena (see refs. ^{/10,11/}), or precritical enhancement effects ^{/12,13/}, in actual experiments are presently subject of theoretical activities as well as experimental proposals. One chapter of these lectures will be devoted to this question. Before facing the full complexity of the pion-nuclear many-body problem, however, it is necessary first to specify the elementary input: mesons, nucleons, $\Delta(1232)$ isobars, and their mutual interactions.

2. Basic meson-baryon effective Lagrangians and elementary processes

2.1 πN Scattering

In our kinematical region of interest (pion energies $\omega \lesssim 3 m_\pi$, pion momenta $|\vec{k}| \lesssim 4 m_\pi$, where $m_\pi = 139$ MeV is the pion mass), the pion-nucleon scattering amplitude, Fig. 1, is dominated by partial waves with $\ell = 0, 1$. The p-wave interactions are of primary importance, because of the presence of the $\Delta(1232)$ resonance in the spin-isospin-3/2 channel. The s-wave interactions play only a minor role in systems with equal numbers of protons and neutrons. The isovector part of the s-wave πN interaction can be thought of as being generated by the exchange of a ρ meson, as shown in Fig. 2a, while the isoscalar s-wave amplitude is small due to cancellations between isoscalar (scalar) 2π exchange and short range repulsive contributions which, among other mechanisms, involve the virtual excitation of intermediate nucleon-antinucleon pairs (see Fig. 2b). We should point out, however, that this cancellation holds only for on-shell pions, and can be unbalanced if the pions move off shell ^{/14/}. We shall not specify here the details of the model illustrated in Fig. 2, but refer to it as a useful way to extrapolate off-shell, e.g. in the $\pi d \leftrightarrow NN$ process, to be discussed later. We would like to mention some alternative models in the literature, among which in particular, is the one of ref. ^{/15/}, which does not make explicit use of ρ exchange. Let us now concentrate on a somewhat more detailed description of the p-wave πN interaction. We wish to construct it from the known intermediate states (the nucleon and its excited states) in the πN s-channel (cf. Fig. 1). A useful way to proceed ^{/16/} is to develop a crossing symmetric K-matrix (reaction matrix), according to Fig. 3, in the form

$$\langle \pi(q') N | K(s) | \pi(q) N \rangle = \frac{1}{4\pi} \sum_X \frac{\langle q' | \delta H_{\pi N X} | X \rangle \langle X | \delta H_{\pi N X} | q \rangle}{M_X - \sqrt{s}} + \text{crossed terms}, \quad (1)$$

where the kinematic variables are explained in Fig. 1 ($\sqrt{s}$ is the πN c.m. energy), and the crossed terms are obtained essentially by the replacements $\sqrt{s} \rightarrow \sqrt{u}$, $q \leftrightarrow -q'$. Here $X = N(939)$, $\Delta(1232)$, $N^*(1470)$ etc. refer to the nucleon and the important πN resonances, M_X are the corresponding physical masses. The $\delta H_{\pi N X}$ are the effective Hamiltonians which couple the πN system to a given intermediate state. Treating these non-relativistically, we have, for example:

$$\delta H_{\pi N N}(q) = i \frac{f(q^2)}{m_\pi} \vec{\sigma} \cdot \vec{q} \tau_\lambda [\pi_\lambda(q) + \pi_{-\lambda}^*(-q)], \quad (2a)$$

$$\delta H_{\pi N \Delta}(q) = i \frac{f^*(q^2)}{m_\pi} \vec{S}^+ \cdot \vec{q} T_\lambda^+ [\pi_\lambda(q) + \pi_{-\lambda}^*(-q)] + h.c., \quad (2b)$$

where $\pi_\lambda(q)$ annihilates a pion of charge λ and four-momentum $q = (\omega, \vec{q})$. In eq. (2b), $\vec{S}^+$ and T^+ are transition spin and isospin operators, respectively, connecting spin-isospin $1/2^-$ and $3/2^-$ states ^{/16/}. The quantities $f(q^2)$ and $f^*(q^2)$ are πNN or $\pi N \Delta$ vertex form factors, to be specified later. Their values at $q^2 = m_\pi^2$ define the relevant π -baryon coupling constants, which we denote by f and f^* , respectively.

The p-wave K matrix, eq. (1), has to be projected onto the different eigen channels $\alpha = (2S, 2I)$ of given spin S and isospin I ($1/2$ or $3/2$ each). The corresponding channel K matrix, denoted by K_α , is related to the partial wave amplitude, expressed in terms of the phase shifts δ_α ,

$$f_\alpha(\omega) = \frac{1}{|\vec{q}|} e^{i\delta_\alpha} \sin \delta_\alpha = \frac{1}{2i|\vec{q}|} (S_\alpha - 1), \quad (3a)$$

by the unitarity condition

$$S_\alpha = \frac{1 + i|\vec{q}|K_\alpha}{1 - i|\vec{q}|K_\alpha} \quad (3b)$$

or simply

$$K_\alpha = \frac{1}{|\vec{q}|} \tan \delta_\alpha. \quad (3c)$$

To illustrate how this scheme works, let us derive the spin-isospin $3/2$ p-wave amplitude f_{33} under the simplifying assumption that only nucleon and $\Delta(1232)$ -isobar intermediate states contribute, and that the baryons are treated in the static limit (i.e. $\sqrt{s} = M + \omega$). Except for isospin factors which we omit here for simplicity, the K-matrix of eq. (1) is then given by

$$\langle q'N | K(s) | qN \rangle = \frac{f^2}{4\pi m_\pi^2} \left[\frac{\vec{\sigma} \cdot \vec{q}' \vec{\sigma} \cdot \vec{q}}{-\omega} + \frac{\vec{\sigma} \cdot \vec{q} \vec{\sigma} \cdot \vec{q}'}{\omega} \right] + \frac{f^{*2}}{4\pi m_\pi^2} \left[\frac{\vec{S} \cdot \vec{q}' \vec{S} \cdot \vec{q}}{\omega_\Delta - \omega} + \frac{\vec{S} \cdot \vec{q} \vec{S} \cdot \vec{q}'}{\omega_\Delta + \omega} \right], \quad (4)$$

where $\omega_\Delta = M_\Delta - M_N = 2.1 m_\pi$. If projected into the spin-isospin- $3/2$ channel, and neglecting the small crossed Δ -isobar piece we obtain

$$K_{33}(\omega) = \frac{1}{|\vec{q}|} \tan \delta_{33} = \frac{1}{3} \frac{|\vec{q}|^2}{4\pi} \left[\frac{4f^2}{\omega} + \frac{f^{*2}}{\omega_{\Delta} - \omega} \right]. \quad (5)$$

Following eqs. (3) and employing the Chew-Low value $f^* = 2f$ for the $\pi N \Delta$ coupling constant, we find, with $\lambda = (4/3) f^2 / 4\pi m_{\pi}^2$:

$$f_{33}(\omega) = \frac{\lambda \vec{q}^2 (\omega_{\Delta}/\omega)}{\omega_{\Delta} - \omega - (i/2) \Gamma_{\Delta}(\omega)}, \quad \Gamma_{\Delta} = \frac{2}{3} \frac{f^{*2}}{4\pi} \frac{|\vec{q}|^3}{m_{\pi}^2} \left(\frac{\omega_{\Delta}}{\omega} \right), \quad (6)$$

which is identical with the result from Chew-Low theory. Note that the decay width Γ_{Δ} of the $\Delta(1232)$ has a factor ω_{Δ}/ω which comes from the crossed nucleon pole term. This factor, which is proportional to $(\vec{q}^2 + m_{\pi}^2)^{-1/2}$, is sometimes misinterpreted as a "long range $\pi N \Delta$ vertex factor", but it has obviously nothing to do with the $\pi N \Delta$ form factor $f^*(q^2)$.

For a quantitative description of the p-wave πN phase shifts, the full relativistic kinematics has to be treated appropriately, and inclusion of the $N^*(1470)$ is required to reproduce the correct energy dependence of δ_{11} . The phase shifts calculated with the model are then shown in Figs. 4, while the explicit form of the partial wave K-matrices, including all factors, is given in table 1. This model, based entirely on π -baryon effective Lagrangians and the physical masses of nucleon, Δ and N^* , is obviously a good starting point for subsequent discussions of pion-nucleus interactions. We emphasize that the separation of the p-wave π -nucleon amplitude into a propagator part and a π -baryon vertex piece will turn out to be useful in the many-body problem, since both ingredients will be renormalized in a nuclear medium.

2.2 Pion-Baryon Vertex Formfactors

The K-matrix scheme developed in the previous chapter determines only the on-shell properties of the pion-baryon vertex functions. Their off-shell extrapolation, which is required in the pion-nuclear many-body problem, has to be obtained from other sources. We would like to parametrize the π -baryon vertex factors, appearing in eqs. (2), in the form, e.g.,

$$f(q^2, s) = f \left(\frac{2M}{M + \sqrt{s}} \right)^{1/2} \left(\frac{\Lambda^2 - m_{\pi}^2}{\Lambda^2 - q^2} \right)^n, \quad (7a)$$

$$f^*(q^2, s) = f^* \left(\frac{2 M_\Delta}{M_\Delta + \sqrt{s}} \right)^{1/2} \left(\frac{\Lambda^{*2} - m_\pi^2}{\Lambda^{*2} - q^2} \right)^n, \quad (7b)$$

where $q^2 = \vec{q}_0^2 - \vec{q}^2$. The additional kinematic correction factor depending on $\sqrt{s}$ becomes obvious from table 1. The coupling constants f, f^* are determined at $q^2 = m_\pi^2$ and $s = M^2$ or $s = M_\Delta^2$, respectively ($f^2/4\pi = 0.08$, $f^{*2}/4\pi = 0.37$). We shall usually employ monopole formfactors with $n = 1$. The cutoff mass Λ , by its inverse, is related to the range of the vertex and has to be determined phenomenologically.

It is clear that the parametrization of eq. (7) hides just another many-body problem, namely the quark dynamics of the $\pi N \leftrightarrow N$ or $\pi N \leftrightarrow \Delta$ transitions. The conceptual hypothesis we have tacitly introduced here is that the pion-nuclear many-body problem, in the limited range of energies and momenta considered, can be described entirely in terms of nucleons, Δ -isobars and mesons, disregarding their underlying quark structure. This is by no means a trivial assumption, and its range of validity is clearly connected with questions about the size of the quark bag. The present status of the problem /18,19/ must be regarded as controversial. However, we would like to emphasize that the picture developed here cannot be easily accommodated with typical MIT-bag radii larger than 1 fm.

2.3 The spin-isospin dependent baryon-baryon interaction

Given the effective Lagrangians of eqs. (2) together with eqs. (7), the one-pion exchange (OPE) NN interaction becomes (Fig. 5 (a)):

$$W_\pi(\omega, \vec{q}) = \frac{f^2(q^2)}{m_\pi^2} V_\pi(\omega, q) \vec{\sigma}_1 \cdot \hat{q} \vec{\sigma}_2 \cdot \hat{q} \vec{\tau}_1 \cdot \vec{\tau}_2, \quad (8)$$

where

$$V_\pi(\omega, q) = \frac{\vec{q}^2}{\omega^2 - \vec{q}^2 - m_\pi^2 + i\delta}, \quad (8a)$$

while the OPE $N\Delta \rightarrow \Delta N$ interaction can be written similarly as (Fig. 5 (b))

$$W_\pi^{N\Delta \rightarrow \Delta N}(\omega, \vec{q}) = \frac{f^{*2}(q^2)}{m_\pi^2} V_\pi(\omega, q) \vec{S}_1^+ \cdot \hat{q} \vec{S}_2 \cdot \hat{q} \vec{T}_1^+ \cdot \vec{T}_2. \quad (9)$$

These will obviously be the driving forces in excitation modes carrying pion

Figure 1:

Kinematic variables in the πN scattering amplitude

Figure 2: Mechanisms for s-wave πN scattering: (a) isoscalar part through σ exchange and short-range repulsion; (b) isovector part through ρ exchange.

Figure 3: Direct and crossed p-wave πN K-matrix. The intermediate states X refer to $N(939)$, $\Delta(1232)$, $N^*(1470)$ etc.

$$K_{11} = \frac{1}{3} \frac{q^2}{m_\pi^2} \frac{M}{\sqrt{s}} \left[\frac{2Mf^2}{4\pi} \left(\frac{9}{M^2 - s} + \frac{1}{M^2 - \bar{u}} \right) + \frac{2M_{N^*}f_{N^*}^2}{4\pi} \left(\frac{9}{M_{N^*}^2 - s} + \frac{1}{M_{N^*}^2 - \bar{u}} \right) + \frac{2M_\Delta f_\Delta^2}{4\pi} \frac{16}{9} \frac{1}{M_\Delta^2 - \bar{u}} \right]$$

$$K_{13} = K_{31} = \frac{1}{3} \frac{q^2}{m_\pi^2} \frac{M}{\sqrt{s}} \left[-\frac{2Mf^2}{4\pi} \frac{2}{M^2 - \bar{u}} - \frac{2M_{N^*}f_{N^*}^2}{4\pi} \frac{2}{M_{N^*}^2 - \bar{u}} + \frac{2M_\Delta f_\Delta^2}{4\pi} \frac{4}{9} \frac{1}{M_\Delta^2 - \bar{u}} \right]$$

$$K_{33} = \frac{1}{3} \frac{q^2}{m_\pi^2} \frac{M}{\sqrt{s}} \left[\frac{2Mf^2}{4\pi} \frac{4}{M^2 - \bar{u}} + \frac{2M_{N^*}^2 f_{N^*}^2}{4\pi} \frac{4}{M_{N^*}^2 - \bar{u}} + \frac{2M_\Delta f_\Delta^2}{4\pi} \left(\frac{1}{M_\Delta^2 - s} + \frac{1}{9} \frac{1}{M_\Delta^2 - \bar{u}} \right) \right]$$

Table 1: Explicit forms of the isobar-model K matrices $K_\alpha = (1/q) \tan \delta_\alpha$ in the p-wave eigenchannels, employing $N(939)$, $\Delta(1232)$ and $N^*(1470)$ intermediate states. Relativistic kinematic factors are included here. Note that $\sqrt{s} = E + \omega$, $E = \sqrt{M^2 + \vec{q}^2}$, in terms of c.m. variables, and $\bar{u} = (E - \omega)^2 - 2q^2$. The masses and coupling constants are

$M = 939$ MeV, $M_\Delta = 1232$ MeV, $M_{N^*} = 1450$ MeV;
 $f^2/4\pi = 0.08$, $f_\Delta^2/4\pi = 0.37$, $f_{N^*}^2/4\pi = 0.02$.

Figure 4: p-wave πN phase shifts calculated within the model presented in table 1. The data are taken from ref. /17/. (The author is grateful to G. Bienek for preparation of these figures.)

Figure 5: (a) and (b): one-pion exchange NN and $\Delta N \rightarrow N\Delta$ interaction
(c) Two-pion exchange interaction in $I=1$ t-channel.

Figure 6: Spectral function (mass distribution) of isovector two-pion exchange. Dashed curve: resonant part corresponding to the ρ meson. Solid curve: full spectral distribution, including non-resonant $\pi\pi$ intermediate states.

quantum numbers. At shorter range, the exchange of two interacting pions in an isovector state has to be considered as well (Fig. 5 (c)). In the NN sector, this interaction can be represented as

$$W_{2\pi}(\omega, \vec{q}) = \left[\int_{4m_\pi^2}^{\infty} dt \frac{p_{2\pi}(t)}{\omega^2 - \vec{q}^2 - t} \right] (\vec{\sigma}_1 \times \vec{q}) \cdot (\vec{\sigma}_2 \times \vec{q}) \vec{\tau}_1 \cdot \vec{\tau}_2. \quad (10)$$

Here $p_{2\pi}(t)$ is related to the square of the $\pi\pi \rightarrow NN$ helicity amplitude f_1^- , as it appears for example in the Paris NN interaction [20]. Its structure is shown in Fig. 6. It exhibits a resonant part related to the ρ meson, but also a non-resonant background which contains, among other pieces, the contribution of iterated OPE to the isovector two-pion exchange. For low momentum transfers, it is useful to approximate the resonant part of $W_{2\pi}$ with zero-width ρ exchange,

$$W_\rho = \frac{f_{\rho NN}^2}{m_\rho^2} \frac{(\vec{\sigma}_1 \times \vec{q}) \cdot (\vec{\sigma}_2 \times \vec{q})}{\omega^2 - \vec{q}^2 - m_\rho^2} \vec{\tau}_1 \cdot \vec{\tau}_2, \quad (11)$$

which defines the ρNN coupling constant. Typical values are $f_{\rho NN}^2/4\pi = 4.5 - 5.0$. An important property of ρ exchange is that its tensor force component has opposite sign as compared to that from OPE, and therefore provides a natural regularisation of the pathologically large OPE tensor interaction at small distance. This is seen by remembering that $\vec{\sigma}_1 \cdot \hat{q} \vec{\sigma}_2 \cdot \hat{q} = \frac{1}{3} \vec{\sigma}_1 \cdot \vec{\sigma}_2 + \frac{1}{3} S_{12}(\hat{q})$, with $S_{12}(\hat{q}) = 3\vec{\sigma}_1 \cdot \hat{q}_2 \vec{\sigma}_2 \cdot \hat{q} - \vec{\sigma}_1 \cdot \vec{\sigma}_2$, whereas $(\vec{\sigma}_1 \times \hat{q}) \cdot (\vec{\sigma}_2 \times \hat{q}) = \frac{2}{3} \vec{\sigma}_1 \cdot \vec{\sigma}_2 - \frac{1}{3} S_{12}(\hat{q})$.

If one of the nucleons in $W_{2\pi}$ is replaced by a Δ , the usual insertions $\vec{\sigma} \rightarrow \vec{S}$ and $\vec{\tau} \rightarrow \vec{T}$ have to be made. Not very much is known about the two-pion exchange $N\Delta$ -interaction. We assume a simple scaling:

$$V_{2\pi}(N\Delta \rightarrow \Delta N) = \alpha_{2\pi} V_{2\pi}(N\Delta \rightarrow NN) = \alpha_{2\pi}^2 V_{2\pi}(NN \rightarrow NN),$$

where $\alpha_{2\pi}$ measures the ratio of $\rho N\Delta$ and ρNN coupling strength. One of the best testing grounds for determining α_ρ , together with the cutoff mass Λ associated with the π -baryon formfactors, is the $\pi d \leftrightarrow pp$ reaction. This is so because the kinematics of this process requires the transfer of large momenta, but little energy, between the two nucleons. In the 3.3 resonance region, the excitation of a $\Delta(1232)$ with subsequent $\Delta N \rightarrow NN$ transition dominates, and the large momentum transfer in the latter process makes it sensitively dependent on $\alpha_{2\pi}$ and Λ . From a detailed analysis of $\pi d \leftrightarrow pp$ total and differential cross sections as well as polarization parameters [21], one finds that $\alpha_{2\pi} = 1.7-2.0$

and $\Lambda = 1.2 - 1.4$ GeV, where the same Λ has been used for both πNN and $\pi N\Delta$ form factors. The lower value of $\alpha_{2\pi}$ would also follow for the quark model ratio of $\rho N\Delta$ and ρNN coupling constants, and is consistent with the $\gamma N\Delta$ transition form factor within the ρ meson dominance model.

3. Pions in Nuclear Matter

3.1 The spectrum of pion-like excitations

Consider now first the response function $R(\omega, q)$ of an infinite nuclear medium with respect to a pion field of frequency ω and momentum q . Following the observations of the previous chapter, the pion field will polarize the medium primarily by exciting nucleon-hole and $\Delta(1232)$ -hole states. A first order picture of the response is then given by the pion self energy $\Pi^{(0)}$, as illustrated in Fig. 7. We write it - somewhat schematically - in the form

$$\Pi^{(0)}(\omega, q) = - \sum_{ph} \frac{|\langle \pi(q) | \delta H | ph \rangle|^2}{E_p - E_h - \omega} + \text{crossed terms}, \quad (12)$$

where $|ph\rangle$ denotes a nucleon- or Δ -hole state carrying pion quantum numbers and δH is the πNN or $\pi N\Delta$ vertex operator given by eqs. (2) (see table 1 for the relevant input parameters). We rewrite eq. (12) as

$$\Pi^{(0)}(\omega, q) = q^2 \chi(\omega, q), \quad (13)$$

which defines the elementary (lowest order) pionic susceptibility χ of the medium. Because of the spin dependence of the p-wave πN interaction, it is very useful to point out ^{123,24/} the analogy of the pionic response problem with that encountered in magnetic materials. In that sense, χ has a "diamagnetic" part χ_Δ which involves high-lying Δ -hole excitations, and "paramagnetic" component χ_N , related to low lying nucleon-hole excitations:

$$\chi = \chi_N + \chi_\Delta. \quad (14)$$

For symmetric nuclear matter at density ρ (Fermi momentum k_F), we find at large frequencies ($\omega \gg \frac{q^2}{2M}$):

$$\chi_N(\omega, q) \cong - \frac{f^2(q^2)}{m_\pi^2} \left[\frac{\rho}{\frac{q^2}{2M^*} - \omega} + \frac{\rho}{\frac{q^2}{2M^*} + \omega} \right] \quad (15a)$$

and

$$\chi_{\Delta}(\omega, q) \approx - \frac{f^{*2}(q^2)}{m_{\pi}^2} \cdot \frac{4}{9} \left[\frac{\rho}{\omega_{\Delta} - \omega - i/2 \Gamma_{\Delta}(\omega)} + \text{crossed term} \right], \quad (15b)$$

where M^* is the nucleon effective mass, and $\omega_{\Delta} = 2.1 m_{\pi}$. Note that $\chi_N(\omega, q) \rightarrow 0$ as $M^* \rightarrow \infty$, so that in this limit the response (for symmetric nuclear matter) is completely determined by $\Delta(1232)$ excitations. At low frequency ω and $q < k_F$,

$$\chi_N(\omega=0, q) = - \frac{f^2(q^2)}{m_{\pi}^2} \frac{2M^*k_F}{\pi^2} \left[1 - \frac{1}{12} \frac{q^2}{k_F^2} + \dots \right] \quad (15c)$$

and

$$\chi_{\Delta}(\omega=0, q) = - \frac{8}{9} \frac{f^{*2}(q^2)}{m_{\pi}^2} \frac{\rho}{\omega_{\Delta}}. \quad (15d)$$

Here χ_N dominates, but the contribution from χ_{Δ} is still sizeable.

For $\omega \geq m_{\pi}$, we have the "pion optics" domain, which is explored by pion elastic scattering. It is convenient there to work with an optical potential. For the lowest order p-wave optical potential, the relation is simply

$$U_{opt}^{(o)}(\omega, q) = \frac{1}{2\omega} \Pi^{(o)}(\omega, q). \quad (16)$$

The response function in this simplest possible model is obtained (see Fig. 8) by the RPA iteration

$$R(\omega, q) = \Pi^{(o)}(\omega, q) + \Pi^{(o)}(\omega, q) \frac{1}{\omega^2 - q^2 - m_{\pi}^2} R(\omega, q). \quad (17)$$

It is convenient to define a "dimesic function" /13,24/ $\epsilon(\omega, q)$ by

$$R(\omega, q) = \frac{\Pi^{(o)}(\omega, q)}{\epsilon(\omega, q)}. \quad (18)$$

From eqs. (17,13),

$$\epsilon(\omega, q) = 1 - \frac{q^2}{\omega^2 - q^2 - m_{\pi}^2} \chi(\omega, q). \quad (19)$$

It is clear that the zeros of $\epsilon(\omega, q)$ determine the spectrum $\omega(q)$ of pion-like excitations of the nuclear medium. Note also that, in the optical part of the spectrum, an index of refraction n for pions can be defined as

$$n^2 = \frac{q^2}{\omega^2 - m_\pi^2} = [1 + \chi(\omega, q)]^{-1}. \quad (20)$$

The T-matrix for pion scattering is simply given by $T=R/2\omega$.

Of particular interest is the question whether the nuclear medium can act as a strong amplifier for the pion field at no cost of energy, because of the strong attraction provided by the p-wave πN interaction in the spin-isospin-3/2-channel. To find this out, let us discuss the dimesic function at zero frequency. Clearly, $\epsilon(\omega=0, q)$ can become small compared to unity at sufficiently large q and at sufficiently high density. In fact, as $\epsilon \rightarrow 0$ for $\omega = 0$, a pionic soft mode instability occurs which indicates the onset of a pion condensate.

For the simple model described above with only OPE as particle-hole interaction, the OPE tensor force is sufficiently attractive at high momentum transfers ($q_c \sim 2-3 m_\pi$) so that, in the absence of repulsive correlations, the condition $\epsilon \rightarrow 0$ is actually met at critical densities ρ_c which are only a fraction of nuclear matter density, $\rho_0 = 0.17 \text{ fm}^{-3}$. Clearly, the critical density ρ_c will depend sensitively on correlations accompanying OPE.

3.2 Nuclear spin-isospin correlations

Guided by the developments of Chapter 2.3, we construct the spin-isospin dependent particle-hole interaction as follows:

$$W_{ph}(\omega, q) = \frac{f^2(q^2)}{m_\pi^2} \left[V_\pi(\omega, q) \vec{\sigma}_1 \cdot \hat{q} \vec{\sigma}_2 \cdot \hat{q} + V_{2\pi}(\omega, q) (\vec{\sigma}_1 \times \hat{q}) \cdot (\vec{\sigma}_2 \times \hat{q}) \right. \\ \left. + g'(\omega, q) \vec{\sigma}_1 \cdot \vec{\sigma}_2 + h'(\omega, q) S_{12}(\hat{q}) \right] \vec{\tau}_1 \cdot \vec{\tau}_2, \quad (21)$$

where $S_{12} = 3 \vec{\sigma}_1 \cdot \hat{q} \vec{\sigma}_2 \cdot \hat{q} - \vec{\sigma}_1 \cdot \vec{\sigma}_2$. Here V_π is again the OPE interaction, by eq. (8a), and

$$V_{2\pi}(\omega, q) = \int_{4m_\pi^2}^{\infty} \frac{q^2 \rho_{2\pi}(t)}{\omega^2 - q^2 - t} dt \bigg/ \frac{f^2(q^2)}{m_\pi^2}, \quad (22)$$

is the isovector two-pion exchange, following eq. (10), including ρ exchange. All other spin-spin and tensor correlations are incorporated by g' and h' , respectively. The h' is quite small in our region of interest, since the leading tensor terms are already contained in the uncorrelated OPE and isovector two-pion exchange (TPE) interactions. On the other hand, $g'(\omega, q)$ represents impor-

tant repulsive correlations. Note that $W_{ph}(\omega=0, q \rightarrow 0) = \frac{f^2(0)}{m_\pi^2} g'(\omega=q=0) \vec{\sigma}_1 \cdot \vec{\sigma}_2 \vec{\tau}_1 \cdot \vec{\tau}_2$, so that $g'(0,0)$ is related, up to a factor, to the Landau parameter G_0' in Migdal's theory of finite Fermi systems. Values $g'(\omega=0, q=0) \gtrsim 0.7$ are deduced from recent phenomenological Migdal theory calculations ^{/25/}, but this is not necessarily inconsistent with smaller values of $g'(\omega=0, q)$ at large momentum transfers q , which we are primarily interested in.

For $\Delta(1232)$ isobars, the replacements $f \rightarrow f^*$, $\vec{\sigma} \rightarrow \vec{S}$ and $\vec{\tau} \rightarrow \vec{T}$ are again to be made, together with the scaling factor $\alpha_{2\pi}$ or $\alpha_{2\pi}^2$ in $V_{2\pi}$. The repulsive correlations for Δ isobars are assumed to be the same as for nucleons. Such a universal g' is favoured by quark model considerations and receives some support by the role which these correlations play in the quenching of Gamow-Teller transitions in various nuclei ^{/26/}. We also note that $g'=1/3$ corresponds to the original value of the Lorentz-Lorenz correction ^{/27/}.

The TPE interaction component $V_{2\pi}$ in W_{ph} of eq. (21) does actually not participate in the pionic response in infinite nuclear matter, because of its transverse nature of the coupling between spin and momentum, as compared to the longitudinal coupling imposed by the pion source operator. We include it here for completeness, both because it will play a role in finite nuclei due to surface effects, and because it becomes important in photon-induced excitations of pion-like states.

With inclusion of correlations, the dimesic function of eq. (19) turns into

$$\epsilon(\omega, q) = 1 - \left[\frac{q^2}{\omega^2 - q^2 - m_\pi^2} + g' \right] \chi(\omega, q). \quad (23)$$

The g' now plays a crucial role in the determination of the threshold for pion condensation. For reference ^{/28/}, we mention that $g' = 0.4$ (in pion mass units) would place the critical density ρ_c at ρ_0 , while $g' = 0.5$ moves ρ_c to about $2\rho_0$.

It is instructive to discuss $\epsilon(q, \omega=0)$ of eq. (23) at normal nuclear matter density as a function of the correlation parameter g' . From Fig. 8 it becomes obvious that the dimesic function at zero frequency carries signatures of the phase transition long before the critical density is actually reached. Note the strong precritical enhancement of $1/\epsilon$ at momenta close to the critical momentum q_c even for values of g' which would place ρ_c well above ρ_0 . The question is then whether such enhancement effects can actually be observed in appropriate experiments. Their presence (or absence) would put constraints on the strength

Figure 7: Lowest order p-wave pion self-energy.

Figure 8: The inverse dimesic function $\epsilon^{-1}(\omega=0, k)$ (see eq. (23) at nuclear matter density $\rho_0 = 0.17 \text{ fm}^{-3}$, for various values of correlation parameter g'

Figure 9: Schematic picture of the various branches of pionic modes of excitation in nuclear matter. Region I: domain of $\Delta(1232)$ propagation; region II: domain of low energy pion-nucleus scattering and pionic atoms; region III: high momentum part of low-frequency pionic particle-hole spectrum, where pionic soft-modes may develop at sufficiently high density.

Figure 10: Resonant first order p-wave π -nucleus scattering and its interpretation in the Δ -hole model (right figure)

of spin-isospin correlations at high momentum transfer. Note also that the behaviour of $\varepsilon(q, \omega=0)$ in the long-wavelength limit ($q \rightarrow 0$) indicates a characteristic quenching (rather than amplification), related to the repulsive correlations governed by g' . This quenching is a well known phenomenon, e.g. in Gamow-Teller and magnetic multipole transitions /28/.

To summarize this section, we would like to present, in Fig. 9, a schematic picture of the spectrum of pionic excitations in nuclear matter, determined by $\varepsilon(\omega, q) = 0$, i.e. by the singularities of the pion-nuclear response function $R(\omega, q)$. Of particular interest in these lectures is the region of $\Delta(1232)$ propagation (I) and the domain of possible pionic soft-mode behaviour (III). The area of low-energy pion-nucleus scattering and pionic atoms (II) is an important topic in itself, but will be touched here only occasionally.

The main objective is now to transcribe the conceptual nuclear matter framework, developed so far, to finite nuclei.

4. Pion-Nucleus scattering and propagation of the $\Delta(1232)$ in nuclei

4.1 The Δ -hole model

We would like to discuss the optical potential, related to the pion self-energy Π by (cf. eq. (16)):

$$\langle \vec{k}' | U(\omega) | \vec{k} \rangle = \frac{1}{2\omega} \langle \vec{k}' | T(\omega) | \vec{k} \rangle \quad (24)$$

for the scattering of pion of frequency ω , with in- and outgoing momenta $\vec{k}$ and $\vec{k}'$, from a given finite nucleus. Instead of the simple geometric series, eq. (17), the pion-nuclear response function, or T matrix, is now determined by the integral equation

$$-\frac{2\pi}{\omega} F_{\pi A}(\theta, \omega) = \langle \vec{k}' | T(\omega) | \vec{k} \rangle = \langle \vec{k}' | U(\omega) | \vec{k} \rangle + 2\omega \int \frac{d^3q}{(2\pi)^3} \frac{\langle \vec{k}' | U(\omega) | \vec{q} \rangle \langle \vec{q} | T(\omega) | \vec{k} \rangle}{\omega^2 - \vec{q}^2 - m_\pi^2 + i\delta}, \quad (25)$$

where $d\sigma/d\Omega = |F_{\pi A}|^2$. It is convenient to split U into a Δ -resonant and a background part:

$$U = U_\Delta + U_{\text{background}}. \quad (26)$$

Let us neglect the background contribution for the moment and concentrate on the 3.3 resonance region, where the pionic response is dominated by the excitation of Δ -hole states,

$$|\alpha\rangle = |(\Delta h) J^\pi, T=1\rangle$$

coupled to angular momentum $J^\pi = 0^-, 1^+, 2^-, \dots$ etc.; we assume that a complete set of such states is chosen, given for example in a harmonic oscillator basis. Following eq. (12), the lowest order resonant optical potential $U_\Delta^{(o)}$, illustrated in Fig. 10, can be written as

$$2\omega \langle \vec{k}' | U_\Delta^{(o)}(\omega) | \vec{k} \rangle = \sum_\alpha \langle \vec{k}' | \delta H_{\pi N \Delta} | \alpha \rangle \frac{1}{\omega - E_\alpha} \langle \alpha | \delta H_{\pi N \Delta} | \vec{k} \rangle \quad (27)$$

where

$$E_\alpha(\omega) = M_\Delta + \epsilon_\Delta - \frac{i}{2} \Gamma_\Delta(\omega) - M_N - \epsilon_h \quad (27a)$$

are the unperturbed Δ -hole energies: ϵ_Δ is the kinetic energy of the $\Delta(1232)$, and ϵ_h (< 0) is the single particle energy of the bound nucleon (note that nucleon binding effects shift the Δ resonance slightly upward). Insertion of $U_\Delta^{(o)}$ into the integral equation, eq. (25), generates multiple scattering (Fig. 11). In the isobar-hole scheme, this is equivalent to the Tamm-Dancoff approximation. In fact the T matrix can be rewritten as

$$2\omega \langle \vec{k}' | T(\omega) | \vec{k} \rangle = \sum_{\alpha\beta} \langle \vec{k}' | \delta H_{\pi N \Delta} | \alpha \rangle G_{\alpha\beta}^{(o)}(\omega) \langle \beta | \delta H_{\pi N \Delta} | \vec{k} \rangle \quad (28)$$

with the Green's function

$$G_{\alpha\beta}^{(o)}(\omega) = \frac{1}{\omega - \mathcal{H}^{(o)}} \quad (28a)$$

where

$$\mathcal{H}_{\alpha\beta}^{(o)} = E_\alpha(\omega) \delta_{\alpha\beta} + \langle \alpha | W_\pi(\omega) | \beta \rangle. \quad (28b)$$

Here W_π is the OPE component of the Δ -hole interaction, described in section 3.2. Note that W_π is complex, because the pion can propagate on-shell. The imaginary part produces a sizeable increase of the Δ width (the well-known elastic broadening). The real part of OPE is attractive and generates a downward shift of the peak in all important partial waves. If small background pieces are added, then the procedure described here is equivalent (except for nucleon binding effects) to a standard first order optical potential calculation. The failure of such a calculation to reproduce the total cross section, and consequently the requirement to go beyond this first order treatment, becomes obvious

from Fig. 13.

In the isobar-hole model, higher order many-body corrections enter at two places:

- (a) The non-static OPE Δ -hole interaction in the elastic channel must be replaced by the full particle-hole interaction described in section 3.2.
- (b) The $\Delta(1232)$, once excited, experiences interactions which couple it to reaction channels (e.g. the quasifree $(\pi, \pi'N)$ channel, the two-body absorption $(\pi, 2N)$ channel etc.) Such couplings modify the mass and width of the Δ -isobar in each individual partial wave (" Δ self-energy corrections").

The authors of refs. ^{/2,3/} have simulated the full diversity of these many-body corrections phenomenologically by a local complex isobar spreading potential. These results have provided clear evidence for the importance of true absorption channels. The position taken in refs. ^{/3,29/} was instead to calculate these corrections microscopically in order to explore their sensitivity with respect to the basic meson-baryon interactions. The discussion in ref. ^{/3/} is based on the full Δ -hole interaction described in section 3.2. The model used to calculate $g'(\omega, \vec{q})$ consists of π and 2π exchange, accompanied by a baryon-baryon correlation function. This gives values $g' = 0.5 - 0.6$ at $q = \omega = 0$, depending on the precise value of the $\rho N\Delta$ coupling strength.

We proceed now to discuss those parts of the $\Delta(1232)$ self energy in nuclei which have turned out to be the most important ones:

(i) Pauli principle corrections

As the pion propagates between pairs of nucleons, processes such as shown in Fig. 12a are obviously forbidden by the Pauli principle. In many-body language, such processes appear as isobar-Fock-terms, as shown in Fig. 12b. The effect of the Pauli blocking is to reduce the phase space for the decay $\Delta \rightarrow \pi N$, since parts of the nucleon states are occupied. This leads to a sizeable reduction of the Δ width ^{/2,3/}, accompanied by a real part which depends on the details of the $\Delta N \rightarrow N\Delta$ interaction. For the model interaction W , this shift is attractive and strongly energy dependent ^{/3/}.

(ii) True absorption corrections

The essential damping mechanism in the propagation of Δ -hole excitations is introduced by their strong coupling to two-nucleon-two-hole continuum configurations (Fig. 13). Let us illustrate briefly how such corrections

increase the Δ width and shift its mass, by investigating, in a much simplified picture, the diagram Fig. 13b. Let $|\lambda\rangle = |2N1h\rangle$ be a two-nucleon-one-hole continuum state. Then the coupling of the Δ to this state leads to a complex shift of the Δ mass,

$$\delta E_{\Delta} = \sum_{\lambda} \frac{|\langle \Delta | W | \lambda \rangle|^2}{E_0 - E_{\lambda} - i\delta} \quad (29)$$

Here the sum over λ is understood to be also an integration over E_{λ} , and E_0 is the starting energy, given essentially by the pion energy (except for binding corrections). Now, E_0 is much larger than the threshold for two-nucleon emission. Using $[E_0 - E_{\lambda} - i\delta]^{-1} = P/(E_0 - E_{\lambda}) - i\pi \delta(E_0 - E_{\lambda})$, we see how the imaginary part gives an energy dependent absorptive width, $\Gamma_{\text{abs}} = -2\text{Im}\delta E_{\Delta}$, which adds to the Δ decay width. This absorptive damping is a substantial effect and overcompensates the reduction of the width by Pauli blocking effects.

Microscopic calculations of all the processes shown in Figs. 12-13 have been performed in ref. /3/ for ^4He and ^{16}O , where the parameters of the interaction W have been chosen in accordance with a calculation of πd absorption which is a very important consistency requirement. We mention that Fig. 13c becomes important in the 3.3 resonance region (sometimes called "reflection term" because it incorporates the resonant rescattering of a pion in the quasi-elastic channel).

The pion-nuclear T-matrix with inclusion of these many-body corrections is now given by

$$2\omega \langle \vec{k}' | T(\omega) | \vec{k} \rangle = \sum_{\alpha\beta} \langle \vec{k}' | \delta H_{\pi N \Delta} | \alpha \rangle G_{\alpha\beta}(\omega) \langle \beta | \delta H_{\pi N \Delta} | \vec{k} \rangle, \quad (30)$$

where $G(\omega) = (\omega - \mathcal{H}(\omega))^{-1}$

is the full isobar-hole Green's function. For simplicity, we write only the direct part of it which defines the Tamm-Dancoff approximation (TDA), while inclusion of crossed terms (the step from TDA to RPA) is shown in detail in ref. /3/. The G is obtained by inversion of the complex matrix

$$\mathcal{H}_{\alpha\beta}(\omega) = \bar{E}_{\alpha}(\omega) \delta_{\alpha\beta} + \sum_{\alpha\beta} (\omega) + W_{\alpha\beta}(\omega), \quad (31)$$

where α and β refer to the Δ -hole basis states. Here $\bar{E}_{\alpha}$ is given by E_{α} of eq.

Figure 11: Multiple scattering in the isobar-hole model with one-pion exchange only.

Figure 12: Effect of the Pauli principle on intermediate pion rescattering between two nucleons: the diagram (a) is Pauli forbidden, if j is an occupied state. (b): representation of the same process as a Fock term in the Δ self-energy.

Figure 13: (a) coupling of the π -nucleus elastic channel to two-nucleon absorption channel. (b): representation of (a) as a Δ -isobar self-energy correction which couples the Δ to 2-nucleon-1-hole continuum configurations. (c): additional contribution from resonant rescattering (reflection term) also incorporated in the Δ -hole model calculation.

Figure 14: Total $\pi^{16}\text{O}$ cross section calculated at different levels of the π -nuclear many-body problem: shown is a standard first order momentum space calculation for reference. The other two curves are calculated within the RPA scheme of ref. /3/.

Pauli + Binding: incorporates isobar Hartree potential, Fock term $\Sigma(\text{Pauli})$ and full interaction W of eq. 21.

Total: includes in addition true absorption and reflection terms $\Sigma(2p2h)$

Input: π and ρ exchange plus baryon-baryon correlation function. (cutoff in $\pi N\Delta$ vertex: $\Lambda = 1.2$ GeV; $\rho N\Delta$ coupling: $f_{\rho}^*/4\pi = 4.1$ ("strong ρ "). Calculation includes all partial waves $J^\pi = 0^-, 1^+, \dots$ up to 8^- . For further details see ref. /3/.

(27a), except that the free Δ isobar kinetic energy can now be replaced by the single particle energies in an isobar Hartree potential, which describes additional binding effects. This Hartree potential is estimated to be roughly of the same (attractive) magnitude as the nucleon single particle potential /3/. The $w_{\alpha\beta}$ are matrix elements of the full isobar-hole interaction, as before, while $\Sigma = \Sigma(\text{Pauli}) + \Sigma(2p2h)$ summarizes the self-energy corrections to the masses and widths of isobar-hole states in each partial wave J generated by the Fock terms ($\Sigma(\text{Pauli})$, Fig. 12) and the absorption and reflection contributions ($\Sigma(2p2h)$, Fig. 13). We show in Fig. 14 the influence of higher order corrections on the total π^{16}_0 cross section /3/. While $\Sigma(\text{Pauli})$ (through its positive imaginary part) leads to an appreciable narrowing of the isobar width in all partial waves, additional inclusion of $\Sigma(2p2h)$ provides the necessary damping. At low energy, the absorptive width comes exclusively from two-nucleon absorption, Fig. 13 (b). The decomposition of σ_{tot} into partial waves is shown in Fig. 15. In the isobar-doorway picture /30,2/, each one of these partial waves is interpreted as an isobar-doorway state of given J^π . Also shown in Fig. 16 are selected examples of differential cross sections.

The apparent success of these and other isobar-hole model calculation, especially in reproducing the important damping mechanisms which the $\Delta(1232)$ encounters in the nucleus, should not obscure some of the basic uncertainties. The shifts and widths of Δ -hole states depend sensitively on details of the $N\Delta$ -interaction which are not very well under control (cutoff factors, $\rho N\Delta$ coupling constant, short range $N\Delta$ correlations etc.) It seems however that the quasifree and absorptive widths are reasonably well reproduced once the parameters of the basic baryon-baryon interactions are fixed in accordance with the $\pi d \leftrightarrow pp$ reaction. On the other hand, the shifts in the positions of isobar-hole states are certainly not better determined than within ~ 30 MeV.

The optical potential U is generally a highly non-local quantity. However, it turns out that at low energy ($T_\pi \lesssim 50$ MeV), it can be converted to a good degree of approximation into an equivalent r -space potential of the Kisslinger type, $\vec{\nabla} F(r) \vec{\nabla}$. This means that connections can be made between the many-body theoretical picture and standard parametrisation of the p -wave optical potential,

$$2\omega U(\vec{r}) = -4\pi \vec{\nabla} L(r) n(r) \vec{\nabla},$$

$$n(r) = c_0 \rho(r) + C_0 \rho^2(r), \quad L(r) = [1 + 4\pi g' n(r)]^{-1}, \quad (32)$$

which has been used with considerable success to describe low energy π -nucleus scattering data /31,32/. The isobar-hole model (for ^{16}O) yields $\text{Im } C_0 = 0.08 m_\pi^{-6}$ /33/, which is close to the "experimental" value of 0.07. The discussion of $\text{Re } C_0$ includes a variety of many-body effects like binding and Pauli corrections as well as dispersive real parts from two-nucleon absorption. The isobar-hole model gives a positive (i.e. attractive) $\text{Re } C_0$ of about the same magnitude as $\text{Im } C_0$, the precise value depending on the treatment of binding effects (but $\text{Re } C_0$ remains positive for a wide variety of model assumptions). We note that a positive $\text{Re } C_0$ seems also to be suggested by recent precision data on shifts and widths of $2p$ levels in pionic atoms /34/. The value of the Ericson-Ericson-Lorentz-Lorentz (EELL) correction is determined by the correlation part g' of the full Δ -hole interaction W . A strong p exchange, accompanied by additional short-range correlations favours a large EELL correction, $g' \gtrsim 0.5$, as compared to the "classical" value $g' = 1/3$.

4.2 Photon-induced excitation of Δ -hole states

There is general agreement about the relevance of true absorption and other higher order processes. In pion-nucleus scattering, however, much of the shift and broadening of Δ -hole states in the dominant partial waves occurs simply because of coherent pion multiple scattering, which then combines with all other damping and dispersive effects. It is therefore useful to investigate processes complementary to pion induced ones, in particular those where the elastic broadening is possibly reduced, in order to obtain independent information about the modification of positions and widths of Δ -hole states. We claim /35/ that total photonuclear cross sections in the 3.3 resonance region are a source of such information. In addition, the coherent photoproduction of π^0 on nuclei has been pointed out /36/ as a useful process, because it is completely dominated by $\Delta(1232)$ excitation.

The $\gamma N \Delta$ transition operator follows from the $\pi N \Delta$ vertex Hamiltonian simply by the replacements

$$f^* \rightarrow f_{\gamma N \Delta}, \quad \vec{S}^* \cdot \vec{q} \rightarrow \vec{S}^* \cdot (\vec{k} \times \hat{E}), \quad T_\lambda^+ \pi_\lambda \rightarrow T_3, \quad (33)$$

where $\vec{k}$ is the photon momentum, $\hat{E}$ the photon polarization vector, and $f_{\gamma N \Delta} = 0.116$. Note that the ratio $f_{\gamma N \Delta}/f^*$ for $f^{*2}/4\pi = 0.37$ is close to the canonical value $(e/g)\mu_V$, where $g^2/4\pi = 14$, the πNN coupling constant, and $\mu_V = (\mu_p - \mu_n)/2 = 3.7$ is

Figure 15: Decomposition of the $\pi^{16}\text{O}$ cross section into partial wave cross sections, once all higher order corrections are included (taken from /3/).

Figure 16: Examples of differential cross sections for $\pi^{16}\text{O}$ including all medium corrections. Taken from ref. /3/.

Figure 17: (a): Lowest order coherent π^0 photoproduction on a nucleus in the Δ -hole model. The $A(\gamma, \pi^0) A_{gs}$ (b) and resonant photon scattering (c) processes in the Δ -hole model, employing the full Δ -hole Green's function.

Figure 18: Differential cross section for coherent π^0 photoproduction on ^{16}O at $\Theta = 10^\circ$ as function of photon energy. A comparison is shown between an impulse approximation calculation (Fig. 17 (a)) and a full Δ -hole model calculation (Fig. 17 (b)). The experimental data are from ref. /6/.

the isovector magnetic moment. The T-matrix for the coherent (γ, π^0) process can now be obtained at once in the Δ -hole model, in analogy with eq. (30) (see Fig. 17 (a)):

$$2\omega \langle \pi^0(\vec{q}) | T(\omega) | \gamma(\vec{k}) \rangle = \sum_{\alpha\beta} \langle \vec{q} | \delta H_{\pi N \Delta} | \alpha \rangle G_{\alpha\beta}(\omega) \langle \beta | \delta H_{\gamma N \Delta} | \vec{k} \rangle, \quad (34)$$

where G is again the full Δ -hole Green's function discussed in section 4.1.

An immediate consequence of the transverse nature of the $\gamma N \Delta$ coupling, as compared to the longitudinal $\pi N \Delta$ coupling, is the suppression of π^0 propagation under extreme forward angles ($\theta = 0$). The $^{16}_0(\gamma, \pi^0)^{16}_0$ angular distribution peaks around $\theta = 20 - 30$ degrees, depending on the photon energy. The damping of Δ -hole modes discussed in section 4.1 should be visible also in the (γ, π^0) reaction. This is evident from Fig. 18, where the impulse approximation result for $^{16}_0(\gamma, \pi^0)^{16}_0$ at $\theta_{CM} = 10^\circ$ is compared with the full Δ -hole model calculation /37/. It appears that the experimental data, obtained at the Bonn synchrotron /6/, clearly require such a strong damping.

Let us now turn to photon scattering and total photon-nucleus cross section $\sigma_{\gamma A}(\omega)$ in the 3.3 resonance region. The amount of experimental data in this area is still quite limited. However, the measurements of the Mainz group /5/ as well as the Bonn data on total cross sections for charged pion photoproduction /6/, show again a considerable damping of the Δ , but almost no shift of the peak in $\sigma_{\gamma A}$, quite unlike the π -nucleus scattering situation. We consider the T matrix for forward Compton scattering, $\langle \vec{k} | T_{\gamma A}(\omega) | \vec{k} \rangle$, and split it again into a Δ -resonant part and a background piece:

$$T_{\gamma A} = T_{\Delta} + T_{back}. \quad (35)$$

The total photonuclear cross section is given by

$$\sigma_{\gamma A}(\omega) = -2 \text{Im} \langle \vec{k} | T_{\gamma A}(\omega) | \vec{k} \rangle. \quad (36)$$

Following the previous developments, the Δ -resonant part T_{Δ} is written again in terms of the full Δ -hole Green's function (see also Fig. 17 (b)):

$$2\omega \langle \vec{k} | T_{\Delta}(\omega) | \vec{k} \rangle = \sum_{\alpha\beta} \langle \vec{k} | \delta H_{\gamma N \Delta} | \alpha \rangle G_{\alpha\beta}(\omega) \langle \beta | \delta H_{\gamma N \Delta} | \vec{k} \rangle. \quad (37)$$

Two major differences to π -nucleus scattering are to be pointed out: first, the Δ -hole states $|\alpha\rangle$ and $|\beta\rangle$ now incorporate also natural parity states with $J^\pi = 1^-, 2^+, \dots$ etc., in addition to the unnatural parity states discussed previously. In fact, about one half of the resonant part of $\sigma_{\gamma A}$ comes from isobar-hole states of natural parity. Second, the transverse γNA Hamiltonian operates now both in the in- and outgoing channel and suppresses the coherent forward propagation of π^0 which appears in the isobar-hole Green's function G through the action of the OPE part of the full Δ -hole interaction W . This means that the strong elastic broadening and the accompanying downward shift of Δ -hole observed in π -nucleus scattering states will be nearly absent in photon scattering. The ρ exchange part of W now acts coherently in natural parity Δ -hole states, but it is of order k^2/m_ρ^2 and therefore not of great importance. This means that resonant photon scattering is selectively sensitive to the damping and shift resulting from Δ -self energy interactions, i.e. the coupling of the $\Delta(1232)$ to many-body excitation channels described in section 4.2. Before we present a calculation of these, some words must be spent on the background pieces of $T_{\gamma A}$ which emerge essentially from non-resonant parts of the charged pion photoproduction amplitude. Their contribution to $\sigma_{\gamma A}$ is parametrized in the form $A^\beta \sigma_{\text{back}}(\gamma N)$, where $\sigma_{\text{back}}(\gamma N)$ is the background part of the elementary γN cross section. Meaningful upper and lower limits are $\beta = 1$ and $\beta = 2/3$, corresponding to no damping or strong damping of background-photo-produced pions within the nucleus. This background is then added to the resonant part calculated in the isobar-hole model. Results for ^{16}O are shown in Fig. 19. The complex isobar self-energies Σ employed in this calculation are exactly the same as those calculated for π -nucleus scattering. The characteristic damping in the resonance region is again evident. The absence of the shift and broadening from elastic multiple π^0 scattering has the consequence that the maximum of $\sigma_{\gamma A}$ is hardly moved compared to the impulse approximation peak: repulsion from the g' part of the Δ -hole interaction, binding effects, and attractive contributions from $\text{Re } \Sigma$, the Δ self-energy corrections, almost cancel each other. It is instructive to associate a "damping exponent" α , defined by $\sigma_{\Delta}(\gamma A) = A^\alpha \sigma_{\Delta}(\gamma N)$, with the resonant part of $\sigma_{\gamma A}$. At the peak, we find $\alpha = 0.84$ for both ^{16}O and ^4He . This appears to be remarkably close to the values α around 0.8 reported by Ziegler ^{15/} for Li and Be. For ^{16}O , a comparison is possible only with the Bonn data for total charged photoproduction (included in Fig. (19)), although this is not quite legitimate, because parts of π^0 photoproduction (those not affected by total absorption of the π^0 leading to emission of charged particles) are not included in the data. This might well give

Figure 19: Total $\gamma^{16}\text{O}$ cross section in the 3.3 resonance region. Shown is a comparison between impulse approximation and a full Δ -hole model calculation, adding non-resonant background, as described in the text. The data correspond to total cross section for photoproduction of charged particles only, taken from ref. /6/.

Figure 20: Pion self-energy in finite nuclei. The pion couples to $T = 1$ nucleon-hole and Δ -hole states with $J^\pi = 0^-, 1^+, 2^-, 3^+, \dots$ etc.

Figure 22: $J^\pi = 2^-$ strength in ^{208}Pb at momentum transfer $q = 2.5 m_\pi$ as seen by an operator $\vec{\sigma} \cdot \vec{q} \exp(i \vec{q} \cdot \vec{r})$. Bottom: results of a standard $3\hbar\omega_0$ P-space calculation using W_{ph} of eq. (7) with $g' = 0.5$ and $h' = 0$, omitting Q-space pionic polarization. Upper three subfigures include Q-space polarization as described in the text, and the variation with the correlation parameter g' is shown. Note different scales in various subfigures.

Figure 21: Role of the spin-isospin tensor $\hat{\mu}_J$ describing Q-space pionic polarization to all orders in a scattering process (e.g. with electrons or protons) leading to a pion-like particle-hole state in P-space. The particle lines in the particle-hole excitations building up $\hat{\mu}_J$ are nucleons or $\Delta(1232)$ isobars.

an additional millibarn or so. The main point we would like to make here is that the general trend of the energy dependence of σ_{YA} predicted by the Δ -hole model, especially the damping of the $\Delta(1232)$ with only a slight shift of the maximum, seems to follow quite well the experimental observations for total charged photoproduction. This then presents an independent test for the $N\Delta$ -interaction mechanisms, on which the calculation of damping widths and dispersive real parts is based.

5. High momentum transfer properties of pion-like nuclear states

We would now like to return to the question whether low-lying nuclear states carrying pion quantum numbers show a characteristic enhancement effect at large momentum transfers as an indication of the proximity of a pion condensate. Such ideas have been developed in refs. /10-13/, and much of the conceptual framework is presented in ref. /38/.

5.1 Response function for pionic low-frequency modes in finite nuclei

Consider pion-like nuclear excited states with angular momentum $J^\pi = 0^-, 1^+, 2^-, \dots$ etc. and isospin $T=1$. These are formed by particle-hole excitations correlated by the spin-isospin dependent interaction, eq. (21), usually employing a small model space (P space) to reproduce the long-wavelength properties of such states. The attraction from the OPE interaction V_π at large q favours strong spin-isospin polarization effects involving a large number of high-lying nucleon-hole and $\Delta(1232)$ -hole states, as long as the repulsion from g' is sufficiently weak. To describe such polarization phenomena, let us return to the finite-nucleus analogue of the lowest order pion self-energy, eq. (12), following refs. /13,39/:

$$\begin{aligned} \langle \vec{q}' | \Pi^{(o)}(\omega) | \vec{q} \rangle &= - \sum_{ph} \frac{\langle \vec{q}' | \delta H | ph \rangle \langle ph | \delta H | \vec{q} \rangle}{E_p - E_h - \omega} + \text{crossed term} \\ &= \sum_J \frac{2J+1}{4\pi} \Pi_J^{(o)}(q', q, \omega) P_J(\hat{q}' \cdot \hat{q}). \end{aligned} \quad (38)$$

(see Fig. 20 for illustration). The partial wave self-energy $\Pi_J^{(o)}$ can be written

$$\Pi_J^{(o)}(q', q, \omega) = \sum_{LL'} a_{JL} [\hat{\Pi}_J(q', q, \omega)]_{LL'} a_{JL'}, \quad (39)$$

where $a_{JL} = (J010|L0)$, and $\hat{\Pi}_J$ is a 2×2 matrix for the possible values of

particle-hole orbital angular momenta, $L, L' = J \pm 1$:

$$[\hat{\Pi}_J(q', q, \omega)]_{LL'} = - \sum_{ph} Q_{ph}^{JL}(q') \left\{ \frac{1}{E_p - E_h - \omega} + \frac{1}{E_p - E_h + \omega} \right\} Q_{ph}^{JL'}(q). \quad (40)$$

The $Q_{ph}^{JL}(q)$ are particle-hole transition form factors. Actual calculations are then performed as follows: the full particle-hole Hilbert space is split into a model space (P space) and a large complementary Q-space. The core polarization arising from virtual excitations in Q-space is incorporated into the model space (P-space) calculation as a renormalization of the relevant interactions and transition operators, as shown in Fig. 21. In this renormalization procedure, the sum in eq. (40) therefore extends over all Q-space ph configurations (typically a 20 - 30 $\hbar\omega_0$ Q-space is required at momentum transfers $q \sim 2-3 m_\pi$). For example, an operator of the type

$$M_{JL}^\lambda(q, r) = i^L q [Y_L \times \sigma]_J j_L(qr) \tau_\lambda, \quad (41)$$

which excites pion-like states, is renormalized according to

$$\tilde{M}_{JL}^\lambda(q, r) = \sum_{L'} \int_0^\infty \frac{dk k^2}{(2\pi)^3} M_{JL'}^\lambda(k, r) [\hat{\mu}_J(k, q)]_{L'L}. \quad (42)$$

Here $\hat{\mu}_J$ is the spin-isospin polarizability tensor, which corresponds to the inverse of the dimesic function ϵ of eq. (23). In terms of the pionic response function $\hat{R}_J$, determined by the equation

$$\hat{R}_J(k, q) = \hat{\Pi}_J(k, q) + \int_0^\infty \frac{dq'}{(2\pi)^3} \hat{\Pi}_J(k, q') \hat{W}_J(q') \hat{R}_J(q', q), \quad (43)$$

the $\hat{\mu}_J$ is given by

$$k^2 [\hat{\mu}_J(k, q)]_{L'L} = (2\pi)^3 \delta(k-q) \delta_{L'L} + [\hat{W}_J(k) \hat{R}_J(k, q)]_{L'L}. \quad (44)$$

Here $\hat{W}_J$ is related to the particle-hole interaction W by

$$[\hat{W}_J(k)]_{L'L} = \left[\frac{1}{3} V_\pi(k) + \frac{2}{3} V_{2\pi}(k) + g' \right] \delta_{LL} + \left[\frac{1}{3} V_\pi(k) - \frac{1}{3} V_{2\pi}(k) + h' \right] (3a_{JL} a_{JL} - \delta_{LL}), \quad (45)$$

where the first and second term on the right hand side corresponds to the spin-spin and tensor part of the interaction, respectively.

Note, that as a pion-like mode of given J becomes soft, $\hat{R}_J$, and hence $\hat{\mu}_J$, tends to become very large in a certain region of momenta around the critical momentum q_c in nuclear matter. The difference between finite nuclei and nuclear matter is that $\hat{R}_J$ is non-local in momentum space for a finite system, the momentum space non-locality being inversely proportional to the nuclear radius

^{/39/}. Thus precritical enhancement effects in small nuclei, if existent, will generally be less pronounced than in heavy nuclei. For sufficiently large nuclei, pionic soft-mode behaviour appears under similar conditions as in nuclear matter: for $g' < 0.4$, all pion-like modes with $J \lesssim q_c R$, where $q_c \sim 2-3 m_\pi$, the critical momentum, tend to become soft. The question is then whether pionic polarization phenomena might be visible even for more realistic values of the correlation parameter, $g'(q \approx q_c, \omega = 0) \gtrsim 0.5$.

We emphasize again that the enhancement of the pion field due to large-space core polarization is a phenomenon restricted to high momentum transfers. In the long wavelength limit, the spin-isospin dependent particle-hole interaction is entirely determined by g' , with the well known consequence of quenching effects observable in transitions related to weak axial currents ^{/26/} and to the spin dependent parts of magnetic multipoles ^{/28/}.

5.2 Applications

Inelastic proton scattering to unnatural parity states in heavy nuclei has been suggested ^{/12,13/} as a possible source of information on the presence or absence of precritical enhancement effects. In fact, proton nucleus scattering provides perturbations of the form $\vec{\sigma} \cdot \hat{q} e^{i\vec{q} \cdot \vec{r}}$, the pion source operator, at high momentum transfer. We would first like to present a schematic example by considering matrix elements of the form $S_J(\vec{\sigma} \cdot \hat{q}) = \langle J^\pi | \vec{\sigma} \cdot \hat{q} e^{i\vec{q} \cdot \vec{r}} | 0^+ \rangle$ from the ground state to some unnatural parity state in ^{208}Pb , mainly to illustrate how soft mode behaviour at high momentum transfer is approached in a finite system ^{/39/}. Here we consider the case of $J^\pi = 2^-$ states in ^{208}Pb at $q = 2.5 m_\pi$, close to the critical momentum for pion condensation in nuclear matter. The P-space consists of all $3\hbar\omega_0$ excitations, while the Q-space ($> 3\hbar\omega_0$) polarization is incorporated by the polarizability tensor $\hat{\mu}_J$, as previously described. Results for the excitation spectrum as seen by a pion source operator are shown in Fig. 22. Note the overall increase of strength with decreasing g' , once the pionic polarization is turned on. Around $g' = 0.4$, the strength is collected into one single low-frequency mode which, upon further reduction of g' , eventually becomes soft. Similar features are observed for all pion-like modes, as long as $J < q_c R$. In heavy nuclei, the precritical enhancement is localized in a narrow band of momentum transfers around $q_c \sim 2.5 m_\pi$. No indication of such behaviour, even for $g' = 0.4$, can be observed in the long-wavelength limit. Instead, there is a quenching of transitions to unnatural parity states because of the repul-

sion from g' at $q \approx 0$.

The situation described above is somewhat idealized, mainly because of the spreading of ph states by their coupling to more complicated (e.g. $2p2h$) configurations. This spreading of strength will work against the formation of collective low-frequency, high-momentum modes. It is not obvious, however, that the spreading at high momentum transfers will be the same as the one observed at low q , because a high momentum mode does not mix easily with low lying $2p2h$ states. This question requires more detailed investigation.

High momentum transfer experiments involving unnatural parity excitations in heavy nuclei are extremely difficult to perform, so that present interest is directed more towards light nuclei, although the polarization effects there are generally expected to be weaker. A particularly well investigated example is the $J^\pi = 1^+$, $T = 1$ state in ^{12}C at 15.1 MeV. Its basic p -shell structure is supposed to be well known, e.g. in terms of a Cohen-Kurath wave function, which we adopt as the model space (P -space) wave function. The interest in this state is motivated by considerable difficulties to reproduce the high momentum transfer behaviour of the $M1$ form factor. In fact, it has been pointed in ref. /40/ that spin-dependent core polarization effects are very important. The first full scale calculation of the ^{12}C $M1$ form factor taking into account polarization effects to all orders has been performed by Delorme et al. /41/. They come to the conclusion that the observed enhancement at large q over the Cohen-Kurath result can be interpreted as an indication of critical opalescence, in the sense that the g' required to reproduce the data is quite small ($g' \approx 0.4$).

We have calculated /42/ the $M1$ form factor together with $d\sigma/d\Omega$ for inelastic proton scattering at $E_p = 800$ MeV leading to the same state (Fig. 24). One might expect that an $M1$ operator, being proportional to $\vec{\sigma} \times \vec{q}$ (i.e. not properly aligned with a pion source function $\vec{\sigma} \cdot \vec{q}$) would not be an appropriate tool for probing into pionic modes. This is true for heavy nuclei, but for light nuclei, such restrictions are not so severe, due to surface effects. The $M1$ form factor is shown in Fig. 23; it demonstrates the obvious failure to reproduce the high momentum transfer behaviour with the Cohen-Kurath wave function. Once large space pionic polarization is included, one observes a shift of the minimum (caused primarily by repulsive correlations proportional to g'). However, we find that the enhancement from V_π alone is only moderate, unless exceedingly small values of g' ($\lesssim 0.4$) are introduced. It is evident that isovector two-pion exchange ($V_{2\pi}$) is of crucial importance, since this component of the particle-hole interaction is properly aligned with the $\vec{\sigma} \times \vec{q}$ nature of the $M1$

operator. In particular, the non-resonant part of $V_{2\pi}$ (other than ρ exchange) gives a large fraction of the enhancement required by the data. Once the full $V_{2\pi}$ is included, the pionic opalescence situation appears not to be encountered as strongly as suggested in ref. /41/.

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Figure 23: M1 form factor for ^{12}C . Short-dashed curve, standard Cohen-Kurath shell model; long-dashed curve: effect of pionic Q-space polarization, using V_π and $g'=0.55$, but $V_{2\pi}=0$ (and $h'=0$); Solid curve: full particle-hole interaction W incorporating, including $V_{2\pi}$. Experimental data: ref. /44/. See ref. /42/ for details.

Figure 24: Glauber calculation of 800 MeV $^{12}\text{C}(p,p')^{12}\text{C}(1^+)$ using the same model as in Fig. 23. Data are from ref. /43/. The spin isospin dependent primary NN amplitude has been adjusted to the low q data. See ref. /42/ for further details.