ON THE POSSIBILITY OF DESIGNING AN ISOCRHNOS ACCELERATOR
WITH CONSTANT BETA/KN FREQUENCIES

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We have investigated, by a method different from that of Teng\(^3\), the possibility of avoiding or delaying the onset of the radial half-integral or integral resonances which limit the energy of constant-frequency FFAG cyclotrons. The machine is divided into \(N\) sectors, each of which consists of a field-free region and a region in which the magnetic field has a constant value of \(k = p/B\) \(dB/d\varphi\) along the orbit. The field region occupies a fraction \(\alpha\) of the sector. Then the focusing depends upon the quantity \(\alpha\), its derivative with respect to radius \(d\alpha/d\varphi\), and the spiraling angle \(\varepsilon\). The value of \(k\) is determined by the isochronism condition; \(k = \alpha\beta\beta^2 - \beta\alpha\), \((\alpha = d\alpha/d\varphi)\) where \(\beta = v/c\), and \(\gamma = E/m_0c^2\).

Using linearized theory and matrix techniques, we find for the radial and vertical betatron sector frequencies the relations

\[
2 \cos \sigma_n = \cos K^{1/2} \left[ 2(1 - \alpha)(T_1 - T_3) \right] - K^{-1/2} \sin K^{1/2} \left[ T_1 - T_3 - 0(1 - \alpha)(T_1 T_3 - K) \right],
\]

\[
2 \cos \sigma_x = \cosh K^{1/2} \left[ 2 - 2(1 - \alpha)(T_1 - T_3) \right] + K^{-1/2} \sinh K^{1/2} \left[ 2 - (T_1 - T_3) - 0(1 - \alpha)(T_1 T_3 - K) \right],
\]

where \(K = 1 + k\), \(0 = 2\pi/N\), \(T_1 = \tan(\varepsilon_1 + \Phi)\), \(T_2 = \tan(\varepsilon_2 - \Phi)\), \(\Phi = (1 - \alpha)(2/2)\), and \(0\beta\alpha = \tan \varepsilon_2 - \tan \varepsilon_1\). \(T_1\) and \(T_2\) are the usual edge-focusing parameters, and \(\varepsilon_1\) and \(\varepsilon_2\) are the angles made by the respective sector edges with the radius.

In the limit of \(\theta \to 0\), the betatron frequencies are

\[\nu_r = \gamma\]

\[\nu_x = -\beta^2 \beta_2 + (1 - \alpha)[1 + 2 \tan \varepsilon_1 \tan \varepsilon_2]/\alpha.\]

The sector flare \(\beta\alpha\) cancels out; the betatron frequencies are not affected by the flare in an isochronous machine. The sector flare is directly interchangeable with \(k\), since the frequencies for a non-isochronous machine can be written

\[\nu_x = (\alpha + k + \beta \alpha)/\alpha\]

\[\nu_x = (\alpha - \beta \alpha + (1 - \alpha)(1 + 2 \tan \varepsilon_1 \tan \varepsilon_2))/\alpha.\]

Thus, e.g., if we set \(k = 0\) (uniform field), the flare becomes \(\beta \alpha = \alpha \beta^2 \beta_2\), and we can obtain a uniform-field isochronous machine, limited only by the usual resonances.

A complete report is in preparation. We are indebted to Dr. T. A. Welton for pointing out an error in the calculations to us.

LIST OF REFERENCES