PROTON ORBITS IN THE BROOKHAVEN LINEAR ACCELERATOR

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After selecting the principal design parameters of the linear accelerator we proceeded to compute some particle trajectories through the machine in order to understand the effect of varying the strengths of the focusing magnets and to try to predict an optimum setting. These computations were done by hand, with the following simplifying assumptions:

1. The longitudinal electric field was taken to be independent of radius and was represented by a $\delta$-function at the center of each r-f gap.

2. The transverse electric and focusing magnetic fields were assumed to vary linearly with displacement, the magnetic field being spread uniformly over the central half of an r-f repeat length, and the electric field occurring as a $\delta$-function at the center of each r-f gap.

3. In the transfer matrices for a repeat length of the focusing lenses (four drift tubes), the trigonometric and hyperbolic functions were expanded to the lowest possible order and the electric impulses retained only in lowest order. The matrix elements were further adjusted slightly to insure unit determinant.

4. Minor complications, such as relativistic mass increase, change in transit-time factor with energy and bore diameter, and r-f magnetic focusing, were omitted.

First, the phase motion was computed for three initial phases, $-45^\circ$, $-5^\circ$, and $+15^\circ$ (synchronous phase = $-25^\circ$), with no initial energy error, for the injector regulation should be good enough and the bunching voltage low enough to produce an energy spread small compared to the energy acceptance of the machine. Next, the quadrupole strengths were selected with the aid of the stability diagram (fig. 1). Initially, the parameter $\pi e W' \sin \varphi / mc^2 \delta$ has the value, $-0.9$, for the synchronous particle, and varies from about $-16$ to $+0.9$ over the range of phase stable particles. The magnetic gradient in the first lens was chosen so that $e H' \lambda \beta / mc$ was 1.3 ($H' = 4.3$ kg/cm) in order that particles at both ends of the phase stable range would be represented by points in the stable area of fig. 1. Since there is little to gain in damping by keeping subsequent lenses strong and something to lose in the amplitudes induced by misalignments, the other magnets were prescribed to lie along the dotted line in fig. 1, as computed for the synchronous particle. Such a prescription, incidentally, should lead to almost constant frequency and amplitude for the synchronous particles, a situation which was verified by the computation.

![Fig. 1.](image)

Sixteen orbits were computed: two independent solutions in each transverse plane for four initial phases, $-45^\circ$, $-25^\circ$, $-5^\circ$, and $+15^\circ$. The result was somewhat discouraging in that the amplitudes varied by as much as a factor of two from those of the synchronous particles and the phases of the transverse oscillations were spread over more than a quarter of a cycle after the first phase oscillation. This behavior is most undesirable, for although the initial volume occupied by the beam in the transverse phase space is preserved, even in the presence of various construction errors, the shape and orientation of this volume at the high energy end will depend on the initial

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** $W'$ = rate of energy gain/$\cos \varphi$; all other symbols have conventional meaning.
longitudinal phase, so that if the beam is to be transferred to the synchrotron by a d-c system the effective volume in phase space is seriously enlarged.

In an effort to produce a radial motion less dependent on longitudinal phase, we computed a second set of 16 orbits, focusing according to the solid line in fig. 1. The focusing fields were increased because the differences in radial motion seemed to be due mainly to the proximity of the lower edge of the stable area. In this case the radial motion was much improved, with the exception of initial phase $+15^\circ$, which deviated still more from the others, apparently because of spending such a long time during the first phase oscillation in an effectively stronger focusing field. We are now converting the result into admittance and emittance figures and computing the effects of misalignments by using the various transfer matrices. We believe that we will want to follow the second prescription and possibly resign ourselves to rejecting in the following focusing system some of the particles injected on the falling side of the voltage wave.