ON THE THEORY OF THE "LUMINOUS" ELECTRONS

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1. An electron moving in a magnetic field $H$ with a relativistic energy $E \gg mc^2$ along a circle of radius $R$ radiates energy in proportion to $E^3$. The formula for the spectral distribution of relativistic radiation derived in our papers, and also by Schwinger, reads:

$$dW = \frac{9\sqrt{3}}{8\pi} W_{dy} \int_0^\infty K_{2/3}(x) \, dx$$

where $y = \frac{2}{3} \sqrt{\frac{me^2}{E}}$, $v$ is the number of the harmonic and $W$ the total radiated energy. The maximum intensity proved to be connected with a very high harmonic

$$v_{\text{max}} \sim \left(\frac{E}{mc^2}\right)^a.$$ 

Further investigations showed a high grade of polarization of the relativistic radiation. In the mean the radiation will be polarized linearly. The circular polarization which vanishes when averaged will be observed only in terms of the angular distribution. Polarization of this kind was discovered visually by Korolev and his collaborators.

2. Let us dwell in greater detail on the quantum theory of the motion of electrons in cyclic accelerators of the betatron and synchrotron types, referring for all details to the papers jointly published with our collaborators in the Soviet Journal of Experimental and Theoretical Physics and the Transactions of the USSR Academy of Sciences during the past few years.

In a general case, the motion of an electron in a magnetic field can be described most simply by means of the cylindrical co-ordinates $r, z, \varphi$.

The azimuthal $l = \pm 1, \pm 2$, radial $s = 0, 1, 2, 3,...$ and axial $k = 0, 1, 2, 3,$ quantum numbers characterize the adiabatic Ehrenfest invariants

$$\oint P_r \, d\varphi = h/l,$$
$$\oint P_r \, dr = hs,$$
$$\oint P_z \, dz = h/k,$$

where $P_r, P_\varphi, P_z$ are the corresponding values of the momenta.

These quantum numbers can also be obtained automatically by solving the corresponding wave equation.

In addition, radial and axial oscillations

$$r - R = A \cos \sqrt{1-q} \frac{C}{R_0} t,$$
$$z = B \cos \sqrt{q} \frac{C}{R_0} t$$

should occur in the direction of the radius and $z$ axis. Here $q$ is the index of the magnetic field.

The amplitudes of radial and axial oscillations are related to the quantum numbers, i.e. the adiabatic invariants, through the formulae

$$A^2 = \frac{2ch_s}{eH(R)\sqrt{1-q}}, \quad B^2 = \frac{2ch_k}{eH(R)\sqrt{q}}.$$

From the last formula, we see that at the condition that $0 < q < 1$, the motion will be stable, especially in view of the fact that when the magnetic field increases, the amplitudes of the radial and axial oscillations are damped in inverse proportion to $\sqrt{H}$, while the radius of the instantaneous stationary orbit tends towards the radius of the equilibrium orbit $R_0$.

3. The above formulae describe the motion of the electron rather well if we disregard radiation.

An analysis of the problem in the classical case shows that it is only the quantum number $n = l + s$ that changes. It begins to decrease, and this leads to a contraction of the radius. It can be proved that in the classical case the quantum number $s$ and $k$ remain unchanged even when radiation takes place.

The formulae for radiation intensity, allowing for quantum effects was obtained by the present writer in collaboration with Klepikov and Ternov:

$$W_{qu} = W_{ce} \left(1 - \frac{55}{16} \frac{\sqrt{3}}{m_e c R \frac{E}{m_e c^2}}\right)$$

The same formula was later also found by Schwinger for a spinless particle.
Incidentally, we have recently shown with Matveyev that allowance for the electron spin can only affect the small terms of the second order $\sim \hbar^2$; it is therefore not surprising that Schwinger, when calculating the radiation intensity for a non-spinning electron in the first approximation, obtained a formula found by us with the aid of the Dirac equation.

It follows from the last formula that quantum corrections for the radiation rate become prominent only in the region of very great energies:

$$E \sim m_0c^2 \left( \frac{m_0cR}{\hbar} \right)^{1/2} = E_{1/2}$$

of approximately the order of hundreds of Bev. In this connection, some investigators have come to the conclusion that quantum corrections can be completely disregarded when $E < E_{1/2}$. However, we concluded in our very first investigation of the quantum theory of the "luminous" electron that this is not altogether the case.

4. Radial and axial oscillations may be affected by quantum corrections at considerably smaller energies.

For instance, an abrupt change in the radius, i.e. of the centre of radial oscillations, may influence the amplitude of radial oscillations if the emission in question is essentially of a quantum nature.

In this case the radius changes by the amount:

$$\Delta R \sim \frac{\Delta E}{E} R,$$

where $\Delta E = \hbar c R \sim \frac{\hbar c}{R} \left( \frac{E}{m_0c^2} \right)^2$ is the energy of the emitted quantum.

As seen from the quantum formula, the square of radial oscillation amplitude changes least when the quantum numbers decreases by a unit. In this case

$$\Delta (A)^2 \sim \frac{\hbar c}{eH} \sim \frac{\hbar cR}{E}$$

If $\Delta R^2 < \Delta (A)^2$, the fluctuations of the radius due to radiation are not capable to change the amplitude of radial oscillations.

On the other hand, when $\Delta R^2 > \Delta (A)^2$, the photon emission will result in an increase of radial oscillations.

The last relation provides us with the condition at which the quantum fluctuations of the radius during radiation lead to an increased amplitude of radial oscillations:

$$E \sim m_0c^2 \left( \frac{m_0cR}{\hbar} \right)^{1/2} = E_{1/6}$$

Thus the critical energy of appearance of quantum corrections is reduced from several hundred Bev to only several hundred Mev.

More accurate calculations with the help of the quantum theory yield the following formula defining both the classical contraction and the quantum broadening amplitudes of radial oscillations:

$$A^2 = A_0 \frac{E(t)}{E(t)} + \frac{55}{24 \sqrt{3}} \frac{e\hbar^2 R}{m_0E(1-q)^3} \int_0^t \left( \frac{E}{m_0c^2} \right)^6 \frac{dt}{R^2}$$

5. As to axial oscillations, these result from the quantum recoil momentum arising during emission.

We have shown thus that beginning with energies somewhat greater than $E_{1/6}$, a quantum excitation of macroscopic radial oscillations ("macroatom") arises which rapidly exceeds the classical damping caused by an increase in the magnetic field $H (a \div (H)^{-1/2})$.

The radiation damping time of these betatron oscillations

$$t_{dam} = \frac{E^3}{W (mc^2)^3}$$

has such a high value that they can be regarded as not being damped.