THE PRODUCTION OF MESONS IN NUCLEON-NUCLEON COLLISIONS

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This introductory talk will be confined to the production of mesons in nucleon-nucleon collisions below 800 Mev. In this region the phenomenological theory for meson production should be useful in interpreting the experimental results, and the multiple production of mesons plays a negligible part. The striking thing about meson production in this energy range is the very rapid rise with energy of the cross-section. Fig. 1 shows the elastic and total cross-sections of protons upon protons. From the production threshold at 290 Mev the cross-section rises until by 800 Mev the nucleon can be looked upon as black with elastic scattering treated as diffraction scattering from an opaque nucleus. This rapid rise can be explained simply if we assume that the matrix elements for meson production are only very slowly varying functions of energy. We can then say that the cross-section for a particular channel is controlled by the statistical weight of the final state; in particular, the statistical weight will be proportional to the volume in momentum space divided by energy. If we have \(N\) particles then we can assign the momenta of \((N-1)\) arbitrarily, so the weight is proportional to \(\gamma^{2(N-1)}/E\gamma^{N-3}\). This applies only to particles emitted in S states; we have an additional \(\gamma^5\) if a particle is emitted in a p-state. In this way we may have excitation functions rising as steeply as \(\gamma^6\).

Limiting the field in this way we have 10 possible reactions, not all of which can be studied directly. The neutron upon neutron reactions, with one exception, can be studied only by n-d collisions. The exception is, of course, the inverse reaction \(\pi^- + d \rightarrow n + n\). Fortunately if charge symmetry holds in this energy region all the neutron-neutron reactions are duplicated in the proton-proton reactions with interchange of protons with neutrons and positive pions with negative pions.

We have experimental information about all these reactions with exception of \(n + n \rightarrow p + n + \pi^-\). On these latter reactions some work has been done by Powell and Knapp \(2\) who have measured angular distributions but no cross-sections unfortunately.

One of the major steps in the simplification of meson-production was taken by Brueckner and Watson \(3\) in 1951 who showed by assuming that the isotopic spin was conserved in this process, the unbound reactions could be described in terms of 3 production amplitudes. The cross-sections for all reactions can then be most simply expressed in terms of these amplitudes which link given initial and final isospin states of the two nucleons. For example in the reaction \(p + p \rightarrow p + p + \pi^0\) both initial and final isospin states must be \(T = 1\) and so only the cross-section \(\sigma_{11}\) would be involved here. The transition \(0 \rightarrow 0\) is forbidden because the meson has \(T = 1\). This refers of course only to total cross-sections at a particular incident nucleon energy and a particular emitted meson energy.

We have now our three cross-sections \(\sigma_{11}, \sigma_{10}, \sigma_{01}\) to which must be added \(\sigma_{10}\) (d) for the case where the final neutron and proton emerge in the bound state; the deuteron, of course, has \(T = 0\). Brueckner and Watson showed that the experimental cross-sections could be formed of linear combinations of these (Table I).*

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* See also Fermi \(3\).
TABLE I

The various possible proton-nucleon reaction expressed in the four fundamental cross-sections \( \sigma_{01}, \sigma_{10}, \sigma_{11} \) and \( \sigma_{10} \) (d).

<table>
<thead>
<tr>
<th>Reactions</th>
<th>Cross-sections</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p + p \rightarrow p + n + \pi^+ )</td>
<td>( \sigma_{10} + \sigma_{11} )</td>
</tr>
<tr>
<td>d + ( \pi^+ )</td>
<td>( \sigma_{10} ) (d)</td>
</tr>
<tr>
<td>p + p + ( \pi^0 )</td>
<td>( \sigma_{11} )</td>
</tr>
<tr>
<td>p + n \rightarrow p + n + ( \pi^0 )</td>
<td>( \frac{1}{4} (\sigma_{00} + \sigma_{11}) )</td>
</tr>
<tr>
<td>d + ( \pi^0 )</td>
<td>( \frac{1}{4} \sigma_{10} (d) )</td>
</tr>
<tr>
<td>p + p + ( \pi^- )</td>
<td>( \frac{1}{4} (\sigma_{01} + \sigma_{11}) )</td>
</tr>
<tr>
<td>n + n + ( \pi^+ )</td>
<td>( \frac{1}{4} (\sigma_{01} + \sigma_{11}) )</td>
</tr>
</tbody>
</table>

The factor \( \frac{1}{4} \) in all the n-p reactions occurs because this system can be formed from either \( T = 1 \) or \( T = 0 \) state. Charge independence has been tested by comparing the angular distributions of \( p + p \rightarrow \pi^+ + d \) and \( p + n \rightarrow \pi^0 + d \) in the experiments of Hildebrand \(^9\) and Wright and Schluter \(^9\). Within the errors the reactions have identical angular distributions and the n-p reaction has \( \frac{1}{2} \) the cross-section of the p-p reaction.

A convenient starting point in the discussion of the production cross-section are the review articles of Rosenfeld \(^9\) and Gell-Mann and Watson \(^9\). Since then a considerable amount of experimental material has appeared and this will be analysed in terms of the phenomenological models they proposed.

\( p + p \rightarrow \pi^+ + d \).

This is by far the best studied reaction because we have the nucleons both initially and finally in known states of isospin \( T = 1 \rightarrow T = 0 \) and the final nucleons are in a known angular momentum state \( ^3S_1 \). Also it is particularly easy to experimentally backwards and forwards.

One has only to guard against observation of \( p + p \rightarrow n + p + \pi^+ \) going forwards

\[ \text{and} \quad \pi^+ + d \rightarrow \pi^+ + n + p \] going backwards.

If we write down the possible spin states of two nucleons in a \( T = 1 \) state with regard to the Pauli principle we have*

\[ ^1S_0, \ ^3P_{0,1,2}, \ ^1D_2. \]

If the meson is emitted in an s-state then the parity of the final state is odd and \( J = 1 \), and hence the contributing state is \( ^3P_1 \). Similarly if the meson is emitted in a p-state, we have even parity and \( J = 0, 1 \) or 2. Hence we can accept \( ^1S_0 \) or \( ^1D_2 \).

Collecting \( \ ^3P_1 \rightarrow \ ^3S_1S \)
\( \ ^1D_2 \rightarrow \ ^3S_1P \)
\( \ ^3S_0 \rightarrow \ ^3S_1P \)

The s-wave part of the reaction cannot interfere with the p-wave part because they come from singlet and triplet states. So we expect a total cross-section of the form

\[ \sigma = \alpha T + \beta \gamma^3. \]

Before going further we would expect the p-wave part to dominate the reaction because we have mesons emitted in p-states and nucleons left in an s-state. We can also say that we expect the \( ^1D_2 \rightarrow \ ^3S_1P \) to be the dominant p-wave reaction on the following grounds. We know that the \( T = \frac{1}{2}, J = \frac{3}{2} \) state is very much enhanced in pion-nucleon scattering. Suppose that this is so here. The nucleon which has not emitted a pion must be in an s-state. If we treat this nucleon as an onlooker then we can say the final system is made up of a system with \( T = \frac{3}{2}, J = \frac{3}{2} \) and even parity, and another with \( T = \frac{1}{2}, J = \frac{1}{2} \) and even parity. So for the final system \( T = 1, 2; J = 1, 2; \) even parity. And so one would expect the \( ^1D_2 \) state alone among the possible initial states to fulfill these conditions. Whereas (3) has an isotropic angular distribution, one would expect an angular distribution of the form \( \left( \frac{1}{3} \cos^2 \theta \right) \) for \( ^1D_2 \rightarrow \ ^3S_1P. \)

It appears in fact that the reaction is swamped by the p-wave part and to measure the s-wave part directly it is necessary to make observations on mesons with less than 20 Mev c.m. energy. This is rather difficult because the cross-sections here are \( \sim \frac{1}{10} \) mb. The only direct experiments are those of Crawford and Stevenson \(^9\). There is, however, an argument which was proposed by Brueckner, Serber and Watson \(^9\) which is very similar to the argument which leads from the Panofsky ratio for the mesonic and radiative capture of negative pions in hydrogen to the phase shifts in pion nucleon scattering. Panofsky and his colleagues \(^6\), besides observing the gamma-ray spectrum from negative pion capture in hydrogen, also observed that from deuterium. By assuming that the rate of capture from the K orbit is the same, they then deduced that the reaction \( \pi^- + d \rightarrow 2n \) was taking place and hence the pion was pseudo-scalar. And so we can start from the experimental result for the ratio

\[ R \left( \frac{\pi^- + d \rightarrow 2n}{\pi^- + d \rightarrow 2n + \gamma} \right) = \frac{7}{3} \]

This measurement has since been repeated by Chinowsky and Steinberger \(^10\).

If we could find the absolute rate for \( \pi^- + d \rightarrow 2n + \gamma \) we should know the absolute rate for \( \pi^- + d \rightarrow 2n \) and

* Using capital letters to denote the orbital angular momentum state of the two nucleons and small letters that of the meson.
hence by charge symmetry $\pi^+ + d \rightarrow 2p$. Brueckner, Serber, and Watson estimate that the ratio

$$\frac{\langle \pi^- + d \rightarrow 2n + \gamma \rangle}{\langle \pi^- + p \rightarrow n + \gamma \rangle} = \frac{s}{t}$$

Then we get at $\langle \pi^- + p \rightarrow n + \gamma \rangle$ from $\gamma + d \rightarrow n + n + \pi^+$

and $\gamma + p \rightarrow n + \pi^+$. And using the most modern values for these quantities we eventually arrive at $\alpha = 0.2$ mb.

There are experiments which could hope to give a value for $\alpha$. If one takes their best measured point at $\eta = 0.58$ then one gets within the errors, after taking away the p-wave part, $\alpha = 0.22$.

If one assumes that $\alpha = 0.2$ and then plots $(\sigma - 0.2 \eta)$ (which should represent the p-wave part) against $\eta^n$ then one gets the following curve (fig. 2).

The striking points about this curve are: 1) the flattening off and fall of the cross-section at a proton bombarding energy which would correspond to the pion-nucleon resonance at 170 Mev.; 2) the good linearity of the plot up to $\eta > 1$. The best fit to the total cross-section then is given by $\sigma = 0.2 \eta + 0.83 \eta^2$.

The angular distribution information is unfortunately nothing like so accurate (see fig. 3). The Pittsburgh points indicate that the value of $\alpha$ remains reasonably constant at approximately 0.23. But there does seem to be genuine disagreement between the Moscow and Pittsburgh points on the one hand and the Liverpool and Chicago points on the other.

The theory here does not give a very good fit to the experimental points.

$$p + p \rightarrow p + p + \pi^0.$$

In this reaction we have a final $T = 1$ state so if the nucleons are in an S-state then it must be a $^1S_0$ state. So with the meson in a p-state we have a final state $T = 1$, $J = 1$, and even parity. The only initial states with even parity are $^1S_0$, $^3D_2$ which do not have $J = 1$. So we should expect the cross-section of this reaction to be well down on $p\pi^+\pi^-$ near threshold.

We can write down $S$s and $P$s and $P$s final states with $T = 1$ for the final nucleus.

<table>
<thead>
<tr>
<th>Initial</th>
<th>Final</th>
<th>$J$</th>
<th>Parity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ss</td>
<td>$^3S_0$</td>
<td>0</td>
<td>$- \sim \eta^8$</td>
</tr>
<tr>
<td>Ps</td>
<td>$^3S_0$</td>
<td>0</td>
<td>+</td>
</tr>
<tr>
<td>none</td>
<td>$^3P_0$</td>
<td>1</td>
<td>$- \sim \eta^8$</td>
</tr>
<tr>
<td>$^1D_2$</td>
<td>$^3P_0$</td>
<td>2</td>
<td>}</td>
</tr>
</tbody>
</table>

And all the reactions for the class Pp which lead to excitation functions proportional to $\eta$.

The experimental information up to this year indicated that an excitation function $\eta$ was probably right. This was based on two measurements, that of Mather and

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* The new measurement at Liverpool is on the reaction $\pi^+ + d \rightarrow p + p$ and has been done by D. Eccleshall and A. W. Merrison and yields $\sigma = 1.58 \pm 0.16 (0.29 \pm 0.02 + \cos^8) \text{mb. at 95 Mev pion laboratory energy.}
Pion production by nucleons

Martinelli \(^{14}\) and of Marshall et al.\(^{19}\). But since then we have the work of Stallwood, Fields, Fox and Kane \(^{19}\) and Tiapkin and Prokoshkin \(^{17}\) which has changed the picture completely. This work indicates that all the experimental points may be fitted by a curve of the form \(A_\gamma^h\) in the range 340-660 Mev. This indicates that the process \(Ps\) is the dominant one which is a little surprising. Mather and Martinelli pointed out, however, that because a meson was emitted with \(l = 1\) with respect to its parent nucleon it does not follow that it has \(l = 1\) with respect to the centre of gravity of the nucleons when there is any appreciable relative velocity of the nucleons. The angular distribution, too, according to Tiapkin and Prokoshkin is largely isotropic which is what one would expect from a \(Pp\) state.

\[
p + p \to n + p + \pi^+ (\sigma_{10} + \sigma_{11})
\]

In this reaction the \(Sp\) states would be given by

\[
\begin{align*}
^3P_1 &\to ^3S_1s \\
^1D_2 &\to ^1S_1p \\
^1S_0 &\to ^1S_0p
\end{align*}
\]

and we can say that the matrix elements connecting given spin states should be the same as with the corresponding deuteron reaction. And the cross-sections now will be given by the deuteron cross-section multiplied by the ratio of numbers of final states in the unbound/bound reactions and by a term \(\frac{\bar{\psi}(R)}{\bar{\psi}_d(R)}\) where the \(\bar{\psi}\) are the final 2-nucleon wave forms at \(R\), the critical distance for meson production. As this is small \((\approx \hbar/mc)\) then a zero range approximation is good enough. Not much can be said about the agreement of this theory \(^{10}\) with experiment at low energies \((\eta < 1)\) until the experiments improve. All the earlier work in this field gave experimental values which were about twice as large as the theory predicts but there has been a tendency recently to reduce. In particular the measurement of Stork and Whitestone \(^{10}\) agrees very well with the theory. The work reported at the Moscow conference seems to suggest that the ratios at higher energies are considerably in excess of the values predicted by this theory.

There are many ways that theory could be in error. In particular at high energies the possibility of large relative velocity of the nucleons is serious. Also it is assumed throughout that the presence of the meson does not upset the nucleon-nucleon force.

The work of Pontekorvo and Selivanov \(^{20}\) on \(n + p \to \pi^0\) gives good agreement with all the above work.

\[
\begin{align*}
n + p &\to \pi^0 \quad 1/(\sigma_{01} + \sigma_{11})
\end{align*}
\]

The only experiments with a bearing on this reaction have been performed by Yodh \(^{21}\) at Chicago. For nucleons in an \(S\)-state we have only the possible reactions

\[
\begin{align*}
^3S_1 &\to ^1S_0p \\
^3D_1 &\to ^1S_0p
\end{align*}
\]

and for these we expect an excitation function varying as \(\eta^4\). For all states involving nucleons in a \(P\)-state we get a momentum dependence on \(\eta^2\). If we average Yodh’s results for positive and negative pions we get a value for the cross-section of this process at \(\approx 400\) Mev of \(0.16\pm 0.04\) mb. If we subtract an extrapolated value for \(\sigma_{11}\) of 0.11 mb for this reaction we arrive at \(\sigma_{01} = 0.05\) which is very small indeed.

LIST OF REFERENCES


