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TWO-DIMENSIONAL KÄHLER EINSTEIN SPACES AND
GRAVITATIONAL INSTANTONS

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A new class of solutions of euclidean Einstein equations with Λ -term ("A-class" is found by solving complex 2-dimensional kahler Einstein equations with the following "realization" of complex metrics. A - class includes two gravitational instantons already known - CP^2 and Eguchi - Hanson metric and allows $U(1)$ -generalized spin structure. It is shown that all Einstein euclidean 2 - axial Bianchi type IX metrics are exhausted by Taub-NUT- de Sitter family and A-class.

The definition of gravitational instantons depends on applications of this notion: 1) it is natural in view of the contribution of gravity to the axial anomaly [1,2] to look for complete non-singular positive definite metrics which are solutions of Einstein equations (generally with Λ - term)

$$R_{\mu\nu} = \Lambda g_{\mu\nu} \quad (\mu, \nu, \dots = 1, 2, 3, 4) \quad (1)$$

allow an ordinary or generalized spin structure [3,4] and lead to non-zero index of Dirac operator (and thus to the chiral symmetry breaking [5]). Similar metrics with nonzero Euler number may play analogous part in the trace anomaly [6].

ii) In the semiclassical calculation of euclidean functional integral in quantum gravity [7] one is interested in the metrics, satisfying classical equations of motion (1) ($\Lambda \neq 0$ in the "volume canonical ensemble" approach [7,8]) and having finite action, thus providing the dominant contribution to the quantum amplitudes. If these metrics are asymptotically locally euclidean (in four dimensions), they may describe the tunnelling between topologically distinct

gravitational vacua [9].

Anyhow the search for various solutions of (1) with non-trivial topology and (or) finite action is of great interest. To obtain a new class of such solutions (A-class for short) we will use the following method. Let $g_{i\bar{j}}$ be kahler metric on $M^2(\mathbb{C})$ [10], i.e. $ds^2 = g_{i\bar{j}} dz^i d\bar{z}^j$ ($i, j, K = 1, 2$) is hermitian positive definite quadratic form and $d\Phi = 0$, $\Phi = \frac{i}{2} g_{k\bar{m}} dz^k \wedge d\bar{z}^m$. First, we solve kahler Einstein equations

$$R_{i\bar{j}} = -\lambda g_{i\bar{j}}, \quad R_{i\bar{j}} = -\frac{\partial^2}{\partial z^i \partial \bar{z}^j} \log \det g_{k\bar{m}}, \quad (2)$$

then carry out the "realization" of $g_{i\bar{j}}$ (i.e. $\{z^i\} \rightarrow \{x^\mu\}$, $g_{i\bar{j}} \rightarrow g_{\mu\nu}$, $ds^2 = ds^2_R = g_{\mu\nu} dx^\mu dx^\nu$) and finally use the theorem (one can get the proof with the help of ref. [11]) that the resulting $g_{\mu\nu}$ satisfies (1) with $\Lambda = -2\lambda$. A-class is obtained if we suppose in addition that $\det g_{k\bar{m}}$ depends only on $\rho = z^i \bar{z}_i = |z_1|^2 + |z_2|^2$ ($U(2)$ -symmetry). In this case from (2) we have

$$g_{i\bar{j}} = A \delta_{ij} + A' \bar{z}_i z_j, \quad [\log A(\rho)]' = \lambda A, \quad (3)$$

where $A = A(\rho)$, $A' = \frac{dA}{d\rho}$. Equation (3) for A can be completely integrated (introducing $t = \log \rho$, $y = (A \cdot \rho)^2$, but we are to use only its first integral

$$\frac{2\gamma^3}{\frac{1}{3}\gamma^6 + \gamma^4 + \alpha} = d(\log \rho), \quad \gamma^2 = A \cdot \rho \quad (4)$$

(here $\mathcal{A} = \text{const}$ and for positive definiteness of $g_{ij}(g_{\mu\nu})$ is assumed). Now let us consider the following "realization" [12]

$$\begin{aligned} Z_1 &= \sqrt{f} \cos \frac{\theta}{2} \cdot \exp \frac{i}{2} (\psi + \varphi) \\ Z_2 &= \sqrt{f} \sin \frac{\theta}{2} \cdot \exp \frac{i}{2} (\psi - \varphi) \end{aligned} \quad (5)$$

where $0 \leq \theta$, $\varphi/2$, $\psi/4 \leq \pi$ are coordinates on the 3-sphere S^3 if $Z^1 = \chi^1 + i\chi^2$, $Z^2 = \chi^3 + i\chi^4$, we have

$$g_{\mu\nu} = A \delta_{\mu\nu} + A' (\chi_\mu \chi_\nu + \tilde{\chi}_\mu \tilde{\chi}_\nu), \quad \tilde{\chi}_\mu = C_{\mu\nu} \chi^\nu, \quad C_{\mu\nu} = \begin{bmatrix} J & 0 \\ 0 & J \end{bmatrix},$$

$$J = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \quad \text{what give } \mathbb{C}P^2 \text{ metric in the form used in [1]}$$

(for $\mathcal{A} = 0$ in (4)). Finally we get (changing f^0 by f with the help of (4))

$$ds_R^2 = f^2(\eta) d\eta^2 + F^2(\eta) (\vec{e}_1^2 + \vec{e}_2^2) + g^2(\eta) \vec{e}_3^2, \quad (6)$$

$$F = f, \quad g = \frac{f}{f}, \quad f^2 = \frac{f^4}{\frac{1}{3} f^6 + f^4 + \mathcal{A}}, \quad (7)$$

i.e. Bianchi type IX metric (here $\{\vec{e}_\alpha\}$ ($\alpha, \beta, \gamma = 1, 2, 3$) is a standart basis of 1-forms on S^3 [13], $\vec{e}_1 = -\frac{1}{2}(\cos\psi d\theta + \sin\psi \sin\theta d\varphi)$, $d\vec{e}_\alpha = \epsilon_{\alpha\beta\gamma} \vec{e}_\beta \wedge \vec{e}_\gamma$, $\int_{S^3} \vec{e}_1 \wedge \vec{e}_2 \wedge \vec{e}_3 = 2\pi^2$).

Introducing an orthonormal basis of 1-forms $\{\theta^a\}$ ($a, b, c = 0, 1, 2, 3$)

$$\theta^0 = f d\eta, \quad \theta^1 = F \vec{e}_1, \quad \theta^2 = F \vec{e}_2, \quad \theta^3 = g \vec{e}_3, \quad (8)$$

one can easily calculate the curvature 2-form $R = d\omega + \omega \wedge \omega$

(with $\omega_{ab} = -\omega_{ba}$ being metrical connection 1-form without torsion, i.e. $d\theta^a + \omega^a_b \wedge \theta^b = 0$)

$$\begin{aligned}
R_{01} &= -R_{23} = -M(\theta^0 \wedge \theta^1 - \theta^2 \wedge \theta^3), \\
R_{02} &= -R_{31} = -M(\theta^0 \wedge \theta^2 - \theta^1 \wedge \theta^3), \\
R_{03} &= -4N\theta^0 \wedge \theta^3 - 2M\theta^1 \wedge \theta^2, \\
R_{12} &= -2M\theta^0 \wedge \theta^3 - 4N\theta^1 \wedge \theta^2,
\end{aligned} \tag{9}$$

where $M = \frac{1}{3} - \frac{2\lambda}{\gamma^2}$, $N = \frac{1}{3} + \frac{\lambda}{\gamma^2}$. Thus we see that: eq. (1) is really satisfied if $\Lambda = -2\lambda (R = *R^*)$; the only anti-self-dual (and at the same time the only asymptotically flat) metric corresponds to the case of $\lambda = 0$ ($R = -*R$); all our metrics have anti-self-dual Weyl tensor $C = -*C$ (that is the "closest" property to anti-self-duality of the curvature when $\Lambda \neq 0$); $\gamma = 0$ is the only possible singularity of the curvature.

The interval $[\gamma_1, \gamma_2]$ of the change of γ in (7) follows from the positive definiteness of the metric ($g_{ij} \geq 0$); the finiteness of the volume of the corresponding spaces $V = \int \theta^1 \wedge \theta^2 \wedge \theta^3 \wedge \theta^0 = \frac{\pi}{\gamma} [\gamma']_{\gamma_1}^{\gamma_2}$, will serve to us as a criterion of compactness of M . Now let us recall two types of removable singularities [8, 14] of the metrics like (6): if for $ds^2 = d\tau^2 + a^2 \epsilon_1^2 + b^2 \epsilon_2^2 + c^2 \epsilon_3^2$, $a^2, b^2, c^2 \rightarrow z^2$ when $z \rightarrow 0$ then $z = 0$ is "nut" (it is removed by adding $z = 0$ to M and introducing Cartesian coordinates); if (for $z \rightarrow 0$) $a^2, b^2 < \infty$ and $c^2 \rightarrow n^2 z^2$ ($n = \text{integer}$) then $z = 0$ is 2-dimensional "bolt" (which is removed by the identification $0 \leq \psi < 2\pi$). In case of non-integer n one cannot remove singularity (which we will call "quasi-bolt") by adjusting the range of ψ to $[0, 2\pi]$ (3-sections will not coincide with $S^3/\mathbb{Z}_n$) and obtain a geodesically complete manifold (the presence of "quasi-bolt").

is not connected with the regularity of the curvature). Let us proceed now to the classification of metrics in A-class:

1a) $\lambda = 0$, $\alpha > 0$, $V = \infty$, $0 \leq \eta^2 < \infty$ - non-compact (NC) singular ($\eta = 0$) space (SS); introducing $R^2 = \frac{\eta^4 + \alpha}{\eta^2}$, we obtain metric I of Eguohi-Hanson [13, 14] (IEH). 1b) $\lambda = 0$, $\alpha = -a^2 \leq 0$, $a^2 \leq \eta^2 < \infty$, $V = \infty$ - non-compact regular space (NCRS), coinciding with II EH metric [13, 14]; $\eta = a$ is bolt ($n=2$), i.e. $0 \leq \psi \leq 2\pi$ (RP^3 as a sections).

When $\lambda \neq 0$ we will use dimensionless variables

$$\xi = \lambda \eta^2, \quad f^2 = \frac{3\xi^2}{\xi^3 + 3\xi^2 + \delta}, \quad \delta = 3\alpha \lambda^2; \quad (10)$$

if $\delta = -(\delta-1)^2(\delta+2)$, then $\xi^3 + 3\xi^2 + \delta = (\xi-a_1)(\xi-a_2)(\xi-a_3)$, where $a_1, a_2 = \frac{1}{2}[-(\delta+2) \pm \sqrt{3(4-\delta^2)}]$, $a_3 = \delta-1$.

2a) $\alpha = 0$ ($\delta = 0$), $\lambda > 0$, $0 \leq \xi < \infty$, $V = \infty$ - NCRS, $\xi = 0$ is nut. 2b) $\alpha = 0$, $\lambda < 0$, $-3 \leq \xi \leq 0$, $V = \frac{9\pi^2}{2\lambda^2}$ - compact regular space (CRS); $\xi = 0$ is nut, $\xi = 3$ is bolt ($n=1$); corresponding complex metric (3) (with $A = \frac{3}{|\lambda|} \cdot \frac{1}{\rho + \rho_0}$) is Fubini-Study metric [10] on 2-dimensional complex projective space CP^2 [1, 12, 14]. 3a) $-4 \leq \delta < 0$ ($1 < \delta \leq 2$), $\lambda > 0$, $a_2 \leq \xi < \infty$, $V = \infty$ - NCRS; $\xi = a_3$ is quasi-bolt ($n=|\delta+1|$) (if f^2 is singular near ξ_0 , $f^2(\xi) \approx \frac{B}{\xi - \xi_0}$, ξ_0 is quasi-bolt with $n = |\xi_0/B|$). 3b) $-4 < \delta < 0$ ($1 < \delta < 2$), $\lambda < 0$, $a_2 \leq \xi \leq a_1$, $V = \frac{\pi^2}{2\lambda^2}(\delta+2)\sqrt{3(4-\delta^2)}$ - CRS; $\xi = a_1, a_2$ are quasi-bolts ($n(a_1) \neq n(a_2)$, so the global identification of ψ does not exist). (It should be stressed that the presence of quasi-bolts may not be the reason for disregard of such metrics as gravitational instantons: for example we must consider in the functional integral all possible metrics without condition

of geodesic completeness. Note that two quasi-bolts are also present in Schwartzschild - de Sitter and Taub-NUT-de Sitter spaces [8,12], but complete manifolds are obtained here only in some limiting cases (for example, when $gM^2\Lambda = 1$

for Schwartzschild- de Sitter metric we get $S^2 \times S^2$ -space).

4a) $\delta > 0$ or $\delta < -4$, $\lambda > 0$, $0 < f < \infty$, $V = \infty - NCSS$.

4b) $\delta > 0$ ($\delta < -2$), $\lambda < 0$, $q_1 \leq f \leq 0$, $V = \frac{\pi^2}{2\lambda^2} (\delta-1)^2 -$

CS^2 , $f=q_1$ is quasi-bolt. Thus we see that: all spaces with $\Lambda > 0$ ($\Lambda < 0$) are compact (non compact); the only

regular and geodesically complete metrics are II EH, CP^2 and

2a (the latter is noncompact, but nonasymptotically flat).

Of all the other cases 3b - metrics are the most interesting (CRS but with quasi-bolts).

Now let us clear up the relation between Λ -class and Taub-NUT- de Sitter (TNUT-g) family of solutions of (1) [12].

One can solve (1) for the metrics of type (6) (with $g = \frac{F}{f}$, that can always be established by a suitable change of η).

From the three coupled equations on F and g , f

$$M_1 = M_2, N_1 = N_2, 2M_1 + 4N_1 = -\Lambda,$$

$$M_1 = \left(\frac{F'}{f}\right)' \frac{1}{Ff}, M_2 = -\frac{1}{Fg} \left(\frac{g^3}{F^2} - \frac{F'g'}{f^2}\right),$$

$$N_1 = \left(\frac{g'}{f}\right)' \frac{1}{fg}, N_2 = \frac{1}{F^2} \left(\frac{F'^2}{f^2} + 3\frac{g^2}{F^2} - 4\right),$$

we get two solutions

$$1) F_1 = \frac{\sinh(q\eta)}{q} (q \neq 0), 2) F_2 = q$$

$$1) g_1^2 = \frac{\cosh(q\eta)}{\sinh^2(q\eta)} \left[\frac{2\lambda}{q^2} \cdot \frac{\cosh^4(q\eta) - 6\cosh^2(q\eta) - 3}{\cosh(q\eta)} + \right. \\ \left. + \frac{4}{q^2} \frac{1 + \cosh^4(q\eta)}{\cosh(q\eta)} + \alpha q^2 \right], \quad (11)$$

$$2) \quad g^2 = \frac{1}{y^2} \left[\frac{1}{3} y^6 + y' + x \right].$$

Here 1) and 2) are correspondingly "regular" and "peculiar" solutions: though $F_2 = \lim_{a \rightarrow 0} F_1$, for g^2 we have $g^2(a \rightarrow 0) \approx \frac{8}{y^2} \left(\frac{1}{a^4} - \frac{21}{3a^6} \right) + g^2$. Solution 2) obviously coincides with A-class (7) and 1) - with TNUT-S in its standard form [12] (after the change of variable $R = L \cosh\left(\frac{y}{2L}\right)$, $L = \frac{1}{2a}$, $x = -2m(4L)^3$, i.e. (6), where $dy \rightarrow dR$ and

$$f^2 = \frac{R^2 - L^2}{\Delta}, \quad F^2 = 4(R^2 - L^2), \quad g^2 = \frac{(4L)^2 \Delta}{R^2 - L^2}, \quad (12)$$

$$\Delta = R^2 - 2mR + L^2 - 21(L' + 2L^2 R^2 - \frac{1}{3} R').$$

We should stress that A-class is independent of TNUT-S: one needs a singular limiting procedure to obtain it from this family. Consequently it is more accurate to speak about A-class and TNUT-S as independent ("peculiar" and "regular") solutions of (1) and it is A-class which naturally includes $\mathbb{CP}^2$ and EH-metrics (compare [12, 14, 15]).

Eguchi-Hanson metrics are known to admit spinor structure [3] (due to their anti-self-duality). Other metrics from A (for example, $\mathbb{CP}^2[3,12]$) lack this property but the existence of complex (kahler) structure signifies the possibility of introducing $\bar{U}(1)$ - generalized spin structure [3,4]. Thus in the presence of electromagnetic field spinors can be defined correctly and the contribution of these metrics in axial anomaly can be discussed. Note that the existence of kahler structure provides a natural definition of electromagnetic field 2-form F in terms of kahler form $\bar{\Phi}$ (after "realization"

(5) ($c = \text{const}$)

$$\Phi = -(\theta^0 \wedge \theta^3 + \theta^1 \wedge \theta^2) = * \Phi, \quad F = 1 \cdot c \cdot \Phi, \quad (13)$$

$$d\Phi = 0$$

(that coincides for CP^2 with the result of [12] but differs from [14] for EH).

While calculating topological invariants for metrics from Λ boundary corrections should be taken into consideration (we will consider as boundaries only S^3 at $\tilde{r} = \infty$, adding the other 2-or-3-surfaces for different boundary values of $\tilde{r}$ to the manifold). Then the 1st Pontryagin number is (for all definitions see [16])

$$P_1 = P_{1V} + P_{1S} = \frac{1}{8\pi^2} \int_{\Lambda} R^{ab} \wedge R^{ab} + \frac{1}{8\pi^2} \int_{\partial M} \Omega^{ab} \wedge R^{ab}, \quad (14)$$

$$P_{1V} = \frac{1}{3} \left[\tilde{r}^2 + \frac{2\delta^2}{\tilde{r}^4} \right]_{\tilde{r}_1}^{\tilde{r}_2}, \quad P_{1S} = \frac{1}{9} \left[-\tilde{r}^2 + \frac{4\delta}{\tilde{r}} - \frac{4\delta^2}{\tilde{r}^4} \right]_{\partial M},$$

($|\tilde{r}_2| > |\tilde{r}_1|$) (Ω - 2nd fundamental form of ∂M),
 i.e. $P_1(\mathbb{R}EH) = -6$ ($= -3$ after identification of ψ),

$P_1(CP^1) = 3$, $P_1(3\theta) = (3/2)[3(4-\delta^2)]^{1/2}$ (here $P_1 = P_{1V}$). In other cases P_1 diverges due to a singularity at $\tilde{r} = 0$ or to the lack of asymptotic flatness when $\Lambda \neq \emptyset$. The dependence of $P_1(3\theta)$ on continuous parameter δ is an unsatisfactory feature for topological invariant and is the evidence of the difficulty in the definition of P_1 in the presence of quasi-bolts (or "conical" singularities). We confront the same problem while

calculating the Euler number $\chi = \chi_V + \chi_S$

$$\chi = \frac{1}{32\pi^2} \int_{\Lambda} R^{ab} \wedge R^{cd} \epsilon_{abcd} + \left(-\frac{1}{32\pi^2} \right) \int_{\partial M} 2(\Omega^{ab} \wedge R^{cd} - \frac{2}{3} \Omega^{ab} \wedge \Omega^{cd} \wedge \Omega^{ef}) \epsilon_{abcd}, \quad \chi_V = \frac{1}{3} \left[\tilde{r}^2 - \frac{\delta^2}{\tilde{r}^4} \right]_{\tilde{r}_1}^{\tilde{r}_2}, \quad (15)$$

$$\chi_S = -\frac{1}{3} \left[\tilde{r}^2 - \frac{\delta^2}{\tilde{r}^4} - 3 \right]_{\partial M},$$

or $\chi(I EH) = 4$ (= 2 after identification of ψ), $\chi(\mathbb{C}P^2) = 3$,
 $\chi(3b) = 2\sqrt{3(4-\delta^2)}$. Again the dependence of χ on δ for 3b)
 and other metrics from Λ speaks for the necessity of intro-
 duction of "corner correction" in Gauss-Bonnet-Chern formula
 (or some special identifications of ψ) in case of quasi-
 bolts.

Now let us proceed to the euclidean action [17] on
 A-class metrics $I = I_V + I_S$

$$I_V = -\frac{1}{\kappa^2} \int_M (R - 2\Lambda) \sqrt{g} d^4x = -\frac{1}{\kappa^2} \int_M R_{ab} \Lambda * \pi^{ab} + \quad (16)$$

$$+ \frac{2\Lambda}{\kappa^2} \cdot V, \quad I_S = \frac{1}{\kappa^2} \int_{\partial M} \Omega_{ab} \Lambda * \pi^{ab},$$

$$\pi^{ab} = \theta^a \wedge \theta^b, \quad \kappa^2 = 16\pi G,$$

where $\Omega_{ab} = \omega_{ab}^i - \bar{\omega}_{ab}^i$ is the second fundamental form of
 ∂M and $\bar{\omega}$ is the connection of the product metric for
 fixed $f(\partial M)$. We get

$$I_V = \frac{\pi}{8G\Lambda} [f^2]_{f_1}^{f_2},$$

$$I_S = \frac{\pi}{4G\Lambda} \left[\frac{4}{3} f^2 + 3f + \frac{\delta}{3f} \right]_{\partial M}. \quad (17)$$

As a result, finite action is obtained in the following cases:

$$I(I EH) = 0, \quad I(\mathbb{C}P^2) = -\frac{3\pi}{8G\Lambda}, \quad I(3b) =$$

$$= -\frac{\pi}{8G\Lambda} (\delta+2)\sqrt{3(4-\delta^2)}, \quad \text{i.e. } \mathbb{C}P^2 \text{ will give maximal contri-}$$

bution to the functional integral as compared to the other
 metrics from Λ (though it is less than that of de Sitter space
 [17], $I(S^4) = -\frac{3\pi}{2G\Lambda} < I(\mathbb{C}P^2)$).

We conclude with the remarks on the perspectives of the
 method of solving (1) with the help of kahler Einstein equa-
 tion (2): a) eq. (2), due to its relatively simple form, pro-

vides the possibility of rigorous mathematical analysis (for example, the proof of the existence and uniqueness theorem [18]); b) the simple differential operator in (2) may find applications in quantum gravity (in integrating over all kahler metrics); c) one can look for the other classes of kahler Einstein spaces, choosing another subsidiary condition (for example, $\det g_{k\bar{m}} = f(z, \bar{z}, z, \bar{z})$) or try to integrate (2) completely using 2-dimensionality of the problem; d) there may be some interest in combining the methods of algebraic geometry and direct solution of (2) while analysing algebraic hypersurfaces $V_m \subset \mathbb{CP}^3$ (for example, K3) as gravitational instantons [4].

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