EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH

CERN-EP/80-151
14 August 1980

VACUUM POLARIZATION IN MUONIC ATOMS

L. Tauscher^{*)}

Institute of Physics, University of Basle,
Basle, Switzerland

Invited talk given at the
Symposium on Present Status and Aims of Quantum Electrodynamics
Mainz, 9-10 May, 1980

^{*)} Visitor at CERN, Geneva, Switzerland.

1. INTRODUCTION

In the course of this meeting it became obvious that the muonic atom is not the only tool for testing vacuum polarization. Although the classical experiments on QED, such as the g-2 experiments, test essentially graphs of type (a) in Fig. 1, their very great accuracy also provides very valuable tests for the much smaller vacuum polarization [type (b) in Fig. 1]. The same is true for the Lamb shift measurement in hydrogen. Table 1 summarizes the accuracy reached in these different measurements¹⁻³). Of course this way of looking at these experiments only makes sense if it is assumed that all other contributions are known theoretically.

The muonic atom is, however, a tool for studying and testing vacuum polarization at stronger fields and higher momentum transfers. Furthermore, the vacuum polarization is the dominant QED correction to the lowest-order Bohr (or Dirac) energies. Moreover, the difficulties of the μHe Lamb shift⁴) measurement, introduced by the finite size of the α -particle, may be overcome by choosing transitions that are much less dependent on the finite size contribution.

It is obvious that in order to test the vacuum polarization, X-ray transitions have to be selected where uncertainties from contributions other than the vacuum polarization are minimized with respect to it. Such transitions are the 3-2 transitions in light elements ($Z \approx 13$), the 4-3 transitions in medium heavy elements ($Z \approx 56$) and the 5-4 transitions in heavy elements ($Z \approx 82$).

In Table 2 the various corrections⁵⁻⁸) contributing to a muonic X-ray transition energy are listed for He, Mg, Ba, and Pb.

It is apparent that the calculations have uncertainties which are comparable to or larger than the experimental errors. The error due to the finite size in the case of μHe amounts to 0.25% of the vacuum polarization. In the case of Mg this error contribution is less than 0.02%, and in Pb it is of the same order of magnitude. The "theoretical" uncertainties in Mg and Pb arise mostly from the electron screening. This uncertainty is not so much due to the calculation of the electron screening potential but stems rather from the badly known number of

s-electrons present when the muonic X-ray transition occurs. Table 2 also shows that the experimental accuracy for Pb is sufficient for it to become sensitive to those contributions which are due to the high fields [such as the ones in which the virtual e^+e^- cloud is polarized, terms of order $(\alpha Z)^3$]. These contributions are not important in the lighter atoms.

It should be noted that the energy of a muonic X-ray transition from one main quantum number to the next lower one is mainly due to the difference of Bohr binding energies, and the QED corrections amount to 0.5% or less of this transition energy. Thus a test of the vacuum polarization to 0.2% accuracy requires a precision of 10 ppm in the determination of the transition energy. This is an accuracy comparable to the one reached in the Lamb shift measurement on hydrogen or the g-2 measurement for the muon.

2. EXPERIMENTS

Two types of experiments were performed so far, using entirely different types of spectrometer [the laser technique of Zavattini⁴⁾ is not a subject of this talk].

At SIN, three elements were measured with the Fribourg *bent crystal spectrometer*. The target is placed in the muon channel, and stop rates of $\sim 0.5 \times 10^6 \text{ s}^{-1}$ may be obtained in a 25 mg/cm^2 thick target. The resolution of this spectrometer is of the order of 35 eV at 56 keV. Details may be found elsewhere⁹⁾. Table 3 shows the measured transitions and their energies as well as the vacuum polarization contribution⁶⁾. The calibration was done relative to γ -ray standards between 50 keV and 90 keV¹⁰⁾.

The accuracy obtained for the Mg $3d_{5/2}-2p_{3/2}$ transition is 9 ppm. The errors are mostly statistical (~ 5 ppm from line shape, 1-2 ppm from geometry, 7 ppm and more from statistics).

In Fig. 2 the deviation of the measured energies from the calculated ones ($E_{\text{exp}} - E_{\text{calc}}$) is displayed in units of percent of the vacuum polarization correction. The error bars are for the experimental errors only. The errors from the calculations are indicated by the broken lines.

The final result is that the crystal spectrometer measurements test the vacuum polarization to 0.2% (from the experimental errors alone).

The usual way of calculating the testing power of several measurements is to add the experimental and the "theoretical" uncertainties (linearly or quadratically) for each experiment and then deduce the weighted average of the deviation and its error. This is wrong since the "theoretical" errors are correlated and systematic. The correct way to determine the "theoretical" uncertainty for several transitions is to calculate the average of the individual "theoretical" uncertainties weighted with the corresponding experimental errors.

Combining the final experimental accuracy of 0.2% with the uncertainty thus obtained for the calculations of 0.17%, the vacuum polarization is tested to 0.26% at 1 σ confidence.

Since 1976, three groups^{7,8,11)} have published precision experiments on heavy muonic atoms for testing the QED contributions. The measurements were done with *Ge diodes*. The spectrometer characteristics are shown in Table 4 (the CERN group used two diodes).

The results of the measurements are listed in Table 5. The energy calibration is with respect to an Au standard of 411,805.2 eV, whereas the final published value is $411,804.41 \pm 0.15$ eV¹²⁾. Thus all experimental values might be too high by about 0.8 eV. (A re-adjustment seems, however, difficult in view of the unknown non-linearities in the different experiments.)

The experimental accuracy has to be considered carefully, since it is only in the CERN measurement that the statistical errors dominate (7-9 eV statistics, ~ 6 eV background and calibration). The SIN group has a statistical error of 3-4 eV and, in linear addition, a 5 eV systematic error. The SREL group claims a statistical error, including background lines and calibration, of 0.7-3 eV, and a constant systematic error of 1.7 eV. Besides pure statistics, uncertainties in these measurements arise from unresolved background lines (X-rays of parallel transitions, nuclear γ -lines, etc.), from geometrical and count rate influences,

etc. All this is discussed carefully in the CERN and SIN experiments, whereas a detailed error discussion is not yet available for the SREL experiment.

The complexity of the experimental spectra is demonstrated in Fig. 3, where the relevant part of the CERN data is displayed.

In Fig. 4 the deviations of the measured energies from the calculated ones are displayed, again in units of percent of the QED contributions. The error bars correspond to the experimental errors, and the broken lines reflect the uncertainties in the calibration.

The measurements reach, in their average, an experimental accuracy of 0.07% for the test of the vacuum polarization. This accuracy is essentially due to the SREL points in Tl and Pb, for which no detailed error discussion is available yet. The weighted average of the uncertainties from the calculation is 0.25%. Thus the total test quality of these experiments is entirely governed by the "theoretical" uncertainties, and amounts to 0.26% at 1 σ confidence.

3. SUMMARY AND OUTLOOK

It is obvious that the quality of the test of QED in muonic atoms depends not only on the experimental precision but also on the correctness of the calculated quantities. Unfortunately the latter is dominating in the case of muonic atoms, as is shown in Table 6.

The uncertainties in the calculation stem from various effects, such as electron screening, nuclear polarization, etc. As long as these effects cannot be fixed by separate experimental checks, any improvement in the experimental accuracy is meaningless.

There exist proposals to study the electron screening by carefully measuring the screening effect in higher orbits (where QED contributions are small), as proposed by the SIN crystal spectrometer group. Similar efforts have to be made for the heavy muonic atoms in order to study and to understand quantitatively the cascade processes. Efforts have also to be made, both experimentally and theoretically, to understand the nuclear polarization.

Experimentally several improvements can be envisaged.

Crystal spectrometer:

- better statistics [factor of $\sim 3(?)$];
- better analysis by eliminating the line shape problem [factor of $2(?)$];
- higher orders (??).

This could lead to a total experimental accuracy of 3-4 ppm or a test quality of 0.05-0.1% of the vacuum polarization.

Ge-diode spectrometers:

- elimination of background problems by better resolution;
- improvement of peak-to-background ratio by an anti-Compton device as used by Beetz et al.¹³⁾;
- reduction of background lines by coincidence measurements of different cascade transitions;
- reduction of calibration systematics by improving the calibration methods used by the CERN group.

A total experimental accuracy of 2-3 ppm seems, however, to be the limit of this technique, corresponding to a test of QED to 0.04-0.06%. In order to reach a similar precision from the "theoretical" side, significant improvements on the electron screening, nuclear polarization, and also from the QED computations are needed.

In summary I think that the crystal spectrometer technique has a good chance to reach a test quality of 0.05-0.1% for the vacuum polarization; whereas for the heavy muonic atoms, even minor improvements will require enormous efforts.

REFERENCES

- 1) H.G. Dehmelt, "Anomalie der magnetischen Momente von Elektron und Positron".
This conference.
- 2) J. Bailey et al., Nucl. Phys. B150, 1 (1979).
- 3) S.R. Lundeen and F.M. Pipkin, Phys. Rev. Lett. 34, 1368 (1975).
- 4) G. Carboni et al., Nucl. Phys. A278, 381 (1977).
- 5) E. Zavattini, Proc. 7th Int. Conf. on High-Energy Physics and Nuclear
Structure, Zurich, 1977 (ed. M. Locher) (Birkhäuser-Verlag, Basle, 1977),
p. 49.
- 6) H.J. Leisi, Proc. 8th Int. Conf. on High-Energy Physics and Nuclear Structure,
Vancouver, 1979 (eds. D.F. Measday and A.W. Thomas), Nucl. Phys. A335, 3
(1980).
- 7) T. Dubler et al., Nucl. Phys. A294, 397 (1978).
- 8) L. Tauscher et al., Z. Phys. A285, 139 (1978).
- 9) H.J. Leisi, Proc. 1st course of the Int. School of Physics of Exotic Atoms,
Erice, 1977 (eds. G. Fiorentini and G. Torelli) (Servizio Documentazione
dei Laboratori Nationali di Frascati, Frascati, 1977), p. 75.
- 10) E.G. Kessler et al., Nucl. Instrum. Methods 160, 435 (1979).
- 11) C.K. Hargrove et al., Phys. Rev. Lett. 39, 307 (1977).
- 12) E.G. Kessler et al., Phys. Rev. Lett. 40, 171 (1978).
- 13) R. Beetz et al., Z. Phys. A286, 215 (1978).

Table 1

Comparison of classical QED tests

Experiment	Exp. value	Vac. pol.	Test of vac. pol. (%)
(g-2) electron ¹⁾	115 965.241 (4) $\times 10^{-8}$	9.4×10^{-8}	0.02
(g-2) muon ²⁾	116 592.2 (9) $\times 10^{-8}$	585.6×10^{-8}	0.15
Lamb shift (H) ³⁾	1057.893 (20) MHz	27.323 MHz	0.07

Table 2

Calculated contributions to various muonic X-ray transitions

	He $2s_{1/2}-2p_{3/2}$ (meV)	Mg $3d_{5/2}-2p_{3/2}$ (eV)	Ba $4f_{7/2}-3d_{5/2}$ (eV)	Pb $5g_{3/2}-4f_{5/2}$ (eV)
Point nucl.	145.6	56 038.9	431 652	429 343
Finite size	-288.9 (4.1)	-0.89 (3)	-55 (1)	-4 (1)
El. screening	-	-0.37 (30)	-17 (1)	-82 (3)
Lamb shift	-11.1	-0.15 ^{a)}	-1.5	-0.9
Recoil	0.3	0.18	3.6	2.2
Nucl. pol.	3.1	0.07 (7)	7.9 (8)	4.5 (5)
Vac. pol.:				
$\alpha(\alpha Z)$	1666.1	177.5	2327.5	2105.0
$\alpha^2(\alpha Z)$	11.6	1.24	16.2	14.5
$\alpha(\alpha Z)^3, 5, \dots$	-	-0.15	-19	-42 (2)
Exp. accuracy	0.3	0.53	8	3.4

a) Contains the anomalous magnetic moment contribution.

Table 3

Crystal spectrometer results (in eV)

Transition	Exp. energy	Calc. energy	Vac. pol.
Mg $3d_{5/2}-2p_{3/2}$	56 216.3 $\pm$ 0.53	56 216.4 $\pm$ 0.30	177.5
$3d_{3/2}-2p_{1/2}$	56 392.7 $\pm$ 0.85	56 391.8 $\pm$ 0.30	179.3
Si $3d_{5/2}-2p_{3/2}$	76 617.6 $\pm$ 1.14	76 617.6 $\pm$ 0.43	273.2
$3d_{3/2}-2p_{1/2}$	76 941.4 $\pm$ 2.10	76 942.3 $\pm$ 0.43	276.8
P $3d_{5/2}-2p_{3/2}$	88 016.2 $\pm$ 2.37	88 015.9 $\pm$ 0.50	330.4
$3d_{3/2}-2p_{1/2}$	88 423.8 $\pm$ 8.2	88 443.2 $\pm$ 0.50	335.4

Table 4

Spectrometer characteristics

	CERN ⁶⁾	SIN ⁷⁾	SREL ¹¹⁾
Type	2 $\times$ Ge(Li) planar	Ge(Li) coax.	Ge planar
Volume (cm ³)	4.9/1.0	50	3.1
Resolution (keV)	1.49/1.31 at 412 keV	1.60 at 412 keV	0.87 at 316 keV

Table 5

Ge spectrometer results (in eV)

	SIN ⁷⁾	SREL ¹¹⁾	CERN ⁸⁾	Calculation	Vac. pol.
Tl $5g_{9/2} - 4f_{7/2}$	420 763 ± 8	420 757.3 ± 3.7		420 768 ± 6	2012 ± 2.2
$5g_{7/2} - 4f_{5/2}$	426 865 ± 8			426 868 ± 6	2091 ± 2.2
Pb $5g_{9/2} - 4f_{7/2}$	431 331 ± 8	431 327.6 ± 3.4	431 360 ± 11	431 337 ± 7	2079 ± 2.2
$5g_{7/2} - 4f_{5/2}$	437 749 ± 8	437 749.4 ± 13.7	437 748 ± 12	437 750 ± 7	2163 ± 2.2
Ba $4f_{7/2} - 3d_{5/2}$	433 897 ± 8	433 904.8 ± 9.6	433 926 ± 8	433 910 ± 3	2326 ± 1.4
$4f_{5/2} - 3d_{3/2}$	441 358 ± 8	441 361.7 ± 5.1	441 374 ± 9	441 361 ± 4	2434 ± 1.4
Ce $4f_{7/2} - 3d_{5/2}$	465 754 ± 8			465 748 ± 6	2547 ± 1.4
$4f_{5/2} - 3d_{3/2}$	474 330 ± 8			474 329 ± 6	2671 ± 1.4

Table 6

Summary of QED test
(% of vacuum polarization)

	Crystal	Ge diodes
Exp. accuracy	0.20	0.07
Calc. uncertainty	0.17	0.25
Test	0.26	0.26

Figure captions

- Fig. 1 : Basic QED corrections.
- Fig. 2 : Deviation of the measured energies from the calculated ones according to Ref. 6. Error bars are experimental errors, broken lines indicate uncertainties from calculations.
- Fig. 3 : X-ray spectrum as obtained in Ref. 8. Background lines are indicated at the bottom with their relative intensities.
- Fig. 4 : Deviations as defined for Fig. 2.

a)

b)

c)

Fig. 1

Fig. 2

Fig. 3

Fig. 4