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CURRENT DIVERGENCES, PION PHOTOPRODUCTION
AND THE RADIATIVE MUON CAPTURE

M. Gmitro *)

CERN -- Geneva

and

A.A. Ovchinnikova

Institute of Nuclear Physics, MSU, Dubna

ABSTRACT

A method of coupling equations based on the usual assumptions about current divergences is suggested, and the radiative muon capture (RMC) elementary amplitude is derived. As usual, non-radiative weak form factors and pion photoproduction amplitude are required as input information. The limit $m_\pi \rightarrow 0$ usual in the current algebra consideration is shown to be helpful in disentangling the structure of the amplitude. The recent description by Hwang and Primakoff of the RMC is found to be inconsistent with the contemporary models of the photoproduction process. Examples of the photon spectra from the $p(\mu^-, \gamma\nu)n$ and ${}^3\text{He}(\mu^-, \gamma\mu){}^3\text{H}$ reactions are calculated.

*) Permanent address: Institute of Nuclear Physics, ČSAV CS 25068 Řež, Czechoslovakia, also at Laboratory of Theoretical Physics, JINR, Dubna, USSR.

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1. INTRODUCTION

The radiative muon capture (RMC) reaction has been known for a long time as a promising source¹⁾ of information on the weak interaction form factors, e.g. the possible renormalization of the induced pseudoscalar form factor. Now the reaction has been observed for the first time²⁾ on a nucleus as light as ^{16}O and remeasured^{2,3)} on ^{40}Ca with results which differ substantially from the earlier ones. Together with a recent theoretical development⁴⁾, it has spurred a lively discussion⁵⁻⁹⁾ about the form of the elementary amplitude to be used in the impulse approximation calculations of these nuclear processes. In the present paper we also analyze this last problem.

The different theoretical approaches to the derivation of RMC elementary amplitude can be characterized as follows:

(i) in the "diagrammatic" treatment^{1,10)} one considers the radiation due to the electric charges and anomalous magnetic momenta of the initial and final particles. The corresponding perturbation theory (PT) diagrams are shown in Fig. 1. Here several additional terms have to be introduced through the minimal electromagnetic coupling in order to guarantee the gauge invariance of the amplitude. Besides the problem of the PCAC violation in this approach (see below, section 5), one also wants to estimate the accuracy of such lowest-order procedure.

(ii) Adler and Dothan¹¹⁾ have turned to the low-energy theorems (LET). Starting with the Born terms of the radiative amplitude, they used the conservation of the electromagnetic current (CEC), CVC, and PCAC conditions to reconstruct the leading terms (in the photon momentum k) of the amplitude. Recently Christillin and Servadio¹²⁾ extended this approach using a different technique. They have presented an expansion consistent up to, and including, terms linear in the transferred momentum q and photon momentum k .

(iii) A more ambitious program based on the current conservation conditions has been undertaken by Hwang and Primakoff⁴⁾. They have tried to reconstruct fully the RMC elementary amplitude using only the CEC, CVC, PCAC conditions, and a dynamical assumption ("linearity hypothesis" - LH), which fixes the functional dependence of the radiative form factors.

As for the numerical results, PT and LET yield RMC amplitudes which lead to very similar photon spectra of the $p(\mu^-, \gamma\nu)n$ reaction. At the same time it was observed⁷⁾ that a photon spectrum reduced by a factor of more than two results if the RMC amplitude of Ref.4) is used in the calculation. The main sources of the discrepancy have been traced back^{5,6)} to the omission of the q^2 -dependence of the form factors and an inappropriate use of the LH in the terms connected with the

pion source. As a result, the Born terms corresponding to the pion pole are left out of the RMC amplitude in Ref. 4). In their⁴⁾ construction it then follows that the pion pole terms are also absent in the pion photoproduction amplitude. It is probably this consequence of Hwang and Primakoff's use of the LH which several authors^{5,7-9)} have observed and mentioned as a LET violation. In Ref. 6) we have demonstrated that the RMC amplitude fully consistent with the PT results is obtained if one avoids the inappropriate use of the LH in the method suggested by Hwang and Primakoff.

In the present work we would like to perform still another program based on the knowledge of the current divergences. We wish to use them in such a way as to show up general restrictions on the RMC amplitude, which are dictated by the physical connection of the radiative muon capture with two simpler and presumably better-known processes, namely the non-radiative μ^- capture and the pion photoproduction reaction. It could be considered as a first step towards an application of the "hard pions"¹³⁾ ideas to RMC analyses. Despite the work already performed, a careful study of the RMC elementary amplitude is still needed since, e.g., the recent calculations^{14,15)} of RMC on ^{40}Ca provide different photon spectra for the same value of the pseudoscalar coupling. In particular those of Ref. 14) show an integral yield of the emitted gamma rays (using the Goldberger-Treiman relation) which is much too high as compared with the experimental data^{2,3)}.

The plan of this work is the following: in Section 2 we discuss the explicit form and some properties of the weak and electromagnetic currents and the off-mass-shell extension of the pion photoproduction amplitude - the building blocks that are needed for the construction of the RMC amplitude. In Section 3 the relations between the three processes are given in the form of the coupling equations for the radiative form factors. The solutions of the equations are found in Section 4. In Section 5 the final expression is given for the RMC elementary amplitude. Here we also compare it with the earlier results. The examples of the photon spectra of $p(\mu^-, \gamma \nu)n$ and $^3\text{He}(\mu^-, \gamma \nu)^3\text{H}$ reactions (in the "elementary particle treatment") are presented in Section 6. Technicalities of the calculation: the Lorentz-covariant expansion of the hadronic matrix elements and the form and solution of the mentioned coupling equations are discussed in Appendices A and B respectively.

2. ORDINARY MUON CAPTURE AND PION PHOTOPRODUCTION

Here we introduce those building blocks which will be used below for the construction of the RMC amplitude. In Fig. 2a is shown the muon capture process

$$\mu^- + p \rightarrow \nu_\mu + n, \quad (1)$$

which is the natural parent of RMC. [Note: $\Lambda(\text{RMC})/\Lambda_\mu \approx 10^{-4}$]. Together with the pion photoproduction (Fig. 2b)

$$\gamma + p \rightarrow \pi^+ + n, \quad (2)$$

and the (nucleon or lepton) electromagnetic vertices (Fig. 2c), they are constituents of the diagrams of Fig. 1 which illustrate some possible mechanisms for RMC. To establish the notation we write the vector and axial-vector weak and electromagnetic currents as

$$\langle n(p') | V_\lambda(0) | p(p) \rangle = \bar{u}(p') \left[F_V(q^2) \gamma_\lambda - F_M(q^2) \sigma_{\mu\lambda} \frac{q_\mu}{2M} \right] u(p), \quad (3)$$

$$\langle n(p') | A_\lambda(0) | p(p) \rangle = \bar{u}(p') \left[F_A(q^2) \gamma_\lambda \gamma_5 + i F_P(q^2) \frac{2M}{m_\pi^2} q_\lambda \gamma_5 \right] u(p), \quad (4)$$

$$\langle N(p') | j_\lambda^{\text{el}}(0) | N(p) \rangle = \bar{u}(p') \left[e_N \gamma_\lambda + \frac{\mu_N}{2M} \sigma_{\lambda\mu} k_\mu \right] u(p), \quad (5)$$

where p (p') are initial (final) nucleon 4-momenta, M and m_π are the nucleon and pion mass respectively. The Pauli metric $[A_\mu \equiv (\vec{A}, iA_0)]$ and the usual Dirac matrices γ and nucleon spinors u are used, $\sigma_{\lambda\mu} = (\gamma_\lambda \gamma_\mu - \gamma_\mu \gamma_\lambda)/(2i)$. Also, k is the photon momentum and $q = p' - p$, $Q = p' + p$. Other symbols have their usual meaning.

The basic assumptions about the current divergences

$$\text{CEC: } \partial_\lambda j_\lambda^{\text{el}}(x) = 0, \quad (6)$$

$$\text{CVC: } \partial_\lambda V_\lambda(x) = 0, \quad (7)$$

$$\text{PCAC: } \partial_\lambda A_\lambda(x) = \frac{a_\pi m_\pi^2}{-\partial_\mu \partial_\mu + m_\pi^2} J^\pi(x), \quad (8)$$

where $a_\pi = 0.94 m_\pi$, and the $SU_2 \otimes SU_2$ current algebra equal-time commutators like

$$\begin{aligned} [V_\lambda(y), j_0^{\text{el}}(x)] \Big|_{x_0=y_0} &= V_\lambda(y) \delta^3(\vec{x}-\vec{y}), \\ [V_0(y), j_\mu^{\text{el}}(x)] \Big|_{x_0=y_0} &= V_\mu(y) \delta^3(\vec{x}-\vec{y}) \end{aligned} \quad (9)$$

will be widely used in our derivation.

Connection with the pion photoproduction reaction is established through Eq.(8) which contains the pion source $J^\pi(x)$. The divergence amplitude

$$\begin{aligned} D_\mu(k, q, Q) &= \int d^4x d^4y e^{-ikx} e^{i(k+q)y} \langle n(p') | T(j_\mu^{\text{el}}(x) \partial_\lambda A_\lambda(y)) | p(p) \rangle \\ &= \frac{a_\pi m_\pi^2}{(q+k)^2 + m_\pi^2} \int d^4x d^4y e^{-ikx} e^{i(k+q)y} \langle n(p') | T(j_\mu^{\text{el}}(x) J^\pi(y)) | p(p) \rangle \end{aligned} \quad (10)$$

is actually just the off-mass-shell extension of the pion photoproduction amplitude. Here T is the symbol of the Wick time-ordered product

$$T(j_\mu(x) K_\lambda(y)) = j_\mu(x) K_\lambda(y) + \theta(y_0 - x_0) [K_\lambda(y), j_\mu(x)]. \quad (11)$$

Using Eqs. (8), (9) and (11), one finds the divergence of D_μ to be

$$\begin{aligned} k_\mu D_\mu &= i \langle n(p') | \partial_\lambda A_\lambda(0) | p(p) \rangle \\ &= i \frac{a_\pi m_\pi^2}{q^2 + m_\pi^2} \langle n(p') | J^\pi(0) | p(p) \rangle \end{aligned} \quad (12)$$

with

$$\langle n(p') | J^\pi(0) | p(p) \rangle = i \bar{u}(p') g_{\pi NN} \gamma_5 u(p).$$

To construct the RMC amplitude, we shall use not only the matrix elements (3) and (4) of the weak nucleonic currents but also the amplitude D_μ . Though it indeed can be constructed more explicitly using some models, (see also Section 5), for the moment we shall only display its analytic structure. Namely, we write

$$\begin{aligned} D_\mu(k, q, Q) = & \frac{g_\pi m_\pi^2}{(q+k)^2 + m_\pi^2} \bar{u}(p') \left[-i g_{\pi NN} \gamma_5 \frac{1}{\not{p} - \not{k} - iM} \right. \\ & \times \left(e_p \gamma_\mu + \frac{\mu_p}{2M} \sigma_{\mu\nu} k_\nu \right) - \left(e_n \gamma_\mu + \frac{\mu_n}{2M} \sigma_{\mu\nu} k_\nu \right) \frac{1}{\not{p}' + \not{k} - iM} i g_{\pi NN} \gamma_5 \\ & \left. + i \bar{g}_{\pi NN} \gamma_5 \frac{e_p - e_n}{q^2 + m_\pi^2} (2q+k)_\mu + C_\mu + \bar{C}_\mu \right] u(p), \end{aligned} \quad (13)$$

where we formally write the proton ($e_p = 1$) and neutron ($e_n = 0$) electric charges and the πNN vertex with the notation $g_{\pi NN} \equiv g_{\pi NN}((q+k)^2)$ and $\bar{g}_{\pi NN} \equiv g_{\pi NN}(q^2)$.

In Eq. (13) we have explicitly displayed the pole terms only, the non-singular part C_μ being assumed in the form

$$C_\mu = \sum_{i=1}^4 A_i R_\mu^i, \quad (14)$$

where the R_μ^i 's are the gauge-invariant combinations of the basic Lorentz covariants:

$$\begin{aligned} R_\mu^1 &= \sigma_{\mu\nu} k_\nu \gamma_5, \\ R_\mu^2 &= 2(k \cdot q Q_\mu - k \cdot Q q_\mu) \gamma_5, \\ R_\mu^3 &= (k \cdot q \gamma_\mu - k q_\mu) \gamma_5, \\ R_\mu^4 &= 2(k \cdot Q \gamma_\mu - k Q_\mu) \gamma_5 - 2M \sigma_{\mu\nu} k_\nu \gamma_5 \end{aligned}$$

This is the most general form of D_μ which satisfies the condition (12). Note that, e.g., the terms proportional to $(q+k)^2 + m_\pi^2$ are not written explicitly, since they should be gauge-invariant and are therefore included in the term C_μ .

The term $\bar{C}_\mu$ is also non-singular and one has to include it when the q^2 dependence of the $g_{\pi NN}$ vertex is taken into account. It is equal to

$$\bar{C}_\mu = -i \gamma_5 2g_\mu g'_{\pi NN}(0)$$

with g' for the derivative of $g_{\pi NN}(0)$. Including $\bar{C}_\mu$ in (13), one ensures that D_μ is gauge-invariant on the mass-shell or, correspondingly, that Eq. (12) holds off-mass-shell. We shall see in Sections 5 and 6 that the q^2 -dependence of the (non-radiative) weak form factors might be important for the form of the RMC amplitude. Then indeed the q^2 -dependence of $g_{\pi NN}$ (and even more so that of $g_{\pi^3\text{He}^3\text{H}}$) cannot be ignored.

3. RADIATIVE MUON CAPTURE

For the RMC reaction on a free proton

$$\mu(p_\mu) + p(p) \rightarrow \nu(p_\nu) + n(p') + \gamma(k, \epsilon) \quad (16)$$

we divide the amplitude into a lepton radiating part $T^{(\ell)}$, which corresponds to Fig. 2a, and the hadronic part $T^{(h)}$: $T = T^{(\ell)} + T^{(h)}$. The usual relativistic reduction technique for the outgoing photon can be used to obtain the amplitude $T^{(h)}$ in the form

$$T^{(h)} = -\frac{Ge}{\sqrt{2}} \frac{\epsilon_\mu^*}{\sqrt{2k_0}} J_{\mu\lambda} \cdot L_\lambda, \quad (17)$$

$$L_\lambda = \bar{u}(p_\nu) \gamma_\lambda (1 + \gamma_5) u(p_\mu), \quad (18)$$

$$\begin{aligned} J_{\mu\lambda} &= V_{\mu\lambda} + A_{\mu\lambda} \\ &= -i \int d^4x d^4y \bar{e}^{ikx} e^{i(k+y)y} \langle n(p') | T(j_\mu^{\text{el}}(x) K_\lambda(y)) | p(p) \rangle, \\ K_\lambda(y) &\equiv V_\lambda(y) + A_\lambda(y), \end{aligned} \quad (19)$$

where G is the weak coupling constant and ϵ_μ^* the photon polarization vector.

Consider now the contractions of the tensor $J_{\mu\lambda}$ with the momenta k_μ and $(k+q)_\lambda$. Using the properties (6) to (9) and the assumption usual in the current algebra about cancellation of the small seagull and Schwinger terms, one gets^{4,11)}

$$\begin{aligned} \text{CEC: } k_\mu J_{\mu\lambda}(k, q, Q) &= \int d^4x d^4y \bar{e}^{-ikx} e^{i(k+q)y} \langle n(p') | \delta(x-y) [K_\lambda(y), j_0^d(x)] | p(p) \rangle \\ &= \langle n(p') | K_\lambda(0) | p(p) \rangle, \end{aligned} \quad (20)$$

$$\text{CVC: } (k+q)_\lambda V_{\mu\lambda}(k, q, Q) = \langle n(p') | V_\mu(0) | p(p) \rangle, \quad (21)$$

$$\text{PCAC: } (k+q)_\lambda A_{\mu\lambda}(k, q, Q) = \langle n(p') | A_\mu(0) | p(p) \rangle + D_\mu(k, q, Q). \quad (22)$$

To derive $T^{(h)}$ we should construct the Lorentz-covariant forms

$$V_{\mu\lambda} = \sum_{ijx} \bar{u}(p') F_{ij}^x(k, q, Q) X_{ij, \mu\lambda}^x(k, q, Q) u(p) \quad (23)$$

and

$$A_{\mu\lambda} = \sum_{ijx} \bar{u}(p') \gamma_5 G_{ij}^x(k, q, Q) Y_{ij, \mu\lambda}^x(k, q, Q) u(p), \quad (24)$$

where the form factors F_{ij}^x and G_{ij}^x are to be determined and X and Y are kinematical covariants. It was shown earlier¹⁵⁾ that for the $\frac{1}{2} \rightarrow \frac{1}{2}$ transition one has 64 independent covariants, $X_{ij, \mu\lambda}^x$ and $Y_{ij, \mu\lambda}^x$. The general expansions (23) and (24) are given in Appendix A.

Now, substituting the explicit forms of V_μ , A_μ , and D_μ given above and Eqs. (A1) and (A2) into (20) - (22), we obtain a set of functional equations connecting the RMC form factors F_{ij}^x and G_{ij}^x with the form factors of ordinary muon capture and those of pion photoproduction. These equations are displayed in Appendix B.

4. RADIATIVE FORM FACTORS

To solve the coupling equations obtained we should assume a definite dynamical structure of the RMC form factors F and G . We expect that the main contribution to the RMC amplitude comes from the nucleon poles. For F and G we assume therefore the form

$$\frac{r^+(k, q, Q)}{p \cdot k} + \frac{r^-(k, q, Q)}{p' \cdot k} + r(k, q, Q). \quad (25)$$

It has already been shown earlier ⁶⁾ that r^+ , r^- , and r can be obtained from the coupling equations if several (4 for the $\frac{1}{2} \rightarrow \frac{1}{2}$ transition) form factors are calculated through the perturbation theory. Here we adopt a simpler procedure: the pole contributions r^+ and r^- can be written directly via the diagrams of Figs. 1b, 1c, and we exclude them from the coupling equations and shall solve these equations for the r terms only.

As usual, we shall consider $F(k, q, Q)$ and $G(k, q, Q)$ as functions of q^2 only. They can be expressed in terms of the electromagnetic and non-radiative weak form factors and their derivatives (with respect to q^2). All these form factors are either constants or smooth functions of q^2 with the important exception of the induced pseudoscalar form factor F_p which behaves as a pole. Correspondingly, we assume the r terms to be constant everywhere except in the case of the axial part, where we expect the following structure dictated by the pseudoscalar pole and the pion propagation pole:

$$r(k, q, Q) = r_0 + \frac{r_1}{q^2 + m_\pi^2} + \frac{r_2}{(q+k)^2 + m_\pi^2} + \frac{r_3}{(q^2 + m_\pi^2)((q+k)^2 + m_\pi^2)}, \quad (26)$$

r_i is assumed to be constant.

Indeed the form (25) is just the simplest reasonable assumption to adopt: it keeps us essentially within the framework of the usual perturbation theory treatment of the RMC amplitude. Already in this case, however, the coupling equations (20) - (22) provide interesting new results, as will be shown below. Much more complicated mechanisms of the RMC reaction can be easily incorporated here. The physical content of the theory is actually controlled by the form of F and G assumed - e.g., one can easily take into account the resonances of the nucleon (nuclear) states involved in RMC by assuming the form

$$\sum_i \frac{r_i^+}{(p-k)^2 + m_i^2} + \sum_j \frac{r_j^-}{(p'-k)^2 + m_j^2} + r \quad (27)$$

instead of (25). Here m_i are the masses of the (ground or excited) states to be included [$m_1 = M$ only has been kept in (25)].

Taking one or another form of the radiative form factors F and G one proceeds to solve the coupling equations and to determine the elements of the RMC amplitude through those of V_μ , A_μ , and D_μ . The details of the solution are given in Appendix B.

5. RMC AMPLITUDE FOR THE $\frac{1}{2} \rightarrow \frac{1}{2}$ TRANSITION

Substituting the results of Appendices A and B into Eqs. (23) and (24), we obtain the RMC amplitude $J_{\mu\lambda}$ in the form

$$J_{\mu\lambda} = -\bar{u}(p') \left[M_{\mu\lambda}^{(pole)} + M_{\mu\lambda}^{(1)} + M_{\mu\lambda}^{(2)} + M_{\mu\lambda}^{(3)} + M_{\mu\lambda}^{(4)} \right] u(p), \quad (28)$$

where in $M_{\mu\lambda}^{(pole)}$ and $M_{\mu\lambda}^{(1)}$ the usual¹⁰⁾ terms are collected:

$$\begin{aligned} M_{\mu\lambda}^{(pole)} = & \left[F_V \gamma_\lambda - \frac{F_M}{2M} \sigma_{\lambda\nu} (q+k)_\nu \right] \frac{1}{\not{p}-\not{k}-iM} (\not{e}_p \gamma_\mu + \frac{\mu_p}{2M} \sigma_{\mu\nu} k_\nu) \\ & + (\not{e}_n \gamma_\mu + \frac{\mu_n}{2M} \sigma_{\mu\nu} k_\nu) \frac{1}{\not{p}'+\not{k}-iM} \left[F_V \gamma_\lambda - \frac{F_M}{2M} \sigma_{\lambda\nu} (q+k)_\nu \right] \\ & + \left[F_A \gamma_\lambda \gamma_5 + iF_P \frac{2M}{m_\pi^2} (q+k)_\lambda \gamma_5 \right] \frac{1}{\not{p}-\not{k}-iM} (\not{e}_p \gamma_\mu + \frac{\mu_p}{2M} \sigma_{\mu\nu} k_\nu) \\ & + (\not{e}_n \gamma_\mu + \frac{\mu_n}{2M} \sigma_{\mu\nu} k_\nu) \frac{1}{\not{p}'+\not{k}-iM} \left[F_A \gamma_\lambda \gamma_5 + iF_P \frac{2M}{m_\pi^2} (q+k)_\lambda \gamma_5 \right], \quad (29) \end{aligned}$$

$$M_{\mu\lambda}^{(1)} = \frac{\bar{F}_M}{2M} \sigma_{\mu\lambda} + i\bar{F}_P \frac{2M}{m_\pi^2} \delta_{\mu\lambda} \gamma_5 - iF_P \frac{2M}{m_\pi^2} \gamma_5 \frac{2q_\mu}{q^2 + m_\pi^2} (q+k)_\lambda, \quad (30)$$

and the terms $M^{(i)}$, $i = 2, 3, 4$ contain certain new information:

$$M_{\mu\lambda}^{(2)} = -\frac{1}{2M} \gamma_5 (\mu_p + \mu_n) \sigma_{\mu\nu} k_\nu (q+k)_\lambda \frac{\bar{F}_A}{(q+k)^2 + m_\pi^2}, \quad (31)$$

$$M_{\mu\lambda}^{(3)} = 2(F'_V - F'_A \gamma_5) (\gamma_\mu k_\lambda + \gamma_\lambda q_\mu - \not{k} \delta_{\mu\lambda}), \quad (32)$$

$$M_{\mu\lambda}^{(4)} = \frac{1}{2M} F'_V (\mu_p - \mu_n) \sigma_{\mu\nu} k_\nu (q+k)_\lambda - \frac{1}{2M} F'_M [\sigma_{\mu\nu} k_\nu (q-k)_\lambda + 2\sigma_{\lambda\nu} (q+k)_\nu q_\mu] - i4M F'_A q_\mu \frac{(q+k)_\lambda}{q^2 + m_\pi^2} - \frac{1}{2M} F'_A (\mu_p + \mu_n) \sigma_{\mu\nu} k_\nu k_\lambda \quad (33)$$

with

$$F_i \equiv F_i((q+k)^2), \quad \bar{F}_i \equiv F_i(q^2), \quad F'_i \equiv \left. \frac{dF_i(q^2)}{dq^2} \right|_{q^2=0}, \quad i = V, M, A, P.$$

Several comments are necessary in order to establish the relation of the amplitude (28) to the earlier results:

(i) The amplitude $J_{\mu\lambda}^{PT} = -\bar{u}(p') [M_{\mu\lambda}^{(pole)} + M_{\mu\lambda}^{(1)}] u(p)$ includes the terms which are usually¹⁰⁾ obtained in the PT with the minimal electromagnetic coupling adopted in order to guarantee the gauge invariance of the amplitude under the assumption that the non-radiative form factors are all constant except the pseudo-scalar one.

(ii) Consider for $J_{\mu\lambda}^{PT}$ the limit of soft pions ($m_\pi \rightarrow 0$) well known in the current algebra. Then the divergence of the axial-vector current is zero, and the Goldberger-Treiman relation becomes

$$F_A(q^2) + \frac{q^2}{m_\pi^2} F_P(q^2) = 0$$

and the condition (22) simplifies to

$$(k+q)_\lambda A_{\mu\lambda}(k, q, Q) = \langle n(p') | A_\mu(0) | p(p) \rangle. \quad (34)$$

It is easy to check that for $J_{\mu\lambda}^{PT}$ Eq. (34) does not hold. To rectify the situation one should consider the amplitude

$$J_{\mu\lambda}^{PT} = \bar{u}(p') M_{\mu\lambda}^{(2)} u(p).$$

Such an additional contribution can also be obtained by LET methods. It should be numerically small in the RMC on the proton and ^3He . We wish to comment here that looking at the $m_\pi \rightarrow 0$ limit has proved to be helpful also in other instances to sort out the structure of the amplitude.

(iii) In a sensitive reaction like RMC the above assumption about the constancy of the non-radiative form factors might prove to be a crude one, especially if RMC on nuclei⁸⁾ treated as elementary particles is considered. In such a case the derivatives of F_V , F_M , and F_A have to be included in $J_{\mu\lambda}$. Some of these terms ($M_{\mu\lambda}^{(3)}$) have already been obtained in Refs. 11), 12) using the LET approach.

(iv) Further details of the RMC amplitude may be easily reconstructed using the present method of coupling equations. They arise from the dynamical assumptions adopted for the physical model of the reaction. E.g., working with the assumptions (25) and (26) we were able to take into account in $M_{\mu\lambda}^{(4)}$ the terms proportional to the derivatives F_M^i of the weak-magnetism form factor and the second order (in k and q) terms proportional to F_V^i . For a rough estimate of the correction terms like $M^{(3)}$ and $M^{(4)}$, one has to recall that they are controlled by a parameter like $(m_\mu/m_\rho)^2 \approx 1/50$.

We have already mentioned in the Introduction that looking for the possible sources of the disagreement between recent experimental data on RMC and the "canonical" calculations which use the Goldberger-Treiman values of F_p , one should probably also reconsider the elementary RMC amplitude. This can easily be done using the method suggested here. One should include the nucleon resonances as in Eq. (27) and implement the vector-meson-dominance hypothesis. Another straightforward way of reconstructing more details of the amplitude $J_{\mu\lambda}$ is connected with the vector D_μ . In expression (14) we have left unknown the coefficients A_1, A_2, A_3 , and A_4 . In Appendix B, we obtain as a bonus a "minimal" solution $A_1 \neq 0, A_2 = A_3 = A_4 = 0$ which is consistent with the idea of coupling RMC with the pion photoproduction reaction. Indeed, one can fix the coefficients A_i from a particular model of the photoproduction reaction. New terms in $A_{\mu\lambda}$ will then be generated through Eqs. (B38) - (B43).

6. NUMERICAL EXAMPLES

In our calculation the numerical results were obtained in a fully relativistic procedure. All algebraic manipulations involving the Dirac spinors and gamma matrices were performed by the analytic manipulation computer program SCHOONSCHIP¹⁷⁾. The result is a formula for

$$\sum_{spins} |T^{(l)} + T^{(h)}|^2$$

expressed as a function of the photon energy k , the angle θ of the photon and neutrino momenta ($\cos \theta = \hat{k} \cdot \hat{\nu}$), and the pseudoscalar form factor

$$F_P = - \frac{m_\pi^2}{q^2 + m_\pi^2} \bar{f}_P.$$

The photon spectrum for any value of $\bar{f}_P$ is then obtained by the numerical integration over the angle θ . (For the explicit formula see, e.g., Eq.(2) of Ref.7)) Our calculated photon spectra are shown in Figs. 3 and 4 for the RMC on proton and in Figs. 5 - 7 for the RMC on ^3He .

6.1 Radiative μ^- capture on the proton

Comparing the individual curves in Fig. 3, one observes that a strong depression of the spectrum results if the q^2 -dependence is dropped as in Ref. 4). Simultaneously, at least two other unwanted effects appear: (i) the pion pole terms disappear not only in the RMC amplitude but in the pion photoproduction as well; the latter is then gauge non-invariant on the mass-shell; and (ii) the nucleon-pole terms of the RMC amplitude differ from the usual Born terms^{11,12)}. We conclude that the q^2 -dependence of F_P cannot be approximated by any constant. On the other hand, taking into account the q^2 -dependence of the form factors F_V , F_M , and F_A changes slightly the spectrum: e.g. the curves a and b of Fig. 3 differ by about 6%: this corresponds to the change in argument of form factors from $q^2 = 0$ to $q^2 = 0.88 m_\mu^2$.

It is well known that the high-energy part of the RMC photon spectrum is sensitive to the value chosen for the induced pseudoscalar. The RMC on proton, though very difficult experimentally, may serve as a check of the Goldberger-Treiman relation. To this end we display in Fig. 4 the gamma spectra for several choices of $\bar{f}_P$. One sees that, e.g., the difference between the curves b (corresponding to the canonical value of F_P) and d ($F_P = 0$) in the high-energy region is about 30 ÷ 40 per cent. It should be noted that the same calculation, if performed with the amplitude of Ref. 4), shows a reversed dependence on the value of the pseudoscalar: the greater is F_P , the lower goes the spectrum.

We have also checked that the numerical effect of the term $M^{(2)}$ of the amplitude (28) is negligible in RMC on proton. We keep this term, however, since it is required to satisfy the PCAC hypothesis.

6.2 Radiative μ^- capture on ${}^3\text{He}$

The analytic results obtained in Section 5 can be immediately applied for any spin $\frac{1}{2} \rightarrow \frac{1}{2}$ RMC transition if the initial and final nuclei are treated as elementary particles (EPT). Though such an approach seems to be very attractive, the actual application to even a light nucleus such as ${}^3\text{He}$ presents several difficulties. First of all, one has to supply reliable values for the nuclear non-radiative weak form factors. In principle we can, using CVC, connect the vector form factors with the better-known electromagnetic ones and, from the experiment on ordinary μ^- capture and β decay, determine the axial form factors. Unfortunately, such data are still only partly available. One is forced then to supply the missing information via impulse-approximation calculations, using some nuclear model¹⁸⁾. Following other authors^{7,8,19,20)} we have used the results of Ref. 18): $F_V(0) = 1$, $F_M(0) = -5.44$, and $F_A(0) = -1.22$. For F_P we have

$$F_P(q^2) = -F_A(0)(1 + q^2/m_\pi^2).$$

In nuclei the q^2 -dependence of all non-radiative form factors can play a numerically non-negligible rôle⁸⁾. In Fig. 5, we compare our result (curve a) with suggestions^{7,19)} to circumvent this problem by taking all form factors at the averaged value $q_L^2 = (p_{3H} - p_{3He})^2$. There we also plot the result obtained with Hwang-Primakoff's amplitude.

As soon as we start to consider the non-radiative form factors to be q^2 dependent, the derivative terms must be included in the RMC amplitude to guarantee the gauge invariance. Here we follow the work by Klieb and Rood⁸⁾ and assume the linear dependence: $F_i(q^2) = F(0) (1 - \frac{1}{6} r_i^2 q^2)$, $i = V, A, M$ with $r_V = 1.94$ fm, and $r_M = r_A = 1.72$ fm. The results are given in Fig. 6, and show that though the inclusion of the term $M^{(2)}$ in amplitude (28) is numerically not important also in ${}^3\text{He}$, the $M^{(3)}$ and $M^{(4)}$ terms which contain the derivatives of form factors contribute almost 10% to the photon yield.

In Fig. 7 we present the results of our calculations for the photon spectrum for different values of the induced pseudoscalar. Derivative terms are always included. Contrary to the case of the RMC on proton, an additional uncertainty arises due to our poor knowledge of the $g_{\pi^3\text{He}^3\text{H}}$ vertex. In fact, one has here no explicit form for the Goldberger-Treiman relation. Recalling the results of Ref. 8) where some possibilities²¹⁾ to fix the value of $g_{\pi^3\text{He}^3\text{H}}$ were tried, one would like to speculate that an RMC experiment can provide information about the π -nuclear vertex as well.

Unfortunately, at the present stage, the uncertainty just mentioned hinders the comparison of the EPT results with those calculated through the impulse approximation. Such an indirect estimation of the mesonic exchange effects would certainly be very interesting. The main aim of the present section was to show in several examples the possible effects to be expected from the RMC amplitude derived through the coupling equation method. We plan to consider the RMC process on ^3He in more detail in a subsequent publication.

7. CONCLUSIONS

We have seen that the method of coupling equations allows us to build up the RMC amplitude systematically in accordance with our physical models for this process. As input assumptions about the dominant reaction mechanisms are required, the output then being the $J_{\mu\lambda}$ amplitude consistent with the gauge invariance and current conservation (partial conservation) conditions.

Though really very difficult, the experimental (and theoretical) studies of RMC on ^3He seem to promise a possibility of gathering more insight into the problems connected with the induced pseudoscalar, the strong π -nucleus vertex and mesonic exchange currents.

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Note added: After this work was completed, the authors saw a paper by Hwang²³⁾. Though he admits that "... the Hwang-Primakoff approach does not comply with Low's prescription", he still suggests that LH may prove useful if higher order diagrams have to be taken into account. Leaving aside the contrast of an ad-hoc-adopted hypothesis with the fruitful methods of current algebra, we wish to point out the following. Technically one does not see why the supposed account of higher-order processes does not change the functional form^{4,6)} of the tensor $V_{\mu\lambda}$. Similarly, as we have shown⁶⁾, the tensor $A_{\mu\lambda}$ also keeps its form suggested by PT if LH is applied just to the form factors G_{ij}^x which define $A_{\mu\lambda}$. One is led to the conclusion that the omission of the pion pole in D_μ is both unwanted (it diminishes the understanding of the pion photoproduction reaction) and unnecessary for the derivation of RMC amplitude (as we have shown in Section 5).

APPENDIX A

In this appendix we list the Lorentz-covariant expansions of tensors $V_{\mu\lambda}$ and $A_{\mu\lambda}$, eqs.(23) and (24). Note that the corresponding expansions given in Ref. 4) contain dependent covariants²²⁾.

$$\begin{aligned}
 V_{\mu\lambda}(k, q, Q) = & \bar{u}(p') \left\{ \sigma_{\mu\lambda} F_{00}^a + i \sigma_{\mu\lambda} \frac{\kappa}{M} F_{00}^b + \delta_{\mu\lambda} F_{00}^c + \frac{Q_\mu}{M^2} (k_\lambda F_{21}^c + Q_\lambda F_{22}^c) \right. \\
 & + \frac{\gamma_\mu}{M} (k_\lambda F_{11}^a + Q_\lambda F_{12}^a + q_\lambda F_{13}^a) + \frac{\gamma_\lambda}{M} (k_\mu F_{11}^b + Q_\mu F_{12}^b + q_\mu F_{13}^b) \\
 & + \frac{\sigma_{\mu\nu} k_\nu}{M^2} (k_\lambda F_{21}^a + Q_\lambda F_{22}^a + q_\lambda F_{23}^a) + \frac{\sigma_{\lambda\nu} k_\nu}{M^2} (k_\mu F_{21}^b + Q_\mu F_{22}^b + q_\mu F_{23}^b) \\
 & + \frac{k_\mu}{M^2} (k_\lambda F_{11}^c + Q_\lambda F_{12}^c + q_\lambda F_{13}^c) + \frac{q_\mu}{M^2} (k_\lambda F_{31}^c + Q_\lambda F_{32}^c + q_\lambda F_{33}^c) \\
 & + i \frac{\kappa}{M} \left[\frac{k_\mu}{M^2} (k_\lambda F_{11}^d + Q_\lambda F_{12}^d + q_\lambda F_{13}^d) + \frac{Q_\mu}{M^2} (k_\lambda F_{21}^d + Q_\lambda F_{22}^d) \right. \\
 & \left. \left. + \delta_{\mu\lambda} F_{00}^d + \frac{q_\mu}{M^2} (k_\lambda F_{31}^d + Q_\lambda F_{32}^d + q_\lambda F_{33}^d) \right] \right\} u(p), \quad (A1)
 \end{aligned}$$

$$\begin{aligned}
 A_{\mu\lambda}(k, q, Q) = & \bar{u}(p') \gamma_5 \left\{ \sigma_{\mu\lambda} G_{00}^a + i \sigma_{\mu\lambda} \frac{\kappa}{M} G_{00}^b + \delta_{\mu\lambda} G_{00}^c + \frac{Q_\mu}{M^2} (k_\lambda G_{21}^c + q_\lambda G_{23}^c) \right. \\
 & + \frac{\gamma_\mu}{M} (k_\lambda G_{11}^a + Q_\lambda G_{12}^a + q_\lambda G_{13}^a) + \frac{\gamma_\lambda}{M} (k_\mu G_{11}^b + Q_\mu G_{12}^b + q_\mu G_{13}^b) \\
 & + \frac{\sigma_{\mu\nu} k_\nu}{M^2} (k_\lambda G_{21}^a + Q_\lambda G_{22}^a + q_\lambda G_{23}^a) + \frac{\sigma_{\lambda\nu} k_\nu}{M^2} (k_\mu G_{21}^b + Q_\mu G_{22}^b + q_\mu G_{23}^b) \\
 & + \frac{k_\mu}{M^2} (k_\lambda G_{11}^c + Q_\lambda G_{12}^c + q_\lambda G_{13}^c) + \frac{q_\mu}{M^2} (k_\lambda G_{31}^c + Q_\lambda G_{32}^c + q_\lambda G_{33}^c) \\
 & + i \frac{\kappa}{M} \left[\frac{k_\mu}{M^2} (k_\lambda G_{11}^d + Q_\lambda G_{12}^d + q_\lambda G_{13}^d) + \frac{Q_\mu}{M^2} (k_\lambda G_{21}^d + q_\lambda G_{23}^d) \right. \\
 & \left. \left. + \delta_{\mu\lambda} G_{00}^d + \frac{q_\mu}{M^2} (k_\lambda G_{31}^d + Q_\lambda G_{32}^d + q_\lambda G_{33}^d) \right] \right\} u(p). \quad (A2)
 \end{aligned}$$

APPENDIX B

In this Appendix we wish to express the radiative form factors F and G of Eqs. (A1) and (A2) through the form factors F_V, F_M, F_A , and F_P and the elements of vector D_μ .

B1. Form factors F_{ij}^x for the $\frac{1}{2} \rightarrow \frac{1}{2}$ transitions

The coupling equations for F_{ij}^x are almost identical with Eqs. (15) and (17) of Ref. 4), therefore we do not display them. Just note¹⁶⁾ that one has to omit, e.g. the terms $Q_\mu q_\lambda F_{23}^c$ and $iQ_\mu q_\lambda F_{23}^d$ in order to avoid the use of dependent covariants erroneously included in the work of Ref. 4).

We now write assumption (25) as

$$F_{ij}^x(k, q, Q) = \frac{1}{p \cdot k} f_{xij}^+(k, q, Q) + \frac{1}{p' \cdot k} f_{xij}^-(k, q, Q) + f_{ij}^x(k, q, Q).$$

The pole terms f^+ and f^- can be constructed directly. Substituting them we are left with the equations

$$\int_{22}^b Q \cdot k + \int_{23}^b q \cdot k - M^2 \int_{00}^a = \frac{M}{2} F_M, \quad (B1)$$

$$\int_{12}^b Q \cdot k + \int_{13}^b q \cdot k = -M(F_V + F_M) + M(\bar{F}_V + \bar{F}_M), \quad (B2)$$

$$\int_{22}^c Q \cdot k + \int_{32}^c q \cdot k = -\frac{i}{2} M(F_M - \bar{F}_M), \quad (B3)$$

$$\int_{21}^d Q \cdot k + \int_{31}^d q \cdot k - M^2 [i(\int_{00}^b + \int_{11}^a) - \int_{00}^d] = 0, \quad (B4)$$

$$\int_{22}^d Q \cdot k + \int_{32}^d q \cdot k - iM^2 \int_{12}^a = 0 \quad (B5)$$

$$\int_{33}^d q \cdot k - i M^2 \int_{13}^a = 0, \quad (B6)$$

$$\int_{21}^c Q \cdot k + \int_{31}^c q \cdot k + M^2 \int_{00}^c = 0, \quad (B7)$$

$$\int_{33}^c q \cdot k = 0, \quad (B8)$$

which are connected with the CEC condition, Eq. (20), and

$$\int_{22}^a Q \cdot k + \int_{21}^a q \cdot k + \int_{23}^a q(q+k) + M^2 \int_{00}^a = -\frac{1}{2} M F_V (\mu_p - \mu_n), \quad (B9)$$

$$(\int_{12}^a - \int_{00}^b) Q \cdot k + (\int_{11}^a + \int_{00}^b) q \cdot k - 2 M^2 \int_{00}^a = M (-F_V + \bar{F}_V + \bar{F}_M), \quad (B10)$$

$$(i \int_{22}^b + \int_{22}^c) Q \cdot k + \int_{21}^c q \cdot k + \int_{23}^c q(q+k) - i M^2 \int_{00}^a = \frac{i}{2} M \bar{F}_M, \quad (B11)$$

$$(\int_{32}^c + i \int_{23}^b) Q \cdot k + \int_{31}^c q \cdot k + \int_{33}^c q(q+k) + M^2 \int_{00}^c = 0, \quad (B12)$$

$$\int_{22}^d Q \cdot k + \int_{21}^d q \cdot k + \int_{23}^d q(q+k) - i M^2 (2 \int_{22}^b + \int_{12}^b + \int_{00}^b) = 0, \quad (B13)$$

$$f_{32}^d q \cdot k + f_{31}^d q \cdot k + f_{33}^d q(q+k) + M^2(f_{00}^d - 2i f_{23}^b - i f_{13}^b) = 0 \quad (B14)$$

which follow from the CVC hypothesis, Eq. (21). It is interesting to note that Eqs. (B4) - (B8) and (B12) - (B14) have kept the same form which they had as equations for F_{ij}^x before the extraction of the pole terms. Also these equations have very simple solutions. Namely, using them, we find

$$f_{21}^d = f_{31}^d = f_{22}^d = f_{32}^d = f_{12}^a = f_{33}^d = f_{13}^a = f_{21}^c = f_{31}^c = f_{00}^c = f_{33}^c = 0,$$

and

$$i(f_{00}^b + f_{11}^a) - f_{00}^d = 0, \quad (B15)$$

$$f_{32}^c + i f_{23}^b = 0, \quad (B16)$$

$$2f_{22}^b + f_{12}^b + f_{00}^b = 0, \quad (B17)$$

$$f_{00}^d - 2i f_{23}^b - i f_{13}^b = 0. \quad (B18)$$

We turn now to the more complicated Eqs. (B1) - (B3) and (B9) - (B11). Substituting into them the expansion

$$F_i((q+k)^2) = F_i(q^2) + 2 q \cdot k F_i'(0), \quad i = V, M$$

one can easily get

$$\begin{aligned} f_{00}^a &= -\frac{1}{2M} \overline{F}_M, & f_{23}^b &= M F_M'(0), & f_{13}^b &= -2M(F_V'(0) + F_M'(0)), \\ f_{32}^c &= -iM F_M'(0), & f_{23}^a &= -\frac{1}{2} M[F_V'(0)(\mu_p - \mu_n) - 2M F_M'(0)], \\ f_{11}^a &= -2M F_V'(0), & f_{21}^a &= -\frac{1}{2} M[F_V'(0)(\mu_p - \mu_n) + 2M F_M'(0)], \end{aligned} \quad (B19)$$

$$f_{22}^b = f_{12}^b = f_{22}^c = f_{00}^b = 0,$$

and from (B15) - (B18) we obtain the last form factor needed:

$$f_{00}^d = -i 2M F'_V(0). \quad (B20)$$

B2. Form factors G_{ij}^x for the $\frac{1}{2} \rightarrow \frac{1}{2}$ transitions

The CEC condition, Eq. (20), leads to the equations

$$G_{12}^b Q.k + G_{13}^b q.k = -M \bar{F}_A, \quad (B21)$$

$$G_{21}^c Q.k + G_{31}^c q.k + M^2 G_{00}^c = 0, \quad (B22)$$

$$G_{23}^c Q.k + G_{33}^c q.k = i 2 \frac{M^3}{m_\pi^2} \bar{F}_P, \quad (B23)$$

$$G_{22}^b Q.k + G_{23}^b q.k - M^2 G_{00}^a = 0, \quad (B24)$$

$$G_{32}^c q.k = 0, \quad (B25)$$

$$G_{21}^d Q.k + G_{31}^d q.k - M^2 [i(G_{00}^b + G_{11}^a) - G_{00}^d] = 0, \quad (B26)$$

$$G_{32}^d q.k - iM^2 G_{12}^a = 0, \quad (B27)$$

$$G_{23}^d Q.k + G_{33}^d q.k - iM^2 G_{13}^a = 0. \quad (B28)$$

Similarly, the PCAC condition, Eq. (22) gives us

$$\begin{aligned} & G_{22}^a Q.k + G_{21}^a q.k + G_{23}^a q(q+k) + M^2 (G_{00}^a - 2 G_{00}^b) \\ & = -M^2 \frac{a_\pi m_\pi^2}{(q+k)^2 + m_\pi^2} \left[q_{\pi NN} ((q+k)^2) \left(\frac{e_p + \mu_p}{2 p.k} - \frac{e_n + \mu_n}{2 p'.k} \right) - A_1 + 2 M A_4 \right], \end{aligned} \quad (B29)$$

$$\begin{aligned}
 & (G_{12}^a - G_{00}^b) Q \cdot k + (G_{11}^a + G_{00}^b) q \cdot k + G_{13}^a q(q+k) \\
 & = -M \bar{F}_A - M \frac{a_\pi m_\pi^2}{(q+k)^2 + m_\pi^2} \left[g_{\pi NN} ((q+k)^2) \frac{\mu_p - \mu_n}{2M} - q \cdot k A_3 - 2Q \cdot k A_4 \right], \quad (B30)
 \end{aligned}$$

$$\begin{aligned}
 & i G_{22}^b Q \cdot k + G_{21}^c q \cdot k + G_{23}^c q(q+k) - i M^2 (2G_{12}^b + G_{00}^a) \\
 & = -M^2 \frac{a_\pi m_\pi^2}{(q+k)^2 + m_\pi^2} \left[i g_{\pi NN} ((q+k)^2) \left(\frac{e_p}{2p \cdot k} - \frac{e_n}{2p' \cdot k} \right) + 2q \cdot k A_2 \right], \quad (B31)
 \end{aligned}$$

$$\begin{aligned}
 & (G_{32}^c + i G_{23}^b) Q \cdot k + G_{31}^c q \cdot k + G_{33}^c q(q+k) + M^2 (G_{00}^c - i 2G_{13}^b) \\
 & = -M^2 \frac{a_\pi m_\pi^2}{(q+k)^2 + m_\pi^2} \left[i g_{\pi NN} ((q+k)^2) \left(\frac{e_p}{2p \cdot k} + \frac{e_n}{2p' \cdot k} \right) \right. \\
 & \quad \left. + \frac{2i}{q^2 + m_\pi^2} g_{\pi NN}(q^2) - 2i g'(0) + 2Q \cdot k A_2 \right] + 2i \frac{M^3}{m_\pi^2} \bar{F}_p, \quad (B32)
 \end{aligned}$$

$$\begin{aligned}
 & G_{21}^d q \cdot k + G_{23}^d q(q+k) - i M^2 (G_{12}^b + G_{00}^b) \\
 & = -\frac{i}{2} M^2 \frac{a_\pi m_\pi^2}{(q+k)^2 + m_\pi^2} \left[g_{\pi NN} ((q+k)^2) \left(\frac{\mu_p}{2p \cdot k} - \frac{\mu_n}{2p' \cdot k} \right) - 4MA_4 \right], \quad (B33)
 \end{aligned}$$

$$\begin{aligned}
 & G_{32}^d Q.k + G_{31}^d q.k + G_{33}^d q(q+k) + M^2 (G_{00}^d - i G_{13}^b) \\
 &= \frac{i}{2} M^2 \frac{a_\pi m_\pi^2}{(q+k)^2 + m_\pi^2} \left[g_{\pi NN} ((q+k)^2) \left(\frac{\mu_p}{2p.k} + \frac{\mu_n}{2p'.k} \right) - 2MA_3 \right]. \quad (B34)
 \end{aligned}$$

Again, extracting the pole terms g^+ and g^- we obtain a new system
 $(G = g^+/p.k + g^-/p'.k + g)$

$$g_{12}^b Q.k + g_{13}^b q.k = M(F_A - \bar{F}_A), \quad (B35)$$

$$g_{21}^c Q.k + g_{31}^c q.k + M^2 g_{00}^c = -2i \frac{M^3}{m_\pi^2} F_P, \quad (B36)$$

$$g_{23}^c Q.k + g_{33}^c q.k = -2i \frac{M^3}{m_\pi^2} (F_P - \bar{F}_P), \quad (B37)$$

$$\begin{aligned}
 & g_{22}^a Q.k + g_{21}^a q.k + g_{23}^a q(q+k) + M^2 (g_{00}^a - 2g_{00}^b) \\
 &= \frac{1}{2} M F_A (\mu_p + \mu_n) + M^2 \frac{a_\pi m_\pi^2}{(q+k)^2 + m_\pi^2} (A_1 - 2MA_4), \quad (B38)
 \end{aligned}$$

$$\begin{aligned}
 & (g_{12}^a - g_{00}^b) Q.k + (g_{11}^a + g_{00}^b) q.k + g_{13}^a q(q+k) \\
 &= M(F_A - \bar{F}_A) + M \frac{a_\pi m_\pi^2}{(q+k)^2 + m_\pi^2} (q.k A_3 - 2Q.k A_4), \quad (B39)
 \end{aligned}$$

$$\begin{aligned}
 & i g_{22}^b Q \cdot k + g_{21}^c q \cdot k + g_{23}^c q(q+k) - i M^2 (2 g_{12}^b + g_{00}^a) \\
 & = -2 M^2 \frac{a_\pi m_\pi^2}{(q+k)^2 + m_\pi^2} q \cdot k A_2, \quad (B40)
 \end{aligned}$$

$$\begin{aligned}
 & (g_{32}^c + i g_{23}^b) Q \cdot k + g_{31}^c q \cdot k + g_{33}^c q(q+k) + M^2 (g_{00}^c - 2 i g_{13}^b) \\
 & = 2 i \frac{M^3}{m_\pi^2} \bar{F}_p - M^2 \frac{a_\pi m_\pi^2}{(q+k)^2 + m_\pi^2} 2 \left[\frac{i}{q^2 + m_\pi^2} g_{\pi NN}(q^2) - i g'(0) + Q \cdot k A_2 \right], \quad (B41)
 \end{aligned}$$

$$\begin{aligned}
 & g_{21}^d q \cdot k + g_{23}^d q(q+k) - i M^2 (g_{12}^b + g_{00}^b) \\
 & = 2 i M^3 \frac{a_\pi m_\pi^2}{(q+k)^2 + m_\pi^2} A_4, \quad (B42)
 \end{aligned}$$

$$\begin{aligned}
 & g_{32}^d Q \cdot k + g_{31}^d q \cdot k + g_{33}^d q(q+k) + M^2 (g_{00}^d - i g_{13}^b) \\
 & = -i M^3 \frac{a_\pi m_\pi^2}{(q+k)^2 + m_\pi^2} A_3, \quad (B43)
 \end{aligned}$$

which should be completed by Eqs. (B24) - (B28) with substitutions of g for G in them. When extracting the pole terms we have used the relation

$$2 M \left[F_A(q^2) + \frac{q^2}{m_\pi^2} F_P(q^2) \right] = \frac{a_\pi m_\pi^2}{q^2 + m_\pi^2} g_{\pi NN}(q^2). \quad (B44)$$

To solve this system we shall assume the form (26) for the form factors g_{ij}^x and the relation

$$\bar{F}_P = F_P(q^2) = -m_\pi^2 \frac{F_A(q^2)}{q^2 + m_\pi^2}.$$

Still unknown are the coefficients A_i of D_μ . We have checked that no contradiction arises if one takes $A_2 = A_3 = A_4 = 0$. The coefficient A_1 is then fixed from the coupling equations themselves. See, however, Section 5 for further discussion.

Again we have two groups of equations. From Eqs. (B24) - (B28) and (B32) - (B34) one obtains

$$g_{22}^b = g_{23}^b = g_{00}^a = g_{21}^d = g_{31}^d = g_{32}^d = g_{12}^d = g_{23}^d = g_{33}^d = g_{13}^a = g_{32}^c = 0 \quad (B45)$$

and from (B35)

$$g_{12}^b = 0, \quad g_{13}^b = 2M F_A'(0). \quad (B46)$$

The second group consists of Eqs. (B36) - (B43). To solve them we write (B36) in the form

$$g_{21}^c Q \cdot k + g_{31}^c q \cdot k + M^2 g_{00}^c = -2i M^3 \left[\frac{\bar{F}_P}{m_\pi^2} + \frac{2q \cdot k}{(q+k)^2 + m_\pi^2} \left(\frac{\bar{F}_A}{q^2 + m_\pi^2} - F_A'(0) \right) \right].$$

It gives

$$\begin{aligned} g_{21}^c &= 0, \\ g_{00}^c &= -2i \frac{M}{m_\pi^2} \bar{F}_P, \\ g_{31}^c &= -4i \frac{M^3}{(q+k)^2 + m_\pi^2} \left[\frac{\bar{F}_A}{q^2 + m_\pi^2} - F_A'(0) \right]. \end{aligned}$$

From (B37) we have

$$\begin{aligned} g_{23}^c &= 0, \\ g_{33}^c &= g_{31}^c. \end{aligned}$$

It is easy to check that Eq. (B41) is consistent with the solutions for g_{00}^c ,

$g_{31}^c, g_{32}^c, g_{33}^c, g_{13}^b$, and g_{23}^b which have been found above. In Eq. (B39) we find only one non-trivial form factor, namely

$$g_{11}^a = 2M F_A'(0)$$

and similarly from (B43):

$$g_{00}^d = 2iM F_A'(0).$$

Now, returning to Eq. (B38) we rewrite it as

$$g_{22}^a Q \cdot k + g_{21}^a q \cdot k + g_{23}^a q(q+k) = \frac{1}{2} M (\mu_p + \mu_n) [F_A(0) + (q^2 + 2q \cdot k) F_A'(0)] + M^2 \frac{a_\pi m_\pi^2}{(q+k)^2 + m_\pi^2} A_1.$$

Let g_{21}^a and g_{23}^a be of the form $\text{const}/[(q+k)^2 + m_\pi^2]$, then

$$g_{23}^a = M^2 \frac{\bar{F}_A}{(q+k)^2 + m_\pi^2},$$

$$g_{21}^a = M^2 \left[\frac{\bar{F}_A}{(q+k)^2 + m_\pi^2} + 2 F_A'(0) \right],$$

and

$$A_1 = -\frac{1}{2M} g_{\pi NN}(q^2).$$

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Figure captions

- Fig. 1 : Diagrams contributing to the radiative muon capture amplitude.
- Fig. 2 : Diagrams corresponding to the a) ordinary muon capture,
b) π photoproduction,
c) γNN vertex.
- Fig. 3 : Photon spectrum due to the RMC on proton:
a) this calculation: $F_V = 1$, $F_M = 3.71$,
 $F_A = 1.25$, $F_P = -1.22/(1 + q^2/m_\pi^2)$;
b) this calculation: $F_V = 0.98$, $F_M = 3.6$, $F_A = 1.2$,
 $F_P = -1.22/(1 + q^2/m_\pi^2)$;
c) Hwang-Primakoff's⁴⁾ RMC amplitude: $F_V = 0.98$, $F_M = 3.6$, $F_A = 1.2$,
 $F_P = -0.8104$.
- Fig. 4 : Dependence of the RMC photon spectrum on the value of the pseudoscalar:
 $F_P = -a/(1 + q^2/m_\pi^2)$
a) $a = 2.44$, b) $a = 1.22$, c) $a = 0.5$, d) $a = 0$, e) $a = -1$.
- Fig. 5 : Photon spectrum due to the RMC on ^3He :
a) this calculation (no derivative terms),
b) Fearing's⁷⁾ approach, form factors are evaluated at $q_L^2 = (p_{\text{He}} - p_H)^2$,
c) RMC amplitude as suggested by Goldstein⁹⁾
d) Hwang-Primakoff's⁴⁾ amplitude.
In all cases we use $F_V = 1$, $F_M = -5.44$ and $F_A = -1.22$.
- Fig. 6 : Role of individual terms in RMC amplitude (28):
a) full amplitude includes $M^{(i)}$, $i = 1 - 4$,
b) $M^{(2)}$, $M^{(3)}$ and $M^{(4)}$ are omitted,
c) $M^{(3)}$ and $M^{(4)}$ are omitted.
- Fig. 7 : Dependence of the RMC photon spectrum on the value of the pseudoscalar
 $F_P(q^2) = -a(-1.22)/(1 + q^2/m_\pi^2)$:
a) $a = 2$, b) $a = 1$, c) $a = 0.5$, d) $a = 0$, and e) $a = -1$.

FIG.1

FIG.2

FIG.3

FIG. 4

FIG.5

FIG. 6

FIG. 7