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THE PROTON INSTABILITY AND THE NEUTRINO MASS IN THE
ASYMPTOTICALLY FREE SU(5)-MODEL

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THE PROTON INSTABILITY AND THE NEUTRINO MASS IN THE
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Abstract⁺⁾

The mass of leptoquark bosons is shown to determine in our version of the $SU(5)$ -theory, not only the proton life-time but also the mass of neutrino associated with the e and μ leptons (to be 0.1eV or less). The possibility of checking this fact is discussed.

⁺⁾ The results of this work were communicated at the Workshop on the Proton Instability , Moscow, January 1980.

As the years pass the attractiveness of the $SU(5)$ Grand unification model [1,2] is becoming more and more manifest. Its popularity does not require now any new supporting arguments and one has every reason to consider this model as a candidate for the future theory of high energy physics. Such status of the $SU(5)$ model is usually associated with a number of circumstances, although it seems that there are only two essential ones among them. Here significant role should be attributed to the success of the $SU(2) \times U(1)$ model of weak and electromagnetic interactions [3], which is a component of the $SU(5)$ theory, as well to the aesthetic attractiveness of unifying strong electromagnetic and weak interactions inside a unique gauge $SU(5)$ group which is a minimum group for such unification. Physical consequences of the $SU(5)$ theory are rather impressive and their investigation is attracting attention [4]. At present a considerable success of the $SU(5)$ -model has been achieved also within the scope of its asymptotically free version [2,5]. Within such model a number of problems were naturally solved [5] [in particular, the problem of the mass spectrum degeneracy (the m_d/m_s -ratio), the n_B/n_γ -ratio etc.] although so far the solution of the same problems by the ordinary $SU(5)$ theory [1,4] meets essential difficulties.

Our $SU(5)$ model proposed previously in ref. [2] is presented here in outline only to the extent needed for understanding what follows below. The complete version is given in ref. [5] where all the parameters connected with it are fixed. The parameters are determined by the low energy phenomenology adopted today although the latter is not yet

sufficient for completely eliminating the arbitrariness from the theory under consideration. The experimental predictions of our model are rather ample and the experimental verification of some of them would be of first-rate importance. For instance, the prediction, following from this $SU(5)$ -theory, on the nonzero mass ($0.1eV$ or less) for the neutrino associated with the e - and μ -leptons must already now become a subject of the experimental check. Also the fact that, from this experiment, the value of the leptoquark boson mass may be extracted which in its turn determines the proton life-time makes this experiment extremely important for clarification of the theory. Here the model also predicts somewhat different phenomenology to the τ -lepton (in particular, the life-time more close to its present experimental value, the absence of its own ^(light)neutrino etc.) and interesting possibility concerning the existence of yet another (apart from the e , μ and τ -leptons) charged δ -lepton belonging to the μ -series whose mass is more 40 GeV. Totally, the low energy phenomenology is presented here by 6 quarks (which may be formed as two sets of (u, d, s) - and (c, b, t) - quarks, as this is done in ref. [6]) and 7 leptons [four (e, μ, τ, δ) being charged and 3 neutral (the 4 components neutrino which is connected with e, μ, τ, δ leptons)]. The rest of the quarks and leptons are heavy and superheavy (with the mass near and above 10^3 GeV), their contribution into the low energy phenomenology being inessential. Under these phenomenology (since e_p is not singlet now) the parity nonconservation in the atomic physics [7] though nonzero, manifests

itself less gravely as compared to the predictions of other $SU(5)$ theories [8] where some disagreement with experiment takes place. The problem of the mass spectroscopy for the charged leptons and quarks (with the charge $\pm 1/3$) is also solved in an original way here. The degeneration between these mass spectra is removed due to the 24-plet of spinor fields which at the same time, is of crucial importance for achieving the asymptotic freedom for the theory as a whole. In this sense, the 45-plet of scalar fields introduced into the theory in ref.[9] to overcome the same mass degeneracy can be hardly thought of as reasonable since this multiplet is 'redundant' if viewed from the point of intrinsic needs of the model. The 5-plet of scalar fields added by us in [2] (to the $SU(5)$ scheme of ref.[1]) proved also to be important. It is due to the latter that one succeeds here in solving another problem of the ordinary $SU(5)$ theory which, though successful otherwise, failed to give correct order of value to the n_s/n_x -ratio. As a whole, the given $SU(5)$ model turned out to be economic and fruitful as it supplied the experiments to come with a number of interesting predictions. Here, we are discussing in detail one of them, namely that which indicates that the mass of leptoquark bosons in the $SU(5)$ theory determines not only the life-time of proton but also the mass of the neutrino associated with the e, μ -leptons.

The Lagrangian of the $SU(5)$ model proposed in ref.[2] may be represented by the three structural blocks

$$L = L_G + L_Y + L_S \quad (1)$$

among which the form of the $\mathcal{L}_Y$ -block responsible for the Yukawa-like interaction for a given composition of the spinor and scalar particles in the model is most important. The model unites three multiplets of scalar fields (Φ , M and H -fields; 24-plet and two 5-plets, respectively) and seven multiplets of spinor fields ($\psi_{L,R}$, $\vartheta_{L,R}$, η , ξ and B -multiplets) which contain in the total, 49 spinor particles. The interaction with the gauge V_M^α -field is given in the usual form (see ref. [5] for the $\mathcal{L}_G$ -block) and is determined by one effective charge. The latter is not, however, arbitrary but is usually fixed by the phenomenology of electromagnetic interactions. In particular, in the present case one has $g = \frac{2\sqrt{3}}{3}e$. The other fitting parameters of the theory (apart from the three masses of the ξ , η and H -multiplets) are connected with the Yukawa-like interaction

$$\begin{aligned} \mathcal{L}_Y = & -(ik_F f^{\alpha\beta\gamma} + k_p d^{\alpha\beta\gamma}) \bar{B}^\alpha \phi_\beta B^\gamma \quad (2) \\ & - [(\bar{B}_2^a)^b (h_1 M_b \eta_R^a + h_2 N_b \eta_L^a + h_3 N^{+,a} \xi_b^R + h_4 M^{+,a} \xi_b^L) + h.c.] \\ & - [\bar{\psi}_{a6}^R M^{ab} (f_1 \vartheta_L^a + q_1 \eta_L^a) + h.c.] - [\bar{\psi}_L^{a6} N_b (f_2 \vartheta_a^R + q_2 \xi_a^R) + h.c.] \end{aligned}$$

which is fixed here following the requirements imposed by the asymptotic freedom and the necessity of removing the degeneracy between the mass spectra of the leptons and quarks. $\mathcal{L}_Y$ totals ten parameters (here k_F, k_p, h_1, f_1 and q_1), six of which are fitting ones whereas the other four (namely k_F, h_1, h_2 and q_2) are needed for realizing the special solution of the renormalisation group equations and are not arbitrary. Confining ourselves to $\mathcal{L}_Y$ as it is given in eq. (2) we

omit here some terms [in particular $\mathcal{E} \dots [\overline{\psi}_R^c] \psi_R M^+$ - terms and others] which might be useful for fixing the needed value of the Cabibbo angle. However, in doing so, we have hopes of finding the latter as an angle which diagonalizes the mass matrix for the U, C -quarks calculated within the one-loop correction following eq.(2). alone. The possibility for these extra terms to be taken into account is reserved in the last resort. The last block $\mathcal{L}_g$ (see refs. [2,5] for it) contains some necessary terms required by the renormalizability of the theory (1). However, it has little influence directly on the physical contents of the model, apart from that it provides the spontaneous symmetry breaking through the Higgs mechanism. For the physical contents, mainly the $\mathcal{L}_Y$ -block is responsible as well as multiplet composition of the spinor and scalar fields. It is from here that the predicted finiteness of the e, μ -neutrino mass originates since $\mathcal{L}_Y$ has been selected so that the degeneracy between the leptonic and quark mass spectra be removed using the B-multiplet of spinor particles to this end.

The asymptotic freedom of the model is proved with the aid of the renormalization group equations using the one-loop approximation [10, 12]. The solutions needed of the renormalization group equations are realized here basing on the ultraviolet unstable fixed points [11, 12], to which possibility there is apparently no alternative [13]. Within the present approach we have fixed four effective charges which give the Yukawa-like interaction (i.e. k_f, h_i - and G_{f_2} - param-

ters, although this possibility is not the only one) as

$$\overline{k}_F^2 = 1.1008, \quad \overline{h}_1^2 = 3.3631, \quad \overline{h}_2^2 = 2.5727, \quad \overline{q}_2^2 = 1.5216 \quad (3)$$

and fixed all the effective charges of the $\mathcal{L}_S$ -block [2,5] as well. Nevertheless, this does not affect the physical contents of the model while its mathematical consistency allows us to take it quite seriously. Moreover, it is only within this theory (i.e. the one with asymptotic freedom), that an accurate discussion of the phenomenology of the high and super-high energies (in particular, the proton instability, various cosmological consequences and other effects) is possible wherein the theories lacking the asymptotic freedom when applied in the corresponding energy range (i.e. of order of 10^{14} Gev) encounter grave difficulties.

The Higgs mechanism which destroys the initial gauge symmetry is realized here in the usual way after the vacuum expectation values of the scalar fields

$$w_a^0 = \left\langle \left(\frac{\lambda^t}{2} \right)_a^b \phi^t \right\rangle ; \quad \left\langle \begin{matrix} M_a \\ N_a \end{matrix} \right\rangle = v_{i,a} \quad (4)$$

have become different from zero. The effective potential of the interaction of the scalar fields to be minimized by appropriate choice of (4) is found here in the tree approximation only

$$\begin{aligned} V[w, v] = & -\frac{M^2}{2} T_2 w^2 - \sum_i m_i (v_i^+ v_i) \\ & + \frac{1}{4} \left(\frac{\lambda_{\phi^1}^2}{2} - \frac{\delta_{\phi^1}^2}{3} \right) (T_2 w^2)^2 + \frac{\delta_{\phi^1}^2}{4} T_2 w^4 + \sum_i \left[\frac{1}{2} \delta_{\phi^1 M^2}^2 (v_i^+ w^2 v_i) + \right. \\ & \left. + \frac{1}{2} \left(\lambda_{\phi^1 M^2}^2 - \frac{1}{3} \delta_{\phi^1 M^2}^2 \right) (T_2 w^2) (v_i^+ v_i) \right] + \end{aligned} \quad (5)$$

$$+ \frac{1}{2} \lambda_M^2 \sum_i (\psi_i^\dagger \psi_i)^2 + \lambda_{M^2 N^2}^2 (\psi_1^\dagger \psi_1)(\psi_2^\dagger \psi_2) + \delta_{M^2 N^2}^2 (\psi_1^\dagger \psi_2)(\psi_2^\dagger \psi_1)$$

Note that the higher quantum corrections are not kept since they do not qualitatively change the result of the one-loop approximation [14]. The form of the matrices (4) is fixed by requiring that the initial $SU(2) \times U(1)$ symmetry should remain and that (4) should commute with the charge operator of the $SU(5)$ theory [2].

$$W = \begin{bmatrix} 8/3 & & & & \\ & 8/3 & & & \\ & & 8/3 & & \\ & & & 2/2-8 & \\ & & & & -2/2 \end{bmatrix} \quad \psi_1 = \begin{bmatrix} \vdots \\ p \end{bmatrix} \quad \psi_2 = \begin{bmatrix} \vdots \\ q \end{bmatrix} \quad (6)$$

As a result, these matrices are determined here by two parameters S and X their action being separated (the complete solution is presented in [5]). Here the parameter X (here $gX \approx 10^2$ GeV; below we consider the case when $p^2 \approx q^2 = X^2$) is responsible for the phenomenology of weak interactions while the parameter S fixes the phenomenology of high and superhigh energies and determines, in particular, the mass of superheavy vector bosons of the theory (the X and Y leptoquark-bosons with the mass $M \approx gS$). The latter is chosen to be sufficiently large since one has to provide at low energy the existing hierarchy of effective charges of strong and electromagnetic interactions using a single effective charge of the theory at $Q^2 \gg M^2$

$$\frac{1}{\alpha(Q^2)} - \frac{8/3}{\alpha_s(Q^2)} = \frac{11}{\pi} \ln \left(\frac{M^2}{Q^2} \right) \quad (7)$$

Estimate obtained within (7) indicate that

$$M \approx 10^{14} \div 10^{15} \text{ GeV} \quad (8)$$

The latter mass, generally speaking, weakly dependent on the choice of Q^2 . For instance, at $Q^2 \approx 10^2 \text{ GeV}$ where $\alpha(Q^2) \approx 1/127$ and $\alpha_s(Q^2) \approx 0.13$ one has $M \approx 1.6 \cdot 10^{15} \text{ GeV}$. The above situation is common for all the SU(5) schemes of Grand unification where the proton is unstable and one must restrict its life-time to fit the experimental data. The estimate of the proton life time made basing on eq.(8) leads us to the value of $10^{31} - 10^{32}$ years which is close to the value found in [15]. The initial (3/8) and the renormalized 0.21 values of the Weinberg angle are also the same in the ordinary SU(5) and our models. A nontrivial peculiarity of our model is the finite mass of the neutrino interacting with the e^- and μ^- leptons which is fixed here by the same parameter, namely by the mass of the leptoquark boson. This prediction is intrinsic only for the SU(5) theory under consideration and is due to the choice of the Yukawa interaction made as in (2). The three additional heavy B, ξ, η -multiplets of spinor particles introduced in ref.[2] also play essential rôle here

$$B = \left[\begin{array}{c|ccc} \text{SU(3)} & h_1 & K_1 & & \\ \text{-quarks} & h_2 & K_2 & & \\ & h_3 & K_3 & & \\ \hline q_1 & q_2 & q_3 & Q & M^+ \\ \tau_1 & \tau_2 & \tau_3 & E^- & N \end{array} \right] \quad \eta = \left[\begin{array}{c} b_1^c \\ b_2^c \\ b_3^c \\ \tau^- \\ N_t \end{array} \right] \quad \xi = \left[\begin{array}{c} t_1 \\ t_2 \\ t_3 \\ \nu^+ \\ N_f \end{array} \right] \quad (9)$$

as well as some changes in the definition of the light multiplets of the model of ref.[1] which are the following (here $\psi_{L,R}$ and $\bar{\psi}_{L,R}$ are $10, 10^*$, and $5, 5^*$ -plets, respectively)

$$\psi_R = \begin{pmatrix} (C_R)_3^c & (C_R)_2^c & C_{L,1} & S_{L,1} \\ U_{R,3} & (C_R)_1^c & C_{L,2} & S_{L,2} \\ U_{R,2} & U_{R,1} & C_{L,3} & S_{L,3} \\ (U_L)_1^c & (U_L)_2^c & (U_L)_3^c & \mu_L^- \\ (d_L)_1^c & (d_L)_2^c & (d_L)_3^c & e_R^- \end{pmatrix} = \psi_L \quad \begin{pmatrix} S_{R,1} \\ S_{R,2} \\ S_{R,3} \\ \mu_R^+ \\ \nu_{\mu R} \end{pmatrix} = \bar{\psi}_R$$

$$\bar{\psi}_L = \left\{ (d_R)_1^c (d_R)_2^c (d_R)_3^c e_L^- \nu_{eL} \right\} \quad (10)$$

The neutrino (ν -lepton) is represented in (10) as one four-component Dirac spinor interacting with the e, μ -leptons via the Zeldovitch-Machaud-Konopinski scheme [16]. Totally, there are four neutral leptons (ν, N_τ, N_μ and N -neutrinos) in this $SU(5)$ theory, the effective mixing taking place, however, only between ν, N_τ and N_μ -leptons (the N -lepton is superheavy with the mass $m_N \approx 10^{14}$ Gev). The nontriviality of the mass matrix responsible for their mass spectroscopy

$$\mathcal{L} = (N, N_\mu, N_\tau, \nu)_L \begin{pmatrix} \tilde{m}_8 & \delta_3 & \delta_1 & 0 \\ \delta_4 & m_\tau & 0 & \mu_\tau \\ \delta_2 & 0 & m_\mu & 0 \\ 0 & 0 & \mu_\mu & 0 \end{pmatrix} \begin{pmatrix} N \\ N_\mu \\ N_\tau \\ \nu \end{pmatrix} + H.C. \quad (11)$$

is entirely owing to the above choice of the Yukawa-like interaction and to the mechanism of spontaneous symmetry breaking that leads, as it follows from (2), to an interference between the heavy and light multiplets present in the theory (in our case they are the multiplets (9) and (10), respectively). In eq. (11), the parameter $\tilde{m}_B$ ($\approx 10^{14}$ GeV) is determined by the matrix $(m)_{\alpha\beta} = m_B \delta_{\alpha\beta} + k_{\alpha\beta}^f \langle \phi^f \rangle$, $\delta_i \approx \frac{h_i}{\sqrt{2}} m_W (\approx 10^2 \text{ GeV})$, the m_η - and m_χ -parameters are order of m_T ($\approx 2 \text{ GeV}$) and m_g ($> 40 \text{ GeV}$), respectively (see ref. [5] for farther explanations, where the other parameters determining eq. (II) are fixed). Thanks to the considerable hierarchy existing among the parameters involved in the mass matrix (11) the spectrum of the latter is readily found in an explicit form where the smallest eigenvalue corresponds to the mass of the neutrino associated with the e, μ -leptons

$$m_\nu \approx \delta^2 / \tilde{m}_B \quad (12)$$

According to (12), the mass m_ν is different from zero in principle, although its value is controlled by the parameter $\tilde{m}_B$. This parameter is estimated in [5] to be about 10^{14} GeV and is as a matter of fact determined by the order of the mass of the leptoquark-bosons. Therefore, the latter is responsible not only for the life-time of the proton but also for the finite mass of the neutrino associated with the e, μ -leptons via the scheme of Zel'dovitch, Machaud and Konopinski [16]

$$J_W \sim \bar{e} \gamma_\mu (1 + \gamma_5) \nu + \bar{\nu} \gamma_\mu (1 - \gamma_5) \mu \quad (13)$$

It is known [17], that with $M_Y = 0$ such scheme (for which eq.(13) holds as long as the symmetry is not broken spontaneously) is identical to the conventional phenomenology of the V-A weak interaction. With $M_Y \neq 0$, however, (i.e. for the theory in the phase of spontaneously broken symmetry) the experiment must distinguish between these two possibilities bearing in mind, however, that the usual V-A -phenomenology remains, certainly, the same in all the cases. For the τ, δ - phenomenology it is important that the ν -neutrino is connected not only with e, μ -leptons but the interference takes place with other leptons also. The present $SU(5)$ model predicts that the ν -neutrino (when the broken symmetry phase takes place) mixes with the other two neutrinos (the heavy N_τ and N_δ -leptons) and thus defines the phenomenology of the τ, δ - leptons as well. As far as the τ -lepton phenomenology is concerned we are found, by fitting the mixing angles, that the life time and the decay rates of the τ -lepton are in agreement with the existing experimental data. Although the own neutrino of the τ -lepton is heavy no evident contradiction is detected and some peculiarities of this scheme may be now considered as a subject of experimental verification. The phenomenology of δ -lepton predicted the present $SU(5)$ model must be taken into account as well when the experimental data are fitting. Here the δ -lepton mass is more then 40 GeV and τ - and δ -lepton phenomenologies are essentially analogous demonstrating that the e, μ - universality holds in the high energy region also.

Thus, our asymptotically free $SU(5)$ -model of Grand unification determined by the 5 fitting parameters (fixed as in ref.[5]) results, after the spontaneous symmetry breaking, in the usual values the masses for the (e, μ, τ) -leptons and (d, s, b) quarks and makes some prediction about the mass phenomenology of other particles (the $\bar{\nu}$ -lepton with $m_{\bar{\nu}} > 40 \text{ GeV}$ for example). The corresponding effective value of the Weinberg angle is equal to 0.21, while the proton life time is about $10^{31}-10^{32}$ years. It is most remarkable that the mass of lepto-quarks is simultaneously responsible for both the instability of the proton and the nonvanishing mass of neutrinos. The usual electronic and μ -mesonic neutrinos join together to form a Dirac spinor with the mass of order of 0.1 eV or less. The finite mass of the latter as well as its interference with the two "heavy" N_{τ} and N_b - neutrinos may well prove to be essential for solving the "solar neutrinos paradox" [18] and also clarify the situation with the $\mu-e\gamma$ decay and other analogous processes.

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