Vladislav V. Kravchenko

Applied

Pseudoanalytic Function Theory

Birkhäuser Verlag
Basel Boston Berlin

Contents

Foreword	xi
1 Introduction	1
 Part I Pseudoanalytic Function Theory and Second-order Elliptic Equations	
2 Definitions and Results from Bers' Theory	9
2.1 Generating pairs and differentiation	11
2.2 Pseudoanalytic functions	13
2.3 Derivatives and integrals of pseudoanalytic functions	13
2.3.1 Equivalent generating pairs	13
2.3.2 Vekua's equation for (F, G) -derivatives	14
2.3.3 Integration	16
3 Solutions of Second-order Elliptic Equations as Real Components of Complex Pseudoanalytic Functions	21
3.1 Factorization of the stationary Schrödinger Operator	23
3.2 Factorization of the operator $\text{divp grad} + q$	27
3.3 Conjugate metaharmonic functions	29
3.4 The main Vekua equation	31
3.5 Cauchy's integral theorem for the Schrödinger equation	32
3.6 p -analytic functions	35
4 Formal Powers	37
4.1 Definition	38
4.2 An important special case	41
4.3 Similarity principle	43
4.4 Taylor series in formal powers	43
4.5 The Runge theorem	45
4.6 Complete systems of solutions for second-order equations	
4.7 A remark on orthogonal coordinate systems in a plane	

4.8	Explicit construction of a generating sequence	46
4.9	Explicit construction of complete systems of solutions of second-order elliptic equations	50
4.9.1	Explicit construction of complete systems of solutions for a stationary Schrödinger equation	51
4.9.2	Complete systems of solutions for the conductivity equation	51
4.10	Numerical solution of boundary value problems	52
5	Cauchy's Integral Formula	
5.1	Preliminary information an the Cauchy integral formula for pseudoanalytic functions	55
5.2	Relation between the main Vekua equation and the system describing p-analytic functions	57
5.3	The transplant operator	58
5.4	Cauchy integral formulas for x^k -analytic functions	60
6	Complex Riccati Equation	
6.1	Preliminary notes	65
6.2	The two-dimensional stationary Schrödinger equation and the complex Riccati equation	67
6.3	Generalizations of classical theorems	69
6.4	Cauchy's integral theorem	71
 Part II Applications to Sturm-Liouville Theory		
7	A Representation for Solutions of the Sturm-Liouville Equation	
7.1	Solving the one-dimensional Schrödinger equation	75
7.2	The "+"-case	76
7.3	Two sets of Taylor coefficients	80
7.4	Solution of the one-dimensional Schrödinger equation	81
7.5	Validating the result	83
7.6	The "—" case	84
7.7	Complex potential	85
7.8	Solution of the Sturm-Liouville equation	86
7.9	Numerical method for solving Sturm-Liouville equations	90
8	Spectral Problems and Darboux Transformation	
8.1	Sturm-Liouville problem as a problem of finding zeros of an analytic function	93
8.1.1	Sturm-Liouville problems with spectral parameter dependent boundary conditions	95

8.2	Numerical method for solving Sturm-Liouville problems	96
8.3	A remark an the Darboux transformation	98

Part III Applications to Real First-order Systems

9 Beltrami Fields

9.1	Description of the result	103
9.2	Reduction of (9.1) to a Vekua equation	104
9.3	Solution in the case when a is a function of one Cartesian variable	105

10 Static Maxwell System in Axially Symmetric Inhomogeneous Media

10.1	Meridional and transverse fields	111
10.2	Reduction of the static Maxwell system to p-analytic functions	112
10.2.1	The meridional case	112
10.2.2	The transverse case	112
10.3	Construction of formal powers	113
10.3.1	Formal powers in the meridional case	113
10.3.2	Formal powers in the transverse case	114

Part IV Hyperbolic Pseudoanalytic Functions

11 Hyperbolic Numbers and Analytic Functions

12 Hyperbolic Pseudoanalytic Functions

12.1	Differential operators	125
12.2	Hyperbolic pseudoanalytic function theory	126
12.3	Generating sequences	130

13 Relationship between Hyperbolic Pseudoanalytic Functions and Solutions of the Klein-Gordon Equation

13.1	Factorization of the Klein-Gordon equation	133
13.2	The main hyperbolic Vekua equation	135
13.3	Generating sequence for the main hyperbolic Vekua equation	138

Part V Bicomplex and Biquaternionic Pseudoanalytic Functions and Applications

14 The Dirac Equation	
14.1 Notation	150
14.2 Quaternionic form of the Dirac equation	151
14.3 The Dirac equation in a two-dimensional case as a bicomplex Vekua equation	153
14.4 Some definitions and results from Bers' theory for bicomplex pseudoanalytic functions	155
14.4.1 Generating pair, derivative and antiderivative	155
14.4.2 Generating sequences and Taylor series in formal powers	156
14.5 The main bicomplex Vekua equation	158
14.6 Dirac equation with a scalar potential	159
15 Complex Second-order Elliptic Equations and Bicomplex Pseudoanalytic Functions	163
16 Multidimensional Second-order Equations	
16.1 Factorization	167
16.2 The main quaternionic Vekua equation	169
Open Problems	171
Bibliography	173
Index	183