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Possible Use of the SNS Synchrotron for Feasibility Tests on Aspects of Heavy Ion Fusion Drivers

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1. INTRODUCTION

Inertial-confinement heavy ion fusion studies originated and have received most attention in the USA. A summary of progress in the field up to 1979 is given in a SLAC/LBL/UC report edited by W Herrmannsfeldt¹. Prior to this report there were study-workshops held at the Claremont Hotel Berkeley (1976), BNL (1977), ANL (1978) and subsequently there has been a 1979 workshop at the Claremont. Proceedings² are available from the first three workshops and will be for the fourth.

There remain a large number of theoretical and practical problems to be solved before a complete accelerator-driver system prototype and a target chamber prototype may be built with any confidence to allow an assessment to be made of the practicality of heavy ion fusion power plants.

Two accelerator-driver systems remain under serious consideration for 1-10 MJ systems of ion kinetic energies approximately 10 GeV:

Scheme 1). A single pass induction linac accelerates the heavy ions in the form of a single bunch which is subsequently split and transported via n transport channels ($n \geq 4$) to the target chamber. Induction linac sections are included in the n beam lines to provide the desired final bunch compression. The process is to be repeated at ~ 10 Hz to relate to power plant operation.

Scheme 2). A system of conventional linacs, harmonic linacs, accumulator storage rings, bunch compression storage rings and n final beam lines, with induction linac sections, provides equivalent final bunches to those of scheme 1).

The low energy stages of the proposed drivers have received the most study and the main areas of uncertainty are considered to be at the subsequent stages:

- a) the dynamics of the single bunch in the induction linac sections.
- b) the practical problem of the bunch splitting in the induction linac case.
- c) the accumulation and the bunch compression in the storage ring scheme.

- d) longitudinal and transverse instabilities in the storage rings.
- e) the practical problems associated with beam loss in the storage rings.
- f) the beam stability in the presence of large transverse and longitudinal space charge forces in the 1 km beam transport lines from the drivers to the target chamber.
- g) the final transport just upstream of the target D-T pellets.
- h) the target chamber design.
- i) the D-T pellet physics, design and yield which determine the basic driver parameters.

It is recognised that further studies are needed before choosing between the alternative driver schemes 1) and 2). Many of the areas of uncertainty require computational models and it is important to obtain experimental checks of such simulations. Prototypes involving intense beams of 10 GeV heavy ions are large scale projects and so it is an advantage if the essential dynamics can be studied experimentally by simpler means. The possible use of the SNS synchrotron for such studies is the basis of this report.

The SNS has a fast cycling synchrotron designed to accelerate $2.5 \cdot 10^{13}$ protons per pulse from 70 MeV to 800 MeV at a 50 Hz rate. The synchrotron may be operated in a proton storage ring mode at 70 MeV when it becomes highly suitable for simulating the conditions in the heavy ion storage ring schemes. This is a consequence of the fact that the velocity of a 70 MeV proton is similar to that of a 10 GeV heavy ion and also that the appropriate intensity level for 70 MeV protons is one to two orders of magnitude less than for the 10 GeV ions. The importance of having comparable velocities is to obtain similar reactive (space charge) components of the longitudinal and transverse beam coupling impedances in the two cases.

The rapid debunching of the beam injected into the accumulator rings may thus be simulated and investigated; as may storage ring longitudinal and transverse instabilities during the maximum storage times envisaged of a few ms. Also of importance is the emittance dilution of the heavy ions in the longitudinal and transverse planes during the storage ring bunch

compression. For longitudinal motion simulation, the same ratio of RF focusing forces to longitudinal space charge forces may be established; for transverse motion, appropriate crossing of integer and half integer betatron resonances may be obtained in the compression.

The 800 MeV proton external beam line of the SNS has increased flexibility at 70 MeV, at which energy it is possible to simulate the beam transport of the heavy ions in the presence of large longitudinal and transverse space charge forces. The external beam line is 150 m in length as compared to 1000 m for the heavy ion case but the reduced length is adequate for a range of studies. In particular, the inclusion and operation of induction linac sections may be investigated.

The range of studies indicated would require additions to be made to the SNS:

- a) improved diagnostics for the synchrotron and beam line.
- b) improved performance for the 70 MeV debuncher system (or momentum scraping).
- c) modifications and additions to the synchrotron RF system.
- d) transverse instability feedback development and (possibly) modifications to the RF shields.
- e) development of induction linac sections for the external beam.

The required improvements have not been costed but are very modest for the facility they provide. It is difficult to envisage an alternative UK project that could contribute to the basic problems of heavy ion fusion drivers for an equivalent outlay. The emphasis of the studies would be directed, however, towards the RF linac-storage ring driver (scheme 2) and to the initial stages of the final compression for both schemes.

2. SIMULATION OF HEAVY ION BUNCH COMPRESSION IN STORAGE RINGS

Parameters for the bunch compression of heavy ions in three reference storage ring designs are given in a report³ written during the 1979 Claremont workshop. In these reference drivers, the storage rings are first assumed to be filled uniformly to their transverse charge limits, $\Delta v = 0.25$. The space charge limit varies approximately as the beam

transverse emittance but is somewhat independent of the storage ring mean radius. A minimum radius leads to a minimum cost and the reference designs thus assume superconducting dipoles of field 5T, for which the ring radii are all less than 100 m. A minimum radius also leads to a minimum number of bunches per ring for a given bunch compression and final bunch length.

The report³ makes approximate estimates for the transverse emittance dilution for a bunch compression of a factor 7. In this process the betatron tune of the machines crosses 2 integer and 4 half-integer resonances. Subsequently, there is betatron and dispersion mismatching at the time of transfer to the external beam line. The emittance blow-up in crossing the resonance lines is dependent on the speed of crossing which, in turn, depends on the RF parameters used for the compression. The RF parameters depend on the allowed level of the peak momentum spread in the beam, on the longitudinal space charge forces and also on practical considerations. The type of RF system required is comparable to that available in the SNS synchrotron, both from the point of view of providing RF focusing and from that of compensating for the longitudinal space charge forces. However, SNS modifications and additions would be necessary to enable simulation of the heavy ion storage ring parameters.

Dilution of the longitudinal emittance may arise from longitudinal instabilities or from the dynamics of bunch formation and rotation. As the bunch compresses, the longitudinal space charge forces increase and the final parameters³ are such that the space charge compression forces are approximately 20 times greater than the RF forces that provide the bunch rotation. There is the added problem that the longitudinal space charge forces vary across the transverse dimensions of the beam. The longitudinal motion of an individual particle is thus dependent on both its azimuthal and transverse position relative to the bunch centre.

To compute the transverse and longitudinal motion numerically is complex because of the scale of the problem and the importance of the wall configuration. Tracking programs that handle both transverse and longitudinal motion have been developed for the case of short linac bunches but with the approximation of neglecting the effect of the walls. There is a

definite need for both computational and experimental work to determine the non-linearities and the emittance dilution introduced in the proposed heavy ion bunch rotation.

The formulae of reference 3 may be used to obtain parameters for simulating the motion of the heavy ions by the SNS 70 MeV proton bunches. In particular, for comparison with the reference case B (> 3 MJ) of report 3:

REFERENCE CASE B		SNS
10 GeV U ⁺		70.44 MeV protons
β	0.29043	0.36713
γ	1.045045	1.075075
R (mean radius)	83.1 m	26.0 m
ν_h (horizontal tune)	8.3	4.31
ν_v (vertical tune)	8.3	3.83
$\Delta\nu_v$ (for $\bar{b} = 1$)	0.25	0.25
ϵ_h (horizontal emittance)	37.5 mrad mm	32.5 to 65 mrad mm
ϵ_v (vertical emittance)	37.5 mrad mm	25 to 50 mrad mm
hN (number of ions)	8.73 10^{14}	0.5 to 1.0 10^{13}
h (harmonic number)	3	2
C (bunch compression)	7	4
$2T_0$ (initial bunch duration)	2000 ns	742 ns
$2T$ (final bunch duration)	285.7 ns	185.5 ns

The total number of protons is in the range 0.5 to 1.0 10^{13} which is well within the SNS linac output specification of 6 10^{13} protons per 500 μ s pulse. The proton bunch compression of a factor 4 is sufficient to depress the betatron tunes through 2 integer and 4 half integer betatron resonances.

From the notation and formulae of Appendix 1, for the longitudinal motion:

	10 GeV U ⁺	70.44 MeV p	70.44 MeV p
h	3	2	2
$h f_0$	0.5	1.3475	1.3475
γt	7.7	5.034	5.034
ϵ_h	37.5 10^{-6}	32.5 10^{-6}	65 10^{-6}
			rad m

	10 GeV U ⁺	70.44 MeV p	70.44 MeV p
ϵ_v	37.5 10^{-6}	25.0 10^{-6}	50 10^{-6}
g	2	4.52	3.83
Z/n	-j(1189)	-j(2008)	-j(1701)
L	-1.134	-0.4743	-0.4019
N	29.1 10^{13}	0.25 10^{13}	0.5 10^{13}
C	7	4	4
D	20	5	5
$\Delta p/p$	$\pm 4 \cdot 10^{-3}$	$\pm 5 \cdot 10^{-3}$	$\pm 5 \cdot 10^{-3}$
$T(\text{min})$	142.8	92.75	92.75
n_q	72.8	95.1	95.1
eV_{sc}	± 3890	± 33.1	± 56.1
eV	± 1690	± 11.23	± 11.23
n	68	92	92
Ratio	6.2	8.5	5.0
			%

The RF requirements for bunching the protons are:

- A maximum saw-tooth RF amplitude for bunch rotation of ± 11.23 keV, and
- A maximum peak voltage per turn for longitudinal space charge compensation of ± 33.1 keV or ± 56.1 keV.

A saw-tooth RF waveform provides linear synchrotron oscillations and the ratio percentages³ refer to the ratio of the saw-tooth rotation forces to the longitudinal space charge forces at the end point of compression. It can be seen from the above parameter list that the range of values proposed for the SNS spans the ratio in the heavy ion reference case.

The RF accelerating system for the SNS consists of six powered ferrite tuned cavities plus two actively excited inductive cavities. The tuned system includes compensation for beam loading and the inductive cavity system provides compensation for the longitudinal space charge forces. To simulate the heavy ion compression the tuned cavity system is not necessary but the inductive cavity system would need to be extended from two to five systems. Each inductive cavity contains eleven ferrite toroids and the resulting 7.2 μ H inductance must be excited by a current 11.2 times

the beam current to provide the correct space charge compensation. The saw-tooth rotation voltage may also be generated in the inductive cavities by the use of additional drive currents of parabolic waveform. There are sufficient ferrites available to produce the five systems but five Class A amplifier drivers would need to be provided. The peak proton current at the end of the simulated compression is 6.48 A and the peak drive currents in the Class A power tubes must be:

	Start of Compression	Mid-Compression	End of Compression
Space Charge Compensation	18 A	36 A	72 A
Current for Saw-Tooth	60 A	15 A	4 A
Total peak current	78 A	71 A	76 A

Also of interest is bunch rotation in the SNS by means of a sine wave RF accelerating field of harmonic number 1 in place of the saw-tooth field of $h = 2$. In this case half the injected beam is first extracted and the rotation of the remaining half evaluated. To generate a $h = 1$ field, one of the ferrite tuned cavities may be used with the ferrite in its remanent state and additional gap capacitance added for resonance.

To enable detailed study of bunch compression would require the following diagnostic equipment:

- beam current monitors of high frequency response to study the bunch shape.
- beam profile monitors to study transverse emittance dilutions.
- beam profile monitors with high frequency response at high and low dispersion points to study the $\Delta p/p$ values along the duration of the bunch.
- monitors in the external beam line to study the beam transport under conditions of high space charge forces.

3. DEBUNCHING AND LONGITUDINAL INSTABILITIES

In the heavy ion driver scheme 2), a 10 GeV RF linac feeds one or more accumulator storage rings with typically a multi-turn injection of ten turns in the horizontal plane. The accumulator rings may or may not feed

buncher rings ahead of the bunch compression storage rings but in either case there is again multi-turn injection. The linac pulse current is of the order 250 mA and the linac pulse length must be of the order of a few ms.

The accumulator rings have a larger radius than that of the bunching rings (R) and do not require superconducting magnets. For reference case B, $R = 83.1$ m and a suitable radius for the accumulator is in the range 2.5 R to 10 R; with such a radius the linac beam debunches within one revolution, even though the momentum spread is small. There is some growth in $\Delta p/p$ during this debunching process and to limit the final $\Delta p/p$ a debuncher is required ahead of the accumulator. The exact requirements of the debuncher are uncertain, firstly because the linac parameters are only estimates and secondly, because the formulae for estimating the growth in $\Delta p/p$ during debunching are only approximate.

At the start of debunching the bunch length is a few times the transverse dimensions of the beam; in this case the usual formulae for estimating the longitudinal space charge forces in a synchrotron is not accurate. Also, the linac computer codes that may be used to estimate the forces suffer from the approximation that they neglect the effect of the surrounding walls.

The debunching mechanism may be investigated with 70.44 MeV protons in the SNS synchrotron. To obtain a $\Delta p/p = \pm 5 \cdot 10^{-3}$, as specified for the protons in section 2, requires a $\Delta p/p$ (before bunching) $= \pm 7 \cdot 10^{-4}$ and debunching then takes approximately 3 revolutions. The growth in $\Delta p/p$ may be measured as a function of the injected bunch length and current. The method proposed for measuring the growth is the following: instead of studying a single bunch over 3 turns, a train of 150 bunches may be injected occupying half the ring circumference, and measurements made of the debunching time taken to fill the ring.

Relevant debunching formulae are given in Appendix 2. These formulae indicate that, for proton pulse currents of 20 mA at 70.44 MeV and for 1 ns bunches, significant growths from $\Delta p/p = \pm 7 \cdot 10^{-4}$ should occur. The growths relative to the initial $\Delta p/p$ are of the same order as in the heavy

ion case for equivalent bunch lengths. The accuracy of the formulae may be studied by the experimental measurements. The topic is important because the heavy ion storage ring systems assume there is little dilution in the longitudinal emittance between the linac and the target.

Multi-turn injection in the accumulator rings takes only a few hundred μ s, after which the heavy ions are extracted to the buncher rings. The longest time that the ions stay in any one of the buncher rings is a few ms; in this interval it is important that longitudinal instabilities do not lead to a significant increase in momentum spread.

The threshold current for coherent longitudinal instabilities of coasting beams is:

$$I = F \left| \gamma^{-2} - \gamma_t^{-2} \right| \left(\frac{\Delta p}{p} \right)_h^2 \beta^2 \gamma (E_p/e) (A/\bar{z}) / \left| \frac{Z}{n} \right| \quad \text{where}$$

F is a factor related to the momentum distribution in the beam,

$(\Delta p/p)_h$ is the momentum spread, full width at half maximum,

(E_p/e) is the proton rest mass in volts,

$(A/\bar{z})$ is the ratio of the mass number to the charge state of the heavy ions,

$\left| \frac{Z}{n} \right|$ is the magnitude of the longitudinal electromagnetic coupling impedance ($\pm n$) and

n is the mode number for angular frequency $n\omega_0$.

In section 2 the reactive component of $\frac{Z}{n}$ due to the longitudinal space charge forces has been estimated at $-j(1189) \Omega$ for the heavy ions and $-j(1701) \Omega$ for the 70.44 MeV protons. Thus both cases are characterised by a high reactive impedance so that even small resistive components are sufficient for instability. The proposed scheme for space charge compensation, using active or passive inductive cavities, reduces the reactive impedance to a low level but only within a restricted frequency range. Above this frequency range the longitudinal instability problem remains.

The total intensity ($\bar{N} = nN$) that corresponds to the threshold current is given by $\bar{N} = 2\pi R \bar{I} B / ZeBc$. Here $\bar{B} = 1$ and the momentum spread values to be used are those existing prior to the commencement of bunching. In the case of the protons the proposed $(\Delta p/p)$ value of $\pm 7 \cdot 10^{-4}$ requires SNS

modifications, either scraping at a high dispersion point in the injection line or alternatively by developments of the linac debuncher.

Threshold intensities, in the frequency range where there is compensation of the longitudinal space charge impedance, are approximately, for an assumed resultant coupling impedance Z/n of 10Ω :

10 GeV U^+	$\bar{N}$	=	1.2	10^{15}
70 MeV protons	$\bar{N}$	=	5	10^{13}

At higher frequencies it is not possible to provide the compensation and instability is expected. Approximate formulae are available for estimating the instability growth rates⁴. In the limit of zero momentum spread, the growth rate is:

$$\frac{1}{\tau} = \frac{n\omega_0}{2\pi} \left[\frac{(\gamma^{-2} - \gamma_t^{-2}) \bar{N} ZeBc}{R \beta^2 \gamma (E_p/e) (A/\bar{z})} \right]^{\frac{1}{2}} \cdot \text{Im} \left[j \frac{Z}{n} \right]^{\frac{1}{2}}$$

$$\text{Im} \left[j \frac{Z}{n} \right]^{\frac{1}{2}} = \text{Im} \left[j (r + jX) \right]^{\frac{1}{2}} \approx r/2 \sqrt{|X|}$$

The parameter r is the resistive component of Z/n , and X , the space charge component, is large and negative. The ratio of the respective growth rates for the protons and heavy ions at frequency $n\omega_0$ is, for equal r values:

$$\frac{1}{\tau} (\text{protons}) + \frac{1}{\tau} (U^+) \approx 3$$

There are no reported experimental results for growth rates of longitudinal instabilities in the regime of large X values. Since comparable values of X , r and $\frac{1}{\tau}$ are to be expected for the protons and heavy ions, the SNS synchrotron appears ideal for studying experimentally the heavy ion longitudinal instability problems. It may be difficult to correlate the experimental results with theory because the space charge impedance X exhibits a large variation over the beam distribution. The dominant contribution to r is expected to arise from the RF cavities used for the compression and compensation.

4. TRANSVERSE INSTABILITIES

Transverse coupling impedances are also characterised by a large reactive space charge component when there are low β -values.

For the case of a circular beam of radius a , uniform transverse density, in a circular chamber of radius b with wall resistivity, ρ , and associated skin depth, δ :

$$Z_L \text{ (reactive)} = -jZ_0 R(a^{-2} - b^{-2}) / \beta^2 \gamma^2 \quad (\Omega m^{-1})$$

$$Z_L \text{ (resistive)} = 2R^2 \rho / b^3 \beta \delta \quad (\Omega m^{-1})$$

Thus, reducing the value b to approach the value a has the effect of decreasing the reactive component of Z_L but increasing the resistive component. There is the added effect of reducing the space charge component of the longitudinal impedance.

In the more general case of an elliptical beam displaced in a rectangular chamber there is a large variation of the transverse coherent forces across the chamber aperture, with a change in sign of the forces as the wall is approached. The average force may be of opposite sign to the case of a circular beam in a circular chamber. In both cases the forces are dependent on the beam transverse distribution.

Coasting beam transverse instabilities should arise for both the 10 GeV U^+ and the 70 MeV proton beams as a consequence of the large reactive components of Z_L . This is so even for low resistive components of Z_L , though a low resistive impedance is important as it is directly related to the instability growth rate.

The threshold intensity, $\bar{N}$, of the coasting beam instability, is:

$$\bar{N} = 4F (E_p/e) (A/\bar{z}) \gamma Q |\Delta Q| / \bar{z} e c Z_L$$

where F is a function related to the beam distribution and ΔQ is the spread in transverse betatron oscillation frequencies due to the momentum spread in the beam, the spread in particle revolution frequencies and also due to the presence of octupoles in the storage ring. Thus the threshold

scales with the mass number A and since the growth rate is proportional to $\bar{N}/A$, there should be comparable growth rates for the U^+ and the protons.

In comparing the transverse and longitudinal motions it may be noted that the reactive space charge components of impedance affect the motions differently. For a longitudinal perturbation the instability growth rate is a function of the resistive and the reactive components whereas in the transverse case the growth rate depends mainly on the resistive term. If a beam is bunched, the longitudinal reactive component modifies the incoherent synchrotron frequencies but not the frequency of the coherent dipole mode. This affects the longitudinal stability as the Landau damping is modified when the distribution of incoherent frequencies is displaced relative to the coherent modes. For transverse motion, unbunched or bunched, the space charge forces contribute to incoherent and coherent frequency shifts; the linear change in the incoherent frequency is usually referred to as the Laslett Q-shift, while the shift in frequency of a coherent mode is obtained from the total reactive components of Z_L minus the transverse incoherent space charge contribution (a^{-2} term).

In the longitudinal case the active excitation of passive inductive cavities provides feed-forward compensation for the incoherent motion. The equivalent in the transverse plane is feed-forward compensation of the storage ring quadrupole focusing; to extend this into the initial bunching phase would entail pulsed excitation of correction quadrupoles. Such schemes leave the coherent frequencies unchanged but correct for the linear part of the incoherent space charge forces.

There do not appear to be suitable passive elements available for compensating the reactive coherent space charge components of Z_L . Passive magnetic and electrode type structures provide too low a reactive impedance to be of interest. Transverse instability feedback systems are usually designed to provide damping for the coherent motion. An extension is to provide both damping and an adjustment of the coherent Q-shift; to realise this in practice it is best to provide a separate feedback system for each mechanism.

Experiments on the SNS that would help to clarify coherent transverse

instability problems in heavy ion storage rings include:

- a) Threshold instability measurements as a function of SNS octupole settings.
- b) Instability growth rates as a function of octupole fields, and also of sextupole fields (to vary the chromaticity).
- c) Studies of combined damping and reactive compensation feedback systems,
- d) Effects of varying beam distributions and momentum spread.
- e) The use of modified RF shields.
- f) Beam transfer function measurements⁵.

5. FINAL TRANSPORT OF A BEAM BUNCH AT HIGH SPACE CHARGE LEVELS

Single bunches of heavy ions are extracted from the storage ring into individual beam lines and directed to the D-T target chamber. During transit, the bunch continues to compress by rotation in $(\Delta p/p, \phi)$ space with a progressive increase in the momentum spread and in the transverse and longitudinal space charge forces. The focusing for the transverse and longitudinal motion must be designed accordingly. For the transverse optics it is best to bring the dispersion to zero well ahead of the target. The longitudinal focusing is provided by the sections of induction linac.

There has been no complete design of a final heavy ion beam line though extensive theoretical and computational studies have been undertaken for unbunched beams⁶. These indicate adverse effects for beams with $K - V$ distributions when the transverse space charge forces reduce the phase shift per focusing cell by a factor approximately 2. Also, initial phase shifts per cell of 60° appear advantageous compared with phase shifts of 90° .

A computational study of the transport of an intense beam bunch is complex and an experimental investigation is preferable. This may be undertaken by extending the simulation of the heavy ion bunch motion from the SNS synchrotron to its external beam line. The extraction kicker pulse length is sufficient to extract one of two 70 MeV proton bunches, and since the external beam line is designed for 800 MeV operation, there is some flexibility in the optics available. At maximum bunch compression in the

synchrotron the space charge detuning reduces the Q-values by a factor approaching 2. The quadrupole focusing strengths are of the same order in the ring and beam line so the required high relative space charge detunings also occur for the proton bunches after extraction.

The length of the beam line is approximately that of the synchrotron circumference and this is insufficient to allow much additional bunch compression. The energy gain (loss) at the head (tail) of the bunch due to the longitudinal space charge forces is thus as indicated in section 2, ± 56 keV for the length of the line. The proton bunch length is 185 ns and induction linac sections providing 200 kV should be adequate for experimental investigations.

6. CONCLUSIONS

Reference designs for heavy ion drivers need to be more closely specified. The present driver specifications are at limits set from target confidence levels, arrived at by the Livermore Group. The entire driver design evolves from these basic parameters so an independent check of the results is highly important.

The choice between the induction linac and the storage ring driver is by no means an obvious one to make. The authors favour the latter because of the severe design problems in the later stages of the induction linac scheme; the final beam lines have to provide 180° bends so there is the complication in the optics of handling the dispersion and, physically, of providing the space for the induction linac sections. Also, the intense single bunch has to be split into a number of separate beam lines and the bunch-splitting is likely to result in some asymmetry of the transverse emittances, further complicating the beam transport.

The heavy ion system is now generally considered as having the greatest potential of the inertial - confinement systems (laser, electron, light-ion and heavy-ion). In the USA the funding levels do not reflect this fact and also the funding for heavy ions is small compared to that for magnetic confinement fusion. As a late starter in the fusion field this is perhaps to be expected. The one advantage that inertial-confinement will have as

compared to magnetic confinement is in the area of the target chamber where the geometry is so much more favourable for power removal.

The proposed studies appear to relate very well to heavy ion development work in other countries and particularly the USA. The five main American Groups involved and their areas of study are:

LBL; ion sources, low energy drift tube stages and induction linacs.
 ANL; ion sources, low- β linacs, multiple charge state storage ring drivers.
 BNL; megalac system, storage ring drivers (now less involved).
 Livermore; design of D-T targets and transport within the target chamber.
 Los Alamos; lead-laboratory, together with Livermore.
 ANL are the most likely of these to undertake an equivalent proposal.
 European and Japanese interests are not so clearly defined as yet.

The parameters given in this report indicate that many of the uncertainties in the dynamics for the heavy-ion fusion storage ring drivers could be resolved by experiments at the SNS with 70 MeV proton bunches. There does not appear to be a facility elsewhere quite as suitable for this purpose.

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NOTATION AND FORMULAE FOR LONGITUDINAL MOTION

Z/n = the reactive impedance due to the longitudinal space charge.
 $L = -jZ/n\omega_0$ is the negative inductance equivalent to the space charge impedance.
 f_0 = the proton revolution frequency, $\omega_0/2\pi$.
 Z_0 = the characteristic impedance of free space.
 g = a factor dependent on the beam and chamber geometry³.
 N = the number of particles per bunch.
 eV_{sc} = the energy gain or loss per turn at the extremities of a parabolic bunch.
 V = the amplitude of the assumed saw-tooth RF waveform for the bunch rotation.
 γ_t = gamma transition.
 n_q = the number of bunch revolutions in a 90° bunch rotation in $(\Delta p/p, \phi)$ space.
 $\Delta p/p$ = the peak momentum spread at the quarter synchrotron point.
 $E = \gamma E_0$ when E_0 is the rest energy of the ion.
 C = the bunch compression factor.
 θ_1 = the ratio of the peak momentum spread after and before compression.
 $D = FD_1$, where F is a bunch form factor (2/3).
 n = the actual number of ion revolutions in the storage ring ($\langle n_q \rangle$) during the bunch rotation process.

$$Z/n \approx -jgZ_0/2B\gamma^2$$

$$eV_{sc} = \pm 3NeL/2T^2$$

$$n_q = \pi/4h (\gamma^{-2} - \gamma_t^{-2}) (\Delta p/p)$$

$$V = 2E\pi^2\beta^2/eh (\gamma^{-2} - \gamma_t^{-2}) (4n_q)^2$$

$$n = n_q (2/\pi) (\cos^{-1} (c^{-1} - D^{-1}))$$

INCREASE IN MOMENTUM SPREAD WHILE DEBUNCHING IN A STORAGE RING

Let the maximum longitudinal space charge electric field at the head or tail of the bunch (non-relativistic case) be: $\bar{E}$
the instantaneous value of the beam peak momentum spread be: $\pm \Delta p/p$
and the instantaneous value of the bunch duration be: $\pm T$

$$\text{Then, } dT/dt = T = (\gamma^{-2} - \gamma_t^{-2}) \Delta p/p$$

$$d(\Delta p/p)/dt = (e\bar{E}rc) / E \beta^2 \gamma^2$$

$$\gamma dV/dt = (\gamma^{-2} - \gamma_t^{-2}) (e\bar{E} \beta c) / E \beta^2 \gamma^2$$

An approximation for $\bar{E}$ may be made by assuming that the bunch remains parabolic in shape during the debunching process. Two separate methods are used to estimate $\bar{E}$ under this approximation.

a) The usual formula for the electric field at the end of a parabolic bunch in a synchrotron is:

$$\bar{E} = 3 \text{ Neg} / 2 \beta^2 c^2 T^2 (4\pi\epsilon_0)$$

where g is a geometrical factor related to the transverse dimensions of the beam and the vacuum chamber. The formula is valid for a bunch of transverse dimensions much less than the bunch length, a condition not strictly true at the commencement of debunching.

b) The effect of the vacuum chamber boundary may be ignored and $\bar{E}$ calculated directly for the assumed parabolic bunch.

$$\bar{E} = 3 \text{ Neg} / 2 \beta^2 c^2 T^2 (4\pi\epsilon_0)$$

$$G = \left[\log \left(\frac{T + \sqrt{D^2 + T^2}}{D} \right) - \frac{4}{3} \frac{\sqrt{D^2 + T^2}}{T} \left(1 + \frac{T^2}{4D^2} \right) + \frac{T^2}{3D^2} + \frac{4D}{3T} \right]$$

$$D = a/2Bc \quad (a = \text{beam radius})$$

Solutions for γ , or $\Delta p/p$, may now be obtained for the cases a) and b).

Define $K = (\gamma^{-2} - \gamma_t^{-2}) \cdot 3Ne^2 Z_O / 2B\gamma^2 \cdot 4\pi E \beta^2$

T_O = initial value of T

For a), $\gamma \cdot d\gamma/dT = Kg/T^2$

$\gamma^2 = C - 2 Kg/T$

$(\Delta p/p)^2 = (\Delta p/p)_O^2 + 2 Kg (T_O^{-1} - T^{-1}) / (\gamma^{-2} - \gamma_t^{-2})^2$

For b), $(\Delta p/p)^2 = (\Delta p/p)_O^2 + 2 KH / (\gamma^{-2} - \gamma_t^{-2})^2$

$$H = \left[\frac{1}{T_O} \log \left(\frac{T_O + \sqrt{D^2 + T_O^2}}{D} \right) - \frac{1}{T} \log \left(\frac{T + \sqrt{D^2 + T^2}}{D} \right) + \frac{T - T_O}{3D^2} + \right.$$

$$\left. \frac{\sqrt{D^2 + T^2}}{3} \left(\frac{2}{T^2} - \frac{1}{D^2} \right) - \frac{\sqrt{D^2 + T_O^2}}{3} \left(\frac{2}{T_O^2} - \frac{1}{D^2} \right) + \frac{2D}{3} \left(\frac{1}{T_O^2} - \frac{1}{T^2} \right) \right]$$

Example:

$T_O = 5 \cdot 10^{-10} \text{ s}$
 $D = 6.25 \cdot 10^{-11} \text{ s}$
 $T = 25 \cdot 10^{-10} \text{ s}$

10 GeV U^+ Case 70 MeV Proton Case

$I = 250 \text{ mA}$ 20 mA
 $(\Delta p/p)_O = \pm 2 \cdot 10^{-4}$ $\pm 7 \cdot 10^{-4}$
 $H = 3.15 \cdot 10^9 \text{ s}^{-1}$ $3.15 \cdot 10^9 \text{ s}^{-1}$
 $g(T_O^{-1} - T^{-1}) = 3.2 \cdot 10^9 \text{ s}^{-1}$ $6.13 \cdot 10^9 \text{ s}^{-1}$
 $(\Delta p/p) = \pm 3.3 \cdot 10^{-4}$ $\pm 1.2 \cdot 10^{-3} \text{ (approx)}$

The relative growth in $(\Delta p/p)$ is comparable in the two cases.

Alternative formulae for debunching may be obtained from the reference 7 which treats the case of a linac beam.