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ABSTRACT

We have computed to first order in α_s the perturbative and non-perturbative contributions to the vacuum polarization from the pseudoscalar current in the unequal mass case.

QCD CONTRIBUTION TO VACUUM POLARIZATION II:
THE PSEUDOSCALAR UNEQUAL MASS CASE

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Recently⁽¹⁾ we have reported on the calculation to lowest non-trivial order in α_s of the vacuum polarization $\Pi(q^2)$ from all currents up to spin $J = 2^{++}$ in the case of equal masses for the quarks. We have followed the methods pioneered by Shifman, Vainshtein and Zakharov⁽²⁾ and included perturbative as well as non-perturbative contributions. The latter originate from higher dimensional operators in the operator product expansion for $\Pi(q^2)$

$$T_{\mu\nu} \dots \Pi(q^2) = \int e^{iqx} \langle 0 | T(j_{\mu} \dots(x) j_{\nu} \dots(0)) | 0 \rangle dx \quad (1)$$

in particular the gluon condensate operator $G_{\mu\nu}^a G^a_{\mu\nu}$ and the two-fermion operator $m_q \bar{q}q$ which get non-vanishing vacuum expectation values due to non-perturbative vacuum fluctuations. The importance of these non-perturbative effects for meson spectroscopy has been shown in numerous applications to light and heavy quark systems by Shifman et al^(2,3). We have extended their methods to include all charmonium states including the P-levels⁽¹⁾ and have reproduced the spectrum to a high degree of accuracy⁽⁴⁾.

To put the ideas of SVZ⁽²⁾ more to the test, it is important to make applications to other systems, in particular to the open charm meson states D, D*, F and F* which contain a heavy quark (m_c) and a light quark (m_u, m_s). We expect non-perturbative contributions from the gluon condensate to be

even more important here than for charmonium but still manageable by the moment methods we have developed⁽⁴⁾.

Therefore, in this letter we report on the calculations of the polarization operator $\Pi(q^2)$ induced by the flavour changing current

$$J_p = i \bar{q}_1 \gamma_5 q_2 \quad (2)$$

with masses m_1 and m_2 for the light q_1 and heavy quark q_2 respectively. The operator product expansion for this current including operators of dimension $d = 4$ is

$$\begin{aligned} \Pi(q^2) = & C_I + C_G \langle 0 | G_{\mu\nu}^a G^a_{\mu\nu} | 0 \rangle + \\ & + C_{m_1} \langle 0 | m_1 \bar{q}_1 q_1 | 0 \rangle + C_{m_2} \langle 0 | m_2 \bar{q}_2 q_2 | 0 \rangle \end{aligned} \quad (3)$$

where the coefficients C_I (perturbative part) and C_G, C_{m_1} and C_{m_2} (non-perturbative) have to be calculated.

1. Calculation of the perturbative part

The diagrams to be calculated to lowest order in α_s are given in fig.1. We have calculated the imaginary parts of these diagrams which gave the result

obtained before^(3,5). For m_1 or $m_2 \rightarrow 0$ there occur no mass singularities as required by the Kinoshita theorem⁽⁶⁾. Another interesting case is $m_2 \rightarrow \infty$ with $m_1, \frac{q^2}{m_2}$ fixed which reduces to the non-relativistic Coulombic limit $\frac{2\pi\alpha_s}{3v}$.

In the for us interesting case $m_1 (=m_u, m_s) \ll m_2 (=m_c)$ we expand to first order in m_1 and obtain the expression (the m_1 correction has only been taken into account for the bare loop).

$$\text{Im} \Pi^{\text{pert}}(q^2) = \frac{3}{8\pi} \frac{(q^2 - m_2^2)^2}{q^2} \left\{ 1 + \frac{2m_1 m_2}{(q^2 - m_2^2)} + \frac{4}{3} \frac{\alpha_s}{\pi} \times \right. \\ \times \left[\frac{13}{4} + 2\ell\left(\frac{m_2^2}{q^2}\right) + \ell n \frac{q^2}{m^2} \frac{q^2}{q^2 - m_2^2} + \frac{3}{2} \ell n \frac{m_2^2}{q^2 - m_2^2} \right. \\ \left. \left. + \ell n \frac{q^2}{q^2 - m_2^2} + \frac{2}{q^2} \ell n \frac{q^2 - m_2^2}{m_2^2} + \frac{2}{q^2 - m_2^2} \ell n \frac{q^2}{m_2^2} \right] \right\} \quad (6)$$

from which the perturbative contribution to the polarization operator can be found via a dispersion relation

$$\Pi^{\text{pert}}(q^2) = \frac{1}{\pi} \int_{m_c^2}^{\infty} \frac{\text{Im} \Pi^{\text{pert}}(s) ds}{s - q^2} \quad (7)$$

2. Calculation of the Non-Perturbative Parts

We will first consider the coefficients C_{m_i} of the two-fermion operators $m_i \bar{q}q$ in the operator expansion(3). The diagrams to be calculated are found by cutting the appropriate

$$\text{Im} \Pi^{\text{pert}}(q^2) = \frac{3}{8\pi} \frac{q^2}{q^2} v \left[1 + \frac{4}{3} \frac{\alpha_s}{\pi v} \right. \\ \times \left[(1 + v^2) \left(\frac{\pi^2}{6} + \ell n \frac{1+v}{1-v} \right) + \ell n \frac{1+v}{1-v} + 2\ell\left(\frac{1-v}{1+v}\right) + \ell\left(\frac{1+v}{2}\right) \right. \\ \left. - \ell\left(\frac{1-v}{2}\right) + \frac{1}{2} \sum_i \left[-4\ell(v_i) + \ell(v_i^2) + \ell\left(\frac{1+v_i}{2}\right) - \ell\left(\frac{1-v_i}{2}\right) \right] \right] \\ + \left(\frac{19}{16} - 3v + \frac{1}{8} v^2 + \frac{3}{16} v^4 \right) \ell n \frac{1+v}{1-v} + \frac{29}{8} v - \frac{3}{8} v^2 + \\ \left. + 6v \ell n \frac{1+v}{2} - 4v \ell n v + M(v, v_1, v_2) \right] \quad (4)$$

where

$$M(v, v_1, v_2) = \frac{1}{2} (1 + v^2) \sum_i \ell n \frac{1 + v_i}{1 - v_i} \ell n \frac{v_i}{v} \\ + \frac{1}{2} v \sum_i \left(\frac{1}{v_i} - \frac{1}{v} \right) \ell n \frac{1 + v_i}{1 - v_i} + 2v \ell n \frac{q}{q} \\ + \frac{3}{2} v \ell n \frac{m_1^2}{m_c^2} + v \left[\frac{(m_1 - m_2)^2}{q^2} v \ell n \frac{1 + v}{1 - v} - \frac{m_1^2 - m_2^2}{q^2} \ell n \frac{m_1}{m_2} \right] \quad (5)$$

$$\text{with } \bar{q}^2 = q^2 - (m_1 - m_2)^2, \quad v^2 = 1 - \frac{4m_1 m_2}{q^2},$$

$$v_1 = \frac{\bar{q}^2 v}{q^2 + m_1^2 - m_2^2}, \quad v_2 = \frac{\bar{q}^2 v}{q^2 - m_1^2 + m_2^2}, \quad m_2^2 = \frac{\max}{\min} (m_1, m_2)$$

$$\text{and the Spence function } \ell(x) = - \int_0^x dt t^{-1} \ell n(1-t).$$

In the limit $m_1 = m_2 = m$ we get $\bar{q}^2 = q^2$, $v = v_1 = v_2$, $M(v, v_1, v_2) = 0$ and eq.(4) reduces to the equal mass case

quark line in the graphs of fig.1(2). The results (to zeroth order in α_s) are given by

$$C_{m_1} = \frac{1}{8m_1^2 q^2} (q^2)^2 (1-v)^2 - \frac{1}{2m_1^2 q^2} (m_1 - m_2)^2 \quad (8)$$

and C_{m_2} the same with m_1 and m_2 interchanged.

Since only light quark operators $m\bar{q}q$ will get a non-vanishing vacuum expectation value, there will be no contribution from $\langle O | m_2 \bar{q}_2 q_2 | O \rangle$ in (3), but the coefficient C_{m_2} will play a role in the contribution to C_G .

There are two contributions to the coefficient C_G of $\langle O | G_{\mu\nu}^a G_{\mu\nu}^a | O \rangle$. The first is given by the diagrams in fig.1 with each gluon line cut. We have calculated these diagrams in two different ways. First, by following the method proposed by SVZ(2) and performing an expansion around $k = 0$ (where k is the gluon momentum). The calculations can then be done completely by computer. Secondly, we have considered the imaginary part of the cut diagrams which can be viewed as gluon scattering off a colorless current

Both methods give the same result which is equal to

$$C_{G1} = \frac{\alpha_s}{48\pi} \cdot \frac{1}{4m_1 m_2} \cdot \frac{q^2}{q^2} \left[\frac{3(3v^2+1)(1-v)^2}{v^4} - \frac{1}{2v} \ln \frac{1-v}{1+v} - \frac{9v^4+4v^2+3}{v^4} \right] \quad (10)$$

with q^2 and v as defined above. From comparison with the result of SVZ in Ref.3 for the equal mass case, it can be seen that (apart from the overall factor $q^2/\bar{q}^2$) eq.(10) can be obtained from the equal mass case by replacing q^2 by $\bar{q}^2$ and m_C^2 by $m_1 m_2$. It can also be seen that (10) is divergent for one or both masses going to zero. These mass singularities are generated by loop momenta $p^2 \sim m_i^2$. They are cancelled by the second contribution to C_G which comes from the operators $m\bar{q}q$, and is equal to

$$C_{G2} = (C_{m_1} + C_{m_2}) \frac{1}{12} \cdot \frac{\alpha_s}{\pi} \quad (11)$$

with C_{m_i} given by (8). Adding (10) and (11), and expanding to first order in the small quark mass m_1 , we find

$$C_G = \frac{\alpha_s}{48\pi} \cdot \frac{1}{2(m_1 - m_2)^2} \left[-8 + 24 \frac{m_1 m_2}{q^2 - m_2^2} \ln \frac{q^2 - m_2^2}{m_1 m_2} - 12 \frac{m_1 m_2}{q^2 - m_2^2} \right] \quad (12)$$

Defining the moments M_n for the polarization operator $\Pi(q^2)$ in eq.(3) in the usual way(2)

$$M_n = \frac{1}{\pi} \int_{m_C}^{\infty} \frac{\text{Im} \Pi(s) ds}{s^{n+2}} = \frac{1}{(n+1)!} \left(\frac{d}{dq} \right)^{n+1} \Pi(q^2) \Big|_{q^2=0} \quad (13)$$

we can derive the moment equation

$$M_n = A_n \left[1 + n \frac{m_1}{m_2} + a_n \alpha_s + b_n \frac{\langle O | \frac{g_s^a}{\pi} G_{\mu\nu}^a G_{\mu\nu}^a | O \rangle}{m_2^4} \right] + C_n \frac{\langle O | m_1 \bar{q}_1 q_1 | O \rangle}{m_2^4} \quad (14)$$

where the bare loop normalization factor A_n and the dimensionless coefficients a_n , b_n and C_n can easily be obtained from (6), (12) and (8) (after expanding to first order in m_1):

$$A_n = \frac{3}{4\pi^2} \cdot \frac{1}{(m_2^2)^n} \frac{1}{n(n+1)(n+2)} \quad (15a)$$

$$a_n = \frac{4}{3\pi} \left\{ \frac{17}{4} + \frac{\pi^2}{3} - \frac{3}{2n} - \frac{5}{2(n+1)} - \frac{9}{2(n+2)} - \frac{23}{2(n+3)} + \frac{12}{n+4} - \frac{6}{(n+3)^2} + 4(\pi^2 - 9) \left(\frac{2}{n+3} - \frac{7}{n+4} + \frac{5}{n+5} \right) - \psi'(n+3) + \left(\frac{3}{2} + \frac{1}{n} + \frac{1}{n+1} + \frac{1}{n+2} - \frac{3}{n+3} \right) (\psi(n+3) - \psi(3)) - 3n \ln 2 \right\} \quad (15b)$$

where ψ and ψ' are derivatives of the gamma function. The last term in (15b) results from the mass renormalization.

$$b_n = \frac{\pi^2}{18} n(n+1)(n+2) \left\{ -4(n+1) + 6 \frac{m_1}{m_2} (n+1)(n+2) + (\psi(n+3) - \psi(3)) + 6(n+1)(n+2) \frac{m_1}{m_2} \ln \frac{m_1}{m_2} + 3(n+1)(n+2) \frac{m_1}{m_2} \right\} \quad (15c)$$

$$C_n = \frac{2\pi^2}{3} n(n+1)(n+2) \left[n+3 - 2 \frac{m_2}{m_1} \right] \quad (15d)$$

Similar calculations can be performed for the vector current and are in progress. We hope to report on the results soon as well as on the applications to the spectroscopy of the open charm states D, D^*, F and F^* .

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FIGURE CAPTIONS

- Fig. 1 Diagrams that contribute to the perturbative part of the vacuum polarization. Curly lines depict gluons, continuous lines quarks, and dashed lines currents. Diagrams that contribute to the non-perturbative parts, are obtained by cutting the appropriate internal lines as indicated in the text.

FIGURE 1