AN INTRODUCTION TO TECHNICOLOUR

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1. **INTRODUCTION**

These lectures will be concerned with an attempt to replace the elementary Higgs scalars of gauge theories with composite ones. Our composite Higgs scalars will be mesonic bound states of a new strong interaction\(^1,2\), analogous to QCD in that it is described by an asymptotically free, unbroken gauge group which acts on a new degree of freedom called Technicolour\(^3\). We will call technifermions the fermions that carry technicolour. We will call technihadrons the technicolour singlet bound states of technifermions, and so on. Our Higgs scalars will be technimesons. The technimesons take vacuum expectation value in analogy with QCD where \((\bar{q}q) \sim (\sigma) = f_\pi\), the pion decay constant; \(f_\pi = 95\) MeV is of order \(\Lambda_C\), the scale of QCD. If we use a technicolour interaction to break the electroweak gauge group \(SU_L(2) \times U_Y(1) \to U_Y(1)\), its scale \(\Lambda_{TC}\) must be of order 1 TeV so that \(m_W = (g/2)F_\pi \approx 80\) GeV, where \(F_\pi\) is the technipion decay constant: \((\bar{\Phi}F) \sim (\Sigma) = F_\pi\). Moreover, the technifermions \(F\) need to have non-trivial \(SU_L(2) \times U_Y(1)\) transformation properties.

Before going into a more detailed description of technicolour, we must answer the reader's likely query as to what causes our dislike of the elementary Higgs scalar.

Higgs scalars are used in gauge theories to cause spontaneous symmetry breakdown\(^4\). If the Higgs scalar in a given gauge theory is elementary, then its self-couplings (parameters in the Higgs potential) and its Yukawa couplings to the fermions are constrained only by the requirements of gauge invariance. As a result, a gauge theory with elementary Higgs scalars has many arbitrary parameters associated with the Higgs fields in addition to those more properly associated with the gauge symmetry, i.e. the gauge coupling constants. In the case of the standard \(SU_L(2) \times U_Y(1)\) electroweak model\(^5\), this arbitrariness of the elementary Higgs scalar couplings translates into the fact that the quark and lepton masses, the weak mixing angles, and the \(W^\pm\) mass cannot be calculated but must be introduced as parameters into the theory. We find this unsatisfactory.
If the standard model is "grand unified" into a simple gauge group G, spontaneous symmetry breaking by elementary Higgs scalars becomes even more unreasonable. Although some new relations among Yukawa couplings (e.g. $m_b = m_t$, ...) can occur as a result of the larger gauge symmetry G, the number of arbitrary parameters is still very high, much too high to accept grand unified theories (GUTs) as the definitive description of all of particle physics but gravity. Moreover, in GUTs an acute problem with regard to elementary Higgs scalars arises, the so-called gauge hierarchy problem, which can be described as follows.

A GUT requires at least two scales of spontaneous symmetry breakdown (SSB), corresponding to $G = SU_L(2) \times U_Y(1) \times SU^C(3)$ at energy scale $M$, and $SU_L(2) \times U_Y(1) \times SU^C(3) + U^{EM}(1) \times SU^C(3)$ at scale $\mu$. The scale $\mu$ is fixed by the weak interaction strength (Fermi coupling) to be of order 250 GeV. The scale M is expected to be of order $10^{14}$ GeV both as a result of the lower limit on the proton lifetime and because of the (almost) successful prediction of $\sin^2 \theta_W$ in the simplest models. If these SSBs are due to elementary Higgs scalars, the ratio $(\mu/M)^2 \approx 10^{-2n}$ has to be introduced by hand and readjusted to 24 decimal places in each order of perturbation theory. That is unsatisfactory.

On the other hand, GUTs are successful in relating gauge coupling constants. Those of the standard model -- $g$, $g'$, and $g_S$ -- can all be determined from knowledge of the value of the GU gauge coupling constant $g_0$ at the GU mass scale $M$, and use of the renormalization group equations. For example,

$$\Lambda^c = M \exp \left[ \frac{-2\pi}{(11 - \frac{2}{3} n_f) \lambda_0} \right] \quad (1.1)$$

where $n_f$ is the number of quark flavours. In GUTs the scale of QCD is calculable in perturbation theory. If technicolour were grand unified with the other gauge interactions into a simple gauge group

$$G \rightarrow SU_L(2) \times U_Y(1) \times SU^C(3) \times G^{TC} \quad (1.2)$$

then the scale $\Lambda^{TC}$ of the technicolour gauge interactions would be similarly calculable. This in turn would determine the scale $\mu$ of the SSB:
SU_L(2) × U_Y(1) + U^{em}(1). In other words, the ratio \((u/M)^2\) would be calculable in such a theory. Like \((Λ^G/M)^2\) it will tend to be a small number.

Let us thus envisage the standard model without elementary Higgs scalar but complemented with a new QCD-like gauge group \(G^TC\), as in the right-hand side of Eq. (1.2). Such a theory would only contain fermions and gauge fields in its Lagrangian. As a consequence it would have only gauge coupling constants as arbitrary parameters, and these will be few in number. Moreover, if there is grand unification, all gauge coupling constants will be calculable in terms of a single number, e.g. \(α_0\) at the Planck mass. Such a framework might truly encompass all of particle physics but gravity in the sense that all physical quantities -- including the masses of the quarks and leptons, those of the \(W^\pm\) and \(Z^0\), the Cabibbo angles, etc. -- would in principle be determined in terms of that single parameter.

By no means has such a successful theory been constructed yet. On the other hand, progress has been made towards the realization of the program, and it is the purpose of these lectures to review some of this progress. The general condition which assures that the technicolour interaction breaks \(SU_L(2) × U_Y(1) + U^{em}(1)\) in the desired way (i.e. \(M_W = M_Z \cos θ_W\)) has been established\(^1\)). This will be discussed in Sections 2 and 3. A mechanism\(^9,10\) has been found to give mass to the quarks and leptons. It will be described in Section 4. In Section 5, we will give the general conditions\(^11\) under which one can have both \(M_u = M_d \cos θ_W\) and \(m_u ≠ m_d\). Section 6 will describe an economical way, "Tumbling"\(^12\), in which a hierarchy could be established in the quark and lepton mass matrices. Finally, in Section 7, we will discuss the experimental implications of technicolour.

2. DYNAMICAL BREAKDOWN OF THE ELECTROWEAK GAUGE SYMMETRIES BY THE COLOUR INTERACTIONS\(^1\)

Most of our ideas about dynamical symmetry breaking are derived from our experience with QCD. To pave the way for technicolour, let us thus review the generally accepted lore about low-energy QCD.
Consider the standard $SU_L(2) \times SU_Y(1) \times SU_C(3)$ gauge theory with its quarks and leptons but without the elementary Higgs scalar. Since there is no Higgs vacuum expectation value to break the electroweak $SU_L(2) \times SU_Y(1)$ symmetry, it seems at first sight that the $W^\pm, Z^0$ gauge fields, the quarks and the leptons must all remain massless along with the photon and the gluons. But that is not so.

Let us for a moment turn off the weak and electromagnetic interactions, keeping only the QCD part of the standard model. For simplicity, we assume only two flavours of quarks: $u$ and $d$. Since these are massless, the QCD Lagrangian has an exact $SU_L(2) \times SU_R(2) \times U_Y(1)$ flavour symmetry, called chiral isospin $\times$ baryon number. The axial $U_A(1)$ symmetry has an anomaly due to instantons. We are in the so-called "chiral limit" of the strong interactions. The successes of a decade of strong interaction physics\textsuperscript{13} under the headings "current algebra", "CVC and PCAC", "chiral perturbation theory", ... teach us that:

$$\langle \bar{u}u \rangle_o = \langle \bar{d}d \rangle_o \neq 0$$

(2.1)

i.e. the QCD vacuum is only isospin $SU_{L+R}(2)$ and baryon number $U_Y(1)$ symmetric.

The spontaneous dynamical breakdown of $SU_L(2) \times SU_R(2) \rightarrow SU_{L+R}(2)$ implies the existence of three Goldstone bosons. These are the pions $\pi^\pm, \pi^0$ which are exactly massless in the chiral limit. Before turning on the weak and electromagnetic interactions, we wish to make the following remarks:

1) The amount of spontaneous symmetry breaking (2.1) is given by $f_\pi$, the pion decay constant, in the following way. Define the scalar and pseudoscalar fields $\sigma$ and $\pi$ which have the quantum numbers of the quark bilinears $\bar{q}q$ and $i\bar{q}Y_2q$, where $q = \begin{pmatrix} u \\ d \end{pmatrix}$. $(\sigma, \pi)$ transform as the $(2,2)$ representation of $SU_L(2) \times SU_R(2)$. The scale of these fields is set by the requirement that their effective Lagrangian\textsuperscript{14} contain the usual kinetic energy term $\frac{1}{2}(\partial_\mu \sigma)^2 + \frac{1}{2}(\partial_\mu \pi)^2$. Then $f_\pi = \langle \sigma \rangle_o \approx 95$ MeV. The spontaneous symmetry breakdown of chiral isospin implies the PCAC relations:

$$i \partial^\mu J^{\sigma\mu} = f_\pi \partial^\mu \partial_\mu \pi$$

(2.2)

for the axial isospin currents.
2) The QCD vacuum is (presumably) exactly isospin and CP invariant. It could have broken isospin by \( \langle \bar{u}u \rangle \neq \langle \bar{d}d \rangle \), and CP by \( \text{Arg} \) \( \langle \bar{u}u\rangle \langle \bar{d}d \rangle \rangle = 4\theta_{\text{QCD}} \neq 0 \). The QCD vacuum is the minimum of an effective potential which is \( SU_L(2) \times SU_R(2) \) and CP invariant but which is otherwise unknown. That the minimum be exactly isospin and CP symmetric is "natural" (has finite probability) but not necessarily true\(^{11}\). However, Coleman and Witten\(^{15}\) have shown that if we replace the strong interaction colour \( SU(3) \) by \( SU(N) \), then the vacuum is necessarily isospin symmetric in the \( N \rightarrow \infty \) limit.

3) Another result which holds in that large \( N \) limit is the following\(^{14}\):

\[
\frac{\mathcal{F}_\pi}{m_\pi} \sim \sqrt{N}, \quad \frac{\langle q \bar{q} \gamma_5 \rangle}{m_\pi^3} \sim N
\]  

\( (2.3) \)

Next we turn on the weak and electromagnetic interactions. The \( \sigma \) and \( \pi^\pm \) fields transform as a doublet of \( SU_L(2) \times U_Y(1) \):

\[
\Phi_{\text{eff}} = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} = \begin{pmatrix} \pi_2 + i\pi_1 \\ \sigma - i\pi_3 \end{pmatrix}
\]  

\( (2.4) \)

The vacuum expectation value \( \langle \sigma \rangle = f_\pi \) thus breaks the electroweak gauge group \( SU_L(2) \times U_Y(1) \rightarrow U^\text{EM}(1) \). The pions \( \pi^\pm, \pi^0 \) become the longitudinal components of the \( W^\pm \) and \( Z^0 \) vector bosons which acquire mass:

\[
M_w = \frac{g}{2} f_\pi \approx 30 \text{ MeV}
\]  

\( (2.5) \)

Because of the isospin symmetry of the strong interactions, the relation

\[
M_w = M_z \cos \theta_w
\]  

\( (2.6) \)

holds to \( \text{all} \) orders of the strong interactions\(^1\). This is a useful result since higher order QCD corrections could not have been calculated in a reliable way.

We will come back to this issue in more detail under the heading "\( M_w = M_z \cos \theta_w \) versus \( m_u \neq m_d \)" (Section 5).

It should already have been clear that one can have spontaneous breakdown of a gauge symmetry without an elementary scalar field. After all, the breakdown of the electromagnetic gauge symmetry in a superconductor is due to the condensation
of Cooper pairs, which act as an effective composite Higgs field. The interesting thing is that a gauge theory like QCD produces spontaneous symmetry breaking through an effective Higgs field, with as a bonus Eq. (2.6), if the right circumstances are met.

3. TECHNICOLOUR

Except for the scale, QCD breaks the electroweak gauge symmetries down in exactly the right way. We thus introduce a new gauge group, technicolour, which is in every respect similar to SU$_C^C$(3) except that its characteristic scale $\Lambda_{TC}$ is such as to give the phenomenologically correct mass for the $W$:

$$M_W = g/2 \; F_\pi \simeq 80 \; \text{GeV}$$

(3.1)

which implies that the technipion decay constant $F_\pi = 250$ GeV, and that $\Lambda_{TC}$ is of order 1 TeV. Technicolour interactions must be asymptotically free, and they must have an isospin-like flavour symmetry group that assures Eq. (2.6) to all orders of the TC interactions$^1)$. We will assume that technicolour is confined: all physical states are TC singlets. Besides the requirements of asymptotic freedom and of an isospin-like flavour symmetry there appear to be (unfortunately) no other a priori restrictions on the technicolour interactions. There is, however, good reason to believe that the colour singlet technifermions should not transform as a pseudoreal representation of the technicolour group. We will come back to this point in Section 7.

Thus consider a gauge theory based on SU$_L$(2) $\times$ U$_Y$(1) $\times$ SU$_C^C$(3) $\times$ SU$_{TC}^C$(3) with all the usual fermions (quarks and leptons) plus a set of technifermions:

$$\begin{pmatrix}
\mathcal{U}_L \\
\mathcal{D}_L
\end{pmatrix} = \begin{pmatrix} 2, & 1/6, & 1, & 3 \end{pmatrix}$$

$$\begin{pmatrix}
\mathcal{U}_R \\
\mathcal{D}_R
\end{pmatrix} = \begin{pmatrix} 1, & 2/3, & 1, & 3 \end{pmatrix}$$

$$\begin{pmatrix}
\mathcal{D}_R
\end{pmatrix} = \begin{pmatrix} 1, & -1/3, & 1, & 3 \end{pmatrix}$$

(3.2)

We choose the TC group to be SU(3) merely as an example. The quarks and leptons are TC singlets. The TC interactions behave in complete analogy with the colour
interactions but for the scale which has changed from $\Lambda^C$ to $\Lambda^{TC}$. So, everything
described in the previous section now applies to the technicolour world. In par-
ticular, $(\Sigma) = F_\Sigma$ where $\Sigma$ is the technisigma field and $F_\Sigma \approx 250 \text{ GeV}$ is the techni-
pion decay constant. The technipions become the longitudinal components of the
$W^\pm$ and $Z^0$ which acquire mass (3.1). The relation $M_W = M_Z \cos \theta_W$ holds true to
all orders of the TC interactions because of isospin conservation in the TC world.

$\Lambda^{TC}$ must be of order 1 TeV. At that energy scale there will be a rich techni-
hadron spectrum with typical 1 TeV splittings. For example, the technirho will
have mass of order

$$m_{\rho} = m_{\rho} \frac{F_\rho}{F_\pi} \sqrt{\frac{3}{N}} \sqrt{\frac{1}{n}}$$

(3.3)

if the TC group is SU(N) -- see Eq. (2.3) -- and $n$ is the number of isospin doub-
lets of technifermions. Even if some technihadrons happen to be light (cf. the
pseudo-Goldstone bosons discussed in Section 7), they will appear elementary un-
less probed at distance scales smaller than about 1 TeV$^{-1}$.

The picture as developed so far has a very serious shortcoming: the leptons
and quarks have remained massless. The quarks of course do acquire a "constituent"
mass from the chiral symmetry breakdown in QCD, but they have failed so far in our
picture to acquire a "current" mass. In the model described above the quarks and
leptons have separate chiral symmetries which remain unbroken even though the
chiral symmetries of the technifermions get broken. On the other hand in the
standard model with the elementary Higgs scalar the Yukawa couplings assure that
the chiral symmetries of the fermions get broken as soon as $\langle \phi \rangle \neq 0$. It thus
appears that we must find a way to produce effective Yukawa couplings between the
ordinary fermions and the technifermions.
4. **EXTENDED TECHNICOLOUR**

In order to give masses to the quarks and leptons we must get rid of the separate chiral symmetries of technifermions and ordinary fermions. Yet the ordinary fermions must remain TC singlets. Also one does not wish to introduce anything else but gauge interactions. The following solution\(^9,10\) was proposed.

The technifermions and ordinary fermions are placed in common multiplets of an "extended technicolour" gauge group. Extended technicolour (ETC) breaks down to technicolour (TC) at some energy scale \(\mu\). The ordinary fermions are singlets of TC (but not ETC). TC interactions are asymptotically free and become strong at \(\Lambda^{TC} \ll \mu\). There the technifermions condense and the electroweak symmetries are broken as before. The multiplet and group structure can be represented as follows:

\[
\begin{pmatrix}
F \\
F \\
F \\
\bar{f}
\end{pmatrix}
\xrightarrow{TC}
\xrightarrow{ETC}
\text{massive } \xi \\
\text{vector bosons}
\]

where \(F\) are technicoloured and \(f\) are ordinary fermions. The \(\xi\) vector bosons which are in ETC but not TC acquire mass of order \(M_\xi \approx \frac{g_{ETC}}{\Lambda^{ETC}}\), and couple to currents of the form \(\bar{F}\gamma_\mu F\). Their effects at energy scales well below \(M_\xi\) can in first approximation be neglected. At \(\Lambda^{TC}\) the technifermions thus condense as before.

Now, the four-fermion interactions mediated by the \(\xi\) vector bosons have the form:

\[
\frac{1}{2} \left( \frac{g_{ETC}}{M_\xi} \right)^2 (\bar{F} \gamma_\mu F) (\bar{F} \gamma^\mu F)
\]

(4.1)

By a Fierz transformation we obtain:

\[
-\frac{1}{2M_\xi^2} \left[ (\bar{F}F)(\bar{F}F) - (\bar{F}\gamma_5 F)(\bar{F}\gamma_5 F) + \cdots \right]
\]

(4.2)

The condensation of technifermions \((\bar{F}F)_0 \neq 0\), then produces a mass for the ordinary fermions:
\[ m_F = \frac{1}{2\mu^2} \langle \bar{F}F \rangle \]

(4.3)

Since \( \langle \bar{F}F \rangle \sim (1 \text{ TeV})^3 \), one needs \( \mu \sim 30 \text{ TeV} \) to produce \( m_F \sim 1 \text{ GeV} \). The exchange of heavy \( \varepsilon \) vector bosons produces effective Yukawa interactions between the technimesons \( \bar{F}F \) (\( \bar{F}F \) bound state) and the ordinary fermions \( f \) (see Fig. 1a). From Eq. (3.2), we can see that the scalar technimesons \( \bar{F}F \) couple to \( \bar{f}f \) whereas the pseudoscalar technimesons \( \bar{F}F \) couple to \( \bar{f}f \). When the scalar technimesons acquire expectation value, the ordinary fermions acquire mass (Fig. 1b).

It was implied in the above description of extended technicolour that the ETC gauge interactions are orthogonal to the other (electroweak and colour) gauge interactions. In that case, the \( \varepsilon \) vector bosons are \( SU_L(2) \times SU_Y(1) \times SU^C(3) \) singlets. To give mass to all the ordinary fermions, an entire family of technifermions is needed, i.e. eight of them:

\[ U_R, U_L, U_B, N, D_R, D_L, D_B \text{ and } E \]

each being a four-component Dirac spinor. One family of technifermions can, however, give mass to several families of ordinary fermions, provided the ETC gauge group breaks down to TC in several successive stages. For example, a three family model could be constructed by having the sequential breakdown:

\[ \begin{align*}
\text{ETC} & \quad \mu \quad \varepsilon' \quad \mu' \quad \varepsilon'' \quad \mu'' \\
SU(6) & \quad \rightarrow \quad SU(5) & \quad \rightarrow \quad SU(4) & \quad \rightarrow \quad SU(3)
\end{align*} \]

(4.4a)

with the fermions

\[ \begin{pmatrix} F & F & F & F & F' & F' & F' \end{pmatrix} \]

(4.4b)

transforming as sextets of ETC. In that case, the first family \( f = \{ e, \nu, u, d \} \) would have mass \( m_F = (1/2\mu^2)(\bar{F}F) \), the second family \( f' = \{ \mu, \nu', c, s \} \) would have mass \( m_{F'} = (1/2\mu'^2)(\bar{F}F) \), and so on. To avoid the intrafamily degeneracies of this overly simple model, one must have more varied choices of ETC fermion representation content and more complicated \( \varepsilon \) vector boson mass matrices than in (4.4).

The question arises, however, of how to avoid isospin degeneracies in the quark
and lepton mass matrices since some isospin-like flavour symmetry was needed to obtain $M_W = M_Z \cos \theta_W$. This problem will be addressed in the next section.

There is another way\(^{17}\) in which the extended technicolour mechanism can be implemented. One unifies colour and technicolour at some energy scale $\mu$, again of order 30 TeV:

$$SU_L(2) \times SU_R(1) \times G^S \quad \mu \rightarrow SU_L(2) \times SU_y(1) \times SU_C(3) \times G^{TC} \quad (4.5)$$

Effective Yukawa interactions between technimesons and ordinary fermions are now produced by exchange of the heavy vector bosons associated with the generators of $G^S/(SU^C(3) \times G^{TC})$. This new type of $S$ vector boson carries colour but is a $SU_L(2)$ singlet. It couples ordinary leptons to techniquarks ($Q = U, D$) and ordinary quarks to technileptons ($L = E, N$) or antitechniquarks, producing the type of effective Yukawa interactions described by Fig. 2. For more details on this type of extended technicolour mechanism, we refer the reader to the model by Farhi and Susskind\(^{17}\).

Whether the $S$ vector bosons are coloured or not, it seems necessary that there be (at least) one complete family of technifermions to give masses to all the ordinary fermions by the ETC mechanism. The flavour symmetry\(^{18}\) of the TC interactions is then $SU_L(8) \times SU_R(8)$ or $SU(16)$, depending on whether the technifermions transform according to a complex or a real representation of $G^{TC}$. The technifermion condensates:

$$\langle \bar{U}_R U_L \rangle = \langle \bar{U}_w U_w \rangle = \langle \bar{B}_B U_B \rangle = \langle \bar{N} N \rangle$$

$$\langle \bar{D}_R D_L \rangle = \langle \bar{D}_w D_w \rangle = \langle \bar{B}_B D_B \rangle = \langle \bar{E} E \rangle \quad (4.6)$$

then break $SU_L(8) \times SU_R(8) \rightarrow SU(8)$ if TC is a complex representation, $SU(16) \rightarrow O(16)$ if TC is a real representation with a symmetric metric, or $SU(16) \rightarrow Sp(16)$ if TC is a real representation with an antisymmetric metric. In any case, a large number of Goldstone and pseudo-Goldstone particles\(^{18}\) are produced when the technicolour spontaneous symmetry breakdown occurs. Three of them become the longitudinal components of the $W^\pm$ and $Z^0$. The others acquire mass when the other interactions $\longrightarrow SU_L(2) \times SU_y(1) \times SU^C(3)$, ETC, etc. $\longrightarrow$ are turned on. Their spectrum, their production and decays will be discussed under the heading "The pseudo-Goldstone bosons of Technicolour" in Section 7.
To give masses to several families of ordinary fermions, a sequential breaking of ETC → E' + TC → ... → TC was required. The reader may ask how these successive SSBs occur if the use of elementary Higgs scalars must be avoided at all costs. Does one need to introduce a new technicolour interaction to perform each one of the successive ETC symmetry breakdowns? Must the technicolour gauge groups therefore proliferate? The answer is no if one decides instead to make use of "tumbling"\(^{12}\), a concept that will be discussed in Section 6.

In the next section we will turn to the question of how to produce isospin splittings in the ordinary fermion mass matrix while keeping some isospin-like symmetry to ensure \(M_W = M_Z \cos \theta_W\).

5. \(M_W = M_Z \cos \theta_W\) \text{ VERSUS } m_u \neq m_d \quad (11)

The general requirements for an interaction to respect the relation

\[
M_W = M_Z \cos \theta_W
\]

are the following:

1) that it conserves electric charge;

2) that it conserves a "custodial" SU(2), which is defined as a SU(2) symmetry of both the Lagrangian and the vacuum, under which the generators (or the gauge fields) of SU\(_L\) transform as a triplet.

Proof: The contributions from that interaction to the SU\(_L\) \(\times U_1\) vector boson mass matrix must be custodial SU(2) symmetric. The vector bosons transform as \((3 + x)\) of the custodial SU(2), \(x\) indicating the undefined transformation property of the hypercharge vector boson. The vector boson (mass)\(^2\)-matrix transforms as

\[
\begin{align*}
\{(3 + x) \times (3 + x)\}^{\gamma \gamma'}_{\gamma \gamma''} &= \{3 \times 3\}^{\gamma \gamma'}_{\gamma \gamma''} + \{x \times x\}^{\gamma \gamma'}_{\gamma \gamma''} \\
&= 1 + s + x' + x''
\end{align*}
\]

(5.2)

The singlet, \(x'\) and \(x''\) are in general different from zero (\(x'\) and \(x''\) may contain singlets), whereas the quintuplet must vanish. The vector boson (mass)\(^2\)-matrix thus has the form:
\[
\begin{pmatrix}
A_3^\mu & A_1^\mu & A_2^\mu & A_0^\mu \\
\mu^\mu & 0 & 0 & \mu^2 \\
0 & m^2 & 0 & 0 \\
0 & 0 & m^2 & 0 \\
\mu^2 & 0 & 0 & m_0^2
\end{pmatrix}
\begin{pmatrix}
A_3^\mu \\
A_1^\mu \\
A_2^\mu \\
A_0^\mu
\end{pmatrix}
\]

(5.3)

The requirement that the photon
\[
A_\mu = \cos \theta_w A_3^\mu + \sin \theta_w A_1^\mu
\]
\[
(Q^5 = Q^3 + \gamma \, \text{tan} \theta_w = g'/g)
\]

be massless then forces the \((\text{mass})^2\)-matrix to have the form
\[
\begin{pmatrix}
q^2 & 0 & 0 & qq' \\
0 & q^2 & 0 & 0 \\
0 & 0 & q^2 & 0 \\
qq' & 0 & 0 & q^4
\end{pmatrix}
\]

(5.5)

which yields Eq. (5.1).

For example, the strong (colour) interactions conserve electric charge and isospin, which is a custodial SU(2), and therefore Eq. (5.1) holds to all orders of the strong interactions\(^1\). On the other hand, the electroweak interactions do not conserve a custodial SU(2) and therefore Eq. (5.1) is violated by order \(\alpha\) corrections\(^1\)

\[
M_w = M_Z \cos \theta_w (1 + O(\alpha))
\]

(5.6)

Also, in the standard model with one elementary Higgs doublet, the Higgs self-interactions conserve a custodial SU(2). Indeed the Higgs potential \(V(\phi^+\phi)\) is automatically \(O(4) = SU_L(2) \times SU(2)\) invariant since

\[
\phi^+\phi = \phi^-\phi^+ \quad \text{where} \quad \phi^- = (\phi_o, \phi, \phi_2, \phi_3)
\]

(5.7)

\[
\phi = \begin{pmatrix}
\phi^o \\
\phi^-
\end{pmatrix}
\]

\(\langle \phi_o \rangle \neq 0\) breaks \(SU_L(2) \times U_Y(1) \rightarrow SU^\text{em}(1)\) and \(SU_L(2) \times SU'(2) + SU(2)\), the diagonal subgroup. Equation (5.1) thus holds to all orders of the Higgs self-interactions. Note that if there are several Higgs doublets, the conservation of a custodial...
SU(2) is not automatic. Unless some special care is taken to impose one, Eq. (5.1) will be violated in higher orders. Finally, because the Yukawa interactions violate isospin, there are additional corrections to Eq. (5.1) which becomes

$$M_w = M_z \cos \theta_w \left(1 + O(\alpha) + O(d \frac{m_u^2 - m_d^2}{M_w^2}) \right)$$

(5.8)

Unless some isospin doublets are very badly split, the various corrections are within the present experimental constraints on allowed deviations (a few percent) from Eq. (5.1). With experiment improving, we can look forward to a test in the not too distant future of the gauge theoretical and renormalizable character of the electroweak interactions.

We are now fully equipped to discuss our question in the context of technicolour models. To assure Eq. (5.1) we must require that the technicolour interactions conserve a custodial SU(2). The latter can (but need not) be the isospin group as in the previously discussed example [Eq. (3.2)]. Now, the ordinary fermions acquire their masses from the TC condensates through the ETC interactions, as discussed in the previous section. It is clear that if isospin is a flavour symmetry of both TC and ETC, then \(m_u = m_d, \ m_c = m_s, \ldots\) will follow. If the custodial SU(2) is TC isospin, it is necessary that isospin be violated by ETC. This requirement can, in general, be implemented since TC is a subgroup of the ETC gauge group and therefore

$$\text{TC} \overset{\text{flavor}}{\rightarrow} \text{ETC} \overset{\text{flavor}}{\leftarrow} \text{symmetry}$$

(5.9)

Equation (5.9) allows the possibility that isospin be a flavour symmetry of TC but not of ETC (see Fig. 3a).

As an example, consider the following gauge group:

$$\text{TC} \overset{SU_L(2) \times U_Y(1) \times SU(3) \times SU(3) \times Sp(6)}{\leftarrow}$$

(5.10)

and the fermion representation content:
\[
\begin{pmatrix}
U_L \\
D_L
\end{pmatrix}
= \begin{pmatrix}
2, \frac{1}{3} , 3 , 3, 1 \\
1, \frac{2}{3} , 3 , 3, 1
\end{pmatrix}_L
\]
\[
\begin{pmatrix}
U_R \\
D_R
\end{pmatrix}
= \begin{pmatrix}
1, \frac{2}{3} , 3 , 3, 1 \\
1, 0 , 1 , \frac{2}{3} , 6
\end{pmatrix}_R
\]
(5.11)

There are no ETC or T'C anomalies. The anomalies involving the electroweak gauge group must be cancelled by the introduction of leptons and techni-leptons. Let us assume the T'C gauge coupling becomes large at \( \Lambda^{T'C} \sim 30 \) TeV. There the following condensate forms

\[
< \tilde{F}^a_{Lij} (-i\sigma_2) \tilde{F}^b_{Lij} \eta^{ij} \varepsilon_{abc} >_0 \approx \delta_{ab} (\Lambda^{T'C})^3
\]
(5.12)

which is a Lorentz scalar, T'C singlet and ETC triplet \((a, b, ... = 1, 2, 3)\) is the ETC index; \(i, j = 1, 2, ..., 6\) is the T'C index; \(\eta\) is the \(6 \times 6\) antisymmetric symplectic metric. It breaks \(SU^{ETC}(3) \times SU^{TC}(2)\). The ETC quark multiplets break up into techniquarks and ordinary quarks:

\[
SU_L(2) \times SU_Y(1) \times SU^e(3) \times SU^T(2) \times SU^T(2)
\]

\[
\begin{pmatrix}
U_L \\
D_L
\end{pmatrix}
= \begin{pmatrix}
U_1 & U_2 \\
D_1 & D_2
\end{pmatrix}_L
= \begin{pmatrix}
2, \frac{1}{3} , 3 , 2+1, 1 \\
1, \frac{2}{3} , 3 , 2+1, 1
\end{pmatrix}_L
\]
(5.13)

The flavour symmetry of the TC interactions includes the chiral isospin group \(SU_L(2) \times SU_R(2)\). Indeed from the point of view of the TC interactions, \(U_L\) and \(D_L\) are equivalent (they are both TC doublets), and so are \(U_R\) and \(D_R\). At the scale \(\Lambda^{TC} \sim \frac{3}{2} \) TeV where the TC gauge coupling constant becomes strong, the techni-fermions condense:

\[
< \bar{U}U > = < \bar{D}D > \sim (\Lambda^{T'C})^3
\]
(5.14)

breaking \(SU_L(2) \times SU_R(2) \rightarrow SU_{L+R}(2)\) (TC isospin) and \(SU_L(2) \times U_Y(1) \rightarrow U^{EM}(1)\).

Since the generators of \(SU_L(2)\) transform as a triplet under TC isospin, we have
\[ M_W = M_Z \cos \theta_W \] to all orders of the TC interactions. However, we will have
\[ m_u \neq m_d \] because isospin is not a symmetry of ETC. Indeed \( U_R \) is a 3 of \( SU^{ETC}(3) \) whereas \( D_R \) is a 3.

Let us calculate the masses of \( u \) and \( d \) in lowest order. The four-fermion
interaction mediated by exchange of the heavy \( SU^{ETC}(3)/SU^{TC}(2) \) vector bosons is

\[
\sum_{i=1,2} \frac{1}{2\mu^2} J^{3\mu}_l J^{3\mu}_i = \frac{1}{2\mu^2} \left( \bar{U}_{L_i} \sigma^\mu u_L + \bar{D}_{L_i} \sigma^\mu d_L + \bar{U}_{R_i} \bar{\sigma}_\mu u_R - \bar{D}_{R_i} \bar{\sigma}_\mu D_R \right) \\
\times \left( u_L^\dagger \sigma^\mu U_{Li}^\dagger + d_L^\dagger \sigma^\mu D_{Li}^\dagger + u_R^\dagger \bar{\sigma}_\mu U_{Ri}^\dagger - D_R^\dagger \bar{\sigma}_\mu D_{Ri}^\dagger \right)
\]  

(5.15)

where
\[ \mu = \frac{M_2}{d_{ETC}} \approx \Lambda^2 \] , \( \sigma_\mu = (1, \bar{\sigma})_\mu = \gamma_0 \gamma_\mu \frac{1-\gamma_5}{2} \]

and
\[ \bar{\sigma}_\mu = (1, -\bar{\sigma})_\mu = \gamma_0 \gamma_\mu \frac{1+\gamma_5}{2} \]

The Fierz transformations are performed by using
\[
\sum_{a=1}^3 (\sigma^a)_{ij} (\sigma^a)^{\ell \kappa} + \delta^i_j \delta^\ell_\kappa = 2 \delta^i_\ell \delta^j_\kappa
\]

(5.16)

One obtains:

\[
-\frac{1}{\mu^2} \left[ (U_{L_i}^\dagger U_{R_i}^\dagger)(u_R^\dagger u_L) + (U_{R_i}^\dagger U_{L_i}^\dagger)(u_L^\dagger u_R) + \ldots \right]
\]

(5.17)

where we have written out only those terms that will become mass terms for the
ordinary quarks when the techniquarks condense. Note that there are no analogous
terms for the down quark. The techniquark condensates have the form

\[
\langle U_{R_i}^\dagger U_{L_i}^\dagger \rangle = \langle U_{L_i}^\dagger U_{R_i}^\dagger \rangle = \langle \bar{e}_i^\dagger D_i^\dagger D_i \rangle = \langle \bar{e}_i^\dagger D_i^\dagger D_i^\dagger \rangle \neq 0.
\]

(5.18)

We find (in lowest order of ETC/TC vector boson exchange):

\[
m_u = \frac{1}{2\mu^2} \langle \bar{U}U \rangle , \quad m_d = 0
\]

(5.19)
The result can also be obtained by comparing the mass generation graphs for $u$ and $d$; see Fig. 4.

One might wonder whether a model which has isospin symmetric ETC interactions must be rejected without further consideration. Indeed in such a model the TC interactions will be isospin symmetric as well [see Eq. (5.9)] and the only way in which we can have $m_u \neq m_d$ is to break isospin spontaneously by the TC condensates. However, this will upset $M_w = M_Z \cos \theta_W$, unless we can impose a custodial SU(2) other than isospin. Thus, in answer to our question, a model with isospin symmetric ETC interactions is viable if and only if one can break isospin spontaneously and still conserve a custodial SU(2). The flavour symmetry of the TC interactions must be sufficiently large in such a model for the situation represented in Fig. 3b to be implemented. Examples of such models can be constructed (see Ref. 11). There will be three Goldstone bosons due to the spontaneous violation of TC isospin. They acquire mass of order $e_F \pi$ because isospin is violated explicitly by the electroweak gauge interactions.

In summary, Fig. 3 represents the two different ways in which $M_w = M_Z \cos \theta_W$ can be made compatible with $m_u \neq m_d$. In the first mechanism (Fig. 3a), isospin is conserved by the TC interactions and the TC condensates, but is not a symmetry of ETC. In the second mechanism (Fig. 3b), isospin is a symmetry of both the ETC and TC interactions but is violated spontaneously by the TC condensates. The custodial symmetry in this case is a SU(2) other than isospin.

6. **Tumbling**

"Tumbling" is a hypothesis as to the behaviour of unbroken asymptotically-free gauge groups with non-real fermion representation content. We adopt here the convention under which all fermions are described by left-handed fields. For example, QCD with $n$ flavours has the fermions $q_{Li}$ and $q^c_{Li} = i \sigma^a q_{Ri}^a$ ($i = 1, 2, ..., n$) and we say its fermion representation content is $n(3) + n(\overline{3})$ of $SU^C(3)$. The fermion representation content of QCD is real. It will be argued below that an asymptotically-free gauge group with non-real fermion representation content breaks itself when the gauge coupling constant becomes large (i.e. in the infrared
region). Several successive breakings of this type can occur before the fermion representation content becomes real under the unbroken subgroup, at which point the "tumbling" stops. Tumbling would therefore be an economical way to implement the successive breakings of ETC down to TC needed to generate interfamily mass splittings (see Section 4).

Consider the unbroken gauge group SU(5) with fermion representation content \((\bar{5}) + (10)\). We represent the \((\bar{5})\) and \((10)\) left-handed two-component fermions by \(\bar{\psi}_i\) and \(\psi_{i j} = -\psi_{j i}\) \((i, j = 1, 2, \ldots, 5)\). This gauge theory is anomaly free like QCD (the anomalies of \(\bar{5}\) and 10 cancel each other) but, unlike QCD, the fermion representation content is not real. What happens when the gauge coupling constant becomes strong? In the case of QCD, quark condensation occurs: \(\langle \bar{q}q \rangle_0 \neq 0\). This must be due in some sense to the fact that the binding energy of a \(\bar{q}q\) bound state becomes so large that it is energetically favourable for the vacuum to be filled up with such pairs. Thus, to try and answer our question in the case of the SU(5) model, let us evaluate the binding energies of the various possible scalar bound states composed of \(\bar{5}\) and 10 fermions. Several combinations can be considered corresponding to the Kronecker products:

\[
\begin{align*}
\bar{5} \times \bar{5} &= 10_A + \bar{15}_S \\
\bar{5} \times 10 &= \bar{5} + 45 \\
10 \times 10 &= \bar{10}_S + 5\bar{15}_S + 45_A
\end{align*}
\tag{6.1}
\]

Note however that only the symmetrical product of two identical fermion multiplets can be a Lorentz scalar since

\[
F^T(-i\sigma_2) F' = -F'^T(-i\sigma_2)^T F = F'^T(-i\sigma_2) F
\tag{6.2}
\]

where \(F\) and \(F'\) are any two two-component left-handed fermions. In the following we will write the Lorentz scalar combination (6.2) simply as \(FF' = F'F\). Thus, the scalar bound state made up of a \(\bar{5}\) and a 10 can either be a \(\bar{5}\) \((\bar{\psi}_{ij}\psi^j)\) or a \(45\) \((\bar{\psi}_{ij}\psi^k - \frac{i}{\sqrt{5}} \delta_{ij}^k \bar{\psi}_{ij}\psi^k)\) of SU(5). On the other hand, the scalar bound state made up of the 10 with itself can be a \(\bar{5}\) or 50 of SU(5) but not a 45. Of course, if there were several 10 present, the 45 can be constructed by anti-symmetrizing over "flavour".
It will be shown below that the potential for such a bound state is given by

\[ V(r) = -\frac{\alpha(\mu)}{2r} \left( C_1 + C_2 - C \right) \]  \hspace{1cm} (6.3)

in the one gluon exchange approximation. \( C_1 \) and \( C_2 \) are the Casimir operators of the constituents and \( C \) is the Casimir operator of the bound state. The channel for which \( C_1 + C_2 - C \) is maximum is called MASC (most attractive scalar channel). The tumbling hypothesis is the following: at energy scale \( \mu \) such that:

\[ \alpha(\mu) \left( C_1 + C_2 - C \right)_{\text{MASC}} = O(1) \]  \hspace{1cm} (6.4)

MASC condenses. In the case of our SU(5) model MASC is two 10 combining into a 5 bound state. According to the tumbling hypothesis, we thus have

\[ \langle \psi_{ij} \psi_{k\ell} \epsilon_{ij} \epsilon_{k\ell} \rho \rangle_0 \approx \mu^3 \delta^{PS} \]  \hspace{1cm} (6.5)

where \( \mu \) is such that

\[ \alpha(\mu) \left( 2 C_{10} - C_5 \right) = \alpha(\mu) \frac{24}{5} = O(1) \]  \hspace{1cm} (6.6)

The tumbling hypothesis is of course consistent with \( \bar{q}q \) condensation in QCD, since among the various possible channels

\[ \begin{align*}
3 \times 3 &= \overline{3} + 6 \\
\overline{3} \times \overline{3} &= 3 + \bar{6} \\
3 \times \bar{3} &= 1 + 8
\end{align*} \]  \hspace{1cm} (6.7)

the channel \( 3 \times \overline{3} \rightarrow 1 \) maximizes \( C_1 + C_2 - C \).

The difference between QCD and an unbroken asymptotically-free gauge group with non-real fermion representation content is that in the latter MASC is not a singlet and therefore breaks the gauge group. In our example, \( \text{SU}(5) \rightarrow \text{SU}(4) \) at scale \( \mu \). The fermion multiplets split up

\[ \overline{5} = \overline{4} + 1 , \quad 10 = 6 + 4 \]  \hspace{1cm} (6.8)

The SU(5)/SU(4) vector bosons acquire mass of order \( g(\mu)\mu \). Also the fermions that participate in the condensate \([\text{in our case, the SU(4) sextet}]\) acquire dynamical mass of order \( \mu \). Their mass term
\[ \sim \mu \psi_{ij} \psi_{kl} \in \mathbf{i}j \mathbf{k} \mathbf{l} \mathbf{5} \]  

is identical in form with the condensate (6.5). Again this is in analogy with QCD where the condensate \((\bar{q}q)_{0} = \mu^3\) implies a dynamical mass term \(\sim \mu \bar{q}q\) for the quarks. These dynamical masses are sharply \(p^2\) dependent and disappear at energy scales above \(\mu\).

Below \(\mu\) we have a theory based on the gauge group SU(4) and the fermion representation content: \(4 + \bar{4} + 1\). The effects due to the heavy vector bosons and the heavy fermion sextet (\(\psi_{ij}, i, j = 1, 2, 3, 4\)) can in first approximation be neglected\(^{(26)}\). The gauge coupling constant now runs according to the SU(4) \(\beta\)-function. The new MASC is 4 and \(\bar{4}\) combining into a singlet. That channel condenses when

\[ \alpha(\mu') \left( C_4 + C_{\bar{4}} - C_1 \right) = \alpha(\mu') \frac{15}{4} = O(1) \]  

The SU(4) gauge group remains unbroken. The 4 and \(\bar{4}\) fermions acquire dynamical mass of order \(\mu'\). The singlet \(\psi_5\) remains massless. This last fact incidentally is consistent with a general set of rules 't Hooft\(^{(21)}\) wrote down which relate the properties of a gauge theory at different mass scales.

Tumbling allows us to establish a hierarchy of mass scales in an economical and natural way. Only two mass scales were produced by our SU(5) example, but it is easy to construct examples that yield several more\(^{(12)}\). Let us thus give a general "treatment" of tumbling. Consider a gauge group \(G\) with fermion representation content: \(n + n' + \ldots\). Several scalar bound states can be formed corresponding to the Kronecker products:

\[ m \times n' = n_1 + n_2 + \ldots \]

\[ m \times n = n_1 + n_3 + \ldots \]

\[ e + c \ldots \]  

with the restriction that bound states must be symmetric in the internal indices.

We have:

\[ \Lambda^a \otimes 1' + 1 \otimes \Lambda^a = \mathbf{U} \begin{pmatrix} \Lambda_1^a & 0 & 0 & \cdots \\ 0 & \Lambda_2^a & 0 & \cdots \\ 0 & 0 & \Lambda_3^a \end{pmatrix} \mathbf{U}^\dagger \]  

(6.12)
where the \( \Lambda^a, \Lambda'^a, \Lambda_1^a, \ldots \) are the generators for the representation \( n, n', n_1, \ldots \) and \( \mathcal{U} \) is a unitary matrix. In the one gluon exchange approximation, the potential for the bound state \( n \times n' \Rightarrow n_k \) (Fig. 5) is given by

\[
\mathcal{V}(r) = \frac{\mathcal{U}(\mu)}{r} \mathcal{P}_k \left( \sum_a \Lambda^a \otimes \Lambda'^a \right) \mathcal{P}_k^\dagger
\]  

(6.13)

where the projection operator \( \mathcal{P}_k \) selects out the scalar bound state \( n_k \) among all the possible ones made up of the fermions \( n \) and \( n' \). Squaring both sides of Eq. (6.13) and summing over \( a \), we find

\[
(C + C') (1 \otimes 1') + 2 \sum_a \Lambda^a \otimes \Lambda'^a
\]

\[
= \mathcal{U} \begin{pmatrix} C, 1, & 0 & \ldots \\ 0 & C_2, 1_2 & \ldots \\ \ldots & \ldots & \ldots \\ \end{pmatrix} \mathcal{U}^\dagger
\]  

(6.14)

where \( C, C', C_1, \ldots \) are the Casimir operators for the representations \( n, n', n_1, \ldots \):

\[
\sum_a \Lambda^a \Lambda'^a = C 1
\]  

(6.15)

Substituting Eq. (6.14) into (6.13), we obtain the result stated in Eq. (6.3).

MASC is the channel for which \( C + C' - C_k \) is maximum. MASC condenses when \( \alpha(\mu)(C + C' - C_k)_{MASC} = 0(1) \). If the condensate is not a singlet, \( G \) breaks down to a subgroup \( G' \). The \( G/G' \) vector bosons and the fermions which participate in the condensate acquire dynamical masses of order \( g(\mu) \mu \) and \( \mu \), respectively. At energy scales below \( \mu \) we have in first approximation a theory based on the gauge group \( G' \) and the fermions which have not yet acquired mass. One determines MASC for this new system, and so on. This process of gauge symmetry breaking stops when the fermion representation content becomes real under the unbroken gauge group. In our example, the point was reached when the unbroken subgroup was \( SU(4) \) and the fermion representation content \( 4 + \bar{4} + 1 \). The condensates that form from then on are singlets of the unbroken subgroup. All fermions which are not singlets under the unbroken subgroup acquire a dynamical mass.
We can now envisage a theory of low energy (≤ 300 TeV) particle physics based on the gauge group

\[ SU_L(2) \times U_Y(1) \times SU(3) \times G^{ETC} \]  

(6.16)

The ETC fermion representation content is complex. From $\mu \sim 300$ TeV, ETC tumbles down to TC at $\mu \sim$ a few TeV. Under TC the fermion representation content is real. At $\mu \sim 1/4$ TeV, the technifermion condensates break $SU_L(2) \times U_Y(1) \rightarrow U^{em}(1)$. The quarks and leptons are TC singlets but are in common ETC multiplets with the techniquarks and technileptons. They acquire mass through the extended technicolour mechanism of Section 4. The quark and lepton masses, the Cabibbo angles, the $W^\pm$ and $Z^0$ masses would all be determined in terms of a single parameter, the ETC gauge coupling constant at some given mass scale.

This sort of scheme is particularly compatible with the requirements, discussed in the previous section, that lead to $M_W = M_Z \cos \theta_W$ and $m_u \neq m_d$. Since the TC fermion representation content is real, TC will be, in general, $SU_L(2) \times SU_R(2)$ invariant. Indeed, under Hermitian conjugation, the handedness of fermion fields is switched $L \leftrightarrow R$ and their representation content is complex conjugated $(n) \leftrightarrow (\bar{n})$. Therefore, if a gauge group has real $(n = \bar{n})$ fermion representation content, the corresponding gauge interactions are left-right symmetric. Since TC is $SU_L(2)$ symmetric and $n_{TC} = \bar{n}_{TC}$, the TC interactions are in general $SU_L(2) \times SU_R(2)$ symmetric. By the same token, ETC has non-real fermion representation content and therefore generally violates isospin. Consequently, we can expect to be in the situation depicted in Fig. 3a which assures $M_W = M_Z \cos \theta_W$ and $m_u \neq m_d$.

A theory of the type described must satisfy a great number of constraints precisely because it is so very predictive. No completely successful candidate has been constructed yet. The various constraints and difficulties involved are discussed in Ref. 22. Let us just list a few.

1) To be self-consistent, the theory must be free of triangle anomalies.

2) There should be no physical Goldstone bosons or light axions. This requires the number of fermion multiplets to be small, which in turn requires some
sort of low-energy (ν 300 TeV) quark-lepton unification\(^{18}\)). Thus, instead of (6.16), one should start with\(^{22}\), for example, SU\(_L(2) \times U_\chi(1) \times SU^{PS}(4) \times G^{ETC}\) where SU\(^{PS}(4)\) is the Pati-Salam group\(^{23}\), or with SU\(_L(2) \times U_\chi(1) \times G^{strong}\) where G\(^{strong}\) unifies\(^{17}\) SU\(^C(3) \times G^{TC}\).

3) The theory should tumble correctly. In particular, the ETC gauge group must be asymptotically free at each stage. At energy scales of order one TeV, the unbroken gauge symmetry should be SU\(_L(2) \times U_\chi(1) \times SU^{C}(3) \times G^{TC}\) and at low energy (below 100 GeV) it should be U\(^{EM}(1) \times SU^{C}(3) \times G^{TC}\).

4) We require \(M_\nu = M_\chi \cos \theta_\nu\), yet isospin splittings in the ordinary fermion mass matrices (cf. Section 5).

5) The low-energy quark-lepton unification should be free of baryon number violation. Also one must make sure ETC instantons do not violate baryon number.

6) The calculated quark and lepton mass matrices should be in reasonable agreement with experiment. Non-trivial Cabibbo angles are required. One must verify that the theory is consistent with such low-energy constraints as the upper limit on \(K_L \rightarrow \mu e\) decays and the small \(K_1 - K_2\) mass difference.

In Ref. 22 a "Mickey Mouse" model was presented which met many but not all of these constraints.

7. THE PSEUDO-GOLDSTONE Bosons of Technicolour

Attempts\(^{17,22}\) at building realistic extended technicolour models suggest the existence of at least one family of technifermions:

\[
\begin{pmatrix}
U_R & U_w & U_\phi & N \\
D_R & D_w & D_\phi & E
\end{pmatrix} = \{ F_{ia} \}
\]

(7.1)

where \(a = 1, \ldots, 4\) and \(i = 1, 2\). These technifermions have the same SU\(_L(2) \times U_\chi(1) \times SU^{C}(3)\) quantum numbers as an ordinary family u, d, ν, e including the right-handed neutrino. Let us assume the existence of one family (7.1). A rich spectrum of pseudo-Goldstone bosons\(^{18}\) (PGBs) will follow, as was already mentioned in Section 4. The left- and right-handed parts of the technifermions transform
according to the same representation \((N)\) of \(G^{TC}\). Thus the TC representation content (as defined in the previous section) is \(8(N) + 8(\bar{N})\), which is real whether or not \((N)\) is real. If \((N)\) is complex \((N \neq \bar{N})\), the flavour symmetry of the TC interactions is \(SU_L(8) \times SU_R(8)\). If \((N)\) is real, the TC representation content is \(16(N)\) and the flavour symmetry of the TC interactions is \(SU(16)\).

The desired technifermion condensates are:

\[
\langle \bar{F}_{ai} F_{bj} \rangle_o = \delta^a_\beta \delta^{i}_j \mu_3^{3} \tag{7.2}
\]

since they would break \(SU_L(2) \times U_Y(1) \times SU^C(3) \times U^{em}(1) \times SU^C(3)\) and yield \(M_\mu = M_\zeta \cos \theta_\mu\). The condensates (7.2) break the total TC flavour symmetry\(^9\)

\[
SU_L(8) \times SU_R(8) \rightarrow SU_{L+R}(8) \quad \text{if} \quad (N) \text{ complex (case } \mathcal{C})
\]

\[
SU(16) \rightarrow O(16) \quad \text{if} \quad (N) \text{ real with a symmetric invariant symbol (case } \mathcal{R}_+ \text{)}
\]

or

\[
SU(16) \rightarrow Sp(16) \quad \text{if} \quad (N) \text{ real with an anti-symmetric (7.3) invariant symbol (case } \mathcal{R}_- \text{)}.
\]

The reader can easily check this for himself. He should just keep in mind [Eq. (6.2)] for the cases \(\mathcal{R}_+\) and \(\mathcal{R}_-\).

Whether the desired TC condensates (7.2) are actually the ones that form under our set of assumptions is of course a question of dynamics. It has been shown\(^{15}\) that in the large \(N\) limit, the TC condensates are indeed such as to break the total TC flavour symmetry according to (7.3). This implies that in the large \(N\) limit, the TC condensates have the form:

\[
\langle \bar{F}_{RI}^{+} F_{RB} \rangle_o \sim (U)_{ai}^{a} \tag{7.4}
\]

where \(U\) is a \(8 \times 8\) unitary matrix. Assuming that the large \(N\) result (7.4) applies, one still needs to show that \(U = I\) to obtain the desired condensates (7.2). This question is decided by turning on the \(SU_L(2) \times U_Y(1) \times SU^C(3)\) gauge interactions and other TC flavour symmetry breaking interactions, and seeing which value of \(U\).
is energetically preferred (from the point of view of the TC interactions alone, the value of $\mathcal{U}_i$ is indifferent). We will come back to this question below. Let us just anticipate by giving the result\(^{2a}\): in cases $C$ and $R_2$, $\mathcal{U}_i = 1$ as desired; in case $R_1$, $\mathcal{U}_i \neq 1$.

There are respectively 63, 135 or 119 (pseudo)-Goldstone bosons depending on whether we are in case $C$, $R_2$ or $R_1$. All these particles are exactly massless in the limit where all other interactions but technicolour are turned off. In the cases $C$ and $R_2$, three of them will be "eaten up" by the $W^\pm$ and $Z^0$. The other acquire masses ranging from a few to 300 GeV (see below) when one turns on the $SU_L(2) \times SU_Y(1) \times SU_C(3)$ gauge interactions, the ETC interactions, the interactions implied by quark-lepton unification, etc. Indeed, these interactions explicitly break various parts of the original TC flavour symmetry. The existence of these relatively light (on a TeV mass scale) PGBs will allow an early experimental test of the ideas of technicolour and extended technicolour. The PGBs will, of course, look elementary when probed at distances larger than $\sim 1$ TeV\(^{-1}\). However, if and when the PGBs are produced in the laboratory, one will be able to check whether their properties (masses, couplings to ordinary particles) are consistent with what one would expect from TC and ETC models. It is the purpose of this section to give a short description of these PGB properties\(^{2a,2a-2b}\).

In case $C$, the (pseudo)-Goldstone bosons have the quantum numbers of

$$\lambda^\alpha F \Lambda^r T^r \gamma_5 F \sim P^{\alpha r}, \quad (\alpha, r) \neq (0,0)$$

(7.5)

where the $\lambda^\alpha$ $(\alpha = 0, 1, \ldots, 15)$ are the 16 $U(4)$ matrices acting on $a, b = 1, \ldots, 4$ and the $\tau^r$ $(r = 0, 1, 2, 3)$ are the 4 $U(2)$ matrices acting on $i, j = 1, 2$. The $\Lambda^\alpha$ and $\tau^r$ are normalized in the usual way: $\text{Tr}(\Lambda^\alpha \Lambda^\beta) = 2 \delta^\alpha \beta$, $\text{Tr}(\tau^r \tau^s) = 2 \delta^r s$.

$P^{00}$ is not a PGB. Rather, it is the techni-eta which is expected to have a mass of order 1 TeV because of the technigluon anomaly. Let us define a set of scalars $S^{\alpha r}$ analogous to the set of pseudoscalars $P^{\alpha r}$:

$$S^{\alpha r} \sim \bar{F} \Lambda^\alpha T^r F$$

(7.6)

Equation (7.2) means that $S^{00}$ takes a vacuum expectation value

$$\langle S^{\alpha r} \rangle_o = \delta^{\alpha 0} \delta^{r 0} F_\Pi$$

(7.7)
Since the generators of SU$_L$(2)

$$Q_L^r = \int d^3x \ F^+ \frac{1}{2} \not\!{\tau} \frac{1 - \gamma_5}{2} \ F$$

(7.8)

satisfy

$$[Q_L^r, S^{oo}] = i \frac{1}{2} P^{or}$$

(7.9)

the SU$_L$(2) currents satisfy the PCAC relations:

$$q^\mu J_L^{\mu} = \frac{1}{2} F_\pi q^2 P^{or}$$

(7.10)

These imply that the $p^0\pi$ will be eaten up by the $w^+ \text{ and } z^0$, which acquire mass

$$M_w = M_z \cos \theta_w = \frac{g}{\sqrt{2}} F_\pi$$

(7.11)

The scale of technicolour should therefore be such that

$$F_\pi = 250 \text{ GeV}$$

(7.12)

The "axial SU(8)" currents satisfy the PCAC relations

$$q^\mu J_{S \mu} = \frac{1}{\sqrt{2}} F_\pi q^2 P^{or}$$

(7.13)

since

$$[Q_{S}^{\alpha r}, S^{oo}] = i \frac{1}{\sqrt{2}} P^{or}$$

(7.14)

where $Q_{S}^{\alpha r}$ are the "axial SU(8)" generators

$$Q_{S}^{\alpha r} = \int d^3x \ F^+ \frac{1}{2} \not\!{\lambda} \not\!{\tau} \frac{1 - \gamma_5}{2} \not\!{\gamma}_S F$$

(7.15)

From Eq. (7.5) one easily reads off the quantum numbers of the $63 - 3 = 60$

PGBs of case C. These are:

- four colour octets, which form a triplet and singlet of isospin, or a doublet of SU$_L$(2) in the manner of Eq. (2.4). These are the $P^{\alpha r}$ for $\alpha = 1, \ldots, 8$ and $r = 0, 1, 2, 3$. We will call them $P_0^3, P_3^3, P_3^+$;
- four colour triplets and four colour antitriplets. These are the $P^{\alpha r}$ for $\alpha = 9, \ldots, 14$ and $r = 0, 1, 2, 3$. They form two doublets of SU$_L$(2). They
are composites made out of a techniquark and an anti-technilepton or vice-versa. We will refer to them as the lepto-quark PGBs;

- four colour singlets, $P_r^{15}, \chi$ for $r = 0, 1, 2, 3$, which also form a triplet and singlet of isospin, and a doublet of $SU_L(2)$. We will call them $P^0$, $P^3$ and $P^\pm$.

In case $\mathcal{K}_+$, there are $72 = 2 \cdot (8.9/2)$ PGBs in addition to the 60 of case $C$. They have the quantum numbers of

$$F_{\{ai}^{\mathcal{C}T} (1 \pm \gamma_5) F_{bj}\}$$

(7.16)

where $\mathcal{C}$ is the charge conjugation matrix and where the $\{\}$ symbol means that one must symmetrize over the pairs of indices $(ai)$ and $(bj)$. The 72 additional PGBs of case $\mathcal{K}_+$ form 3 colour sextets with electric charges $4/3, 1/3, -2/3$; 5 colour triplets with electric charges $5/3, -1/3$ (twice), $-1/3$; 3 colour singlets with electric charges 0, -1, -2; plus all the corresponding antiparticles.

In case $\mathcal{K}_-$ there are $56 = 2 \cdot (8.7/2)$ PGBs in addition to the 60 of case $C$. They have the quantum numbers of

$$F_{[ai}^{\mathcal{C}T} (1 \pm \gamma_5) F_{bj]}$$

(7.17)

where the symbol $[\ ]$ means that one must anti-symmetrize over the pairs of indices $(ai)$ and $(bj)$.

One calculates the PGB masses from Dashen's formula$^2$9):

$$\left( M^2 \right)_{PP'} = \left( \frac{\sqrt{2}}{F_{\tau}} \right)^2 \left< 0 \left| \left[ Q_p , \left[ Q_{p'}, \delta H(0) \right] \right] \right| 0 \right>$$

(7.18)

where $\delta H(0)$ is the TC flavour symmetry breaking perturbation in the Hamiltonian density due to the $SU_L(2) \times U_Y(1) \times SU^C(3)$ gauge interactions, the interactions mediated by broken Pati-Salam generators, etc. We refer the reader to the original papers$^{18,22,26,27}$ for the details of the calculations. The results are the following:
- For case C:
  - Color octet: $P_8^{\pm, 0.3}$
    
  - Lepto-quark PGBs
    
  - Color singlet: $P^\pm$
    
  - $P_0^{0.3}$
    
    \[ 245 \text{ GeV} \sqrt{\frac{4}{N}} \]
    
    \[ \sim 160 \text{ GeV} \sqrt{\frac{4}{N}} \]
    
    \[ 5 \text{ to } 8 \text{ GeV} \sqrt{\frac{4}{N}} \]
    
    \[ \leq 3 \text{ GeV} \]

- For case $R_+$, the additional PGBs:
  - Color sextet
    
  - Color triplet
    
  - Color singlet
    
    \[ \sim 260 \text{ GeV} \sqrt{\frac{4}{N}} \]
    
    \[ 145 \text{ to } 165 \text{ GeV} \sqrt{\frac{4}{N}} \]
    
    \[ 50 \text{ to } 80 \text{ GeV} \sqrt{\frac{4}{N}} \]

In case $R_-$ it was found\(^2\) that the PGB (mass)\(^2\)-matrix (7.14) calculated by turning on the $SU_L(2) \times U_Y(1) \times SU_C(3)$ gauge interactions, is not positive-definite. This means that the electroweak and/or strong interactions prefer a vacuum other than the desired one (7.2). It turns out that the true (lowest energy) vacuum for the case $R_-$ is:

\[ \langle \bar{U}_R U_R \rangle = \langle \bar{D}_R D_R \rangle = \langle \bar{U}_w U_w \rangle = \langle \bar{D}_w D_w \rangle \]

\[ = \langle \bar{U}_B U_B \rangle = \langle \bar{D}_B D_B \rangle = \langle E^T C \frac{1 + Y_5}{2} N \rangle = \langle E^T C \frac{1 - Y_5}{2} N \rangle \]

(7.19)

which breaks $U_{em}(1)$ and gives an intolerably large mass to the photon. The reason why the vacuum (7.19) is preferred over (7.2) is easy to see. The technilepton condensates in (7.19) break $SU_L(2) \times U_Y(1) \rightarrow SU_L(2)$ (remember that the combination $EN$ must be antisymmetrized in the case $R_-$), whereas those in (7.2):
\[ \left< \tilde{F}_{ai} F_{bj} \right>_0 = \delta^a_b \delta^i_j \mu_{\tau c}^3 \]  \hspace{1cm} (7.20)

break SU_L(2) \times SU_Y(1) \times U^{em}(1). Clearly (7.19) is preferred because \( g > g' \). In conclusion, a \( \tilde{R} \) type TC representation seems unacceptable for the technifermions, unless other radiative corrections (ETC?) are large enough to change qualitatively the above picture obtained from the strong and electroweak radiative corrections.

The couplings of PGBs to the gauge fields \((W^\pm, Z^0, \gamma, \text{gluons})\) are mainly of two types. First, there are the minimal couplings implied by \( SU_L(2) \times SU_Y(1) \times SU_C(3) \) gauge invariance and determined by the quantum numbers of the PGBs under that gauge group. These couplings are always of the type \( g(q^u_{B_i})P_1^{-}P_2 \) and \( g^2(B_{1u} B_{2u})P_1 P_2 \), where \( P \) represents a PGB, \( B \) a gauge vector boson and \( g \) the relevant gauge coupling constant. Second there are \( P - B_1 - B_2 \) couplings due to PCAC (7.15) and the anomalous triangle diagrams for VVA and AAA type three-current vertices. They are completely analogous to the \( \pi^0 \gamma \gamma \) coupling\(^{30}\), i.e.

\[ \frac{1}{2} k_{1\alpha} k_{2\beta} \varepsilon_{1\gamma} \varepsilon_{2\delta} \varepsilon^{\alpha\beta\gamma\delta} \left( \frac{S_{12}}{4\pi^2 F_\pi} \right) \]  \hspace{1cm} (7.21)

and are calculated in the same way. \( k \) and \( \varepsilon \) designate the momenta and polarization of \( B_1 \) and \( B_2 \), and

\[ S_{PB_1B_2} = q_1 q_2 \text{Tr} \left( Q_P \{ Q_1, Q_2 \}_{\frac{1}{2}} \right) \]  \hspace{1cm} (7.22)

where \( Q_1 \) and \( Q_2 \) are the charges \( B_1 \) and \( B_2 \) couple to and \( Q_P \) is the charge for which \( P \) is the Goldstone boson [Eq. (7.15)]. The contributions from left- and right-handed parts of the technifermions are summed separately in the trace of Eq. (7.22). In this convention \( S_{\pi^0 \gamma \gamma} = e^2 \). All the possible vertices (7.19) have been calculated\(^{26-28}\) for the case \( C \). The anomaly factors \( S \) are proportional to the number of technicolours. A measurement of one of these couplings (7.19) would in principle determine \( N \).

The effective Yukawa couplings of the PGBs to ordinary quarks and leptons are due to the exchange of massive ETC/TC vector bosons [see Figs. 1 and 2 and Eq. (4.2)]. Unfortunately, there is no definite model of ETC yet and the effective Yukawa couplings of the PGBs to quarks and leptons are of course ETC model dependent.
There is, however, a natural requirement\textsuperscript{28}) on the structure of the extended technicolour theory which greatly reduces this model dependence. Because some of the PGBs (\(P^3, P^0, P^2\)) are so very light, we must require in view of the tiny \(K_1 - K_2\) mass difference that their couplings be diagonal in flavour. In electroweak models with several explicit Higgs doublets, this is natural only if all quarks of the same charge get their masses from the vacuum expectation value of the neutral member of the same Higgs doublet, which may, however, be different for the different types \((u, c, t, \ldots)\) and \((d, s, b, \ldots)\) of quarks\textsuperscript{31}). The natural analogue in extended technicolour models of the required structure in explicit Higgs models is that each quark of the same charge gets its mass from the same condensate of technifermions — "monophagy". It was found\textsuperscript{28}) that in a "monophagic" ETC model the effective Yukawa couplings of the \(P^0, P^3, P^+\) and \(P_8^0, P_8^1, P_8^2\) PGBs are completely specified in terms of the quark and lepton masses and the Kobayashi-Maskawa matrix of mixing angles, modulo some discrete ambiguities in the choice of electroweak and colour quantum numbers of the ETC/TC gauge bosons.

For example, if the ETC/TC vector bosons are \(SU_L(2) \times SU_Y(1) \times SU^C(3)\) singlets, then a model with one technifamily is necessarily "monophagic" since the \((u, c, t, \ldots)\) quarks can only get their masses from the \(\bar{q}u\) condensate, the \((d, s, b, \ldots)\) quarks only from the \(\bar{q}d\) condensate, etc. The effective Yukawa couplings of the case C, colour octet and singlet, PGBs are in this case\textsuperscript{28}):

\[
\begin{align*}
\frac{-i}{F_\pi} P_0^0 &\left[ \left( \bar{u} \, m^u \, Y_s \, u + \bar{d} \, m^d \, Y_s \, d \right) \frac{1}{\sqrt{3}} \right] - \frac{\sqrt{3}}{\sqrt{3}} \left( \bar{e} \, m^e \, Y_s \, e \right) \\
\frac{-i}{F_\pi} P_3^3 &\left[ \left( \bar{u} \, m^u \, Y_s \, u - \bar{d} \, m^d \, Y_s \, d \right) \frac{1}{\sqrt{3}} \right] - \frac{\sqrt{3}}{\sqrt{3}} \left( \bar{e} \, m^e \, Y_s \, e \right) \\
\frac{-i}{F_\pi} P_+^+ &\left[ \bar{u} \left( U_{km} \, m^d \, \frac{1 + Y_s}{2} - m^u \, U_{km} \, \frac{1 - Y_s}{2} \right) d \sqrt{2} \right] - \frac{\sqrt{3}}{\sqrt{3}} \left( \bar{e} \, m^e \, \frac{Y_s}{2} \right) \\
\frac{-i}{F_\pi} P_8^0 &\left[ \bar{u} \, m^u \, \lambda^\alpha \, Y_s \, u + \bar{d} \, m^d \, \lambda^\alpha \, Y_s \, d \right] \sqrt{2} \\
\end{align*}
\]
\[ \frac{-i}{F_\pi} P_{\pi \alpha} \left[ \bar{u} m^u \chi^\alpha \gamma_5 u - \bar{d} m^d \chi^\alpha \gamma_5 d \right] \gamma^\mu \gamma^\nu \]

\[ \frac{-1}{F_\pi} P_{\pi \alpha} \left[ \bar{u} (U_{K_M} m^d \frac{1+\gamma_5}{2} - m^u U_{K_M} \frac{1-\gamma_5}{2}) \chi^\alpha d \right] \gamma^\mu \gamma^\nu \]

(7.23)

where \( u, d, e, \nu \) are generic symbols for \((u, c, t, \ldots), (d, s, b, \ldots), (e, \bar{e}, \nu, \bar{\nu}, \tau, \ldots)\), and \( m^u, m^d, m^e \) are the diagonal mass matrices of these fermions. \( U_{K_M} \) is the Cabibbo-Kobayashi-Maskawa matrix of mixing angles.

In monopagic ETC models, the contributions to the \( K_1 - K_2 \) mass difference from PGB exchange are safely small. The reader should beware, however, that ETC models can still violate the tiny \( K_1 - K_2 \) mass difference in other ways than by PGB exchange, notably by the exchange of single heavy ETC vector bosons that directly mediate \((\bar{d}d)^2\) type effective interactions\(^{32}\).

The couplings (7.23) are proportional to the fermion masses as in the case of an explicit Higgs doublet. However, they differ from the Yukawa couplings of an explicit Higgs in that the couplings are pseudoscalar\(^{33}\) rather than scalar and through the appearance of characteristic factors of \( \sqrt{3} \).

PGBs can be produced in pairs through their minimal gauge couplings to the photon, the gluons, the \( Z^0 \), \( \ldots \). For example, \( P^2 \) (mass: 5-8 GeV) will be produced in pairs in \( e^+e^- \) collisions and should contribute one quarter unit to \( R_{e^+e^-} \).

But PGBs can also be produced singly through the anomaly vertices. For example, \( P^0 \) and \( P^0 \) will be produced singly in \( pp \) or \( p\bar{p} \) collisions\(^{18,25,27}\) through the \( P^0 \) and \( P^0 \) anomaly vertices (see Fig. 6; \( G \) is a gluon); \( P^0 \) and \( P^0 \) will be produced singly in \( e^+e^- \) collisions\(^{18,28}\) through the \( P^0 \) and \( P^0 \) anomaly vertices (Fig. 7). Neutral PGBs can also be produced in the decays of heavy oniums, e.g. \( \bar{t}t \to P^0, P^0 \to \gamma \). Charged PGBs will be produced in the decays of heavy quarks, e.g. \( t \to b + P^* \).

The dominant decay modes of PGBs are of two types: \( P + f_1 f_2 \) through the effective Yukawa couplings or \( P \to B_1 + B_2 \) through the anomaly vertices. One finds for example:
\[ \Gamma (p^0 \to 2 \text{ gluons}) = \frac{m_{p^0}^3 G_F}{12 \sqrt{2} \pi} \left( \frac{\alpha_s}{\pi} \right)^2 N^2 \]

\[ \Gamma (p^0 \to q \bar{q}) = \frac{G_F m_q^2 m_{p^0}}{4 \sqrt{2} \pi} \]

(7.24)

where the \( \bar{q}q \) decay rate was calculated in the ETC model that yields Eqs. (7.23).

More details about PCB phenomenology can be found in Refs. 26, 27 and 28.
REFERENCES AND FOOTNOTES

1) The present renaissance of interest in dynamical symmetry breaking is due to
   the papers of L. Susskind, Phys. Rev. D 20 (1979) 2619 and S. Weinberg,
   Phys. Rev. D 19 (1979) 1277, which showed how to dynamically break the
   electroweak gauge group in a satisfactory way. Dynamical symmetry breaking
   had, of course, been proposed before that. See Ref. 2 for a partial list
   of contributors.


3) The same concept has also been called hypercolour or primecolour.


   A. Salam, in Elementary particle physics (ed. N. Svartholm) (Almqvist and

   (1978) 66.
21) G. 't Hooft, Lectures given at the Cargèse Summer Institute, August 26 - September 8, 1979.
24) M.E. Peskin, Saclay preprint (May 1980).


33) See Ref. 25 for some consequences of the mainly pseudoscalar character of the FGB couplings to fermions.
Figure captions

Fig. 1: a) Effective Yukawa couplings between technimesons and ordinary
ermions induced by the exchange of heavy ETC/TC vectors bosons $\xi$.
b) The ordinary fermions acquire small masses when the technifermions
condense.

Fig. 2: Effective Yukawa couplings induced by the exchange of heavy
$G^S/[G^TC \times SU^C(3)]$ vector bosons.

Fig. 3: The two flavour symmetry geographies that assure simultaneously
$M_W = M_Z \cos \theta_W$ and $m_u \neq m_d$. The shaded areas represent the flavour
symmetries which are spontaneously broken by the TC condensates.

Fig. 4: Lowest-order mass generation diagrams for the $u$ and $d$ quarks in the
model described in Section 5. The diagram for the $d$ quark vanishes.

Fig. 5: Formation of a scalar bound state $(n_k)$ composed of two left-handed
spin $\frac{1}{2}$ fermions $(n)$ and $(n')$, in the one gluon exchange approximation.
$(n)$, $(n')$, and $(n_k)$ refer to the colour representation of the par-
ticles.

Fig. 6: $P^0_s$ and $P^0$ can be produced singly in hadron collisions through the
anomalous $P^0_{(s)}$ (gluon)(gluon) vertex.

Fig. 7: Single PGB production in $e^+e^-$ collisions.
Fig. 1
Fig. 2
Fig. 3
Fig. 4
Fig. 5

Fig. 6
Fig. 7