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PICTURE OF QCD JETS: LEADING LOG
APPROXIMATION AND BEYOND IT[†]

BY

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ABSTRACTS

We review the picture of QCD jets as it emerged from the analyses based on perturbative QCD in the leading log approximation (LLA) and beyond it. Part I (by K. Konishi) gives a brief review of the intuitive (fractal) picture and quantitative description of QCD jets in LLA, and contains discussions on some salient aspects of QCD corrections beyond the leading order. Part II (by J. Kalinowski) is a summary of the construction of the jet calculus beyond the leading order, and of calculations to next-to-leading order of jet calculus vertices and generalized Altarelli-Parisi parton decay functions.

Explicit calculation suggests important corrections beyond leading order to the previous analyses of quantities which are sensitive to the multiplicity of soft gluons. On the other hand, for the measures which depend on the effects of soft gluons only indirectly (e.g. through the observation of fast quarks), the explicit results to next-to-leading order are consistent with the general arguments given by several authors. Corrections to inclusive P_T -correlations among fast particles inside a jet, are found to be small beyond LLA.

(1.1) PICTURE OF QCD JETS IN LIA

At this meeting Söding has presented beautiful data from PETRA, which show that the Quantum Chromodynamics (QCD) has passed a first series of tests in the physics of jets. Among others, three jet-like events were observed with a rate consistent with the expectation from the lowest-order QCD diagrams (with a reasonable value of α_s). A closer look into the analyses based on perturbative QCD, however, suggests that the picture of hadronic jets produced in hard processes (typically in $e^+e^- \rightarrow \text{hadrons}$) is more complicated than naively suggested from lowest order Feynman diagrams.

Consider a typical event in $e^+e^- \rightarrow \text{hadrons}$ shown in Fig.1. How many jets are there? This question can be answered in two different ways. According to the by-now standard approach à la Sterman-Weinberg, one must give precise criteria as to what should be called a jet in terms of opening angle δ of the cone and the energy fraction outside it, and then each event would be either two-jet or three-jet type (or more). The two-jet cross section, for instance, can be computed from the lowest order Feynman diagrams of Fig.2.

An alternative way to answer the same question is to define a mini-jet as a (small) calorimeter with half-opening angle δ , which records the hadronic energy fraction x such that

$$x \geq x_0 \quad (x_0 \text{ is fixed, } x_0 \ll 1).$$

Covering the whole solid angle of events by many small calorimeters of the same size, we can simply study the average number of mini-jets. This number, as a function of experimental resolutions δ and x_0 , can be computed in LIA. From this second viewpoint, relevant Feynman diagrams look like the one in Fig.3. QCD jets are fractals.

Let us now compare Fig.2 and Fig.3. Are these two pictures of QCD jets compatible? The key to understanding the relationship between these two different pictures, is the Kinoshita-Lee-Nauenberg (KLN) theorem of mass singularities. The same $O(\alpha_s)$ diagrams contribute both to $(d\sigma/dx) e^+e^- \rightarrow q(x) + X$ and $(d\sigma/d\Omega) e^+e^- \rightarrow 2 \text{ jets } (\delta, \epsilon)$ the only difference being the relative weights of diagrams in two cases. In the latter case where only the group property of the final states is fixed by experimental observation, the cross section is free of mass singularities in accord with the KLN theorem. In $(d\sigma/dx) e^+e^- \rightarrow q(x) + X$, however, the observation of quark momentum destroys the KLN cancellation and the result contains $\log Q^2/m^2$ (m = the quark mass) to order $\alpha_s(Q^2)$. Notice that the smallness of $\alpha_s(Q^2) \sim 1/\log Q^2/\Lambda^2$ is compensated by a large $\log Q^2/m^2$: a resummation to all orders of $\alpha_s(Q^2)$ is necessary. The full answer is

$$(d\sigma/dx) e^+e^- \rightarrow q(x) + X \propto \left(\frac{\log Q^2/\Lambda^2}{\log m^2/\Lambda^2} \right)^{\gamma_n} \quad (1)$$

(γ_n = calculable constants = anomalous dimensions), which is well known.

A simple argument similar to the one given above for the lowest order, shows that the higher order effects in $\alpha_s(Q^2)$ are always there and only for a limited class of measures do the lowest-order calculations make sense. (It should be borne in mind that even for those "infrared-safe" cases, in general there appear large $\log \delta$'s or $\log \epsilon$'s which are just as bad: the resummation of e.g. $(\alpha_s(Q^2) \log 1/\delta)^n$ is necessary in many practical cases).

Any physical cross section in a hard process is of the form,

$$\sigma = \frac{1}{Q^2} f(\alpha_s(Q^2), Q^2/m^2, x) \quad (2)$$

(Q^2 = the large mass scale of the process) because of the renormalisability. A possible way to make use of perturbative QCD is to reorganise the series as

$$= \sum_{m=0}^{\infty} (\alpha_s(Q^2))^m f_m \quad (3)$$

where $f_m = f(\alpha_s(Q^2) \log Q^2/m^2; x)$, or

$$f_m = f(\alpha_s(Q^2) \log Q^2/m^2 \log \delta; x), \text{ or}$$

something else, depending on the situation, but in general f_m contains whole higher order terms in $\alpha_s(Q^2)$ already. Keeping the first term of Eq.(3) (LLA), the subsequent terms are - presumably - small by factors of $1/\log Q^2/\Lambda^2$.

Quantitative description of evolution of jets in LLA can be summarised in a simple parton-probabilistic algorithm,

the jet calculus. The m-parton inclusive spectra are computed from the sum of effective tree diagrams:

$$\frac{d\sigma}{dx_1 \dots dx_m} = \sum_{\substack{\text{(trees)} \\ \text{internal} \\ \text{masses} \\ \text{parton species}}} \int dy_i \quad (4)$$

where $dy_i = (dq_1^2/q_1^2) \cdot \alpha_s(q_1^2)/2\pi$; $D(q_1^2; q_2^2; x)$ = inverse Mellin transform of $(\log q_1^2/\log q_2^2)^{\gamma_n}$ and the vertices are given by

$$\begin{aligned} \hat{P}_{Q,Q}(z) &= \hat{P}_{G,Q}(1-z) = C_F(1+z^2)/(1-z) ; \\ \hat{P}_{Q,G}(z) &= (N_F/2) \{z^2 + (1-z)^2\} ; \\ \hat{P}_{G,G}(z) &= 2C_A \{(1-z)/z + z/(1-z) + z(1-z)\} \end{aligned} \quad (5)$$

The evolution of jets in LLA is formally a (Markov) branching process with $\gamma \propto \log \log Q^2$ as a "time" variable.

The predictions of QCD in the physics of jets which

I find particularly interesting, include:

1. Two-particle inclusive energy-energy correlation (the comparison with experiments in e^+e^- annihilation was presented by Söding in this school);
2. Measurements to anomalous dimensions of QCD by a calorimeter-experiment (this could possibly be done at PETRA, PEP and LEP);
3. Energy antenna pattern of Basham and others. (This seems to be, among the so-called infra-red safe measures, a relatively clean one);

4. Effective quark and gluon form factors (this was discussed by Petronzio in this school);
5. Multiplicity and multiplicity moments of off-shell partons (see also Part II)/hadrons.
6. Jet evolution in colour, space-time and rapidity. Preconfinement and a consistent overall picture of jet evolution.
7. Monte Carlo models of jet evolution based on the parton branching picture in LLA.

BIBLIOGRAPHY TO PART (I.1)

K. Konishi, "Delving into QCD jets", CERN-TH-2853 (1980), to be published in Survey in High Energy Physics, and references therein, and CERN-TH-2897 (1980) and references therein; see Yu. I. Dokshitzer, D.I. Dyakonov and S.I. Troyan, Physics Reports, 58 (1980) 271, for a somewhat complementary review on the physics of jets; see also S.D. Ellis, Univ. Washington RLO-1388-811 (1980) and S. Wolfram, Caltech preprint, CALT-68-778 (1980).

(I.2) EVOLUTION OF JETS BEYOND LEADING LOGS: GENERALITIES Motivation

Calculations to higher orders in the series Eq.(3) in some hard processes have resulted in large corrections, as discussed by Altarelli in this school. It is evidently an interesting question whether the jet calculus predictions in LLA will be modified substantially by higher order corrections. Especially, a class of problems, 4-7 above, has been studied in an approximation which involves certain conjectures about non-leading corrections. In view of their phenomenological relevance in the near-future experiments, it is important to check the consistency of approximations used. See the end for a comment related to this point.

Methods and Main Results

Our method of analysis of jet evolution beyond leading log approximation, is a direct calculation of cross sections in an axial gauge. The main results of the work done in collaboration with J. Kalinowski, P. Scharbach and T.R. Taylor are: the general formalism for constructing jet calculus to any given order, the calculation of complete set of quark decay probabilities to next-to-leading order, among which the generalized Altarelli-Parisi quark decay functions are special cases, and some phenomenological implication to the structure of jets. Leaving the details of construction and calculations to next-to-leading order to the talk by Jan Kalinowski, I will concentrate on general aspects of the work. I shall also mention a particularly interesting feature

of our results related to strong soft-divergences in QCD.

Ideas

The idea behind our work is that the factorization theorem of mass singularities contains implications wider than usually acknowledged. The diagrammatic derivation of the factorization theorem does not only lead to the parton-model results (with calculable logarithmic corrections to structure or fragmentation functions). It also gives in principle precise predictions about the properties of associated jets. It was the purpose and results of our work to find a practical way to extract the information about the jets, based on a diagrammatic approach.

Final States in Hard Processes and Mass Singularities

A convenient way to make manifest the mass-singularity structure of final states in a hard process, is to combine the off-shell unitarity equations of Polyakov with the use of an axial gauge. For instance the total cross section for e^+e^- annihilation into hadrons is written as,

$$\sigma_{\text{tot}} \propto \text{Disc}_{Q^2} \left\{ \text{---} \text{---} \text{---} + \text{---} \text{---} \text{---} + \dots \right\} \quad (6)$$

where the crossed lines are cut exact propagators, uncut propagators also exact ones, and vertices are one-particle irreducible (lpI). By rewriting

$$\begin{aligned} \text{(A)} \quad q \text{---} &= \text{---} \text{---} \text{---} + \text{---} \text{---} \text{---} + \dots \quad \text{if } \sqrt{q^2} \geq M_0 \\ &= \text{---} \text{---} \text{---} : \quad \text{if } \sqrt{q^2} < M_0, \end{aligned}$$

everywhere,

$$\begin{aligned} \text{(B)} \quad & \text{---} \text{---} \text{---} \rightarrow \text{---} \text{---} \text{---} ; \\ & \text{---} \text{---} \text{---} + \text{---} \text{---} \text{---} + \dots \rightarrow \text{---} \text{---} \text{---} ; \text{ etc.} \\ \text{(C)} \quad & \rho \text{---} \text{---} \text{---} = \text{---} \text{---} \text{---} + \text{---} \text{---} \text{---} ; \text{ etc.} \end{aligned} \quad (7)$$

where ρ projects just the collinear log term (see Part II), the right hand side of Fig.(6) becomes of diagonal form. In axial gauge QCD, all collinear logs arise from integration over mass of intermediate parton connecting two effective vertices. The problem is then reduced to the computation of these vertices $V_{a \rightarrow bc \dots}$ to a given order in α_s . The jet calculus rules are as simple as in Eq.(4): $\hat{P}$ is replaced by $V_{a \rightarrow bc \dots}$ which now contain three-body decays as well, and D should be computed from the generalized Altarelli-Parisi kernel $\pi_{b,a}(\alpha_s; x) \equiv V_{a \rightarrow b}^{(1)}$.

It is possible, because of the KLN theorem and factorization property of mass singularities, to choose a convention such that $V_{a \rightarrow bc \dots}$'s can be treated (formally) as inclusive probabilities. The parton probabilistic picture of jet evolution found in LLA can thus be maintained to all order of logs and of α_s .

Comparison with OPE (or similar) Approach

To elucidate the similarity and difference with other more conventional types of approach (for instance the talk by Petronzio and Furmanski in this school; also a paper by Floratos et al), consider $e^+e^- \rightarrow$ a parton $(x) + X$. We shall schematically review the analogy and differences. The cross section is given, by the factorization theorem, by

$$d\sigma/dx = \sum_a C(\alpha_s(Q^2); Q^2/M^2; x) \otimes D(M^2; M_O^2; x) \quad (8)$$

$\otimes$ (non-perturbative fragmentation function)

$$\text{where } D(M^2; M_O^2; x) = \left(\begin{array}{c} \text{inverse} \\ \text{moment of} \end{array} \right) \exp \int_{M_O^2}^{M^2} \frac{dq^2}{q^2} \pi_n(\alpha_s(q^2)) \quad (9)$$

and

$$\pi_n(\alpha_s(q^2)) = \int_0^1 dx x^n \pi(\alpha_s(q^2); x) = \int dx x^n \left[\frac{\alpha_s(q^2)}{2\pi} P^{AP}(x) + \dots \right] \quad (10)$$

Analogy

Our Approach
(Kalinowski et al)

OPE-like Approach
(Curci et al, Floratos et al)

$$\pi_n(\alpha_s) = \text{moment of} \quad \leftrightarrow \quad \gamma_n(\alpha_s) = \text{anomalous dimensions}$$

generalized Altarelli-Parisi functions

$$\frac{d\sigma}{dx} = \text{independent of } M^2 \quad \leftrightarrow \quad \text{of } \mu_O^2 = \text{renormalization scale of operator matrix elements}$$

$$\frac{d\sigma}{dx} = \text{independent of } \mu^2 \text{ (renormalization scale for } \alpha_s) \quad = \quad \mu_g^2$$

$$\begin{array}{ll} \text{factorization of mass} & \leftrightarrow \text{multiplicative renormalization} \\ \text{singularities} & \text{of operator matrix element} \\ M \frac{\partial}{\partial M} D_n = D_n \pi_n & \leftrightarrow \left[\mu \frac{\partial}{\partial \mu} + \beta \frac{\partial}{\partial g} - \gamma_n \right] D_n = 0 \\ \text{Altarelli-Parisi equation} & \text{R-G equation (with } \mu_O = \mu_g) \end{array}$$

Physics Differences

$$\begin{array}{ll} M^2 = \text{upper limit of} & \leftrightarrow \mu_O^2 \text{ not observable} \\ \text{intermediate parton mass} & \text{(only a single particle observed; unitarity used)} \\ \sim P_T\text{-spread of jets} & \\ = \text{observable} & \end{array}$$

$$\begin{array}{ll} \text{Possible to resolve} & \leftrightarrow \text{difficult} \\ \text{associated jets} & \end{array}$$

$$\pi(\alpha_s(q^2); x) \text{ in principle observable} \quad \leftrightarrow \quad \gamma_n(\alpha_s) \text{ not a physical quantity in itself}$$

Strong Soft Divergences in QCD

Finally, we make a comment on an interesting feature of the result of next-to-leading calculations just completed. $\pi_{b,a}(\alpha_s; x)$ reads up to next-to-leading order,

$$\pi_{b,a}(\alpha_s; x) = \frac{\alpha_s(q^2)}{2\pi} P_{b,a}^{AP}(x) + \left(\frac{\alpha_s(q^2)}{2\pi} \right)^2 P_{b,a}^{NL}(x) + \left[\left(\frac{\alpha_s(q^2)}{2\pi} \right)^3 \right] \quad (11)$$

where $P_{b,a}^{AP}$ is the usual Altarelli-Parisi parton decay function (closely related to $\hat{P}_{ba}$ in Eq.(5)), and $P_{b,a}^{NL}$ the next-to-leading corrections to it.

For small x , $P_{G,Q}^{NL}(x)$ was found to behave as (Kalinowski et al; Furmanski et al), (see Eq.(29)),

$$P_{G,Q}^{NL}(x) \underset{x \ll 1}{\approx} -4C_F C_A \log^2 x / x + \text{less singular terms} \quad (12)$$

Analogous behaviour was also found for $P_{G,G}^{NL}$ (Furmanski et al; Floratos et al)

$$P_{G,G}^{NL}(x) \underset{x \ll 1}{\approx} -4C_A^2 \log^2 x / x + \text{less singular terms} \quad (13)$$

It is easy to see that half of $\log^2 x / x$ term has a trivial kinematic origin, due to the correct kinematic limits over which the integrations of ladder variables are performed.

The other half in Eq.(12) and Eq.(13) comes from new diagrams containing an extra three-gluon coupling, and their origin can be traced to certain double soft divergence pointed out in Kalinowski et al.

In the analysis of problems 5-7 listed at the end of Part (I.1) it has so far been assumed or argued that the only important non-leading corrections to the leading log results,

$$P_{G,Q}^{AP} \underset{x \ll 1}{\approx} -2C_F/x + \text{less singular terms} \quad (14)$$

and

$$P_{G,G}^{AP} \underset{x \ll 1}{\approx} -2C_A/x + \text{less singular terms} \quad (15)$$

are of kinematical nature as mentioned above. The results

of the explicit calculation, Eq.(12) and Eq.(13), indicate that this may not be the case. Indeed, for sufficiently small x the $\alpha_s^2(q^2)$ term in Eq.(11) dominates over the first even after the "kinematical" terms are subtracted. Thus unless an ingenious cancellation occurs between $C(\alpha_s(Q^2); Q^2/M^2; x)$ and $D(M^2; M_O^2; x)$ in Eq.(8), the re-summation to all orders of $\alpha_s(q^2)$ in $\prod(\alpha_s(q^2), x)$ and calculation of C would be required for studying the small x behaviour of hard processes. A careful examination is needed to see whether the results of analyses of problems 5-7 (they are all sensitive to the multiplicity of soft gluons) performed so far would be substantially affected by corrections beyond the leading log approximation.

EFFECTS OF SOFT GLUONS FELT INDIRECTLY

There is a class of quantities which is also sensitive to the effects of soft gluons but only indirectly: they are felt through the observation of fast quarks. Examples are quark form factor and the $x+1$ behaviour of structure/fragmentation functions in QCD. For these, there is an argument (see the talk by Marchesini in this school) that the dominant higher order corrections are summarised by the replacement

$$\alpha_s(q^2) P^{AP}(x) \xrightarrow{x \rightarrow 1} \left[\alpha_s(q^2(1-x)) \hat{P}^{AP}(x) \right] +$$

of the parton decay functions. In contrast to the situation where soft gluons are counted or observed directly, here the explicit calculation to next-to-leading order (see Eq.(29)) confirms the above mentioned argument.

(II) JET CALCULUS TO ALL ORDERS AND RESULTS TO
NEXT-TO-LEADING ORDER

(II-1) PROJECTION OPERATOR

We work in a light-like axial gauge, and in an infinite momentum frame of a decaying parton with momentum q

where p_{Tj} are transverse momenta of decay products in this

Diagrams for $i(q) \rightarrow a_1(p_1) + a_k(p_k)$ look like

which projects out only those terms which give rise to collinear singularities. With the choice of the gauge fixing vector n^μ , Eq.(16), the projector $\mathcal{P}_{ph}^{(in)}$ which extracts quark physical spin

states is defined as follows

$$\mathcal{P}_{ph}^{in}(A) \equiv \frac{A}{4n \cdot q} \cdot T_r(A), \quad (19)$$

where A is a square of a sum of amplitudes as in Eq. (17), including propagator q .

According to operation (A) of Eq. (7), only evolution above $q^2 > M_0^2$ should be taken into account, and therefore

$$\mathcal{P}_{M_0}^{(in)} = \delta^4(q - q') \sigma(q'^2 - M_0^2) \quad (20)$$

Typically, the propagator q is attached to another 2PI vertex with the parent parton with off-shell mass squared q_p^2 . The integration over internal line q^2 introduces also non-logarithmic contributions arising from kinematical bounds and phase space in $4-2\epsilon$ dimensions. Therefore, the third projector

$$\mathcal{P}_{\log}^{(out)} \equiv \int d\epsilon \delta(\epsilon) \log \frac{q_p^2}{M_0^2} \log \frac{q_p^2}{q_p^2(\frac{3}{2})} \quad \text{and sets } \epsilon = 0.$$

extracts only term linear in $\log \frac{q_p^2}{M_0^2}$

(II-2) RESULTS FOR QUARK DECAY PROBABILITIES TO THE NEXT-TO-LEADING ORDER

- a) Three-body and two-body inclusive decay probabilities.
To this order there are two possible (exclusive) three-body decays

$$V_{Q_1}^{(3)} \rightarrow Q_1(x_3) + G(x_1) + G(x_2) = C_F^2(A+B) + \frac{1}{2} C_F C_A(C+D-B) \quad (22)$$

$$V_{Q_1}^{(3)} \rightarrow Q_1(x_1) + Q_j(x_2) + \bar{Q}_j(x_3) = C_F(C_F - \frac{C_A}{2}) \delta_{ij} E + C_F T_R(F + \delta_{ij} F), \quad (23)$$

where $x_3 = 1-x_1-x_2$, $T_R = \frac{1}{2}$, $C_F = \frac{4}{3}$, $C_A = 3$ for $SU(3)$ colour, i, j are quark flavour indices and functions $A(x_1, x_2)$ to $F(x_1, x_2)$ are given in Ref. (Kalinowski et al).

The two-body inclusive decay probabilities $V^{(2)}$ are calculated via the energy-momentum sum rule

$$\sum_{d=0}^{1-x_1-x_2} dx_3 x_3 V_{a+bcd}^{(3)}(x_1, x_2, x_3) = (1-x_1-x_2) V_{a+bc}^{(2)}(x_1, x_2) \quad (24)$$

plus the term $\sim \delta(1-x_1-x_2)$ coming from exclusive two-body decays.

- b) One-parton inclusive vertices (generalised Altarelli-Parisi probabilities)

They are calculated from the energy-momentum sum rule

$$\sum_{c=0}^{1-x_1} dx_2 x_2 V_{a+bc}^{(2)}(x_1, x_2) = (1-x_1) V_{a+b}^{(1)}(x_1) \equiv (1-x_1) P_{b,a}^{NL}(x_1) \quad (25)$$

up to terms proportional to $\delta(1-x_1)$. These terms (corrections to quark propagator) can be fixed using the charge sum rule

$$\int_0^1 dx \left[P_{Q_j Q_1}^{NL}(x) - P_{\bar{Q}_j Q_1}^{NL}(x) \right] = 0 \quad (26)$$

Then the energy-momentum sum rule

$$\int_0^1 dx \, x \, p_{b,a}^{NL}(x) = 0 \quad (27)$$

serves as a non-trivial check of the above construction. As a result we get

$$p_{q,Q_1}^{NL}(x) = \delta_{ij} \left\{ \left[C_F^2 P_1 + \frac{1}{2} C_F C_A P_2 + C_F N_F^T P_3 \right] + C_F (C_F - C_A/2) \pi \delta(1-x) \right\} + C_F^T P_4 \quad (28)$$

$$p_{G,Q_1}^{NL}(x) = C_F^2 P_5 + \frac{1}{2} C_F C_A P_6 + C_F N_F^T P_7$$

$$p_{Q,Q_1}^{NL}(x) = \delta_{ij} C_F (C_F - C_A/2) P_8 + C_F^T P_4$$

where

$$\begin{aligned} P_1 &= 2 \frac{1+x^2}{1-x} \log x \log(1-x) + \left[\frac{3}{1-x} - 5 - 3x \right] \log x + \\ &\quad + \left[\frac{1}{2}(1+x) - 2 \frac{1+x^2}{1-x} \right] \log^2 x - 5(1-x), \\ P_2 &= \frac{1+x^2}{1-x} \left[\log^2 x - \frac{1}{3} \log(1-x) + \frac{67}{9} - \frac{\pi^2}{3} \right] + 2(1+x) \log x \\ &\quad + \frac{29}{3} (1-x), \\ P_3 &= \frac{2}{3} \frac{1+x^2}{1-x} \left[\log(1-x) - \frac{5}{3} \right] - \frac{2}{3} (1-x), \\ P_4 &= (1+x) \log^2 x + \frac{\log x}{x} \left[\frac{4}{3} - x^2 \right] + \frac{1}{9x} [13 - 45x + 45x^2 - 13x^3], \end{aligned}$$

$$P_5 = -\frac{1+(1-x)^2}{x} \left[2 \operatorname{Li}_2(x) + \frac{\pi^2}{3} + \frac{3}{2} \log(1-x) \right] + (1 - \frac{1}{2}x) \cdot \log^2 x + x \log x + \frac{1}{2x} [5 - 9x + 2x^2],$$

$$P_6 = \frac{1+(1-x)^2}{x} \left[\pi^2 - 4 \operatorname{Li}_2(1-x) \right] - 2 \frac{\log^2 x}{x} \left[4 + 2x + 3x^2 \right] - \frac{\log x}{x} \left[\frac{62}{3} - 2x^2 \right] + \frac{1}{9x} [-45 + 118x - 14x^2 + 26x^3] + \frac{2(2+2x+x^2)}{x} \left[2 \operatorname{Li}_2(-x) + 2 \log x \log(1+x) + \frac{\pi^2}{6} \right],$$

$$P_7 = -\frac{10}{9} \cdot \frac{1+(1-x)^2}{x},$$

$$P_8 = 2(1+x) \log x + 4(1-x) + \frac{1+x^2}{1+x} \left\{ \log^2 x - 4 \log x \log(1+x) - 4 \operatorname{Li}_2(-x) - \frac{\pi^2}{3} \right\},$$

$$\pi = \int_0^1 dx \, p_8(x) = 13/4 - \pi^2/2 + 2\zeta(3); \quad \zeta(3) = \sum_{k=1}^{\infty} 1/k^3 = 1.20205\dots$$

$$(11-3) \quad \underline{\text{SCHEME DEPENDENCE}} \quad (29)$$

To the next-to-leading order the quantity

$$c(x) - \frac{1}{2\pi b} p^{NL}(x), \quad b = (11N_C - 2N_F)/12\pi \quad (30)$$

is scheme independent, where c and p^{NL} are the coefficients of $\frac{\alpha}{2\pi}$ in the hard subprocess cross section $C_{\gamma^*}(\alpha(Q^2), x)$ and the coefficient of $\left(\frac{\alpha}{2\pi}\right)^2$ in $\pi(\alpha(Q^2), x)$, respectively.

We find the following difference between our result for $p_{Q,Q}^{NL}(x)$ and the result of Curci, Furmanski and Petronzio

$$P_{QQ}^{NL}(x) - P_{QQ}^{CFP}(x) = -2\pi b \left\{ \frac{1+x}{1-x} \left[\log x(1-x) - \log 4\pi + \gamma \right] + (1-x) \right\} + \quad (31)$$

The origin of Eq.(31) is in the different definitions adopted for the projection operator. However, this also causes difference in $C_{\gamma^*}(\alpha(Q^2), x)$. In our convention $C_{\gamma^*}(\alpha(Q^2), x)$ for $M^2 = Q^2$ does not contain contribution from mass singular term $\sim dq^2/q^2$ from one rung diagram (contains all contributions from non-singular terms). In the convention of Ref.(Curci et al), $C_{\gamma^*}(\alpha(Q^2), x)$ is defined as a finite part after q^2 -integration. Therefore, we find that the only difference between our scheme and CFP comes from the finite part they pick up after q^2 -integration of the dq^2/q^2 term of a one rung ladder, i.e.

$$C_{\gamma^*} - C_{\gamma^*}^{CFP} = -C_F \left\{ \frac{1+x^2}{1-x} \left[\log x(1-x) - \log 4\pi + \gamma \right] + (1-x) \right\} + \quad (32)$$

which cancels the effect of Eq.(31) in physical combination Eq.(30).

(II-4) NEXT-TO-LEADING CORRECTIONS TO P_T -DISTRIBUTIONS

The complete discussion of the next-to-leading corrections to the jet evolution would require the knowledge of the gluon decay probabilities, which have not yet been calculated. However, using the quantities we have already computed we can get some estimates on how important these corrections are.

Consider two-parton inclusive distribution inside a quark jet $i \rightarrow a(x_a) + b(x_b)$ produced e.g. in e^+e^- annihilation or in leptonproduction. We fix the relative transverse momentum

P_T , $\Lambda^2 \ll P_T^2 \ll Q^2$, take the n^{th} moment in x_a , the first moment in x_b and sum over parton species b . Taking sufficiently large n we can assume that parton a is a quark produced in the hard vertex $a \equiv i$. Using the jet calculus rules one can easily derive (Kalinowski et al) the corresponding cross section

$$\left(\frac{d\sigma}{dp_T} \right)_{n,l} \sim \frac{\partial}{\partial p_T} \left\{ D_{QQ}(p_T^2, M_O^2; n) D_{QQ}(Q^2, p_T^2; n+1) \right\}, \quad (33)$$

where the process dependent "hard subprocess cross section" has been omitted since with a quark dominating in the intermediate state it gives a p_T -independent multiplicative factor.

The formula Eq.(33) has been studied numerically to next-to-leading accuracy. The results for $\left(\frac{d\sigma}{dp_T} \right)_{n,l}$, normalized to LLA value at $P_T = 6$ GeV, for $n = 3$ and $\overline{MS}$ and $\overline{MS}$ renormalization schemes, are shown in Fig.4. The smallness of next-to-leading corrections is mainly due to the fact that only differences of n^{th} and $(n+1)^{th}$ moments of $\Pi(\alpha, x)$ enter and that α^2 term in Π tends to cancel the two-loop corrections to $\alpha_s(q)^2$. Thus we find that the next-to-leading corrections to such highly inclusive measurements are quite small and the jet calculus at LLA already provides a good approximation.

BIBLIOGRAPHY TO PART(II)

As in Bibliography to Part (I-2).

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Fig. 1

Fig. 2

Fig. 3

FIG. 4