Picture of QCD Jets: Leading Log Approximation and Beyond it

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PICTURE OF QCD JETS: LEADING LOG APPROXIMATION AND BEYOND IT†

BY

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ABSTRACTS

We review the picture of QCD jets as it emerged from the analyses based on perturbative QCD in the leading log approximation (LLA) and beyond it. Part I (by K. Konishi) gives a brief review of the intuitive (fractal) picture and quantitative description of QCD jets in LLA, and contains discussions on some salient aspects of QCD corrections beyond the leading order. Part II (by J. Kalinowski) is a summary of the construction of the jet calculus beyond the leading order, and of calculations to next-to-leading order of jet calculus vertices and generalized Altarelli-Parisi parton decay functions.

Explicit calculation suggests important corrections beyond leading order to the previous analyses of quantities which are sensitive to the multiplicity of soft gluons. On the other hand, for the measures which depend on the effects of soft gluons only indirectly (e.g. through the observation of fast quarks), the explicit results to next-to-leading order are consistent with the general arguments given by several authors. Corrections to inclusive $p_T$-correlations among fast particles inside a jet, are found to be small beyond LLA.
\[
\left( 2^{m} \right)^{1/2} \log \frac{1}{0.05} \to \frac{x \times (x) + b + e^{x}(xp/\alpha)}{x^{2} + d}
\]

This equation is used to determine the energy fraction of the decay, which is a crucial component of the analysis of the decay process. The energy fraction is important for understanding the dynamics of the decay, particularly in the context of nuclear physics. The equation is derived from the conservation of energy and momentum, and it allows for the calculation of the energy fraction from the experimental data. The parameters in the equation are determined through fitting procedures, which involve minimizing the difference between the calculated and experimental energy fractions. This process is essential for validating the theoretical models and for improving our understanding of the decay process.
A simple argument similar to the one given above for the lowest order, shows that the higher order effects in $\alpha_s(Q^2)$ are always there and only for a limited class of measures do the lowest-order calculations make sense. (It should be borne in mind that even for those "infrared-safe" cases, in general there appear large log $\delta$'s or log $\epsilon$'s which are just as bad: the resummation of e.g. $(\alpha_s(Q^2) \log 1/\delta)^n$ is necessary in many practical cases).

Any physical cross section in a hard process is of the form,

$$\sigma = \frac{1}{Q^2} f(\alpha_s(Q^2), Q^2/m^2, x)$$

($Q^2$ = the large mass scale of the process) because of the renormalisability. A possible way to make use of perturbative QCD is to reorganise the series as

$$= \sum_{m=0}^{\infty} (\alpha_s(Q^2))^m f_m$$

where $f_m = f(\alpha_s(Q^2), \log Q^2/m^2; x)$, or

$$f_m = f(\alpha_s(Q^2), \log Q^2/m^2 \log \delta; x),$$

something else, depending on the situation, but in general $f_m$ contains whole higher order terms in $\alpha_s(Q^2)$ already.

Keeping the first term of Eq. (3) (LLA), the subsequent terms are - presumably - small by factors of $1/\log Q^2/\Lambda^2$.

Quantitative description of evolution of jets in LLA can be summarised in a simple parton-probabilistic algorithm, the jet calculus. The $m$-parton inclusive spectra are computed from the sum of effective tree diagrams:

$$\frac{d^m\sigma}{dx_1 \cdots dx_m} = \sum_{\text{trees}} \int dy_i \quad \text{(trees) internal masses}$$

$$\text{parton species}$$

where $dy_i = (dq_i^2/dq_i^2) \cdot \alpha_s(q_i^2)/z_i^2$; $D(q_i^2; q_i^2; x)$ = inverse Mellin transform of $(\log q_i^2/\log q_i^2)^n$ and the vertices are given by

\begin{align*}
\hat{P}_{Q,Q}(z) &= \hat{P}_{G,Q}(1-z) = C_P(1+z^2)/(1-z) ; \\
\hat{P}_{Q,Q}(z) &= (N_c/2) \left\{ z^2 + (1-z)^2 \right\} ; \\
\hat{P}_{G,Q}(z) &= 2C_A \left\{ (1-z)/z + z/(1-z) + z(1-z) \right\} 
\end{align*}

The evolution of jets in LLA is formally a (Markov) branching process with $y \propto \log Q^2$ as a "time" variable.

The predictions of QCD in the physics of jets which I find particularly interesting, include:

1. Two-particle inclusive energy-energy correlation (the comparison with experiments in $e^+e^-$-annihilation was presented by Söding in this school);
2. Measurements to anomalous dimensions of QCD by a calorimeter-experiment (this could possibly be done at PETRA, PEP and LEF);
3. Energy antenna pattern of Hasham and others. (This seems to be, among the so-called infra-red safe measures, a relatively clean one);
I shall also mention a particularly interesting feature. Kalnitsky, I will concentrate on general aspects of the work, calculations to next-to-leading order to the talk by Jan

structure of jets, leaving the details of construction and some phenomenological implications to our model of amplitudes of jet evolution beyond leading order.

In an earlier gauge, the main results of the work done in the approximation of cross sections, now approximated, is a direct calculation of next-to-leading order contributions to any given order, the calculation of complete set

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Near evolution in color, space-time, and rapidity.

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of our results related to strong soft-divergences in QCD.

**Ideas**

The idea behind our work is that the factorization theorem of mass singularities contains implications wider than usually acknowledged. The diagrammatic derivation of the factorization theorem does not only lead to the parton-model results (with calculable logarithmic corrections to structure or fragmentation functions). It also gives in principle precise predictions about the properties of associated jets. It was the purpose and results of our work to find a practical way to extract the information about the jets, based on a diagrammatic approach.

**Final States in Hard Processes and Mass Singularities**

A convenient way to make manifest the mass-singularity structure of final states in a hard process, is to combine the off-shell unitarity equations of Polyakov with the use of an axial gauge. For instance the total cross section for $e^+e^-$ annihilation into hadrons is written as,

$$
\sigma_{\text{tot}} = \text{Disc}\left\{ -\bigotimes + \bigotimes + \ldots \right\}
$$

(6)

where the crossed lines are cut exact propagators, uncut propagators also exact ones, and vertices are one-particle irreducible (1PI). By rewriting

(A) $q \rightarrow \phi + \chi + \ldots$ if $q^2 \geq M_0$,

\[ \text{etc.} \]

(B) $\quad \phi + \chi + \ldots \rightarrow \chi$ ; etc.

(C) $\quad \phi \rightarrow \chi + \chi$ ; etc.

where $\phi$ projects just the collinear log term (see Part II), the right hand side of Fig.(6) becomes of diagonal form. In axial gauge QCD, all collinear logs arise from integration over mass of intermediate parton connecting two effective vertices. The problem is then reduced to the computation of these vertices $V_{a-bc\ldots}$ to a given order in $\alpha_s$. The jet calculus rules are as simple as in Eq.(4): $\phi$ is replaced by $\phi_{a-bc\ldots}$ which now contain three-body decays as well, and $D$ should be computed from the generalized Altarelli-Parisi kernel $\eta_{b,a}(\alpha_s; \chi) \equiv V_{a-b}^{(1)}$.

It is possible, because of the KLN theorem and factorization property of mass singularities, to choose a convention such that $V_{a-bc\ldots}$ can be treated (formally) as inclusive probabilities. The parton probabilistic picture of jet evolution found in LLA can thus be maintained to all order of logs and of $\alpha_s$. 

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next-to-leading corrections to it. 

\[ \left( \frac{Z}{\beta} \right) \text{part of the universal partition decay} \]

\[ \frac{d}{d_v} \left( \frac{Z}{\beta} \right) + \left( x \right) \frac{d}{d_v} \left( x^{\bar{v}} \right) = \left( x \right)^{\bar{v}} \]

\[ \sum \text{ terms up to next-to-leading order,} \]

\[ \text{or the result of next-to-leading calculations just computed.} \]

Finally, we make a comment on an interesting feature

Strong soft divergences in QCD.

\begin{align*}
\text{quantity in tensor} & \quad \text{not a physical} \\
\langle \sigma^{\mu} \rangle & \quad \langle \sigma^{\mu} \rangle \\
\text{associated jets} & \quad \text{possible to resolve} \\
\text{observable} & \quad \text{not observable}
\end{align*}

\[ \left( \begin{array}{c}
\text{unphysical quantity} \\
\text{not a separate particle}
\end{array} \right) \]

\[ \text{immediate partition} \quad \text{not an observable} \\
\text{apparent limit of mass} = \frac{Z}{\beta} \]

\[ \text{physically measurable} \]

\[ \langle \sigma^{\mu} \rangle = u \quad \text{with} \quad \langle \sigma^{\mu} \rangle \\
\text{Algebra of operators} \quad \text{with} \quad \text{operators} \quad \text{are} \]

\[ \text{signature of operator matrix element} \quad \text{expression for correlation function} \]

\[ \text{leading to the analityc and difference. The cross} \]

\[ \text{not much more common type of approach for this case.} \]

\[ \text{comparison with QCD or artificial approach} \]
For small \( x \), \( P_{G, Q}^{NL}(x) \) was found to behave as (Kalinowski et al; Furmanski et al; (see Eq.(29)),

\[
P_{G, Q}^{NL}(x) \rightarrow -4C_A \log \frac{x}{x + \text{less singular terms}} \quad x < 1
\]  
(12)

Analogous behaviour was also found for \( P_{G, G}^{NL} \) (Furmanski et al; Floratos et al)

\[
P_{G, G}^{NL}(x) \rightarrow -4C_A \log \frac{x}{x + \text{less singular terms}} \quad x < 1
\]  
(13)

It is easy to see that half of \( \log^2 \frac{x}{x} \) term has a trivial kinematic origin, due to the correct kinematic limits over which the integrations of ladder variables are performed.

The other half in Eq.(12) and Eq.(13) comes from new diagrams containing an extra three-gluon coupling, and their origin can be traced to certain double soft divergence pointed out in Kalinowski et al.

In the analysis of problems 5-7 listed at the end of Part (I.1) it has so far been assumed or argued that the only important non-leading corrections to the leading log results,

\[
P_{G, Q}^{AP} \rightarrow -2C_F/x + \text{less singular terms} \quad x < 1
\]  
(14)

and

\[
P_{G, G}^{AP} \rightarrow -2C_A/x + \text{less singular terms} \quad x < 1
\]  
(15)

are of kinematical nature as mentioned above. The results of the explicit calculation, Eq.(12) and Eq.(13), indicate that this may not be the case. Indeed, for sufficiently small \( x \) the \( \alpha_s^2(q^2) \) term in Eq.(11) dominates over the first even after the "kinematical" terms are subtracted. Thus unless an ingenious cancellation occurs between \( C(\alpha_s(q^2); \frac{q^2}{m^2}; x) \) and \( D(\alpha_s^2; \frac{M^2}{m^2}; x) \) in Eq.(8), the re-summation to all orders of \( \alpha_s(q^2) \) in \( T_0[q^2, x] \) and calculation of \( C \) would be required for studying the small \( x \) behaviour of hard processes. A careful examination is needed to see whether the results of analysis of problems 5-7 (they are all sensitive to the multiplicity of soft gluons) performed so far would be substantially affected by corrections beyond the leading log approximation.

**EFFECTS OF SOFT GLUONS FELT INDIRECTLY**

There is a class of quantities which is also sensitive to the effects of soft gluons but only indirectly: they are felt through the observation of fast quarks. Examples are quark form factor and the \( x \to 1 \) behaviour of structure/fragmentation functions in QCD. For these, there is an argument (see the talk by Marchesini in this school) that the dominant higher order corrections are summarised by the replacement

\[
\alpha_s(q^2) P_{G, Q}^{AP}(x) \rightarrow \left[ \alpha_s(q^2(1 - x)) \right] P_{G, Q}^{AP}(x)
\]

of the parton decay functions. In contrast to the situation where soft gluons are counted or observed directly, here the explicit calculation to next-to-leading order (see Eq.(29)) confirms the above mentioned argument.
We define the projection operator

Diagram for (1): \( \frac{x}{d} \cdot y + \frac{x}{d} \cdot y + \frac{x}{d} \cdot y + \frac{x}{d} \cdot y \)

where \( \frac{x}{d} \cdot y \) are transverse momenta of decay products in the

(16)

\[ o \cdot o = o, \quad o \cdot u = 0, \quad o \cdot v = 0, \quad o \cdot n' = 0, \quad u \cdot n' = 0, \quad n \cdot u = 0, \quad n \cdot n' = 0,\]

momentum frame of a decay product with momentum \( u \) and momentum frame of a light-like axial gauge, and in an

II-1) PROJECTION OPERATOR

Next-to-leading order

II) Derivatives to All Orders and Results to
states is defined as follows

$$P_{\text{in}} \left( \mathbf{d} \right) = \frac{1}{4 \pi q} \cdot T_R \left( \mathbf{d} \right),$$

where $\mathbf{d}$ is a square of a sum of amplitudes as in Eq. (17), including propagator $q$.

According to operation (A) of Eq. (7), only evolution above $q^2 > M_0^2$ should be taken into account, and therefore

$$P_{\text{in}} \left( \mathbf{d} \right) = \delta^4 \left( q - q' \right) \sigma \left( q^2 - M_0^2 \right)$$

Typically, the propagator $q$ is attached to another 2PI vertex with the parent parton with off-shell mass squared $q_p^2$.

The integration over internal line $q^2$ introduces also non-logarithmic contributions arising from kinematical bounds and phase space in 4-$\pi$ dimensions. Therefore, the third projector

$$P_{\text{out}} \left( \mathbf{d} \right) = \int \frac{d^4 \mathbf{d}}{M_0^2} \log \frac{q_p^2}{M_0^2} \frac{q^2}{M_0^2} \frac{3}{2} \frac{1}{P \left( q^2 \right)}$$

extracts only term linear in $\log \frac{q_p^2}{M_0^2}$ and sets $c = 0$.

(II-2) RESULTS FOR QUARK DECAY PROBABILITIES TO THE NEXT-TO-LEADING ORDER

a) Three-body and two-body inclusive decay probabilities.

To this order there are two possible (exclusive) three-body decays

$$V^{(3)}_{Q_1 + Q_1 \left( x_3 \right) + G \left( x_1 \right) + G \left( x_2 \right)} = \frac{1}{2} C_F^2 \left( A + B \right) + \frac{1}{2} C_F^2 \left( C + D - B \right)$$

$$V^{(3)}_{Q_2 + Q_1 \left( x_1 \right) + Q_2 \left( x_2 \right) + \bar{Q}_3 \left( x_3 \right)} = C_F^2 \left( \frac{C_F^2}{A} \right) \delta_{13} \delta_{12} \left( x \right) + C_F^2 \left( \frac{C_F^2}{A} \right) \delta_{13} \delta_{12} \left( x \right)$$

where $x = 1 - x_1 - x_2$, $T_R = \frac{1}{2}$, $C_F = \frac{4}{3}$, $C_A = 3$ for SU(3) colour, $i,j$ are quark flavour indices and functions $A(x_1, x_2)$ to $F(x_1, x_2)$ are given in Ref. (Kalinowski et al).

The two-body inclusive decay probabilities $V^{(2)}$ are calculated via the energy-momentum sum rule

$$\int dx_1 x_1 V^{(2)}_{a+b+c} \left( x_1, x_2, x_3 \right) = \left( 1 - x_1 - x_2 \right) V^{(2)}_{a+b+c} \left( x_1, x_2 \right)$$

plus the term $\delta \left( x_1 - x_2 \right)$ coming from exclusive two-body decays.

b) One-parton inclusive vertices (generalised Altarelli-Parisi probabilities)

They are calculated from the energy-momentum sum rule

$$\int dx_1 x_1 V^{(2)}_{a+b+c} \left( x_1, x_2 \right) = \left( 1 - x_1 \right) V^{(1)}_{a+b+c} \left( x_1 \right)$$

up to terms proportional to $\delta \left( x_1 \right)$. These terms (corrections to quark propagator) can be fixed using the charge sum rule

$$\int_0^1 dx \left[ \frac{P_{\text{NL}} \left( x \right)}{Q \left( x \right)} - \frac{P_{\text{NL}} \left( x \right)}{G \left( x \right)} \right] = 0$$

(26)
We find the following difference between our result for

\[ y_{12} = \frac{Q^2}{x^2} - \frac{Q^1}{x^1} = q \cdot (x) \]

and the result of current, parametric and perturbative

\[ \left( \frac{Q^2}{x^2} - \frac{Q^1}{x^1} \right) = q \cdot (x) \]

allowing to write the quantity

\[ (x) \]

in scheme independent, where \( c \) and \( \frac{Q^1}{x^1} \) are the coefficients

\[ (10) \]

\[ \text{GOV: } (x) \]

\[ \text{DEF: } (x) \]

where

\[ \text{Equation } (3) \]

\[ \text{Equation } (4) \]

\[ \text{Equation } (5) \]

\[ \text{Equation } (6) \]

\[ \text{Equation } (7) \]

\[ \text{Equation } (8) \]

\[ \text{Equation } (9) \]

\[ \text{Equation } (10) \]

\[ \text{Equation } (11) \]

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\[ \text{Equation } (16) \]

\[ \text{Equation } (17) \]

\[ \text{Equation } (18) \]

\[ \text{Equation } (19) \]

\[ \text{Equation } (20) \]

\[ \text{Equation } (21) \]

\[ \text{Equation } (22) \]

\[ \text{Equation } (23) \]
\[ p_{\text{NL}}(x) - p_{\text{CPP}}(x) = -2eB \left\{ \frac{\log(x) - 4\pi + \gamma}{x} \right\} + (1-x) \]

The origin of Eq. (31) is in the different definitions adopted for the projection operator. However, this also causes a difference in \( C_{\gamma^*}(a(Q^2), x) \). In our convention \( C_{\gamma^*}(a(Q^2), x) \) for \( n^2 = Q^2 \) does not contain contributions from mass singular terms \( \sim dq^2/q^2 \) from one rung line diagram (contains all contributions from non-singular terms). In the convention of Ref. (Curci et al.), \( C_{\gamma^*}(a(Q^2), x) \) is defined as a finite part after \( q^2 \)-integration. Therefore, we find that the only difference between our scheme and CPP comes from the finite part they pick up after \( q^2 \)-integration of the \( dq^2/q^2 \) term of a one rung ladder, i.e.

\[ C_{\gamma^*} - C_{\text{CPP}} = -C_F \left\{ \frac{\log(x) - 4\pi + \gamma}{x} \right\} + (1-x) \]

which cancels the effect of Eq. (31) in physical combination Eq. (30).

(II-4) NEXT-TO-LEADING CORRECTIONS TO \( p_T \)-DISTRIBUTIONS

The complete discussion of the next-to-leading corrections to the jet evolution would require the knowledge of the gluon decay probabilities, which have not yet been calculated. However, using the quantities we have already computed we can get some estimates on how important these corrections are.

Consider two-parton inclusive distribution inside a quark jet \( i \to a(x_a) + b(x_b) \) produced e.g. in \( e^+e^- \) annihilation or in leptoproduction. We fix the relative transverse momentum \( p_T, \Lambda^2 < p_T^2 < Q^2 \), take the \( n \)th moment in \( x_a \), the first moment in \( x_b \) and sum over parton species \( b \). Taking sufficiently large \( n \) we can assume that parton \( a \) is a quark produced in the hard vertex \( a \equiv 1 \). Using the jet calculus rules one can easily derive (Kalinowski et al) the corresponding cross section

\[ \left( \frac{d\sigma}{dp_T^2} \right) \sim \frac{3}{p_T^2} \left\{ D_{\text{QQ}}(p_T^2, M^2, n), D_{\text{QQ}}(Q^2, E_T^2, n+1) \right\} \]

where the process dependent "hard subprocess cross section" has been omitted since with a quark dominating in the intermediate state it gives a \( p_T \)-independent multiplicative factor.

The formula Eq. (33) has been studied numerically to next-to-leading accuracy. The results for \( \left( \frac{d\sigma}{dp_T} \right) \), normalized to LLA value at \( p_T = 6 \text{ GeV} \), for \( n = 3 \) and \( \text{MS} \) and \( \overline{\text{MS}} \) renormalization schemes, are shown in Fig. 4. The smallness of next-to-leading corrections is mainly due to the fact that only differences of \( n \)th and \( (n+1) \)th moments of \( H(a, x) \) enter and that \( a^2 \) term in \( H \) tends to cancel the two-loop corrections to \( a_s(q^2) \).

Thus we find that the next-to-leading corrections to such highly inclusive measurements are quite small and the jet calculus at LLA already provides a good approximation.

BIBLIOGRAPHY TO PART (II)

As in Bibliography to Part (I-2).
and at the same time for admiring the beauty of the Driem.

ing us with an opportunity for eno the physical phenomena
we thank Prof. A. Schilling for inviting us, and provid-

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\[ \left( \frac{d\sigma}{dp_T} \right)_{3,1}^{LL + NL} / \left( \frac{d\sigma}{dp_T} \right)_{3,1}^{LL} \]

\[ \Lambda = 0.5 \text{ GeV} \]

\[ p_T \text{ (GeV/c)} \]

\[ \overline{\text{MS}} \]

\[ \text{MS} \]

FIG. 4